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Rank of ρ = # of nonzero λ_i's

$$\rho = U \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{pmatrix} U^\dagger$$

Rank = 1 means Pure State

Rank = K means min. number of states necessary to describe a mixed state

(Von Neumann) Entropy of a Matrix

$$S(\rho) = \sum_{i=1}^{n} \lambda_i \log_2\left(\frac{1}{\lambda_i}\right)$$

Pure State $S = 0$ $\left(S(|\psi\rangle\langle\psi|) = 0\right)$

Entangled State Entropy

$$|\psi\rangle = \sum_{i=1}^{K} \alpha_i \, |u_i\rangle_A \otimes |v_i\rangle_B$$

$$\rho_B = \sum_{i=1}^{K} |\alpha_i|^2 \, |v_i\rangle\langle v_i|$$ (Bob's Density Matrix)

$$\rho_A = \sum_{i=1}^{K} |\alpha_i|^2 \, |u_i\rangle\langle u_i|$$

$$S(\rho_A) = \sum_{i=1}^{K} |\alpha_i|^2 \log_2 \frac{1}{|\alpha_i|^2} = S(\rho_B) = \text{\# ebits}$$