School of Computer Science
Faculty of Science
National University of Engineering

Solution of Test 0 Exam

Subject: Machine Learning for Data Science **Period: 2019.2**

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1. Show that
$$\int_{-1}^{1} (x^2-1)^n dx = (-1)^n \frac{2n \cdot (2n-2) \cdots 2}{(2n+1)(2n-1) \cdots 3} \cdot 2.$$

2.
$$\begin{aligned}
\int_B \exp\left(x^2+y^2\right) dxdy &= \int_A f\left(x(u,v), y(u,v)\right) |JF(u.v)| dudv \\
&= \int_2^3 \int_{\pi/6}^{\pi/3} \exp\left(u^2\right) ududv \\
&= \frac{\pi}{6} \int_2^3 u \exp\left(u^2\right) du \\
&= \frac{\pi}{6} \left(\frac{1}{2} \exp\left(u^2\right)\right)_2^3 \\
&= \frac{\pi}{12} \exp(9/4).
\end{aligned}$$

3.

   (a) Indeed, if A and B are disjoint, then $\mathbf{P}(A \cap B) = \mathbf{P}(A)\,\mathbf{P}(B)$ if and only if $\mathbf{P}(A) = 0$ or $\mathbf{P}(B) = 0$.

   (b)

$$\begin{aligned}
\mathbf{P}(A \cup B \,|\, C)\,\mathbf{P}(C) &= \mathbf{P}((A \cup B) \cap C) \\
&= \mathbf{P}(A \cap C) + \mathbf{P}((B \setminus A) \cap C) \\
&= \mathbf{P}(A \cap C) + \mathbf{P}(B \cap C) - \mathbf{P}((B \cap A) \cap C) \\
&= \ldots
\end{aligned}$$

$$\mathbf{P}(A \cup B \,|\, C) = \mathbf{P}(A \,|\, C) + \mathbf{P}(B \,|\, C) - \mathbf{P}(A \cap B \,|\, C).$$

(c) Assume that $\mathbf{P}(C) > 0$ and $A_1, \ldots, A_n$ are all pairwise disjoint. Show that

$$\mathbf{P}\left(\bigcup_{j=1}^{n} A_j \,\middle|\, C\right) = \sum_{j=1}^{n} \mathbf{P}(A_j \,|\, C).$$

(d) Let A be any event and $0 < \mathbf{P}(B) < 1$. Show that

$$\mathbf{P}(A) = \mathbf{P}(A \,|\, B)\,\mathbf{P}(B) + \mathbf{P}\left(A \,|\, B^{\complement}\right)\mathbf{P}\left(B^{\complement}\right).$$

4. $\mathbf{P}\{X > 0\} = \mathbf{P}\{X < 0\} = 1/2$. $\mathbf{P}\{|X| = 1 \,|\, X > 0\} = \alpha$ and $\mathbf{P}\{|X| = 1 \,|\, X < 0\} = \beta$

(a)

$$\begin{aligned}
\mathbf{P}\{|X| = 1\} &= \mathbf{P}\{|X| = 1, X > 0\} + \mathbf{P}\{|X| = 1, X < 0\} \\
&= \mathbf{P}\{|X| = 1 \,|\, X > 0\}\,\mathbf{P}\{X > 0\} + \mathbf{P}\{|X| = 1 \,|\, X < 0\}\,\mathbf{P}\{X < 0\} \\
&= \frac{1}{2}\beta + \frac{1}{2}\alpha \\
&= \frac{1}{2}(\alpha + \beta).
\end{aligned}$$

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(b)

$$\begin{aligned}
\mathbf{P}\{|X_n| = 1, |X_{n+1}| = 1\} &= \mathbf{P}\{|X_n| = 1, |X_{n+1}| = 1, X > 0\} + \mathbf{P}\{|X_n| = 1, |X_{n+1}| = 1, X < \\
&= \mathbf{P}\{|X_n| = 1, |X_{n+1}| = 1 \,|\, X > 0\}\,\mathbf{P}\{X > 0\} + \mathbf{P}\{|X_n| = 1, |X_{n+} \\
&= \frac{1}{2}\beta^2 + \frac{1}{2}\alpha^2 \\
&= \frac{1}{2}(\alpha^2 + \beta^2).
\end{aligned}$$

5.

(a) `mystery(A,k)` finds the `k`-th element in `A` in ascending order. This is a divide-and-conquer selection algorithm, similar to quicksort.

(b) The worst case complexity is $O(n^2)$ because of the fixed choice of pivot, as in quicksort.

(c) A typical worst case input would be, for instance, where `A` is in ascending order and `k = n`.