OPERACIONES ENTRE MATRICES, VECTORES (TENSORES)

$$A = \begin{pmatrix} -5 & 2 \\ 3 & -1 \\ 1 & 0 \end{pmatrix} \quad 3 \text{ filas}$$

2 columnas

$$B = \begin{pmatrix} 1 & 2 \\ 0 & 5 \\ 3 & -1 \end{pmatrix}$$

Sumar o restar: A y B tienen las mismas dimensiones:

$$A+B = \begin{pmatrix} -5+1 & 2+2 \\ 3+0 & -1+5 \\ 1+3 & 0-1 \end{pmatrix} = \begin{pmatrix} -4 & 4 \\ 3 & 4 \\ 4 & -1 \end{pmatrix}$$

$$A - B = \begin{pmatrix} -5-1 & 2-2 \\ 3-0 & -1-5 \\ 1-3 & 0+1 \end{pmatrix} = \begin{pmatrix} -6 & 0 \\ 3 & -6 \\ -2 & 1 \end{pmatrix}$$

Multiplicación de un escalar por una matriz:

$x = -2$

$$xA = (-2)\begin{pmatrix} -5 & 2 \\ 3 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 10 & -4 \\ -6 & 2 \\ -2 & 0 \end{pmatrix}$$

Multiplicación de matrices: (# de columnas C = # de filas A)

$$C = \underbrace{\begin{pmatrix} 2 & -1 & 3 \\ 1 & 0 & -2 \end{pmatrix}}_{3 \text{ columnas}} \qquad A = \left.\begin{pmatrix} -5 & 2 \\ 3 & -1 \\ 1 & 0 \end{pmatrix}\right\} 3 \text{ filas}$$

$$CA = \begin{pmatrix} (2)(-5)+(-1)(3)+(3)(1) & (2)(2)+(-1)(-1)+(3)(0) \\ (1)(-5)+(0)(3)+(-2)(1) & (1)(2)+(0)(-1)+(-2)(0) \end{pmatrix}$$

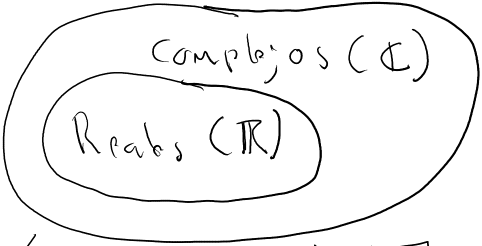
$$CA = \begin{pmatrix} -10-3+3 & 4+1+0 \\ -5+0-2 & 2+0+0 \end{pmatrix} = \begin{pmatrix} -10 & 5 \\ -7 & 2 \end{pmatrix}$$

Vectores = Matriz columna:

Ejm: $\begin{pmatrix} -5 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ } 4 dimnsiones

Ejm: $\begin{pmatrix} -1+5i \\ 2-i \end{pmatrix}$

$\vec{z}_1 = \begin{pmatrix} -i \\ 1+i \\ 0 \\ 1+2i \end{pmatrix}$ $\vec{z}_2 = \begin{pmatrix} 0 \\ i \\ 1-2i \\ 3+i \end{pmatrix}$



Matrices → Vectores

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i = -\sqrt{-1}$

$i^4 = (-1)(-1)$

$i^4 = 1$

$$\vec{z}_1 + \vec{z}_2 = \begin{pmatrix} -i \\ 1+2i \\ 1-2i \\ 4+3i \end{pmatrix}$$

Multiplicación de vectores!

$$\vec{z}_1 = \underbrace{\begin{pmatrix} -i \\ 1+i \\ 0 \\ 1+2i \end{pmatrix}}_{\text{1 columna}} \qquad \vec{z}_2 = \left.\begin{pmatrix} 0 \\ i \\ 1-2i \\ 3+i \end{pmatrix}\right\} \text{4 filas}$$

$$\vec{z}_1 \vec{z}_2 =$$ 

Transpuesta de $\vec{z}_1 \rightarrow$ Matriz Fila

$$(\vec{z}_1)^T = \underbrace{\begin{pmatrix} -i & 1+i & 0 & 1+2i \end{pmatrix}}_{\text{4 columnas}}$$

Producto punto:

$$\vec{z}_1 \cdot \vec{z}_2 = (\vec{z}_1)^T \vec{z}_2 = \begin{pmatrix} -i & 1+i & 0 & 1+2i \end{pmatrix} \begin{pmatrix} 0 \\ i \\ 1-2i \\ 3+i \end{pmatrix}$$

$$= (-i)(0) + (1+i)(i) + (0)(1-2i) + (1+2i)(3+i)$$

$$= 0 + i + i^2 + 0 + 3 + i + 6i + 2i^2$$

$$= i - 1 + 3 + i + 6i - 2$$

$$= \boxed{0+8i}$$

Producto tensorial:

$$\vec{z}_1 \otimes \vec{z}_2 = \begin{pmatrix} -i \\ 1+i \\ 0 \\ 1+2i \end{pmatrix} \otimes \begin{pmatrix} 0 \\ i \\ -2i \\ 3+i \end{pmatrix} = \begin{pmatrix} -i \begin{pmatrix} 0 \\ i \\ -2i \\ 3+i \end{pmatrix} \\ (1+i) \begin{pmatrix} 0 \\ i \\ -2i \\ 3+i \end{pmatrix} \\ 0 \begin{pmatrix} 0 \\ i \\ -2i \\ 3+i \end{pmatrix} \\ (1+2i) \begin{pmatrix} 0 \\ i \\ -2i \\ 3+i \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 0 \\ -i^2 \\ 2i^2 \\ -3i-i^2 \\ 0 \\ i+i^2 \\ -2i-2i^2 \\ 3+i+3i+i^2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ i+2i^2 \\ -2i-4i^2 \\ 3+i+6i+2i^2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -2 \\ 1-3i \\ 0 \\ -1+i \\ 2-2i \\ 2+4i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -2+i \\ 4-2i \\ 1+7i \end{pmatrix}$$