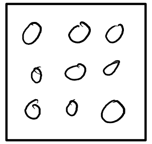
Sistema con 9 partículas:

T



Energía promedio total:

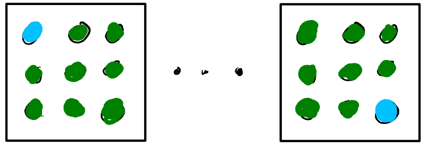
$$U = \underbrace{n_1 \varepsilon_1 + n_2 \varepsilon_2}_{\text{energías}} = 20$$

$\varepsilon_1 = 2$ $\qquad$ $\varepsilon_2 = 4$

Si: $n_1 = 8$, $n_2 = 1$:

$U = 8 \cdot 2 + 1 \cdot 4 = 9$ ✓

Posibles configuraciones:



$\Omega = 9$

*Principio de Boltzmann:

$$S = k_B \ln \Omega \quad (1.1)$$

Microestados: # configs. compatible con factores externos

$\Omega \geq 1 \rightarrow S \geq 0$ siempre

Si $n_1 = 9$ y $n_2 = 0$:

$\Omega = 1 \rightarrow S = k_B \ln(1)$

$S = 0$

*Postulado: El sistema debe

$$S_{max} \longleftrightarrow \Omega_{más} \quad (1.2)$$

Si el sistema puede estar en todos los estados posibles:

$\Omega_1 < \Omega_2 < \Omega_3$

$\downarrow$

Ω_{max}

es más probable que lo encontremos en Ω_{max}

partículas iguales

$\downarrow$

*Permutaciones con repetición:

Partículas con energ. $E_1 = n_1$

" $\quad\vdots$

" $\quad E_M = n_M$

$+$

N

$$\Omega = \frac{N!}{n_1!\, n_2! \ldots n_M!} \quad (1.3)$$

$$n_1 + \ldots + n_M = N \quad (1.4)$$

$\rightarrow$ Macroestados

$$E_1 n_1 + \ldots + E_M n_M = U \quad (1.5)$$

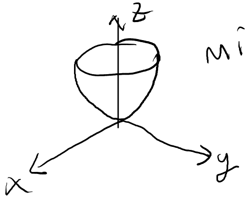
*Multiplicadores de Lagrange para encontrar máximos o mínimos entre 3 superficies: $z(x,y)$, $g(x,y) = 0$, $h(x,y) = 0$

$$\mathcal{L} = z(x,y) - \alpha\, g(x,y) - \beta\, h(x,y) \quad (1.6)$$

$$\left.\begin{aligned} \frac{\partial \mathcal{L}}{\partial x} &= 0 \\ \frac{\partial \mathcal{L}}{\partial y} &= 0 \end{aligned}\right\} x_m, y_m \quad (1.7)$$

Ejm:

Paraboloide: $f = x^2 + y^2$



mínimo:

$x = 0$

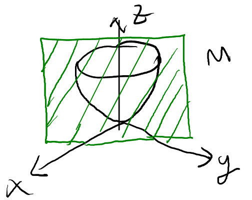
$y = 0$

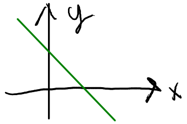
Si ahora agregamos el plano:

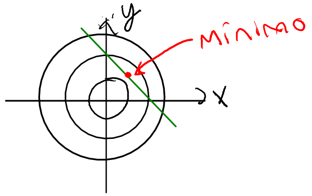
$x + 2y = 8 \rightarrow y = \frac{-x+8}{2}$

$\downarrow$

$g(x,y) = x + 2y - 8 = 0$







$$\mathcal{L} = (x^2+y^2) - \alpha(x+2y-8)$$

$$\to \frac{\partial \mathcal{L}}{\partial x} = 2x_m - \alpha = 0 \qquad x_m = \alpha/2$$

$$\to \frac{\partial \mathcal{L}}{\partial y} = 2y_m - 2\alpha = 0 \qquad y_m = \alpha$$

$$\to x_m + 2y_m = 8$$

$$\alpha = 16/5$$

Solución:

$$x_m = \frac{\alpha}{2} = \frac{8}{5}$$

$$y_m = \alpha = 16/5$$

* Formula de Stirling:

$$\ln n! \approx n\ln n - n \quad (1.8) \qquad n \to \infty$$

Para: $n_1 + n_2 = N$ cte

$n_1\varepsilon_1 + n_2\varepsilon_2 = U$ cte

$$\left.\begin{matrix} n_1 = ? \\ n_2 = ? \end{matrix}\right\} S_{max}$$

$$S = k\ln\Omega = k\ln\left(\frac{N!}{n_1!\,n_2!}\right)$$

$$S = k[\ln(N!) - \ln(n_1!) - \ln(n_2!)]$$

$$\frac{S}{k} = N\ln N - \cancel{N} - n_1\ln n_1 + \cancel{n_1} - n_2\ln n_2 + \cancel{n_2}$$

Intentando maximizar S:

$$\mathcal{L}(n_1,n_2) = k(N\ln N - n_1\ln n_1 - n_2\ln n_2) - \alpha(\underbrace{\text{Restricción 1}}_{n_1+n_2-N}) - \gamma(\underbrace{\text{Restricción 2}}_{n_1\varepsilon_1+n_2\varepsilon_2-U})$$

$$\to \frac{\partial \mathcal{L}}{\partial n_1} = -k\ln n_1 - k - \alpha - \gamma\varepsilon_1 = 0$$

$$n_1 = \underbrace{\exp\left(-1-\frac{\alpha}{k}\right)}_{A}\exp\left(-\underbrace{\frac{\gamma}{k}}_{\beta}\varepsilon_1\right)$$

$$\to \frac{\partial \mathcal{L}}{\partial n_2} = 0$$

$$n_2 = \underbrace{\exp\left(-1-\frac{\alpha}{k}\right)}_{A}\exp(-\beta\varepsilon_2)$$

$$\to n_1 + n_2 = N$$

$$A\left(e^{-\beta\varepsilon_1} + e^{-\beta\varepsilon_2}\right) = N$$

$$\to n_1\varepsilon_1 + n_2\varepsilon_2 = U$$

$$A\left(\varepsilon_1 e^{-\beta\varepsilon_1} + \varepsilon_2 e^{-\beta\varepsilon_2}\right) = U$$

$$\to \frac{U}{N} = \frac{\varepsilon_1 e^{-\beta\varepsilon_1} + \varepsilon_2 e^{-\beta\varepsilon_2}}{e^{-\beta\varepsilon_1} + e^{-\beta\varepsilon_2}}$$

$$U = \underbrace{\left(N\frac{e^{-\beta\varepsilon_1}}{e^{-\beta\varepsilon_1} + e^{-\beta\varepsilon_2}}\right)}_{n_1}\varepsilon_1 + \underbrace{\left(N\frac{e^{-\beta\varepsilon_2}}{e^{-\beta\varepsilon_1} + e^{-\beta\varepsilon_2}}\right)}_{n_2}\varepsilon_2$$

Probabilidad de que una partícula tenga la energía ε_1:

$$P_1 = \frac{n_1}{N} = \frac{e^{-\beta\varepsilon_1}}{e^{-\beta\varepsilon_1} + e^{-\varepsilon_2\beta}}$$

Probabilidad de que una partícula tenga la energía ε_2:

$$P_2 = \frac{n_2}{N} = \frac{e^{-\beta\varepsilon_2}}{e^{-\beta\varepsilon_1} + e^{-\varepsilon_2\beta}}$$

* Función de partición de una partícula (1.9)

$$Z_1 = e^{-\beta E_1} + \dots + e^{-\beta E_M} \qquad \beta = \frac{1}{k_B T}$$

* Probabilidad de tener n_i con energía E_i a S_{max}:

$$P_i = \frac{n_i}{N} = \frac{e^{-\beta E_i}}{Z_1} \qquad (1.10)$$

$$\to -\frac{1}{Z_1}\frac{\partial Z_1}{\partial \beta} = -\frac{1}{Z_1}\frac{\partial}{\partial \beta}\left(e^{-\beta\varepsilon_1} + e^{-\beta\varepsilon_2}\right) = \frac{\varepsilon_1 e^{-\beta\varepsilon_1} + \varepsilon_2 e^{-\beta\varepsilon_2}}{e^{-\beta\varepsilon_1} + e^{-\beta\varepsilon_2}}$$

$$-\left(\frac{\partial \ln Z_1}{\partial \beta}\right) = \left(\frac{U}{N}\right)$$

$$U = -\frac{\partial N\ln Z_1}{\partial \beta} = -\frac{\partial \ln Z_1^N}{\partial \beta}$$

* Función de partición para N partículas:

$$Z = Z_1^N \qquad (1.11)$$

* Energía interna total:

$$U = -\frac{\partial}{\partial\beta}\ln Z \qquad (1.12)$$

$$\to \frac{S}{K} = N\ln N - \underbrace{n_1\ln n_1}_{Np_1} - \underbrace{n_2\ln n_2}_{Np_2}$$

$$\frac{S}{KN} = \cancel{\ln N} - p_1\cancel{\ln N} - p_1\ln p_1 - p_2\cancel{\ln N} - p_2\ln p_2$$

$$(\ln N)(1 - p_1 - p_2) \to 0$$

$$S = Nk_B(-p_1\ln p_1 - p_2\ln p_2)$$

$$\boxed{(1.10) \to p_i = \frac{e^{-\beta\varepsilon_i}}{Z_1}}$$

$$S = \frac{Nk_B}{Z_1}\left(\beta\varepsilon_1 e^{-\beta\varepsilon_1} + e^{-\beta\varepsilon_1}\ln Z_1 + \beta\varepsilon_2 e^{-\beta\varepsilon_2} + e^{-\beta\varepsilon_2}\ln Z_1\right)$$

$$\to Z_1\ln Z_1$$

$$S = Nk_B\left(\beta\underbrace{\frac{\varepsilon_1 e^{-\beta\varepsilon_1} + \varepsilon_2 e^{-\beta\varepsilon_2}}{Z_1}}_{U/N} + \ln Z_1\right)$$

$$S = k_B\beta U + k_B\ln Z_1^N \to Z$$

* Entropía: (1.13) ← (1.12)

$$S = -k_B\beta\frac{\partial\ln Z}{\partial\beta} + k_B\ln Z$$

* Primer principio de la termo. con N cte:

$$dU = TdS - p\,d\forall \qquad (1.14)$$

$$\to T = \left(\frac{\partial U}{\partial S}\right)_{\forall} \qquad \to p = -\left(\frac{\partial U}{\partial\forall}\right)_S \qquad (1.15)$$

cte   cte

Usando (1.13):

$$S = k_B\beta U + k_B\ln Z$$

$$U = \frac{S - k_B\ln Z}{k_B\beta}$$

Si $\forall$ es cte:

$$\to \left(\frac{\partial U}{\partial S}\right)_{\forall} = \frac{1}{k_B\beta} = T \leftarrow (1.15)$$

$$\beta = \frac{1}{k_B T} \checkmark$$

* Energía libre de Helmholtz:

$$A = U - TS \qquad (1.16)$$

$$U \to -\cancel{\frac{\partial\ln Z}{\partial\beta}} \qquad TS \to -k_B\beta\cancel{\frac{\partial\ln Z}{\partial\beta}} + k_B\ln Z$$

$$\to 1/k_BT$$

$$A = -\frac{\ln Z}{\beta} \qquad (1.17)$$

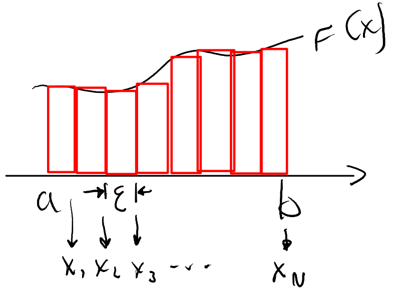
* Cambio de la energía libre de Helmholtz:

$$dA = -SdT - p\,d\forall \qquad (1.18)$$

Si T es cte:

$$p = -\left(\frac{\partial A}{\partial\forall}\right)_T \qquad (1.19)$$

$$p = \frac{1}{\beta}\left(\frac{\partial\ln Z}{\partial\forall}\right)_T \qquad (1.20)$$



$$\to \varepsilon = \frac{b-a}{N} \qquad \to x_n = a + n\varepsilon \qquad \to x_N = a + N\varepsilon$$

$$x_N = a + (b-a) = b \checkmark$$

$$\to A \simeq \sum_{n=0}^{N}\varepsilon F(x_n) = \varepsilon\sum_{n=0}^{N}F(a+n\varepsilon)$$

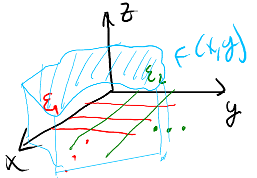
Cuando $\varepsilon \to 0$

$$\int_a^b F(x)\,dx \simeq \varepsilon\sum_{n=0}^{N}F(a+n\varepsilon)$$

$$\sum_{n=0}^{N}F(a+n\varepsilon) \simeq \frac{1}{\varepsilon}\int_a^b F(x)\,dx \qquad (2.1)$$

Para el ∀ bajo la superficie:

$F(x,y)$:



$$\to \varepsilon_1 \varepsilon_2 \sum_{n=0}^{N} \sum_{m=0}^{M} F(x_n, y_m) \simeq \int_{a_x}^{b_x} \int_{a_y}^{b_y} F(x,y)\,dx\,dy$$

$$\sum_{n=0}^{N} \sum_{m=0}^{M} F(x_n, y_m) \simeq \frac{1}{\varepsilon_1 \varepsilon_2} \int_{a_x}^{b_x} \int_{a_y}^{b_y} F(x,y)\,dx\,dy \qquad (2.2)$$

Approx. de series con integrales; n, m y $p = 0, 1, 2, \dots, N$

$N \to \infty$

$\varepsilon_1 = x_1 - x_0 = x_2 - x_1 = \dots$

$\varepsilon_2 = y_1 - y_0 = y_2 - y_1 = \dots$

$\varepsilon_3 = z_1 - z_0 = z_2 - z_1 = \dots$

$$\sum_{n}^{N} \sum_{m}^{N} \sum_{p}^{N} F(x_n, y_m, z_p) \qquad (2.3)$$

$$\approx \frac{1}{\varepsilon_1 \varepsilon_2 \varepsilon_3} \int_{x_0}^{x_N} \int_{y_0}^{y_N} \int_{z_0}^{z_N} F(x,y,z)\,dx\,dy\,dz$$

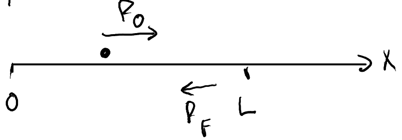
→ Energía de una partícula en un campo conservativo:

$$E = \frac{p^2}{2m} + E_p(x) \qquad (3.1)$$

→ Espacio de fases:

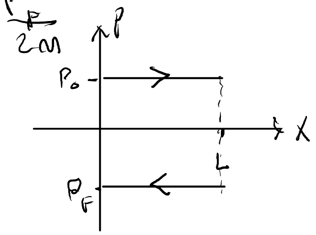
Cant. de mov (posición) $= p(x)$

Ex: partícula q' se mueve a p cte y rebota en L elásticamente!



$$\to E_0 = \frac{p_0^2}{2m} = E_F = \frac{p_F^2}{2m}$$

$$\to p_F = -p_0$$



→ No interactúa con las fronteras

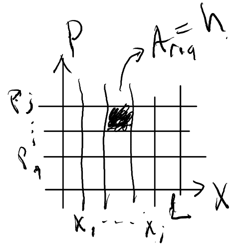
→ Partícula libre en una caja de tamaño L en 1D:

$\to \varepsilon = \sqrt{h}$

$\varepsilon^2 = h$

$\to x_i = i\varepsilon = i\sqrt{h}$

$\to p_j = j\varepsilon = j\sqrt{h}$



$$\to E_{ij}^{1D} = \frac{p_j^2}{2m} \qquad (3.2)$$

$$\to Z_1 = \sum_i \sum_j e^{-\beta E_{ij}} = \sum_i \sum_j e^{-\beta \frac{p_j^2}{2m}}$$

De discreto a continuo: ← (2.3)

$$Z_1 \approx \underbrace{\frac{1}{\varepsilon^2} \int_0^L dx}_{L/h} \int_{-\infty}^{\infty} dp\, e^{-\beta p^2/2m}$$

$L \gg h$

Si: $t = \sqrt{\frac{\beta}{2m}}\, p$

$$dp = \sqrt{\frac{2m}{\beta}}\, dt$$

→ Integral de Gauss:

$$\int_{-\infty}^{\infty} dt\, e^{-t^2} = \sqrt{\pi} \qquad (3.3)$$

$$\to Z_1^{1D} = \frac{L}{h} \sqrt{\frac{2m\pi}{\beta}} \qquad (3.4)$$

$$\to U = -\frac{\partial}{\partial \beta} \ln Z \quad \leftarrow (1.12)$$

Una partícula:

$$U_1 = -\frac{\partial}{\partial \beta} \ln Z_1$$

$$\ln\left(\frac{L}{h}\right) + \frac{1}{2} \ln(2m\pi) - \frac{1}{2} \ln \beta$$

$$U_1^{1D} = \frac{1}{2\beta} = \frac{1}{2} k_B T \qquad L \gg \sqrt{h} \qquad -\infty < p < \infty$$

→ Energía promedio (3.5)

→ Partícula libre en una caja de tamaño V en 3D:

$$\underbrace{\begin{matrix} x \to i_1 & p_x \to j_1 \\ y \to i_2 & p_y \to j_2 \\ z \to i_3 & p_z \to j_3 \end{matrix}}_{\text{Phase space}}$$

$$\to E_{ij} = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} \quad (4.1)$$

$$\to h^3 = \underbrace{(\Delta p_x \Delta x)(\Delta p_y \Delta y)(\Delta p_z \Delta z)}_{\text{Phase space}} \quad (4.2)$$

$$\to z_1^{3D} = \sum_{i_1}\sum_{i_2}\sum_{i_3}\sum_{j_1}\sum_{j_2}\sum_{j_3} e^{-\beta E_{ij}}$$

$$\approx \frac{1}{h^3} \underbrace{\int dx \int dy \int dz}_{V} \int_{-\infty}^{\infty} dp_x\, e^{-\beta \frac{p_x^2}{2m}} \int_{-\infty}^{\infty} dp_y\, e^{-\beta \frac{p_y^2}{2m}} \int_{-\infty}^{\infty} dp_y\, e^{-\beta \frac{p_y^2}{2m}}$$

$$z_1^{3D} = z_1 = \frac{V}{h^3}\left(\sqrt{\frac{2m\pi}{\beta}}\right)^3 \quad (4.3)$$

$$\to U_1^{3D} = -\frac{\partial}{\partial \beta} \ln z_1^{3D}$$

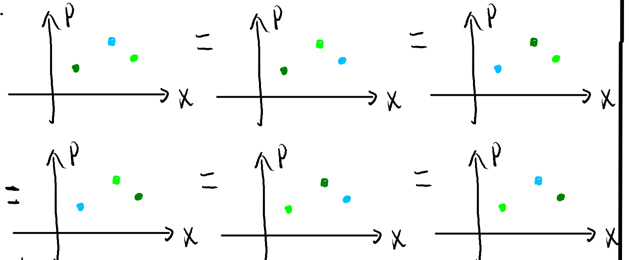
$$= -\frac{\partial}{\partial \beta}\left[\ln\left(\left(\frac{V}{h^3}\sqrt{2m\pi}\right)^3\right) - \ln \beta^{3/2}\right]$$

$$U_1 = U_1^{3D} = \frac{3}{2\beta} = \frac{3}{2} k_B T \quad (4.4)$$

↳ Energía promedio

→ N partículas libres e idénticas no interactuantes
↳ IDEAL GASES

Posible permutations among 3 particles in the phase space:



Same states: $3!$

Posible permutations among N particles: $N!$

$$Z_N = \frac{1}{N!}\sum_{\tau_1}\sum_{p_1} \cdots \sum_{\tau_N}\sum_{p_N} \exp\left(-\frac{\beta p_1^2}{2m} + \cdots + -\frac{\beta p_N^2}{2m}\right)$$

τ_1: x, y, z ; p_1: p_x, p_y, p_z

$$\approx \frac{1}{N!}\frac{1}{h^3}\underbrace{\int d\tau_1}_{V}\int dp_1\, e^{-\beta \frac{p_1^2}{2m}} \times \cdots \times \frac{1}{h^3}\underbrace{\int d\tau_N}_{V}\int dp_N\, e^{-\beta\frac{p_N^2}{2m}}$$

$$= \frac{1}{N!} z_1 \times \cdots \times z_1 = \frac{(z_1)^N}{N!}$$

$$\to Z_N^{1D} = \frac{(z_1^{1D})^N}{N!} \quad (4.5)$$

$$\to Z_N^{3D} = Z_N = \frac{(z_1^{3D})^N}{N!} \quad (4.6)$$

$$Z = \frac{V^N}{N!}\left(\frac{2m\pi}{h^2\beta}\right)^{3N/2}$$

$$\ln Z = N \ln V + \frac{3N}{2}\ln\left(\frac{2m\pi}{h^2}\right) - \ln N! - \frac{3N}{2}\ln \beta$$

$$\to P = \frac{1}{\beta}\left(\frac{\partial \ln Z}{\partial V}\right)_T \leftarrow (1.20)$$

$$P = \frac{k_B T N}{V} \quad (5.1)$$

Usando $PV = nRT$:

$$nR = k_B N \quad (5.2)$$

$$\to U = -\frac{\partial \ln Z}{\partial \beta} \leftarrow (1.12)$$

$$U = \frac{3N}{2}(k_B T) = \frac{3}{2} nRT \quad (5.3)$$

$$\to S = \underbrace{k_B \beta}_{1/T} U + k_B \ln Z \quad (1.13)$$

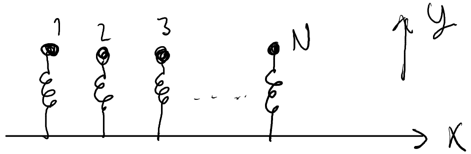
$$S = \frac{3nR}{2} + nR \ln V + \frac{3nR}{2}\ln\frac{2m\pi}{h^2} - k_B \ln N! + \frac{3}{2} nR \underbrace{\ln k_B T}_{\ln k_B + \ln T} \quad (5.4)$$

Total differential:

$$\to dS = \frac{\partial S}{\partial V} dV + \frac{\partial S}{\partial T} dT$$

$$dS = nR \frac{dV}{V} + \frac{3}{2} nR \frac{dT}{T} \quad (5.5)$$

Función de partición para N osciladores en equilibrio térmico T:



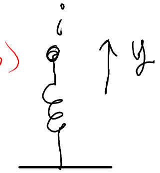
$$\to Z = Z_1^N \quad (6.1)$$

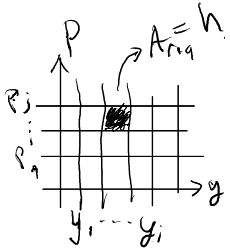
$\underline{1} \to$ Son distinguibles

posición X determinadas

Para un oscilador:

$$\to E_{ij} = \frac{p_j^2}{2m} + \frac{m\omega^2}{2} y_i^2 \quad (6.3)$$





$$\to Z_1 = \sum_i \sum_j E_{ij}$$

$$\simeq \frac{1}{h} \int_{-\infty}^{\infty} dy\, e^{-\beta \frac{m\omega^2}{2} y^2} \int_{-\infty}^{\infty} dp\, e^{-\beta \frac{p^2}{2m}}$$

$$Z_1 = \frac{1}{h} \sqrt{\frac{2\pi}{m\omega^2 \beta}} \sqrt{\frac{2m\pi}{\beta}}$$

$$Z_1 = \frac{1}{h} \frac{2\pi}{\beta\omega} \quad (6.4)$$

$$\to \ln Z_1 = \ln\left(\frac{2\pi}{h\omega}\right) - \ln\beta$$

$$\to U_1 = -\frac{\partial}{\partial\beta} \ln Z_1 = \frac{1}{\beta}$$

$$U_1 = k_B T \quad (6.5)$$

For N oscillators:

$$Z = Z_1^N = \left(\frac{2\pi}{h\beta\omega}\right)^N$$

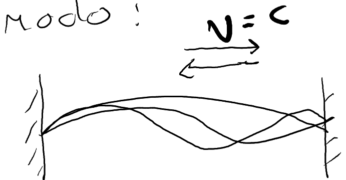
$$\ln Z = N \ln\left(\frac{2\pi}{h\omega}\right) - N\ln\beta$$

$$\to U = -\frac{\partial}{\partial\beta} \ln Z = \frac{N}{\beta}$$

$$U = N k_B T \quad (6.2)$$

Onda estacionaria en n modo:

$$v = c$$

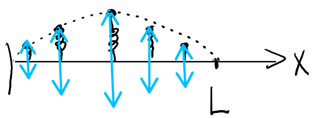


$$\to y = y_0(t) \sin k_n x \quad (6.6)$$

$$\to k_n = \frac{n\pi}{L} \quad (6.7) \qquad \to \omega_n = k_n c \quad (6.8)$$

Para N osciladores vibrando a n modo normal:

Ex: n = 1



$$\to y(x,t) = \underbrace{(A\cos\omega_n t)}_{\text{The amplitude changes over time}} \sin(k_n x)$$

For oscillator at x_i:

$$\to y = \left(A \sin \frac{n\pi}{L} x_i\right) \cos \omega_n t \quad (6.9)$$

For every vibrational modes:

$$k_n = \frac{n\pi}{L} \qquad (6.10)$$

$$n = 1, 2, \ldots, \infty$$

$$\omega_n = \frac{n\pi}{L} c$$

At temperature T:

$$U_{Total} = \sum_{\omega_n = 1}^{\infty} N k_B T$$

UV catastrophe!

$$\to U_{total} = N k_B T \sum_{\omega_n = 1}^{\infty} 1 \quad (6.11)$$

infinite vibrational modes

EM waves into a box $\to$ stationary waves at T

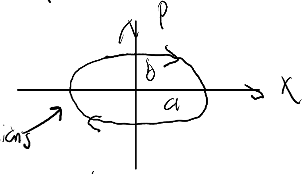
Para un oscilador clásico a energía cte E: (7.1)

$$1 = \left(\frac{x}{\frac{1}{\omega}\sqrt{\frac{2E}{m}}}\right)^2 + \left(\frac{p}{\sqrt{2mE}}\right)^2$$

Eq. elipse:

$$1 = \left(\frac{x}{a}\right)^2 + \left(\frac{p}{b}\right)^2 \quad (7.2)$$

En el espacio de fases:

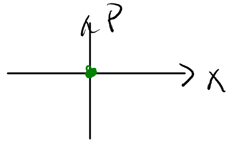


$$A = \pi a b$$

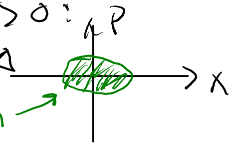
$$A = \frac{2\pi E}{\omega} \quad (7.3)$$

$$E = \frac{A\omega}{2\pi}$$

No oscillations:



Quantum system at $T>0$:



E_{min}

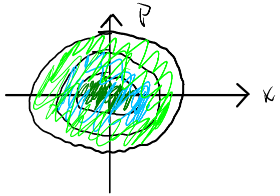
planck cte ← $A_{min} = h$

Hipótesis de Planck (a nivel quant.):

$A_{min} = h$

$A_2 = 2h$

$A_3 = 3h$



$$E_n = n\frac{h}{2\pi}\omega = n\hbar\omega \quad n = 1,2,3,\dots \quad (7.4)$$

Realmente es un oscilador armónico cuántico: (7.5)

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) \quad n = 0,1,2,\dots$$

modos de vibrac.

For quantum oscillator at T:

$$Z_m = \sum_{n=0}^{\infty} e^{-\beta\hbar\omega_m n}$$

vibrational modes

↳ $\omega_m = \frac{m\pi}{L}c$ (6.10) δ

$$= 1 + e^{-\beta\hbar\omega_m} + \left(e^{-\beta\hbar\omega_m}\right)^2 + \dots$$

↳ Geometric series.

$$Z_m = \frac{1}{1 - e^{-\beta\hbar\omega_m}} \quad (8.1)$$

Mean energy for each mode

$$U_m = -\frac{\partial}{\partial\beta}\ln Z = \frac{\hbar\omega_m}{e^{\beta\hbar\omega_m} - 1} \quad (8.2)$$

Classic oscillator: $T \to \infty$

$\downarrow$

$\beta \to 0$

$$e^{\beta\hbar\omega} \underset{\text{Taylor}}{\simeq} 1 + \beta\hbar\omega \quad (8.4)$$

$$\to U = \frac{\hbar\omega_m}{(1 + \beta\hbar\omega_m) - 1}$$

$$U = k_B T \quad (8.3)$$

Oscilador quantum a $T \simeq 0$:

$\to \beta \to \infty \quad \to U = 0 \quad (8.5)$

mínima U realmente es

$$U = \frac{1}{2}\hbar\omega_m \quad (8.6)$$

schrödinger

Cavidad electromagnética:

En las paredes:

$$E_T = 0 \quad B_n = 0 \quad (8.7)$$

Transversal, normal

$\vec{E}, \vec{B}$ ← Maxwell

$$\vec{E} = \vec{E}(\vec{r}, t, \omega, \underbrace{m_x, m_y, m_z})$$

It is a stationary wave (6.6) → $\underbrace{\text{Vibrational Modes}}_{\vec{m}}$ (8.8)

Para el modo de vibración m:

$$\to \vec{K}_m = \left(\frac{M_x\pi}{L}, \frac{M_y\pi}{L}, \frac{M_z\pi}{L}\right) \quad (6.10)$$

$$\to \omega_m = \underbrace{\frac{\pi}{L}\sqrt{M_x^2+M_y^2+M_z^2}}_{|\vec{K}_m| = K_m}\cdot c \quad (8.9)$$

$$\to U_m = \frac{\hbar\omega_m}{e^{\beta\hbar\omega_m}-1}$$

$$U_m = \frac{\hbar K_m c}{e^{\beta\hbar K_m c}-1} \quad (8.10)$$

Every $\vec{K}$

Total mean energy

$$U = \sum_{M_x}\sum_{M_y}\sum_{M_z} U_M$$

$$\to \Delta M_x = \Delta M_y = \Delta M_z = 1$$

$$\to \Delta K_x \Delta K_y \Delta K_z = (\pi/L)^3$$

$$\downarrow \quad \frac{\Delta M_x \pi}{L}$$

In the K-space:

$$U = \frac{1}{(\pi/L)^3}\int d^3K \, \frac{\hbar K c}{e^{\beta\hbar K c}-1}, \qquad d^3K \to dK_x dK_y dK_z$$

IF: $\vec{r} = \beta\hbar\vec{K}c \to dr_x = \beta\hbar c\, dK_x$

$$dr_x dr_y dr_z = (\beta\hbar c)^3 dK_x dK_y dK_z$$

$$d^3r = (\beta\hbar c)^3 d^3K$$

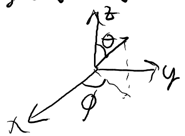
$$\to \hbar K c = \frac{\beta\hbar c|\vec{K}|}{\beta} = \frac{|\vec{r}|}{\beta} = \frac{1}{\beta} r$$

$$\to U = \frac{L^3}{\pi^3}\left[\frac{1}{(\beta\hbar c)^3}\right]\left(\frac{1}{\beta}\right)\int d^3r \, \frac{r}{e^r-1}$$

$$\int d^3r = \int dV = \int r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$\to \frac{r}{e^r-1} \neq F(\theta,\phi)$$

$$\int dV = \int \sin\theta \, d\theta \int d\phi \int dr\, r^2$$



$\to M_x, M_y, M_z > 0$

$\downarrow$

$K_x, K_y, K_z > 0$

$\downarrow$

$r_x, r_y, r_z > 0 \to$

1/8 from sphere

$\downarrow$

$0 \le \theta \le \pi/2, \quad 0 \le \phi \le \pi/2$

$$\to dV = \int_0^{\pi/2}\sin\theta \, d\theta \int_0^{\pi/2} d\phi \int_0^\infty dr\, r^2$$

$$dV = \pi/2 \int_0^\infty dr\, r^2$$

$$\to U = \dots \left(\frac{\pi}{2}\right) 2 \int_0^\infty dr \, \frac{r^3}{e^r-1}$$

2 polarizations of EM wave

$$U = L^3 \, \frac{1}{\pi^2\beta^4(\hbar c)^3} \underbrace{\int_0^\infty dr \, \frac{r^3}{e^r-1}}_{\pi^4/15}$$

$$U = \frac{L^3\pi^2}{15(\hbar c)^3\beta^4} \neq \infty \quad (8.11)$$

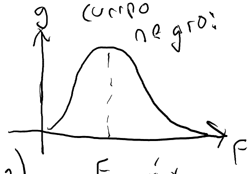
↳ No UV catastr...

Ley de Stefan-Boltzmann:

Densidad de energía:

$$u = \frac{U}{L^3} = \frac{U}{V} = \frac{\pi^2 k_B^4}{15(\hbar c)^3} T^4 \quad (8.12)$$

Ley de Planck:



$$g = \frac{2\pi h}{c^2}\,\frac{F^3}{e^{\beta h F}-1} \quad (9.1)$$

$$\frac{dg}{dF}\Big|_{F_{max}} = 0 \quad (9.2) \qquad \to [g] = \frac{J}{m^2}$$

$$\to F_{max} = \frac{2.822}{h} k_B T \quad (9.3)$$

$$\to \lambda_{max} = \frac{c}{F_{max}} = \frac{0.0028976\, mK}{T} \quad (9.4)$$

Potencia emisiva hemisférica total:

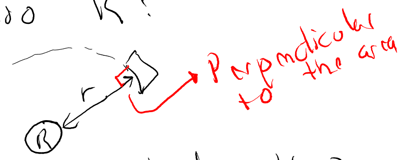
$$W = \frac{c}{4} U = \frac{2\pi^5 k_B^4}{15 c^2 h^3} T^4$$

(8.12)

$$W = \sigma T^4 \quad (9.5)$$

$$[W] = \frac{Watt}{m^2}$$

Intensidad a una distancia r de un cuerpo negro con radio R:



$$I = \frac{\text{Power from body with } R}{\text{Area receiving radiation}}$$

$$I = \frac{\sigma T^4 \, 4\pi R^2}{4\pi r^2}$$

$$I = \frac{\sigma T^4 R^2}{r^2} < \sigma T^4 \quad (9.6)$$