APPLICATION OF FUZZY LOGIC TO APPROXIMATE REASONING USING LINGUISTIC SYNTHESIS

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Summary

This paper describes an application of fuzzy logic in designing controllers for industrial plants. A Fuzzy Logic is used to synthesise linguistic control protocol of a skilled operator. The method has been applied to pilot scale plants as well as in a practical industrial situation. The merits of this method in its usefulness to control engineering are discussed. This work also illustrates the potential for using fuzzy logic in modelling and decision making. An avenue for further work in this area is described where the needis to go beyond a purely descriptive approach and explore means by which a prescriptive system may be implemented.

Introduction

The fact that mathematics as a whole is taken to be synonymous with precision has caused many scientists and philosophers to show considerable concern about its lack of application to real world problems. This concern arises because in logic as well as in science there is constantly a gap between theory and the interpretation of results from the inexact real world. Many eminent thinkers have contributed to the discussion on vagueness, occasionally holding human subjectivity as the culprit.

In an excellent analysis of the subject Black[1] says ... "that with the provision of an adequate symbolism the need is removed for regarding vagueness as a defect of language". In his paper he strongly argues that vagueness should not be equated with subjectivity. Briefly, his argument may be summarised by noting that the colour 'Blue', say, is vague but not subjective since its sensation among all human beings is roughly similar. It is possible to deal with colour precisely by considering the e.m. radiation producing it but in doing so the important human sensation of colour, as it happens to be vague, has to be sacrificed. Furthermore, it may be argued that vagueness is not a defect of language, but also an important source of creativity. Analogies are extremely important to creative thinking and vagueness plays a dominant role in such thought processes.

Black's motivation to symbolise vagueness appears to be at the back of all investigations of "Deviant Logics"[2]. An important contribution in the past 10 years has been that of Zadeh's fuzzy-set-theory and fuzzy-logic[3]. In his recent writings Zadeh[4,5] states clearly his motivation which is to use fuzzy sets to symbolise Approximate Reasoning (AR). Whereas there are many applications of fuzzy-set-theory, this paper describes one of the first results in the application of AR and linguistic synthesis.

An Outline of the Paper's Content

The intention in this paper is to review the whole program of investigation concerning the application of Approximate Reasoning and to analyse the findings in order to offer insightful comments and conclusions. The original work in this program was done in early 1974[6] and first published later that year[7,8]. This was the control of a pilot scale steam-engine using fuzzy-logic to interpret linguistic rules which qualitatively express the control strategy. This work is briefly reviewed in the next section of this paper.

Since the publication of the above work several researchers elsewhere have also implemented the approach using different pilot scale plants. This together with the continuing work as part of this programme have produced results which throw more light on the usefulness of applying fuzzy-logic to linguistic synthesis. Below, comments are offered on some of the key findings of these studies.

One of the comments that has been made about fuzzy-logic is that in its present form it is essentially descriptive and does not offer a prescriptive approach to reasoning. In the first place, it should be noted that fuzzy-logic, like any other form of logic, can only be a system for inferring consequences from previously stated premises and only from these premises. A prescriptive system is possible, however, if a hierarchical decision making approach is used so that the strategy at a lower level is derived as a consequence of a description at a higher level. Early results of such an experiment of a prescriptive method (some might term this a learning or an adaptive approach) is discussed later in this paper. To conclude this paper the last section examines the future trend in this field in the light of experience being gained from current investigations described here.

An Experiment in Linguistic Synthesis

A Brief Review of Fuzzy Logic

The point of view adopted here is that the variables are associated with universes of discourse which are non-fuzzy sets. These variables take on specific linguistic values. These linguistic values are expressed as fuzzy subsets of the universes.

Given a subset A of X ($A \subset X$) A can be represented by a characteristic function: $\chi_A: X \to \{0,1\}$. If the above mapping is from X to a closed interval $[0,1]$ then we have a fuzzy subset. Thus if A were a fuzzy subset of X it could be represented by a membership function: $\mu_A: X \to [0,1]$

Note that X is a non-fuzzy support set of a universe of discourse, say height of people. A can then be equated to a linguistic value such as tall people. Given two such linguistic values A_1 and A_2 on the same support set X, logical combinations: $\overline{A}_1$; $A_1 \wedge A_2$; $A_1 \vee A_2$; can be formed as:

$\overline{A}_2$ is formed by taking $(1-\mu_{A_2})$ as its membership value at each element of the support set.

$A_1 \wedge A_2$ is formed by taking $\min(\mu_{A_1}, \mu_{A_2})$ at each element of the support set, and

$A_1 \vee A_2$ is formed by taking $\max(\mu_{A_1}, \mu_{A_2})$ at each element of the support set.

It is in the definition of implication that this logic may be found to differ from other logics. Given $A \rightarrow B$ (If A then B), then it can happen that A and B are linguistic values of two disparate universes of support, say X and Y. Note here that the implication is between individual values and not the underlying variables. Thus the relation R between A and B is a fuzzy subset of the universe of support $X \times Y$, the cross-product of X and Y. $\mu_R: X \times Y \rightarrow [0,1]$. $\mu_R(x,y)$ is related to $\mu_A(x)$ and $\mu_B(y)$ (in the present application) by the following:

$$\mu_R(x,y) = \min(\mu_A(x),\ \mu_B(y)).\ x\varepsilon X,\ y\varepsilon Y$$

If the relation R represents a "nested" implication (i.e. If A then (If B then C) or $A \rightarrow B \rightarrow C$), then R will have a corresponding higher order cross-product support set.

Now if some relation R between A and B is known and so is some value A^1 then the idea is to infer B^1 from R and A^1; $B^1 = A^1 o R$, where A^1 is composed with R. This has the effect of reducing the dimensionality of the support set of R to that of B^1. In this work, the compositional rule of inference used to relate μ_{B^1} to μ_R and μ_{A^1} is:

$$\mu_{B^1}(y) = \max_x \min(\mu_{A^1}(x),\ \mu_R(x,y)).$$

Application to a Fuzzy-controller

As stated earlier the linguistic synthesis approach was initially applied to control a pilot scale steam-engine, a more detailed description of which is given elsewhere[6,7,8]. A concise summary of this work is presented here. One aspect of control in this system is the regulation of pressure in the boiler around a prescribed set-point. The control is achieved by measuring the pressure at regular intervals and inferring from this the heat setting to be used during that interval. The essence of this work is simply that if an experienced operator can provide the protocol for achieving such a control in qualitative linguistic terms, then fuzzy logic as described above can be used to implement successfully this strategy.

The protocol obtained from the operator in this case considers pressure error (PE) and change in the pressure error (CPE) to infer the amount of change in the head (HC). The protocol consists of a set of rules in terms of specific linguistic values of these variables and is shown in figure 1*. Now it can be seen that these rules are in the form of If...Then statements (implications) and thus, from above, each rule i will translate into a relation R_i. The overall protocol is then a relation R formed by 'oring' together the R_i's: $R = R_1 V R_2 \ldots V R_i \ldots V R_n$.

Let us say now that each rule R_i represents an implication $A_i \rightarrow B_i \rightarrow C_i$. The decision making algorithm that is implemented contains two phases:

(a) The initial setting up phase when the protocol R is formed from two sets of data:

(i) The individual linguistic values A_i, B_i, C_i given as fuzzy subsets.

(ii) The rules as in fig. 1 which specify the actual combination of these values to form each R_i.

*The abbreviations used for these linguistic values here are: ZE-zero; PZ-positive zero; PS-positive small, PM-positive medium; PB-positive big and the same for negative values NZ, NS, NM and NB. Change in Error negative is taken as movement towards set-point and positive as away from set-point.

(b) The decision making phase is invoked at each sampling instant during run-time with the exact measured values A^1 and B^1 supplied to it. This phase then is nothing but the use of a compositional rule of inference to derive C^1 as follows:

$$C^1 = A^1 o (B^1 o R).$$

Note that A^1, B^1 can be non-fuzzy, whereas since C^1 is a fuzzy subset of the set of all possible actions a procedure is required to determine the actual action to be taken from the knowledge of C^1. Also there is a certain advantage in deferring the computation of R until the second phase. Because then this provides a means of altering the control strategy on-line by altering the data structures containing the rules during run-time. However, what need concern us at present is the results obtained from the application of this method to the pilot scale plant. In repeated trials it was found that the results compared favourably with those from a straight application of classical methods from control engineering practice (i.e. 2 or 3 term controllers).

Comments on Fuzzy-logic Controller Studies

Two main conclusions have been drawn from this work. First, that the results vindicate the approach advocated by Zadeh and demonstrate its potential. Second, it can be asserted that the method can easily be applied to many practical situations. This assertion is supported by considering a practical instance, that of cement kiln operation, in which a similar control protocol applies. In a book on cement kilns, Peray and Waddell[9] list a collection of rules for controlling a kiln. Examples of these rules are shown in figure 2. From this it is immediately apparent that the method as described, can be used for translating these rules.

In recent months, this method has been further tested by various research workers[11-17] on other pilot scale plants such as batch chemical reactors[12], heat-exchangers[15] and so on. In some of these studies[19] the plant is characterised by time lag between the applied action and the observed output necessitating a modification of the method detailed above. The controller is used in conjunction with a fuzzy plant model without time lag to determine the actual action to be applied to the plant itself. In one study different control policies are compared analogous to the classical 3 term controller. Worth noting is the application of this method to a real plant - a sinter plant at the British Steel Corporation[16,17]. Results show that fuzzy control achieves useful reduction in the standard deviation of the measured output when compared with a manual operation of the plant. Furthermore it also compares favourably with a classical 2-term control of the plant.

In many of these studies, rules exactly as those given in fig. 1 are used with only minor changes. This is not surprising as the rules indicate the relationship between error, change in error and control action that exists in most dynamical plants. This relation is mainly one of monotonicity between the outputs of a plant and the input applied to it. What is more of interest is that in most studies it is found that this form of controller is far less sensitive to parameter changes within the plant than the classical 2-term controller. At this stage only a qualitative explanation can be offered for this. It appears that the former is a reasonable controller as it relies on the underlying relationships between the plant outputs and inputs whereas the latter is a pedantic controller in which the action is computed as a linear combination of the measurements and thus more susceptible to parameter changes.

These results and observations have motivated a further analysis of the fuzzy controller[19]. It is easy to see that because of the incremental nature of the action (heat change instead of actual heat value) the fuzzy controller is analogous to a proportional plus integral controller, (i.e. integral control is achieved without the need for memory in the controller which could not be easily accommodated in purely fuzzy logic context). It has also been noted that just as a classical controller can be tuned by adjusting the gains of the terms, the tuning of the fuzzy controller can be achieved both by adjusting the gains in a similar way and also by adjusting the protocol (see below). This has prompted many to suggest that in a fuzzy controller one has a 'non-linear' controller. While such analogies are useful, in the view of this author they can also sometimes mislead. For example, the term 'non-linear' in a control sense implies that one views the model of the plant as an integro-differential system. In a truly system sense the word 'non-linear' can have no meaning. Partly to emphasise this, the fuzzy controller has been applied to a system which is not commonly modelled using integro-differential equations - a traffic junction[21]. In this application the controller considers the waiting queue in one leg of the junction and the rate of flow in the other to determine the extension of the green phase of the traffic lights. Results indicate that the method is obviously applicable in areas other than industrial plant controllers. To put it succinctly therefore, with complex humanistic systems as with difficult plants (difficult in the sense that they are difficult to model accurately using integro-differential equations), heuristic decision making and control can be used very effectively. The point therefore is that fuzzy logic is a powerful method for implementing heuristics.

To implement heuristics using this method the likely source of difficulty, then, is that the quality of decision is only as good as the relation R from which it is inferred. R is in turn affected by three factors. First, it is affected by the set of rules in the protocol. With more complex situations a good protocol is not easy to derive. What is referred to as human factors research is devoted to exactly such matters[18]. Unlikely as it may seem the human being does not always find it easy to verbalise his considerations during decision making. As a hypothetical example an operator may say he took an action because the system felt 'sticky'. Thus he may attribute an action to variables that cannot be related to physical quantities let alone measurable ones (colour of the flame in some process for another example). The human operator also finds it difficult to control either too fast a system (because he cannot integrate his action) or too slow a system (because he cannot determine rates of change) or too complex a system (because he has too many variables to control). Thus it is important that the future work in this area must consider the derivation of the initial protocol very carefully.

The second factor affecting the quality of decision (though not R itself explicitly) is the underlying range of elements in the support set which provides the context for interpreting the linguistic rules. This can be illustrated by noting that 'tall people' in a land of pygmies is likely to have the support set of range of height from 3 to 5 ft. 6in, whereas the more normal range of height may be from 4ft to 7ft. Such considerations are implicit in any application and are equivalent to what a control engineer would term the gains applied to each variable.

Finally, R is affected by the membership values in the fuzzy subsets defining the linguistic values. This is perhaps the least important of all the factors because the degree of change permitted here is limited as too much change in the membership values of a fuzzy subset is likely to affect the linguistic meaning ascribed to it. This is illustrated in figure 3 in which the effect of a given linguistic value (bold line) is altered by using a different linguistic value (as in a), increasing the gain, thus decreasing the range of the support set (as in b), and lastly in a minor way by adjusting the defining values of the fuzzy subset.

A Recipe for a Prescriptive Approach

As mentioned earlier, the main difficulty that is likely with this method is that a good decision requires that a good set of rules are described at the beginning. And yet the goal in any application and a set of assumptions regarding that application are much easier to state. In the control situation the goal is simply to regulate the plant output around the set-point and the only assumption is that the plant input and output are monotonically related. If the output is high then too much input was applied and vice versa. Therefore the proper amount of input required can usually be inferred backward from the stated goal. An early attempt at implementing such a prescriptive approach is described here[20].

The overall schematic diagram for the control system is shown in figure 4*. The goal is effectively a band within which the output is to be maintained. It is specified as fuzzy rules which give the corrections needed to keep the output within the band. The input to these rules are time from start and the set-point deviation. The rules specify the change to be made in the controller. Examples of these rules are:

(a) IF *Time* is *Small* AND *deviation* is *negative big* THEN *desired change* is *big*.

(b) IF *TIME* is *big* AND *deviation* is *positive zero* THEN *desired change* is *zero*.

The result of this set of rules is used to alter the lower level control rules appropriately. Since these control rules are of the form $A_i \rightarrow B_i \rightarrow C_i$ the modification is affected by first finding the linguisting values A_i and B_i that best describe the plant state for which a change in action is required. This search is simply done by a supremum operation on the range of linguistic values. The action C_i in the controller rules at A_i, B_i value is then altered by the desired amount. The results of applying this policy are shown in figure 5.

The tables in fig. 5 are a method of displaying all the linguistic rules of the controller. The measurements error and change in error are given on the axes and the table entries give the action to be applied. The abbreviations are as stated in the footnote on page 2.

The rules in figure 5(a) (those without asterisk) are the stated protocol of an experienced operator. On applying the above procedure the extra rules marked with an asterisk are created improving the

*The system in this experiment is a simulation of a batch chemical reactor.

performance slightly. When the experiment is started with no rules at all then the rules of fig. 5(b) are created. The difference between the two strategies merely imply two feasible solutions within the prescribed band. Now it is possible to narrow this band but in the extreme case this results in a lack of convergence of rules, the quality of control however remains satisfactory.

The convergence property itself is a matter of further investigation. Analysis suggests that the lack of convergence may be attributed to two factors. Firstly it can arise because the controller is required to make a decision based on not enough variables. Secondly, since the measured state of any system is the consequences of a whole history of past inputs it is not enough to alter the rule affecting only the single previous state. This is the familiar 'bootstrapping' notion found in learning control application.

To conclude therefore, the advantage of the prescriptive method is that it reduces the difficulties with deriving a good protocol mentioned earlier providing the relevant variables can be identified. The protocol is then derived by a process of accumulation or integration of past experience. Present work is aimed at extension of this method to a multi-variable situation.

Conclusions

The prescriptive approach described above is very much an ad hoc implementation. It illustrates what needs to be done to advance beyond a simply descriptive system. What is desired is that such an approach should appear naturally within a suitably improved fuzzy logic theory itself. If, as is suggested here, hierarchical statements are a main requirement of such a theory then this means that fuzzy logic should have an auto-descriptive property found in multiple valued logics[10]. From the application point of view both a learning situation described here as well as decision making in complex systems are best framed in terms of hierarchical structures. This is very much the direction in which the theory of Fuzzy logic and approximate reasoning is likely to go. The work described here demonstrates the great usefulness of applying AR using fuzzy logic to management and other humanistic systems.

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PRESSURE ERROR = PE, CHANGE IN PRESSURE ERROR = CPE
AND HEAT INPUT CHANGE = HC

IF PE = (NB OR NM) THEN IF CPE = NS THEN HC = PM

OR

IF PE = NS THEN IF CPE = PS THEN HC = PM

OR

IF PE = NO THEN IF CPE = (PB OR PM) THEN HC = PM

OR

IF PE = NO THEN IF CPE = (NB OR NM) THEN HC = NM

OR

IF PE = PO OR NO THEN IF CPE = NO THEN HC = NO

OR

IF PE = PO THEN IF CPE = (NB OR NM) THEN HC = PM

OR

IF PE = PO THEN IF CPE = (PB OR PM) THEN HC = NM

OR

IF PE = PS THEN IF CPE = (PS OR NO) THEN HC = NM

OR

IF PE = PB OR PM) THEN IF CPE = NS THEN HC = NM

Fig. 1 A list of rules used in the steam-engine control system

BACK-END TEMPERATURE = BE, BURNING ZONE TEMPERATURE = BZ
PERCENTAGE OF OXYGEN GAS IN THE KILN EXIT GAS = OX

CASE	CONDITION	ACTION TO BE TAKEN
1	BZ LOW OX LOW BE LOW	WHEN BZ IS DRASTICALLY LOW A. REDUCE KILN SPEED B. REDUCE FUEL WHEN BZ IS SLIGHTLY LOW C. INCREASE I.D. FAN SPEED D. INCREASE FUEL RATE
2	BZ LOW OX LOW BE OK	A. REDUCE KILN SPEED B. REDUCE FUEL RATE C. REDUCE I.D. FAN SPEED
3	BZ LOW OX LOW BE HIGH	A. REDUCE KILN SPEED B. REDUCE FUEL RATE C. REDUCE I.D. FAN SPEED

TOTAL OF 27 RULES

Fig. 2 Examples of rules used for controlling a cement kiln

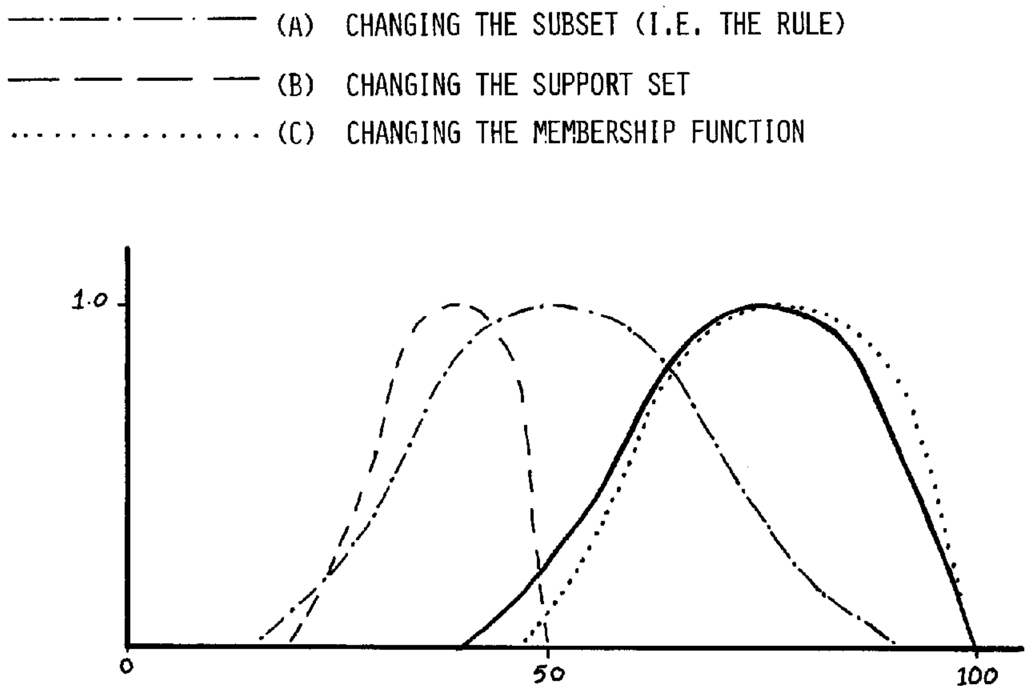

Fig. 3 Modification of fuzzy control

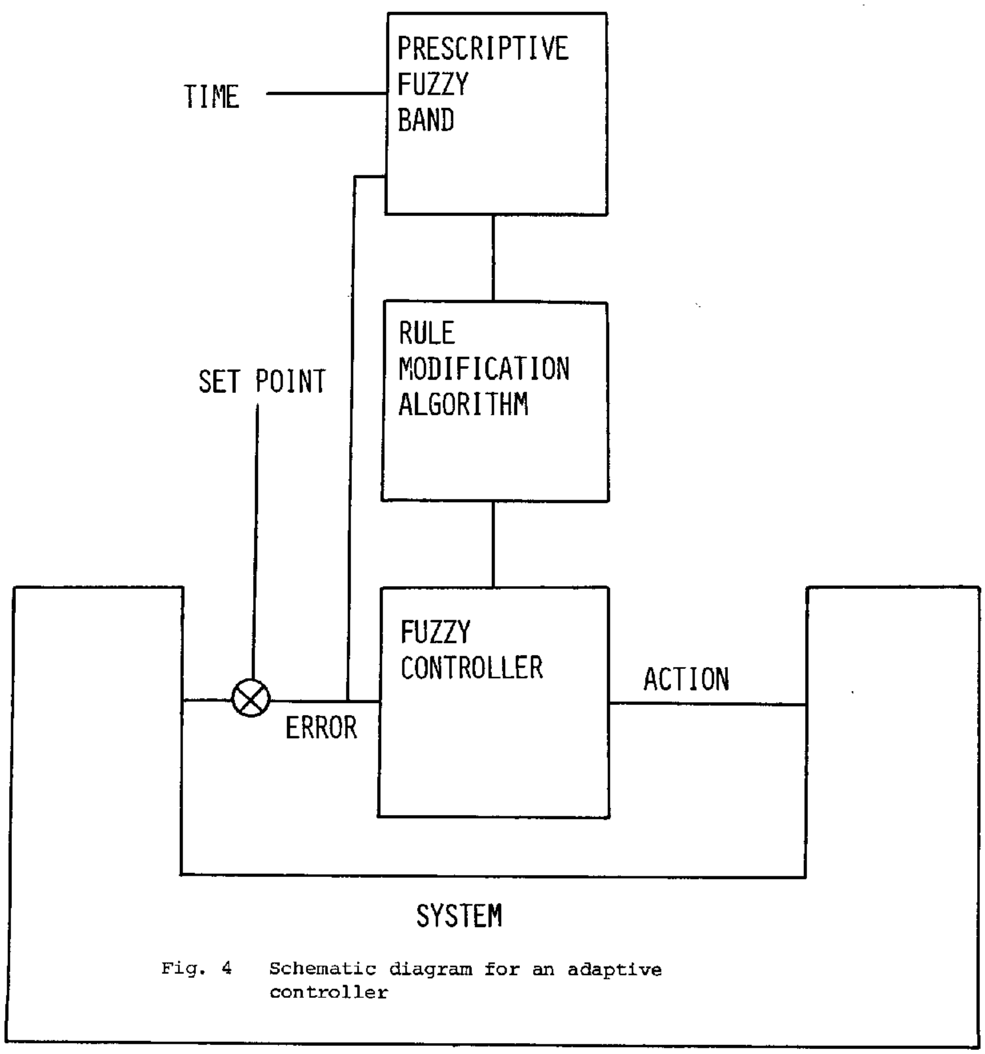

Fig. 4 Schematic diagram for an adaptive controller

ERROR

CHANGE IN ERROR	NB	NM	NS	NZ	PZ	PS	PM	PB
NB	PB*	PB*	NS	NM	PM	PS		NB*
NM	PM*		NS	NM	PM	PS		
NS	PM	PM	ZE	NS	PS	ZE	NM	NM
ZE	PB	PB	PM	ZE	ZE	NM	NB	NB
PS	PB	PB	PM	PS	NS	NM	NB	NB
PM			PB	PM	NM	NB		
PB	PB	PB	PB	PM	NM	NB	NB	NB

(A)

ERROR

CHANGE IN ERROR	NB	NM	NS	NZ	PZ	PS	PM	PB
NB	PB	PB	NB	PB	PS	PB		
NM	PB		ZE	ZE	PB	NS		NB
NS	PS		PS	ZE	PM	NM		NB
ZE	PB		ZE	ZE	PS	NM		
PS		PS	PS	PB	ZE	NB		
PM			ZE	ZE	NM	NS		
PB	PB	PS	PB	PS	NS	NB	NB	NB

(B)

Fig. 5 Rules resulting from a learning process