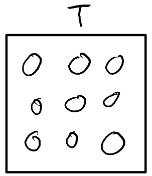
Sistema con 9 partículas:



Energía promedio total:

$U = n_1 \varepsilon_1 + n_2 \varepsilon_2 = 20$

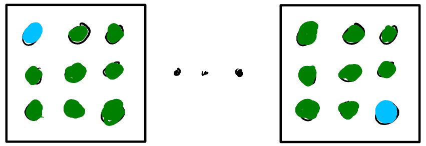
energías

$\varepsilon_1 = 2$ $\qquad$ $\varepsilon_2 = 4$

Si: $n_1 = 8$, $n_2 = 1$:

$U = 8 \cdot 2 + 1 \cdot 4 = 9$ ✓

Posibles configuraciones:



$\Omega = 9$

*Principio de Boltzmann:

$$S = K_B \ln \Omega \quad (1.1)$$

Microestados: # configs. compatible con factores externos

$\Omega \geq 1 \rightarrow S > 0$ siempre

Si $n_1 = 9$ y $n_2 = 0$:

$\Omega = 1 \rightarrow S = k_B \ln(1)$

$S = 0$

*Postulado: El sistema debe

$$S_{max} \longleftrightarrow \Omega_{más} \quad (1.2)$$

Si el sistema puede estar en todos los estados posibles:

$$\Omega_1 < \Omega_2 < \Omega_3$$

$\downarrow$

Ω_{max}

es mas probable que lo encontremos en Ω_{max}

partículas iguales

$\downarrow$

*Permutaciones con repetición:

Partículas con energ. $E_1 = n_1$

" $\vdots$

" $E_M = n_M$

$+$

N

$$\Omega = \frac{N!}{n_1!\, n_2! \ldots n_M!} \quad (1.3)$$

$$n_1 + \ldots + n_M = N \quad (1.4)$$

$\rightarrow$ Macroestados

$$E_1 n_1 + \ldots + E_M n_M = U \quad (1.5)$$

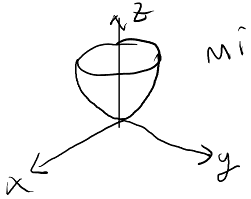
*Multiplicadores de Lagrange: para encontrar máximos o mínimos entre 3 superficies: $z(x,y)$, $g(x,y) = 0$, $h(x,y) = 0$

$$\mathcal{L} = z(x,y) - \alpha\, g(x,y) - \beta\, h(x,y) \quad (1.6)$$

$$\left.\begin{aligned} \frac{\partial \mathcal{L}}{\partial x} &= 0 \\ \frac{\partial \mathcal{L}}{\partial y} &= 0 \end{aligned}\right\} x_M, y_M \quad (1.7)$$

Ejm:

Paraboloide: $f = x^2 + y^2$



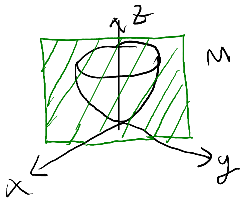
mínimo:
$x = 0$
$y = 0$

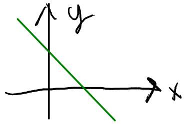
si ahora agregamos el plano:

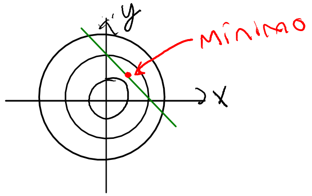
$x + 2y = 8 \rightarrow y = \frac{-x + 8}{2}$

$\downarrow$

$g(x,y) = x + 2y - 8 = 0$







$$\mathcal{L} = (x^2+y^2) - \alpha(x+2y-8)$$

$$\to \frac{\partial \mathcal{L}}{\partial x} = 2x_m - \alpha = 0 \qquad x_m = \alpha/2$$

$$\to \frac{\partial \mathcal{L}}{\partial y} = 2y_m - 2\alpha = 0 \qquad y_m = \alpha$$

$$\to x_m + 2y_m = 8$$

$$\alpha = 16/5$$

Solución:

$$x_m = \frac{\alpha}{2} = \frac{8}{5}$$

$$y_m = \alpha = 16/5$$

* Formula de Stirling:

$$\ln n! \approx n\ln n - n \quad (1.8) \qquad n \to \infty$$

Para: $n_1+n_2=N$ cte

$n_1\varepsilon_1 + n_2\varepsilon_2 = U$ cte

$$\left.\begin{matrix} n_1 = ? \\ n_2 = ? \end{matrix}\right\} S_{max}$$

$$S = k\ln\Omega = k\ln\left(\frac{N!}{n_1!\,n_2!}\right)$$

$$S = k[\ln(N!) - \ln(n_1!) - \ln(n_2!)]$$

$$\frac{S}{k} = N\ln N - \cancel{N} - n_1\ln n_1 + \cancel{n_1} - n_2\ln n_2 + \cancel{n_2}$$

Intentando maximizar S:

$$\mathcal{L}(n_1,n_2) = k(N\ln N - n_1\ln n_1 - n_2\ln n_2) - \alpha(\underbrace{\text{Restricción 1}}_{n_1+n_2-N}) - \gamma(\underbrace{\text{Restricción 2}}_{n_1\varepsilon_1+n_2\varepsilon_2-U})$$

$$\to \frac{\partial \mathcal{L}}{\partial n_1} = -k\ln n_1 - k - \alpha - \gamma\varepsilon_1 = 0$$

$$n_1 = \underbrace{\exp\left(-1-\frac{\alpha}{k}\right)}_{A}\exp\left(-\underbrace{\frac{\gamma}{k}}_{\beta}\varepsilon_1\right)$$

$$\to \frac{\partial \mathcal{L}}{\partial n_2} = 0$$

$$n_2 = \underbrace{\exp\left(-1-\frac{\alpha}{k}\right)}_{A}\exp(-\beta\varepsilon_2)$$

$$\to n_1+n_2=N$$

$$A\left(e^{-\beta\varepsilon_1}+e^{-\beta\varepsilon_2}\right) = N$$

$$\to n_1\varepsilon_1+n_2\varepsilon_2=U$$

$$A\left(\varepsilon_1 e^{-\beta\varepsilon_1}+\varepsilon_2 e^{-\beta\varepsilon_2}\right) = U$$

$$\to \frac{U}{N} = \frac{\varepsilon_1 e^{-\beta\varepsilon_1}+\varepsilon_2 e^{-\beta\varepsilon_2}}{e^{-\beta\varepsilon_1}+e^{-\beta\varepsilon_2}}$$

$$U = \underbrace{\left(N\frac{e^{-\beta\varepsilon_1}}{e^{-\beta\varepsilon_1}+e^{-\beta\varepsilon_2}}\right)}_{n_1}\varepsilon_1 + \underbrace{\left(N\frac{e^{-\beta\varepsilon_2}}{e^{-\beta\varepsilon_1}+e^{-\beta\varepsilon_2}}\right)}_{n_2}\varepsilon_2$$

Probabilidad de que una partícula tenga la energía ε_1:

$$P_1 = \frac{n_1}{N} = \frac{e^{-\beta\varepsilon_1}}{e^{-\beta\varepsilon_1}+e^{-\varepsilon_2\beta}}$$

Probabilidad de que una partícula tenga la energía ε_2:

$$P_2 = \frac{n_2}{N} = \frac{e^{-\beta\varepsilon_2}}{e^{-\beta\varepsilon_1}+e^{-\varepsilon_2\beta}}$$

* Función de partición de una partícula (1.9)

$$Z_1 = e^{-\beta E_1}+\dots+e^{-\beta E_M} \qquad \beta = \frac{1}{k_B T}$$

* Probabilidad de tener n_i con energía E_i a S_{max}:

$$P_i = \frac{n_i}{N} = \frac{e^{-\beta E_i}}{Z_1} \quad (1.10)$$

$$\to -\frac{1}{Z_1}\frac{\partial Z_1}{\partial \beta} = -\frac{1}{Z_1}\frac{\partial}{\partial \beta}\left(e^{-\beta\varepsilon_1}+e^{-\beta\varepsilon_2}\right)$$

$$= \frac{\varepsilon_1 e^{-\beta\varepsilon_1}+\varepsilon_2 e^{-\beta\varepsilon_2}}{e^{-\beta\varepsilon_1}+e^{-\beta\varepsilon_2}}$$

$$-\left(\frac{\partial \ln Z_1}{\partial \beta}\right) = \left(\frac{U}{N}\right)$$

$$U = -\frac{\partial N\ln Z_1}{\partial \beta} = -\frac{\partial \ln Z_1^N}{\partial \beta}$$

* Función de partición para N partículas:

$$Z = Z_1^N \qquad (1.11)$$

* Energía interna total:

$$U = -\frac{\partial}{\partial\beta}\ln Z \qquad (1.12)$$

$$\rightarrow \frac{S}{K} = N\ln N - \underbrace{n_1\ln n_1}_{Np_1} - \underbrace{n_2\ln n_2}_{Np_2}$$

$$\frac{S}{KN} = \cancel{\ln N} - p_1\cancel{\ln N} - p_1\ln p_1 - p_2\cancel{\ln N} - p_2\ln p_2$$

$$(\ln N)(\underbrace{1-p_1-p_2}_{0})$$

$$S = Nk_B(-p_1\ln p_1 - p_2\ln p_2)$$

$$\boxed{(1.10)\ \rightarrow p_i = \frac{e^{-\beta\varepsilon_i}}{Z_1}}$$

$$S = \frac{Nk_B}{Z_1}\left(\beta\varepsilon_1 e^{-\beta\varepsilon_1} + e^{-\beta\varepsilon_1}\ln Z_1 + \beta\varepsilon_2 e^{-\beta\varepsilon_2} + e^{-\beta\varepsilon_2}\ln Z_1\right)$$

$\rightarrow Z_1\ln Z_1$

$$S = Nk_B\left(\beta\underbrace{\frac{\varepsilon_1 e^{-\beta\varepsilon_1} + \varepsilon_2 e^{-\beta\varepsilon_2}}{Z_1}}_{U/N} + \ln Z_1\right)$$

$$S = k_B\beta U + k_B\ln Z_1^N \rightarrow Z$$

* Entropía: (1.13) ← (1.12)

$$S = -k_B\beta\frac{\partial\ln Z}{\partial\beta} + k_B\ln Z$$

* Primer principio de la termo. con N cte:

$$dU = TdS - Pd\forall \qquad (1.14)$$

$$\rightarrow T = \left(\frac{\partial U}{\partial S}\right)_{\forall} \qquad \rightarrow P = -\left(\frac{\partial U}{\partial \forall}\right)_S \qquad (1.15)$$

cte   cte

Usando (1.13):

$$S = k_B\beta U + k_B\ln Z$$

$$U = \frac{S - k_B\ln Z}{k_B\beta}$$

Si $\forall$ es cte:

$$\rightarrow \left(\frac{\partial U}{\partial S}\right)_{\forall} = \frac{1}{k_B\beta} = T \leftarrow (1.15)$$

$$\beta = \frac{1}{k_B T} \checkmark$$

* Energía libre de Helmholtz:

$$A = U - TS \qquad (1.16)$$

$$\cancel{-\frac{\partial\ln Z}{\partial\beta}} \qquad -\underbrace{k_B\beta}_{1/k_BT}\cancel{\frac{\partial\ln Z}{\partial\beta}} + k_B\ln Z$$

$$A = -\frac{\ln Z}{\beta} \qquad (1.17)$$

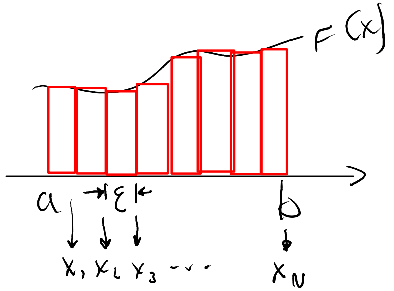
* Cambio de la energía libre de Helmholtz:

$$dA = -SdT - Pd\forall \qquad (1.18)$$

Si T es cte:

$$P = -\left(\frac{\partial A}{\partial\forall}\right)_T \qquad (1.19)$$

$$P = \frac{1}{\beta}\left(\frac{\partial\ln Z}{\partial\forall}\right)_T \qquad (1.20)$$



$$\rightarrow \varepsilon = \frac{b-a}{N} \qquad \rightarrow x_n = a + n\varepsilon$$

$$\rightarrow x_N = a + N\varepsilon$$

$$x_N = a + (b-a) = b \checkmark$$

$$\rightarrow A \simeq \sum_{n=0}^{N}\varepsilon F(x_n) = \varepsilon\sum_{n=0}^{N}F(a+n\varepsilon)$$

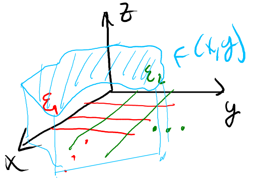
Cuando $\varepsilon \rightarrow 0$

$$\int_a^b F(x)\,dx \simeq \varepsilon\sum_{n=0}^{N}F(a+n\varepsilon)$$

$$\sum_{n=0}^{N}F(a+n\varepsilon) \simeq \frac{1}{\varepsilon}\int_a^b F(x)\,dx \qquad (2.1)$$

Para el $\forall$ bajo la superficie:

$F(x,y)$:



$\to \varepsilon_1 \varepsilon_2 \sum_{n=0}^{N} \sum_{m=0}^{M} F(x_n, y_m)$

$\simeq \int_{a_x}^{b_x} \int_{a_y}^{b_y} F(x,y)\,dx\,dy$

$$\sum_{n=0}^{N} \sum_{m=0}^{M} F(x_n, y_m) \simeq \frac{1}{\varepsilon_1 \varepsilon_2} \int_{a_x}^{b_x} \int_{a_y}^{b_y} F(x,y)\,dx\,dy \qquad (2.2)$$

Approx. de series con integrales; n, m y $p = 0, 1, 2, \ldots, N$

$N \to \infty$

$\varepsilon_1 = x_1 - x_0 = x_2 - x_1 = \ldots$

$\varepsilon_2 = y_1 - y_0 = y_2 - y_1 = \ldots$

$\varepsilon_3 = z_1 - z_0 = z_2 - z_1 = \ldots$

$$\sum_{n}^{N} \sum_{m}^{N} \sum_{p}^{N} F(x_n, y_m, z_p) \qquad (2.3)$$

$$\approx \frac{1}{\varepsilon_1 \varepsilon_2 \varepsilon_3} \int_{x_0}^{x_N} \int_{y_0}^{y_N} \int_{z_0}^{z_N} F(x,y,z)\,dx\,dy\,dz$$

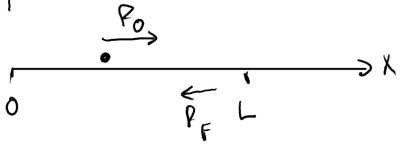
→ Energía de una partícula en un campo conservativo:

$$E = \frac{p^2}{2m} + E_p(x) \qquad (3.1)$$

→ Espacio de fases:

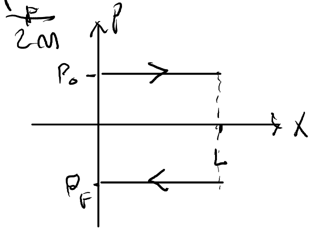
Cant. de mov (posición) = $p(x)$

Ex: partícula q' se mueve a p cte y rebota en L elásticamente:



$\to E_0 = \frac{p_0^2}{2m} = E_F = \frac{p_F^2}{2m}$

$\to p_F = -p_0$



→ No interactúa con las fronteras

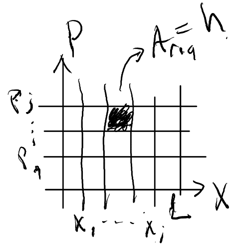
→ Partícula libre en una caja de tamaño L en 1D:

$\to \varepsilon = \sqrt{h}$

$\varepsilon^2 = h$

$\to x_i = i\varepsilon = i\sqrt{h}$

$\to p_j = j\varepsilon = j\sqrt{h}$



$$\to E_{ij}^{1D} = \frac{p_j^2}{2m} \qquad (3.2)$$

$$\to Z_1 = \sum_i \sum_j e^{-\beta E_{ij}} = \sum_i \sum_j e^{-\beta \frac{p_j^2}{2m}}$$

De discreto a continuo: ← (2.3)

$$Z_1 \underset{L \gg h}{\approx} \underbrace{\frac{1}{\varepsilon^2} \int_0^L dx}_{L/h} \int_{-\infty}^{\infty} dp\, e^{-\beta p^2/2m}$$

Si: $t = \sqrt{\frac{\beta}{2m}}\, p$

$dp = \sqrt{\frac{2m}{\beta}}\, dt$

→ Integral de Gauss:

$$\int_{-\infty}^{\infty} dt\, e^{-t^2} = \sqrt{\pi} \qquad (3.3)$$

$$\to Z_1^{1D} = \frac{L}{h} \sqrt{\frac{2m\pi}{\beta}} \qquad (3.4)$$

$\to U = -\frac{\partial}{\partial \beta} \ln Z$ ← (1.12)

Una partícula:

$U_1 = -\frac{\partial}{\partial \beta} \ln Z_1$

$\ln\left(\frac{L}{h}\right) + \frac{1}{2}\ln(2m\pi) - \frac{1}{2}\ln\beta$

$U_1^{1D} = \frac{1}{2\beta} = \frac{1}{2} k_B T \qquad L \gg \sqrt{h}$

$-\infty < p < \infty$

↳ Energía promedio (3.5)

→ Partícula libre en una caja de tamaño V en 3D:

$x \to i_1$ $\quad p_x \to j_1$

$y \to i_2$ $\quad p_y \to j_2$

$z \to i_3$ $\quad p_z \to j_3$

Phase space

→ $E_{ij} = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m}$ (4.1)

→ $h^3 = \underbrace{(\Delta p_x \Delta x)(\Delta p_y \Delta y)(\Delta p_z \Delta z)}_{\text{Phase space}}$ (4.2)

→ $z_1^{3D} = \sum_{i_1}\sum_{i_2}\sum_{i_3}\sum_{j_1}\sum_{j_2}\sum_{j_3} e^{-\beta E_{ij}}$

$\approx \frac{1}{h^3}\underbrace{\int dx \int dy \int dz}_{V} \int_{-\infty}^{\infty} dp_x\, e^{-\beta \frac{p_x^2}{2m}} \int_{-\infty}^{\infty} dp_y\, e^{-\beta \frac{p_y^2}{2m}} \int_{-\infty}^{\infty} dp_y\, e^{-\beta \frac{p_y^2}{2m}}$

$z_1^{3D} = z_1 = \frac{V}{h^3}\left(\sqrt{\frac{2m\pi}{\beta}}\right)^3$ (4.3)

→ $U_1^{3D} = -\frac{\partial}{\partial \beta}\ln z_1^{3D}$

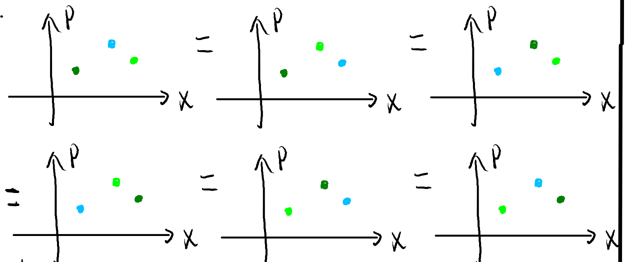
$= -\frac{\partial}{\partial \beta}\left[\ln\left(\left(\frac{V}{h^3}\sqrt{2m\pi}\right)^3\right) - \ln \beta^{3/2}\right]$

$U_1 = U_1^{3D} = \frac{3}{2\beta} = \frac{3}{2}k_B T$ (4.4)

Energía promedio

→ N partículas libres e idénticas no interactuantes → IDEAL GASES

Posible permutations among 3 particles in the phase space:



Same states: 3!

Posible permutations among N particles: N!

$Z_N = \frac{1}{N!}\sum_{\tau_1}\sum_{p_1}\cdots\sum_{\tau_N}\sum_{p_N}\exp\left(-\frac{\beta p_1^2}{2m} + \cdots + -\frac{\beta p_N^2}{2m}\right)$

τ_1: x, y, z $\quad p_1$: p_x, p_y, p_z

$\simeq \frac{1}{N!}\frac{1}{h^3}\underbrace{\int d\tau_1}_{V}\int p_1\, e^{-\beta \frac{p_1^2}{2m}} \times \cdots$

$\times \cdots \frac{1}{h^3}\underbrace{\int d\tau_N}_{V}\int dp_N\, e^{-\beta \frac{p_N^2}{2m}}$

$= \frac{1}{N!} z_1 \times \cdots \times z_1 = \frac{(z_1)^N}{N!}$

→ $Z_N^{1D} = \frac{(z_1^{1D})^N}{N!}$ (4.5)

→ $Z_N^{3D} = Z_N = \frac{(z_1^{3D})^N}{N!}$ (4.6)

$Z = \frac{V^N}{N!}\left(\frac{2m\pi}{h^2 \beta}\right)^{3N/2}$

$\ln Z = N \ln V + \frac{3N}{2}\ln\left(\frac{2m\pi}{h^2}\right) - \ln N! - \frac{3N}{2}\ln \beta$

→ $P = \frac{1}{\beta}\left(\frac{\partial \ln Z}{\partial V}\right)_T$ ← (1.20)

$P = \frac{k_B T N}{V}$ (5.1)

Usando $PV = nRT$:

$nR = k_B N$ (5.2)

→ $U = -\frac{\partial \ln Z}{\partial \beta}$ ← (1.12)

$U = \frac{3N}{2}(k_B T) = \frac{3}{2}nRT$ (5.3)

→ $S = \underbrace{k_B \beta}_{1/T} U + k_B \ln Z$ (1.13)

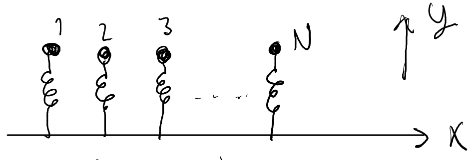
$S = \frac{3nR}{2} + nR\ln V + \frac{3nR}{2}\ln\frac{2m\pi}{h^2} - k_B \ln N! + \frac{3}{2}nR\underbrace{\ln k_B T}_{\ln k_B + \ln T}$ (5.4)

Total differential:

$$\to dS = \frac{\partial S}{\partial V} dV + \frac{\partial S}{\partial T} dT$$

$$dS = nR\frac{dV}{V} + \frac{3}{2} nR \frac{dT}{T} \quad (5.5)$$

Función de partición para N osciladores en equilibrio térmico T:



$$\to Z = \underline{Z_1}^N \quad (6.1)$$

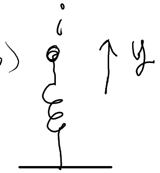
$1 \to$ Son distinguibles

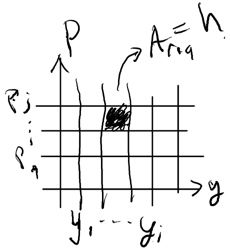
↑

posición x determinadas

Para un oscilador:

$$\to E_{ij} = \frac{p_j^2}{2m} + \frac{m\omega^2}{2} y_i^2 \quad (6.3)$$





$$\to Z_1 = \sum_i \sum_j E_{ij}$$

$$\simeq \frac{1}{h} \int_{-\infty}^{\infty} dy\, e^{-\beta \frac{m\omega^2}{2} y^2} \int_{-\infty}^{\infty} dp\, e^{-\beta \frac{p^2}{2m}}$$

$$Z_1 = \frac{1}{h} \sqrt{\frac{2\pi}{m\omega^2 \beta}} \sqrt{\frac{2m\pi}{\beta}}$$

$$Z_1 = \frac{1}{h} \frac{2\pi}{\beta\omega} \quad (6.4)$$

$$\to \ln Z_1 = \ln\left(\frac{2\pi}{h\omega}\right) - \ln\beta$$

$$\to U_1 = -\frac{\partial}{\partial\beta} \ln Z_1 = \frac{1}{\beta}$$

$$U_1 = k_B T \quad (6.5)$$

For N oscillators:

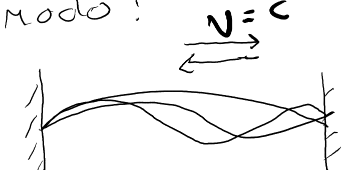
$$Z = Z_1^N = \left(\frac{2\pi}{h\beta\omega}\right)^N$$

$$\ln Z = N \ln\left(\frac{2\pi}{h\omega}\right) - N\ln\beta$$

$$\to U = -\frac{\partial}{\partial\beta} \ln Z = \frac{N}{\beta}$$

$$U = N k_B T \quad (6.2)$$

Onda estacionaria en n modo:

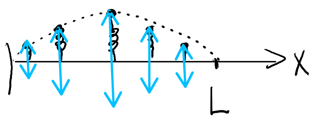


$$\to y = y_0(t) \sin k_n x \quad (6.6)$$

$$\to k_n = \frac{n\pi}{L} \quad (6.7) \qquad \to \omega_n = k_n c \quad (6.8)$$

Para N osciladores vibrando a n modo normal:

Ex: n = 1



$$\to y(x,t) = \underbrace{(A\cos\omega_n t)}_{\text{The amplitude changes over time}} \sin(k_n x)$$

For oscillator at x_i:

$$\to y = \left(A \sin\frac{n\pi}{L} x_i\right) \cos\omega_n t \quad (6.9)$$

For every vibrational modes:

$$k_n = \frac{n\pi}{L} \quad (6.10) \qquad n = 1, 2, \ldots, \infty$$

$$\omega_n = \frac{n\pi}{L} c$$

At temperature T:

$$U_{Total} = \sum_{\omega_n = 1}^{\infty} N k_B T$$

UV catastrophe!

$$\to U_{total} = N k_B T \underbrace{\sum_{\omega_n = 1}^{\infty} 1}_{\text{infinite vibrational modes}} \quad (6.11)$$

EM waves inside a box $\to$ stationary waves at T

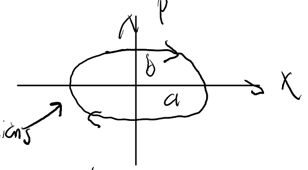
Para un oscilador clásico a energía cte E: (7.1)

$$1 = \left(\frac{x}{\frac{1}{\omega}\sqrt{\frac{2E}{m}}}\right)^2 + \left(\frac{p}{\sqrt{2mE}}\right)^2$$

Ec. elipse:

$$1 = \left(\frac{x}{a}\right)^2 + \left(\frac{p}{b}\right)^2 \quad (7.2)$$

En el espacio de fases:

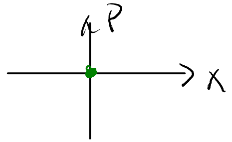


$A = \pi a b$

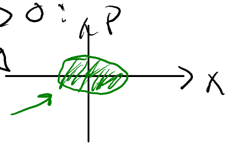
$$A = \frac{2\pi E}{\omega} \quad (7.3)$$

$$E = \frac{A\omega}{2\pi}$$

No oscillations:



Quantum system at $T>0$:



E_{min}

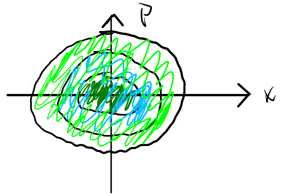
planck cte ← $A_{min} = h$

Hipotesis de Planck (a nivel quant.):

$A_{min} = h$

$A_2 = 2h$

$A_3 = 3h$



$$E_n = n\frac{h}{2\pi}\omega = n\hbar\omega \quad n = 1,2,3,\ldots \quad (7.4)$$

Realmente es un oscilador armónico cuántico: (7.5)

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) \quad n = 0,1,2,\ldots$$

modos de vibrac.

For quantum oscillator at T:

$$Z_m = \sum_{n=0}^{\infty} e^{-\beta\hbar\omega_m n}$$

vibrational modes

$\omega_m = \frac{m\pi}{L} c$ (6.10) δ

$$= 1 + e^{-\beta\hbar\omega_m} + \left(e^{-\beta\hbar\omega_m}\right)^2 + \ldots$$

Geometric series.

$$Z_m = \frac{1}{1 - e^{-\beta\hbar\omega_m}} \quad (8.1)$$

Mean energy for each mode

$$U_m = -\frac{\partial \ln Z}{\partial \beta} = \frac{\hbar\omega_m}{e^{\beta\hbar\omega_m} - 1} \quad (8.2)$$

Classic oscillator: $T \to \infty$

$\beta \to 0$

$$e^{\beta\hbar\omega} \underset{\text{Taylor}}{\simeq} 1 + \beta\hbar\omega \quad (8.4)$$

$$\to U = \frac{\hbar\omega_m}{(1 + \beta\hbar\omega_m) - 1}$$

$$U = k_B T \quad (8.3)$$

Oscilador quantum a $T \approx 0$:

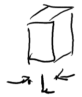
$\to \beta \to \infty \quad \to U = 0 \quad (8.5)$

mínima U realmente es

$$U = \frac{1}{2}\hbar\omega_m \quad (8.6)$$

schrödinger

Cavidad electromagnética:



En las paredes:

$E_T = 0 \quad B_n = 0 \quad (8.7)$

Transversal normal

$\vec{E}, \vec{B} \leftarrow$ Maxwell

$$\vec{E} = \vec{E}(\vec{r}, t, \omega, m_x, m_y, m_z)$$

It is a stationary wave → Vibrational modes

(6.6)

$\vec{m}$ (8.8)

Para el modo de vibración m:

$$\to \vec{K}_m = \left(\frac{M_x\pi}{L}, \frac{M_y\pi}{L}, \frac{M_z\pi}{L}\right) \quad (6.10)$$

$$\to \omega_m = \underbrace{\frac{\pi}{L}\sqrt{M_x^2+M_y^2+M_z^2}}_{|\vec{K}_m| = K_m}\cdot c \quad (8.9)$$

$$\to U_m = \frac{\hbar\omega_m}{e^{\beta\hbar\omega_m}-1}$$

$$U_m = \frac{\hbar K_m c}{e^{\beta\hbar K_m c}-1} \quad (8.10)$$

Every $\vec{K}$

Total mean energy

$$U = \sum_{M_x}\sum_{M_y}\sum_{M_z} U_M$$

$$\to \Delta M_x = \Delta M_y = \Delta M_z = 1$$

$$\to \Delta K_x \Delta K_y \Delta K_z = (\pi/L)^3$$

$$\downarrow$$

$$\frac{\Delta M_x \pi}{L}$$

In the K-space:

$$U = \frac{1}{(\pi/L)^3}\int \underbrace{d^3K}_{dK_x dK_y dK_z} \frac{\hbar K c}{e^{\beta\hbar K c}-1}$$

If: $\vec{r} = \beta\hbar\vec{K}c \to dr_x = \beta\hbar c\, dK_x$

$$dr_x dr_y dr_z = (\beta\hbar c)^3 dK_x dK_y dK_z$$

$$d^3r = (\beta\hbar c)^3 d^3K$$

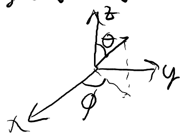
$$\to \hbar K c = \frac{\beta\hbar c|\vec{K}|}{\beta} = \frac{|\vec{r}|}{\beta} = \frac{1}{\beta} r$$

$$\to U = \frac{L^3}{\pi^3}\left[\frac{1}{(\beta\hbar c)^3}\right]\left(\frac{1}{\beta}\right)\int d^3r \frac{r}{e^r-1}$$

$$\int dr^3 = \int dV = \int r^2 \sin\theta\, d\theta\, d\phi\, dr$$

$$\to \frac{r}{e^r-1} \neq f(\theta,\phi)$$

$$\int dV = \int \sin\theta\, d\theta \int d\phi \int dr\, r^2$$



$$\to M_x, M_y, M_z > 0$$
$$\downarrow$$
$$K_x, K_y, K_z > 0$$
$$\downarrow$$
$$r_x, r_y, r_z > 0 \to$$

$\frac{1}{8}$ from sphere

$$\downarrow$$
$$0 \le \theta \le \pi/2, \quad 0 \le \phi \le \pi/2$$

$$\to dV = \int_0^{\pi/2}\sin\theta\, d\theta \int_0^{\pi/2} d\phi \int_0^\infty dr\, r^2$$

$$dV = \pi/2 \int_0^\infty dr\, r^2$$

$$\to U = \ldots \left(\frac{\pi}{2}\right) 2 \int_0^\infty dr \frac{r^3}{e^r-1}$$

2 polarizations of EM wave

$$U = L^3 \frac{1}{\pi^2\beta^4(\hbar c)^3} \underbrace{\int_0^\infty dr \frac{r^3}{e^r-1}}_{\pi^4/15}$$

$$U = \frac{L^3\pi^2}{15(\hbar c)^3\beta^4} \neq \infty \quad (8.11)$$

↳ No UV catastr.

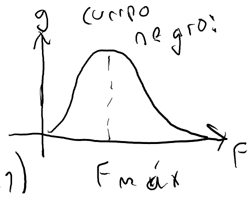
Ley de Stefan-Boltzman:

Densidad de energía:

$$u = \frac{U}{L^3} = \frac{U}{V} = \frac{\pi^2 k_B^4}{15(\hbar c)^3} T^4 \quad (8.12)$$

Ley de Planck:

$$g = \frac{2\pi h}{c^2}\frac{f^3}{e^{\beta h f}-1} \quad (9.1)$$



$$\frac{dg}{df}\Big|_{f_{max}} = 0 \quad (9.2) \qquad \to [g] = \frac{J}{m^2}$$

$$\to f_{max} = \frac{2.822}{h} k_B T \quad (9.3)$$

$$\to \lambda_{max} = \frac{c}{f_{max}} = \frac{0.0028976\, mK}{T} \quad (9.4)$$

Potencia emisiva hemisférica total:

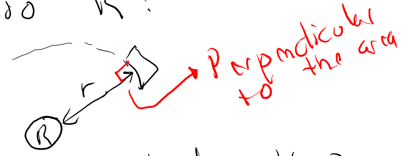
$$W = \frac{c}{4} U = \frac{2\pi^5 k_B^4}{15 c^2 h^3} T^4$$

(8.12)

$$W = \sigma T^4 \quad (9.5)$$

$$[W] = \frac{\text{Watt}}{m^2}$$

Intensidad a una distancia r de un cuerpo negro con radio R:



$$I = \frac{\text{Power from body with } R}{\text{Area receiving radiation}}$$

$$I = \frac{\sigma T^4 \, 4\pi R^2}{4\pi r^2}$$

$$I = \frac{\sigma T^4 R^2}{r^2} < \sigma T^4 \quad (9.6)$$