Test 4

Question 1

(a) Let $X \sim \text{Unif}([0,1])$.

$$\left(X : (\Omega, \mathcal{F}, P) \to \mathbb{R} \right)$$

Let $\{X(\omega_1), \dots, X(\omega_N)\}$ be our set of observations and let $t \in [0,1]$ be our test point. Then

Case 1: $t < 0.05$

Then we shall use observations in the range $[0, 0.1] =: I_1$

Case 2: $0.05 \leq t \leq 0.95$

Then ... range $[x-0.05, x+0.05] =: I_2$

Case 3: $0.95 < t$

Then ... range $[0.9, 1] =: I_3$

(2)

Hence, as intervals I_1, I_2 and I_3 have length 0.1 and $X \sim Unif([0,1])$, 'on average' the fraction

$$\frac{\#\text{ of observations used to make prediction}}{N} \quad \cdots (1)$$

is $\frac{\text{length}(I_1)}{\text{length}([0,1])} = \frac{0.1}{1} = 0.1$.

(b) Let $X \sim Unif([0,1]\times[0,1])$

Let $\{X(\omega_1), \dots, X(\omega_N)\}$ be our set of observations and let $t := (t_1, t_2) \in [0,1]\times[0,1]$ be our test point.

Then, analogously to item (a) we shall use observations in a certain set $I_1 \times I_2$ to make a prediction for t, where

$$\text{length}(I_1) = \text{length}(I_2) = 0.1.$$

Then, 'on average' the fraction (1) is

$$\frac{\text{area}(I_1 \times I_2)}{\text{area}([0,1]\times[0,1])} = \frac{0.1 \times 0.1}{1} = 10^{-2}.$$

(c) Analogously to the items above, the fraction (1) is

$$\frac{\text{volume}(I_1 \times \dots \times I_p)}{\text{volume}(\underbrace{[0,1]\times\dots\times[0,1]}_{\text{'p times'}})} = \frac{(0.1)^p}{1} = 10^{-p}.$$

(d) Indeed, since 10^{-p} is the fraction of training observations 'near' any given test observation and it is close to zero when p is large.