Matriz identidad (cuadrada):

$$I = \underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots \\ 0 & 1 & 0 & \cdots \\ 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}}_{\text{Diagonal principal}} \Bigg] \text{ Matriz diagonal}$$

Para los $\mathbb{R}$ el elemento identidad de la multiplicación es **1**:

$$(-5)(1) = -5$$

$$(3)(1) = 3$$

Para una matriz cuadrada

$$MI = M$$

Ejm:

$$M_{4\times4} = \begin{pmatrix} -1 & 2 & 3 & 0 \\ 0 & 1 & -2 & 1 \\ 3 & -1 & 0 & 1 \\ 2 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 2 & 3 & 0 \\ 0 & 1 & -2 & 1 \\ 3 & -1 & 0 & 1 \\ 2 & 1 & 0 & -1 \end{pmatrix} = M$$

Inversa de una matriz (cuadrada)

$$M M^{-1} = I$$

$$\begin{pmatrix} -1 & 2 & 3 & 0 \\ 0 & 1 & -2 & 1 \\ 3 & -1 & 0 & 1 \\ 2 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} & & \\ & ? & \\ & & \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$M^{-1} = \frac{(M^*)^T}{|M|}$$

← Transpuesta de la matriz adjunta

← Determinante

Inverso de un $\mathbb{R}$:

$$-3 = (-3)(1)$$

$$-3\left(\frac{1}{-3}\right) = \left(\frac{1}{\cancel{-3}}\right)(1)(\cancel{-3})$$

$$-3\underbrace{\left(\frac{1}{-3}\right)} = 1$$

Inverso de $-3 = -0.3\overline{3}$

Determinante de una matriz (cuadradas)

$$A_{2\times 2} = \begin{pmatrix} -1 & 1 \\ 3 & 2 \end{pmatrix}$$

$$|A| = \begin{vmatrix} -1 & 1 \\ 3 & 2 \end{vmatrix} = (-1)(2) - (1)(3) = -2 - 3 = -5$$

Ejm:

$$M_{4\times 4} = \begin{pmatrix} -1 & 2 & 3 & 0 \\ 0 & 1 & -2 & 1 \\ 3 & -1 & 0 & 1 \\ 2 & 1 & 0 & -1 \end{pmatrix}$$

$$|M| = \begin{vmatrix} -1 & 2 & 3 & 0 \\ 0 & 1 & -2 & 1 \\ 3 & -1 & 0 & 1 \\ 2 & 1 & 0 & -1 \end{vmatrix} = (-1)\begin{vmatrix} 1 & -2 & 1 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \end{vmatrix} - (2)\begin{vmatrix} 0 & -2 & 1 \\ 3 & 0 & 1 \\ 2 & 0 & -1 \end{vmatrix} + (3)\begin{vmatrix} 0 & 1 & 1 \\ 3 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

$$- (0)\begin{vmatrix} 0 & 1 & -2 \\ 3 & -1 & 0 \\ 2 & 1 & 0 \end{vmatrix}$$

$$\rightarrow \begin{vmatrix} 1 & -2 & 1 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \end{vmatrix} = (1)\underbrace{\begin{vmatrix} 0 & 1 \\ 0 & -1 \end{vmatrix}}_{0-0=0} - (-2)\underbrace{\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix}}_{1-1=0} + (1)\underbrace{\begin{vmatrix} -1 & 0 \\ 1 & 0 \end{vmatrix}}_{0-0=0}$$

$$= 0 - 0 + 0 = 0$$

$$\begin{vmatrix} 0 & -2 & 1 \\ 3 & 0 & 1 \\ 2 & 0 & -1 \end{vmatrix} = (0)\underbrace{\begin{vmatrix} 0 & 1 \\ 0 & -1 \end{vmatrix}}_{0} \underbrace{- (-2)\underbrace{\begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix}}_{(3)(-1)-(1)(2)=-5}}_{2(-5)=-10} + (1)\underbrace{\begin{vmatrix} 3 & 0 \\ 2 & 0 \end{vmatrix}}_{0}$$

$$= -10$$

$$\begin{vmatrix} 0 & 1 & 1 \\ 3 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 0\underbrace{\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix}}_{0} - (1)\underbrace{\begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix}}_{-5} + (1)\underbrace{\begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix}}_{5} = 0 - 1(-5) + (1)(5) = 10$$

$$\begin{vmatrix} 0 & 1 & -2 \\ 3 & -1 & 0 \\ 2 & 1 & 0 \end{vmatrix} = 0 - 0 + (-2)\begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix} = (-2)(5) = -10$$

$$\rightarrow |M| = \cancel{(-1)(0)} - (2)(-10) + (3)(10) - \cancel{(0)(-10)}$$

$$= 20 + 30$$

$$|M| = 50$$

Ejm:

$$M_{4x4} = \begin{pmatrix} -1 & 2 & 3 & 0 \\ 0 & 1 & -2 & 1 \\ 3 & -1 & 0 & 1 \\ 2 & 1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix}$$

Adjunto de $M_{ij} = (-1)^{i+j} \, |M$ sin la fila i y la columna $j|$

Ejm:

Adjunto de $M_{23} = M^{*}_{23} = (-1)^{2+3} \begin{vmatrix} -1 & 2 & 0 \\ 3 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix}$

$$M_{4\times4} = \begin{pmatrix} -1 & 2 & 3 & 0 \\ 0 & 1 & -2 & 1 \\ 3 & -1 & 0 & 1 \\ 2 & 1 & 0 & -1 \end{pmatrix}$$

$$M^{*}_{23} = (-1)^{5} \{ (-1)[(-1)(-1) - (1)(1)] - (2)[-3-2] + (0)[3+2] \}$$

$$= -1 \{ (-1)[0] - (2)[-5] + 0 \}$$

$$M^{*}_{23} = -10$$

$$M = \begin{pmatrix} -1 & 2 & 3 & 0 \\ 0 & 1 & -2 & 1 \\ 3 & -1 & 0 & 1 \\ 2 & 1 & 0 & -1 \end{pmatrix}$$

$$\rightarrow M^{*}_{42} = (-1)^{4+2} \begin{vmatrix} -1 & 3 & 0 \\ 0 & -2 & 1 \\ 3 & 0 & 1 \end{vmatrix}$$

$$= 1 \left\{ (-1)[-2] - (3)[-3] + 0 \right\}$$

$$M^{*}_{42} = 11$$

$$\rightarrow M^{*} = \begin{pmatrix} M^{*}_{11} & M^{*}_{12} & M^{*}_{13} & M^{*}_{14} \\ M^{*}_{21} & M^{*}_{22} & M^{*}_{23} & M^{*}_{24} \\ M^{*}_{31} & M^{*}_{32} & M^{*}_{33} & M^{*}_{34} \\ M^{*}_{41} & M^{*}_{42} & M^{*}_{43} & M^{*}_{44} \end{pmatrix} = \begin{pmatrix} 0 & 10 & 10 & 10 \\ 0 & 15 & -10 & 15 \\ 10 & -4 & 6 & 16 \\ 10 & 11 & -4 & -19 \end{pmatrix}$$

$$M^* = \begin{pmatrix} 0 & 10 & 10 & 10 \\ 0 & 15 & -10 & 15 \\ 10 & -4 & 6 & 16 \\ 10 & 11 & -4 & -19 \end{pmatrix}$$

Transpuesta de M^*:

$$(M^*)^T = \begin{pmatrix} 0 & 0 & 10 & 10 \\ 10 & 15 & -4 & 11 \\ 10 & -10 & 6 & -4 \\ 10 & 15 & 16 & -19 \end{pmatrix}$$

Matriz inversa

$$M^{-1} = \frac{(M^*)^T}{|M|} = (M^*)^T \cdot (1/|M|) = \begin{pmatrix} 0 & 0 & 10 & 10 \\ 10 & 15 & -4 & 11 \\ 10 & -10 & 6 & -4 \\ 10 & 15 & 16 & -19 \end{pmatrix} \cdot \left(\frac{1}{50}\right)$$

Como $|M| \neq 0$

↳ M tiene inversa

$$M^{-1} = \begin{pmatrix} 0 & 0 & 0.2 & 0.2 \\ 0.2 & 0.3 & -0.08 & 0.22 \\ 0.2 & -0.2 & 0.12 & -0.08 \\ 0.2 & 0.3 & 0.32 & 0.38 \end{pmatrix}$$

$$M = \begin{pmatrix} -1 & 2 & 3 & 0 \\ 0 & 1 & -2 & 1 \\ 3 & -1 & 0 & 1 \\ 2 & 1 & 0 & -1 \end{pmatrix}$$

Sistemas de ecuaciones lineales: $a, b, c, d = ?$

$$\begin{aligned} -a + 2b + 3c \quad &= 1 \\ +b - 2c + d &= -1 \\ 3a - b \quad + d &= 2 \\ 2a + b \quad - d &= -2 \end{aligned}$$

$$\begin{pmatrix} -1 & 2 & 3 & 0 \\ 0 & 1 & -2 & 1 \\ 3 & -1 & 0 & 1 \\ 2 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \\ -2 \end{pmatrix}$$

$$\rightarrow M\vec{x} = \vec{b}$$

$$M^{-1}M\vec{x} = M^{-1}\vec{b}$$

$$\underline{I}\,\vec{x} = M^{-1}\vec{b}$$

$$\vec{x} = M^{-1}\vec{b}$$

$$\vec{x} = \begin{pmatrix} 0 & 0 & 0.2 & 0.2 \\ 0.2 & 0.3 & -0.08 & 0.22 \\ 0.2 & -0.2 & 0.12 & -0.08 \\ 0.2 & 0.3 & 0.32 & 0.38 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 2 \\ -2 \end{pmatrix}$$

$$\vec{x} = \begin{pmatrix} 0 \\ -0.7 \\ 0.8 \\ 1.3 \end{pmatrix}$$

$a = 0,\ b = -0.7,\ c = 0.8,\ d = 1.3$