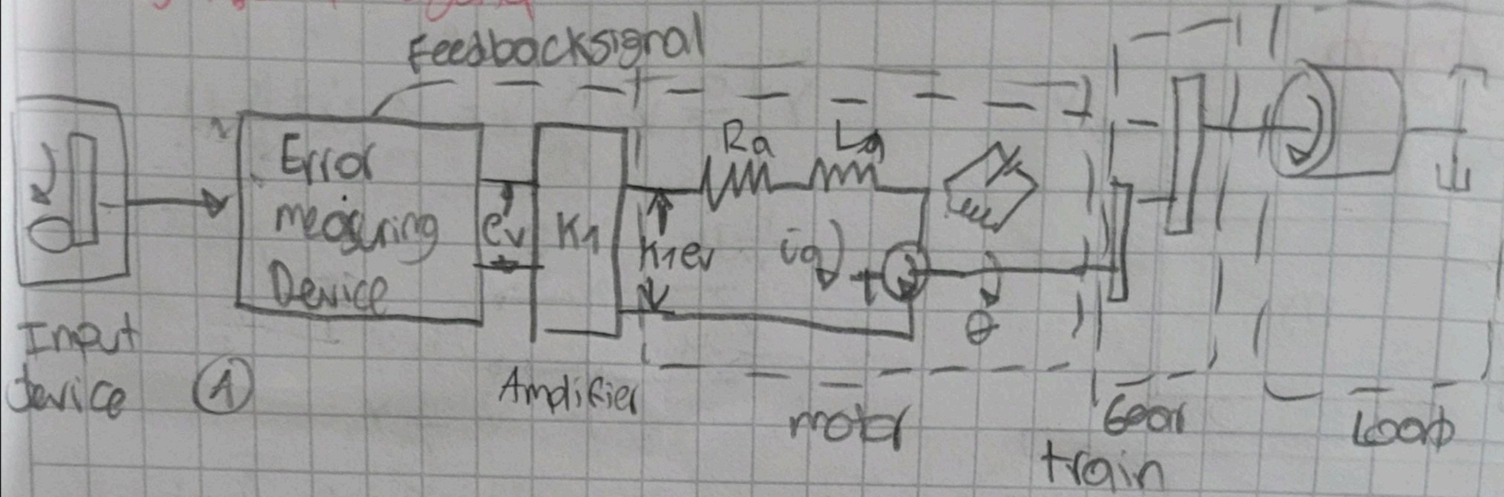
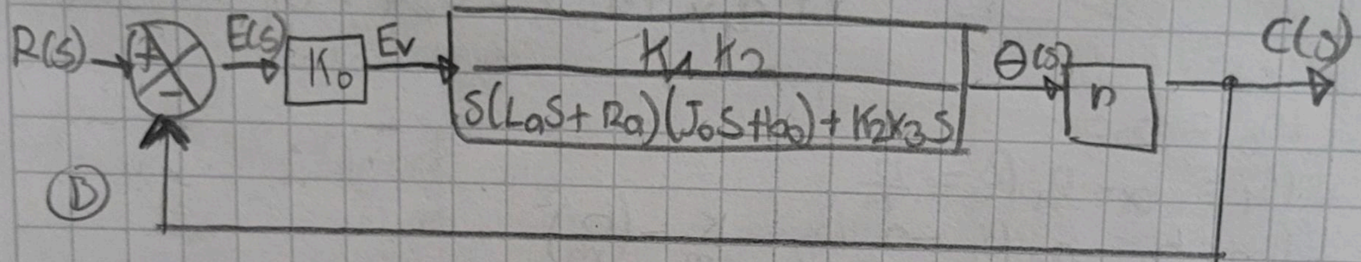
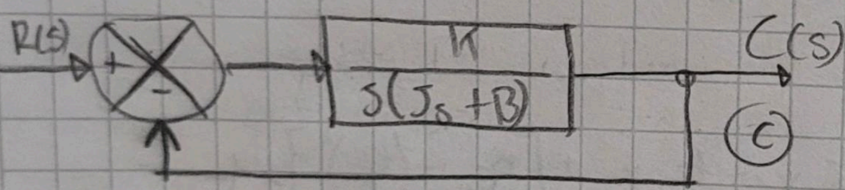
2. Ej A.3-9 Ogata





block diagram of the system



Simplified block diagram

Eq. armature circuit:

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a$$

$$L_a \frac{di}{dt} + R_a i_a + K_3 \frac{d\theta}{dt} = K_1 e_v$$

Eq. torque equilibrium

$$J_0 \frac{d^2\theta}{dt} + b_0 \frac{d\theta}{dt} = T = K_2 i_a$$

$$\frac{\theta(s)}{E_v(s)} = \frac{K_1 K_2}{s(L_a s + R_a)(J_0 s + b_0) + K_2 K_3 s}$$

$$C(s) = n\,\theta(s)$$

$$E_v(s) = K_0[R(s) - C(s)] = K_0 E(s)$$

$$G(s) = \frac{C(s)}{\theta(s)} \cdot \frac{\theta(s)}{E_v(s)} \cdot \frac{E_v(s)}{E(s)} = \frac{K_0 K_1 K_2 n}{s[(L_a s + R_a)(J_0 s + b_0) + K_2 K_3]}$$

(B)

L_a is small, it can be neglected

$$G(s)=\frac{K_0K_1K_2n}{s[R_a(J_0 s+b_0)+K_2K_3]}=\frac{K_0K_1K_2n/R_a}{J_0 s^2+\left(b_0+\frac{K_2K_3}{R_a}\right)s}$$

$J=J_0/n^2$ = moment of inertia refered to the output shaft

$B=[b_0+(K_2K_3/R_a)]/n^2$ = viscous friction coefficient refered to the output shaft

$K=K_0K_1K_2/nR_a$

(c) $G(s)=\dfrac{K}{Js^2+Bs}$ or $G(s)=\dfrac{K_m}{s(T_m s+1)}$ $\quad K_m=\dfrac{K}{B}$ $\quad T_m=\dfrac{J}{B}=\dfrac{R_aJ_0}{R_ab_0+K_2K_3}$

$$\frac{\theta}{E_v}=\frac{K}{Js^2+Bs}$$

$$Js^2\theta+Bs\theta=E_vK$$

$$J\ddot{\theta}+B\dot{\theta}=e_vK \qquad \ddot{\theta}=\frac{e_vK}{J}-\frac{B\dot{\theta}}{J}$$

$x_1=\theta$

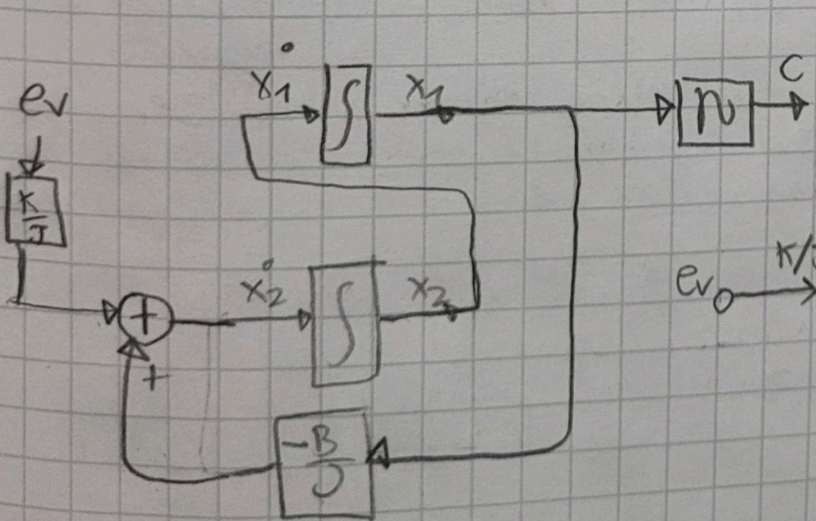
$x_2=\dot{x}_1=\dot{\theta}$

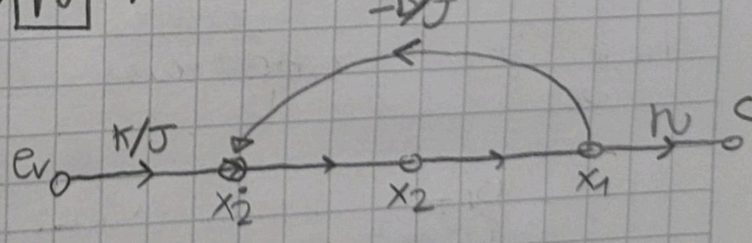
$\dot{x}_2 \quad \ddot{\theta}$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -\frac{B}{J} & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ \frac{K}{J}\end{bmatrix}\begin{bmatrix}e_v\end{bmatrix}$$

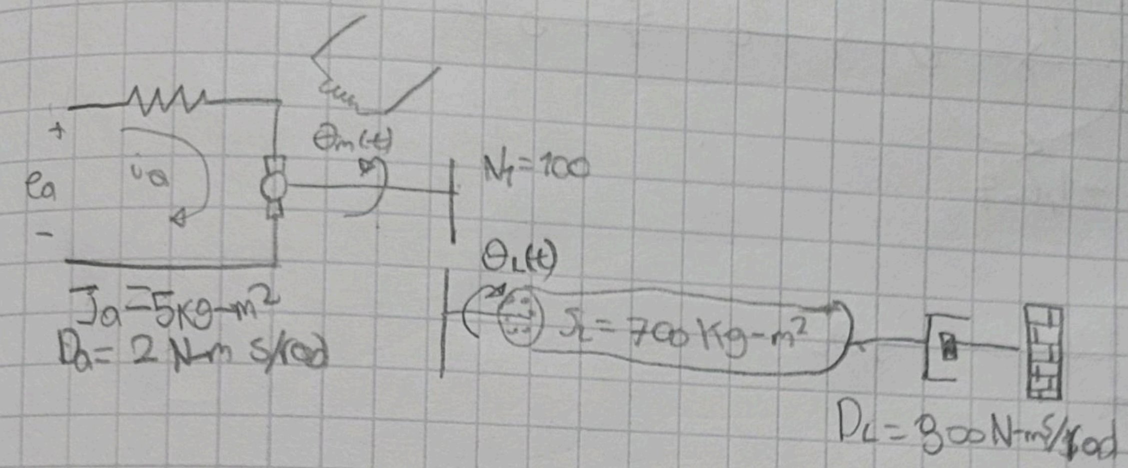
$C(s)=n\theta(s)$

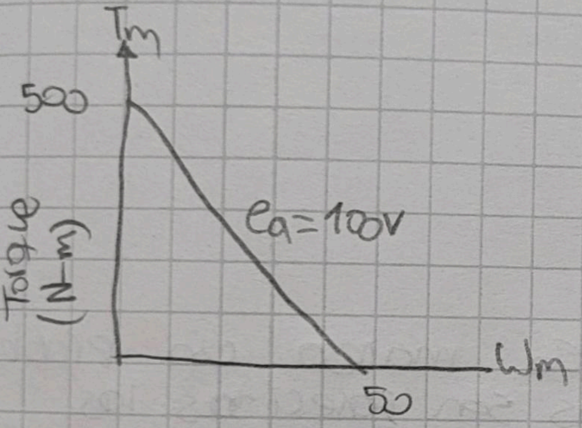
$$C=\begin{bmatrix}n & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\end{bmatrix}\begin{bmatrix}e_v\end{bmatrix}$$





3. Ej 2.23 Norman Nise





$$E_a(s) \rightarrow \boxed{\frac{0,0417}{s(s+1,667)}} \rightarrow \theta_L(s)$$

$T_{stall} = 500$
$W_{no\text{-}load} = 50$
$e_a = 100$

$$\frac{K_t}{R_a} = \frac{T_{stall}}{e_a} = \frac{500}{100} = 5 \ , \quad K_b = \frac{e_a}{W_{no\,load}} = \frac{100}{50} = 2$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{5/12}{s\left(s+\frac{1}{12}(10+5\cdot 2)\right)} = \frac{0,417}{s(s+1,667)}$$

we use the gear ratio $\frac{N_1}{N_2} = \frac{1}{10}$, and find

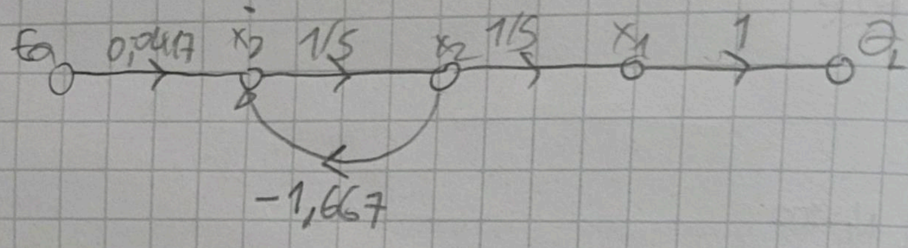
$$\frac{\theta_L(s)}{E_a(s)} = \frac{0,0417}{s(s+1,667)} \qquad \ddot{\theta}_L + 1,667\,\dot{\theta}_L = 0,0417\,E_a$$

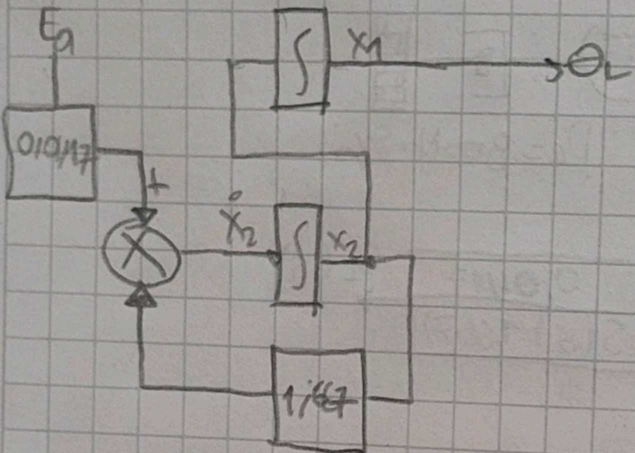
$$\ddot{\theta}_L = 0,0417\,E_a - 1,667\,\dot{\theta}_L$$

$x_1 = \theta_L$
$x_2 = \dot{\theta}_L = \dot{x}_1$
$\ddot{\theta}_L = \dot{x}_2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1,667 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0,0417 \end{bmatrix} E_a$$

$$\theta_L = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} E_a$$





Comparación En ambos casos se maneja un ejerc relacionado con motores. ambos son parecidos los relaciones que tienen sus modelos de motor entre el modelo electrico y el modelo mecanico. Ambos ejemplos difieren en que el de Norman se maneja el comportamiento de la posición angular de salida en un motor DC normal, en cambio, el otro ejemplo de Ogata se maneja un sistema de realimentacion a través de engranajes y potenciometros, lo cual permite al sistema controlar la posición de salida y su error.

Por lo tanto ambos ejercicios son parecidos, pero uno presenta realimentación a la salida, por lo que se trata de ejercicios distintos.

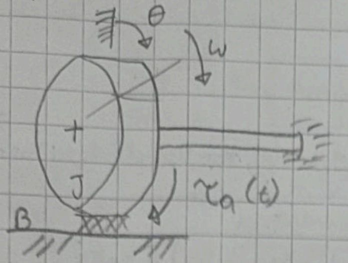
Nombre Jhon Esteban Moreno Saenz 20211005052 Fecha día 29 mes 05 año 24

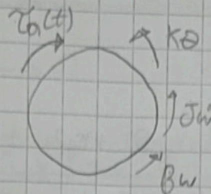
Profesor Henry Borrero Guerrero Materia Sistemas Dinamicos

Institución Universidad distrital Francisco Jose de Caldas Curso 005-1 Nota

Segundo parcial

1.





$\tau_a = K\theta + J\dot{w} + Bw$

$\dot{\theta} = w$

$\tau_a = K\theta + J\ddot{\theta} + B\dot{\theta}$

Funcion de transferencia

$\tau_a(s) = K\theta + J\theta s^2 + B\theta s$

$\tau_a(s) = \theta(Js^2 + Bs + K)$

$$\boxed{H(s) = \frac{\theta(s)}{\tau_a(s)} = \frac{1}{Js^2 + Bs + K}}$$

$x_1 = \theta$

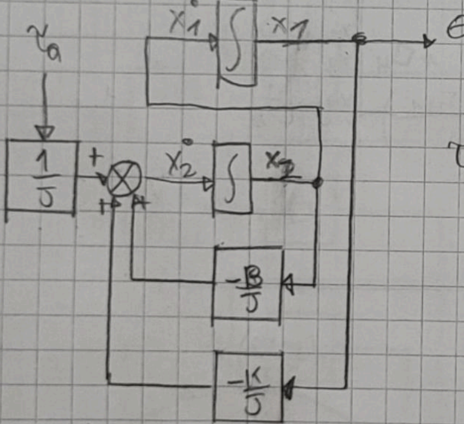
$x_2 = \dot{x}_1 = \dot{\theta}$

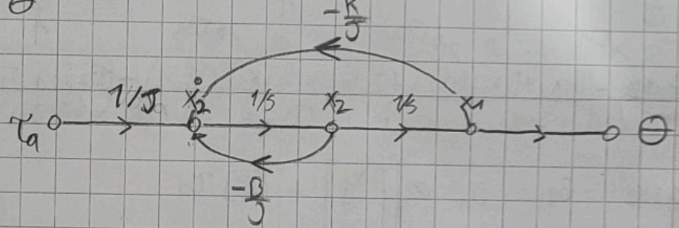
$\dot{x}_2 = \ddot{\theta}$

$\dot{x}_2 = \frac{1}{J}(\tau_a - Kx_1 - Bx_2)$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{K}{J} & -\frac{B}{J}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{J}\end{bmatrix}\tau_a$$

$$\theta = [1\ 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix} + [0]\tau_a$$





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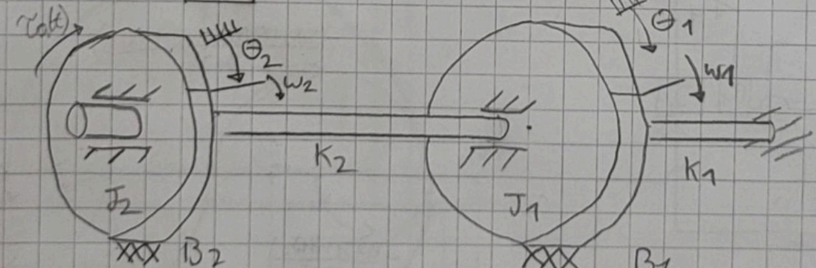
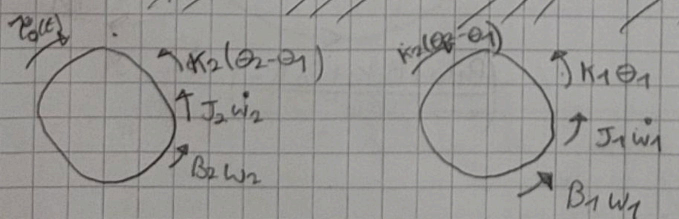




① $\tau_a = K_2(\theta_2-\theta_1) + J_2\dot{w}_2 + B_2w_2$

$\tau_a = K_2(\theta_2-\theta_1) + J_2\ddot{\theta}_2 + B_2\dot{\theta}_2$

② $K_2(\theta_2-\theta_1) = K_1\theta_1 + J_1\dot{w}_1 + B_1w_1$

$K_2(\theta_2-\theta_1) = K_1\theta_1 + J_1\ddot{\theta}_1 + B_1\dot{\theta}_1$

De ① $\theta_1 = \frac{1}{K_2}(J_2\ddot{\theta}_2 + B_2w_2 + K_2\theta_2 - \tau_a)$

De ② Reescribo

$J_1\ddot{\theta}_1 + B_1\dot{\theta}_1 + (K_1+K_2)\theta_1 - K_2\theta_2 = 0$

Reemplazo en ②

$$0 = \frac{J_1}{\cancel{K_2}}\left(J_2\ddddot{\theta}_2 + B_2\dddot{\theta}_2 + K_2\ddot{\theta}_2 - \ddot{\tau}_a\right) + \frac{B_1}{\cancel{K_2}}\left(J_2\dddot{\theta}_2 + B_2\ddot{\theta}_2 + K_2\dot{\theta}_2 - \dot{\tau}_a\right) + \frac{K_1+K_2}{\cancel{K_2}}\left(J_2\ddot{\theta}_2 + B_2\dot{\theta}_2 + K_2\theta_2 - \tau_a\right) + \cancel{K_2}\theta_2 = 0$$

$$\boxed{\begin{aligned}&\ddddot{\theta}_2 J_1J_2 + \dddot{\theta}_2(J_1B_2 + J_2B_1) + \ddot{\theta}_2(J_1K_2 + B_1B_2 + K_1J_2 + K_2J_2) + \dot{\theta}(B_1K_2 + B_2K_1 + B_2K_2) + \theta_2(K_2K_1 + \cancel{K_2} - \cancel{K_2})\\ &= J_1\ddot{\tau}_a + B_1\dot{\tau}_a + \tau_a(K_1+K_2)\end{aligned}}$$

- Funcion de transferencia

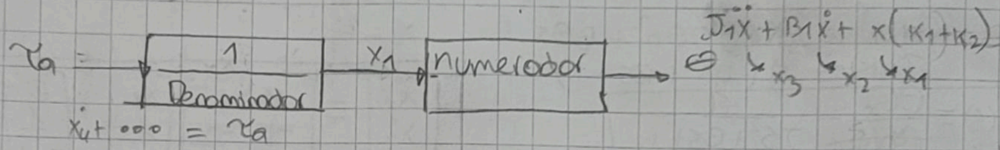
$$\tau_a(J_1s^2 + B_1s + K_1 + K_2) = \theta\left(J_1J_2s^4 + (J_1B_2 + J_2B_1)s^3 + s^2(J_1K_2 + B_1B_2 + K_1J_2 + K_2J_2) + s(B_1K_2 + B_2K_1 + B_2K_2) + K_2K_1\right)$$

$$\boxed{\frac{\theta_2}{\tau_a} = \frac{J_1s^2 + B_1s + K_1 + K_2}{s^4J_1J_2 + s^3(J_1B_2 + J_2B_1) + s^2(J_1K_2 + B_1B_2 + K_1J_2 + K_2J_2) + s(B_1K_2 + B_2K_1 + B_2K_2) + K_1K_2}}$$

numerador

Denominador

$x_1 = \theta_2$
$x_2 = \dot{\theta}_2 = \dot{x}_1$
$x_3 = \ddot{\theta}_2 = \dot{x}_2$
$x_4 = \dddot{\theta}_2 = \dot{x}_3$
$\ddddot{\theta}_2 = \dot{x}_4$

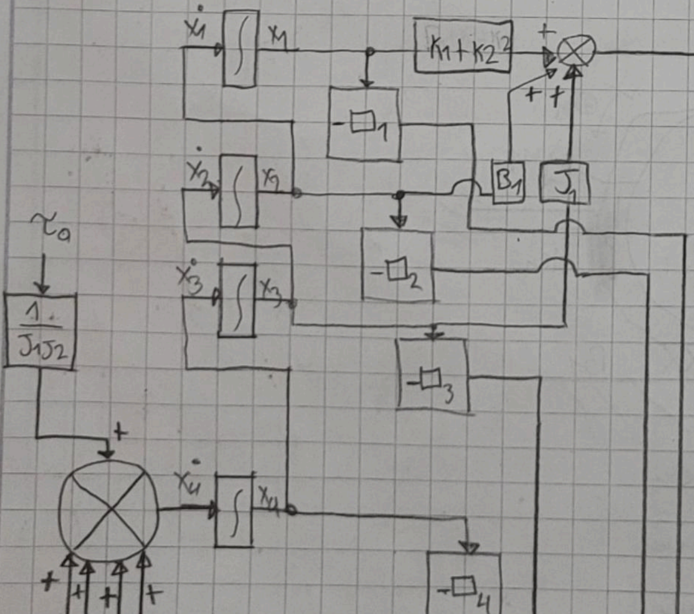


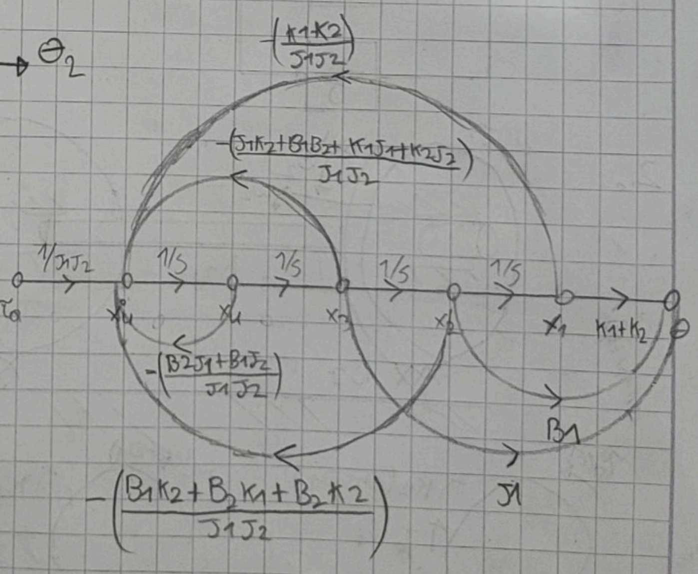
$$\dot{x}_4 = \frac{1}{J_1J_2}\left(\tau_a - x_4(J_1B_2 + J_2B_1) - x_3(J_1K_2 + B_1B_2 + K_1J_2 + K_2J_2) - x_2(B_1K_2 + B_2K_1 + B_2K_2) - x_1(K_1K_2)\right)$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -\left(\frac{K_1K_2}{J_1J_2}\right) & -\left(\frac{B_1K_2 + B_2K_1 + B_2K_2}{J_1J_2}\right) & -\left(\frac{J_1K_2 + B_1B_2 + K_1J_2 + K_2J_2}{J_1J_2}\right) & -\left(\frac{B_2J_1 + J_2B_1}{J_1J_2}\right)\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\0\\0\\ \frac{1}{J_1J_2}\end{bmatrix}\tau_a$$

$\square_1$ $\square_2$ $\square_3$ $\square_4$

$$\theta = \begin{bmatrix}K_1+K_2 & B_1 & J_1 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} + [0]\tau_a$$





3. Partiendo del punto anterior, A partir de la funcion de transferencia, y teniendo en cuenta que $K_1 = 0$ entonces tenemos:

$$\boxed{\frac{\theta_2}{\tau_a} = \frac{\overbrace{J_1 s^2 + B_1 s + K_2}^{\to Num}}{\underbrace{s^4 J_1 J_2 + s^3(J_1B_2 + J_2B_1) + s^2(J_1K_2 + B_1B_2 + K_2J_2) + s(K_2B_1 + K_2B_2)}_{\to Den}}}$$

$x_1 = \theta_2$

$x_2 = \dot{\theta}_2 = \dot{x}_1$

$x_3 = \ddot{\theta}_2 = \dot{x}_2$

$x_4 = \dddot{\theta}_2 = \dot{x}_3$

$\ddddot{\theta}_2 = \dot{x}_4$

$$\tau_a \to \boxed{\frac{1}{Den}} \xrightarrow{x_1} \boxed{Num} \to \theta_2$$

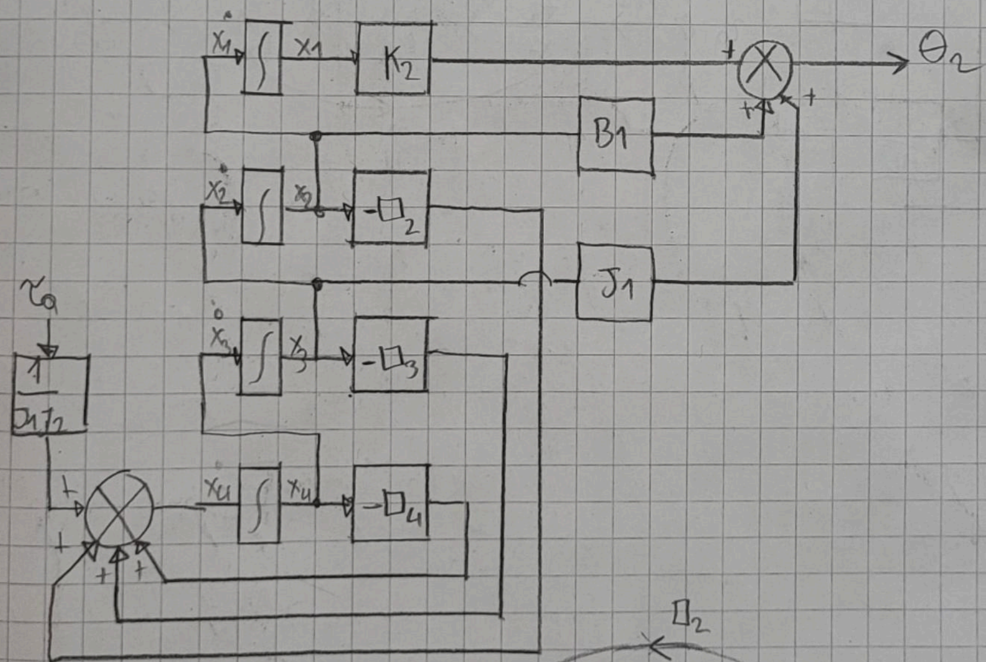
$$\underbrace{J_1 \ddot{x}}_{\to x_3} + \underbrace{B_1 \dot{x}}_{\to x_2} + \underbrace{K_2 x}_{\to x_1} = \theta \quad \text{(Salida)}$$

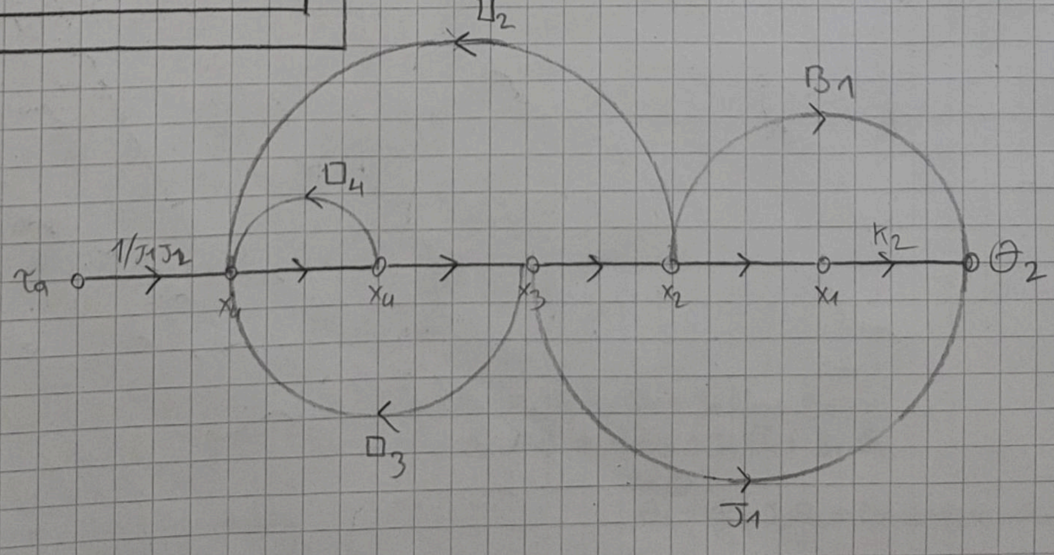
$\dot{x}_4 + \dots = \tau_a$

$$\dot{x}_4 = \frac{1}{J_1J_2}\left(\tau_a - x_4(J_1B_2 + J_2B_1) - x_3(J_1K_2 + B_1B_2 + K_2J_2) - x_2(K_2B_1 + K_2B_2)\right)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \underbrace{-\left(\frac{K_2B_1 + K_2B_2}{J_1J_2}\right)}_{\to \square_2} & \underbrace{-\left(\frac{J_1K_2 + B_1B_2 + K_2J_2}{J_1J_2}\right)}_{\to \square_3} & \underbrace{-\left(\frac{J_1B_2 + J_2B_1}{J_1J_2}\right)}_{\to \square_4} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_1J_2} \end{bmatrix} \tau_a$$

$$\theta = \begin{bmatrix} K_2 & B_1 & J_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [0]\,\tau_a$$





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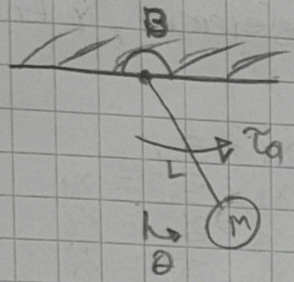

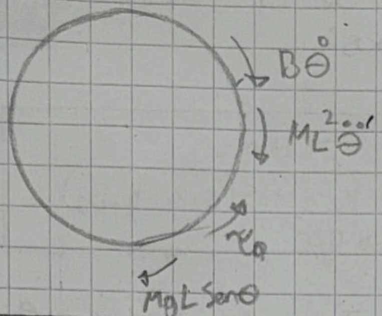

$$\tau_a = ML^2\ddot{\theta} + B\dot{\theta} + MgL\,\mathrm{Sen}\,\theta$$

$$\mathrm{Sen}\,\theta \approx \theta$$

$$\tau_a = ML^2\ddot{\theta} + B\dot{\theta} + MgL\,\theta$$

$$\tau_a = \theta\left(ML^2S^2 + BS + MgL\right)$$

$$\boxed{\frac{\theta}{\tau_a} = \frac{1}{ML^2S^2 + BS + MgL}}$$

$$x_1 = \theta$$
$$x_2 = \dot{\theta} = \dot{x}_1$$
$$\ddot{\theta} = \dot{x}_2$$

$$\ddot{\theta} = \frac{1}{ML^2}\left(\tau_a - B\dot{\theta} - MgL\,\theta\right)$$

$$\dot{x}_2 = \frac{1}{\cancel{ML^2}}\left(\tau_a - Bx_2 - \cancel{MgL}\,x_1\right)$$

$$\boxed{\begin{aligned}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} &= \begin{bmatrix}0 & 1\\ -\frac{g}{L} & -\frac{B}{ML^2}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{ML^2}\end{bmatrix}\tau_a\\ \theta &= \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\end{bmatrix}\tau_a\end{aligned}}$$