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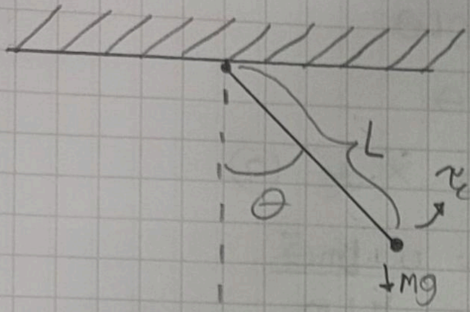
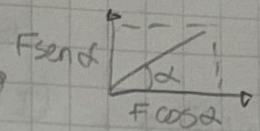
4a) Tarea Pendulo Simple



momento inercia



$$I = mL^2$$

$$I\ddot{\theta} = \tau_c - m \cdot g L \operatorname{Sen}\theta$$

$$\ddot{\theta} = \frac{\tau_c}{I} - \frac{mgL}{I} \operatorname{Sen}\theta$$

linealizo para ángulos pequeños. $\operatorname{Sen}\theta \approx \theta$

$$\ddot{\theta} = \frac{\tau_c}{I} - \frac{mgL}{I}\theta$$

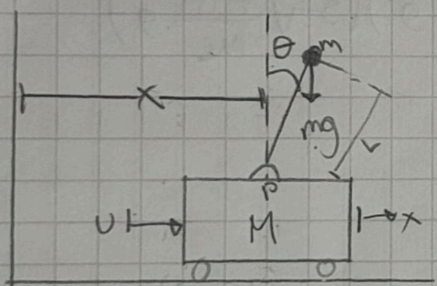
$$\ddot{\theta} = \frac{\tau_c}{mL^2} - \frac{g}{L}\theta$$

$$q_1 = \theta$$
$$q_2 = \dot{q}_1 = \dot{\theta}$$
$$\dot{q}_2 = \ddot{\theta}$$

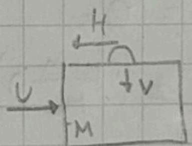
$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{L} & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{mL^2} \end{bmatrix} \tau_c$$

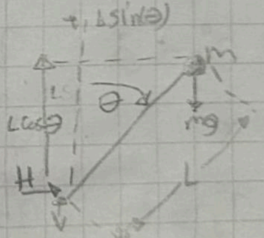
$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + [0]\tau_c$$

4b) Tarea Pendulo invertido



M: Masa del carro
m: " " Pendulo
L: Longitud " "
U: Fuerza aplicada al carro
P: Punto de articulación
θ: Ángulo del péndulo respecto a la normal





$$\Sigma F_c = m a_i$$
$$\Sigma F_j = m a_j$$
$$\Sigma F_G = I a_G$$

$$X_G = X + L\sin(\theta)$$
$$Y_G = L\cos(\theta)$$

① $I\ddot{\theta} = VL\sin\theta - HL\cos\theta$

(masa del pendulo)

② $\Sigma F_x = ma = m\ddot{x} = m\frac{d^2}{dt^2}\left(x + L\sin(\theta)\right) = H$

③ $\Sigma F_y = m\frac{d^2}{dt^2}\left(L\cos(\theta)\right) = V - mg$

(masa del carro)

④ $\Sigma F_x = M\frac{d^2x}{dt^2} = U - H$

⑤ $\Sigma F_y = Mg = V$

Suponemos θ pequeño, $\sin(\theta) \approx \theta$; $\cos(\theta) = 1$
reemplazo en ①, ②, ③.

⑥ $I\ddot{\theta} = VL\theta - HL$

⑦ $m(\ddot{x} + L\ddot{\theta}) = H$

⑧ $0 = V - mg$

de ④ y ⑤ :

⑨ $(M+m)\ddot{x} + mL\ddot{\theta} = u$

de ⑥, ⑦, ⑧:

$(I + mL^2)\ddot{\theta} + mL\ddot{x} = mgL\theta$

⑩ $mL^2\ddot{\theta} + mL\ddot{x} = mgL\theta$

$I \approx 0$ debido al centro de gravedad

Eliminando $\ddot{x}$ y $\ddot{\theta}$ de ⑨ y ⑩

⑪ $ML\ddot{\theta} = (m+M)g\theta - u$

⑫ $M\ddot{x} = u - mg\theta$

despejando $\ddot{x}$ de ⑨

⑬ $\ddot{x} = \dfrac{u - Lm\ddot{\theta}}{M+m}$

Aplicando ⑬ en ⑩

⑭ $ML\ddot{\theta} = g\theta(M+m) - u$

Aplico laplace en ⑭

$MLs^2\theta = (M+m)g\theta - u$

La funcion de transferencia resultante es:

$$H(s) = \frac{\theta(s)}{U(s)} = \frac{1}{MLs^2 - (M+m)g} = \frac{1}{ML\left(s + \sqrt{\frac{M+m}{ML}g}\right)\left(s - \sqrt{\frac{m+M}{ML}g}\right)}$$

$$s = \pm\sqrt{\frac{M+m}{ML}g}$$

$y_1 = \theta = q_1$

$y_2 = x = q_3$

$q_1 = \theta$

$q_2 = \dot{q}_1 = \dot{\theta}$

$q_3 = x$

$q_4 = \dot{q}_3 = \dot{x}$

Reemplazo en ⑪ y ⑫

$\dot{q}_1 = q_2$

$\dot{q}_2 = \left(\frac{M+m}{ML}g\right)q_1 + \left(\frac{-1}{ML}\right)u$

$\dot{q}_3 = q_4$

$\dot{q}_4 = \left(-\frac{m}{M}g\right)x_1 + \left(\frac{1}{M}\right)u$

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\\ \dot{q}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ \frac{M+m}{ML}g & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ -\frac{m}{M}g & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}q_1\\ q_2\\ q_3\\ q_4\end{bmatrix} + \begin{bmatrix}0\\ -\frac{1}{ML}\\ 0\\ \frac{1}{M}\end{bmatrix}u$$

$$\begin{matrix}y_1\\ y_2\end{matrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}q_1\\ q_2\\ q_3\\ q_4\end{bmatrix}$$