Random Motion of Colloidal Particles Suspended in Water

Juan Gonzalez De Mendoza
Physics Department, University of Texas at Austin

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Abstract

The understanding that matter is made of indivisible particles is the base of all of modern physics and has led us to our current standard model of particle physics. By analyzing the erratic motion of $1\mu m$ in diameter polystyrene spheres in water with computer software, we calculated Boltzmann's constant to be $k = (1.35 \pm .05) * 10^{-27} \frac{J}{K}$, which agrees within error bounds to the accepted value of $1.38 * 10^{-23} \frac{J}{K}$. This upholds Einstein's calculations on Brownian motion and verifies Dalton's atomic theory.

1 Introduction

The idea that matter is composed of indivisible particles called atoms is the base of modern physics. It has been experimentally verified multiple times and has helped the development of the fields of thermodynamics and quantum physics.

This idea of dimensionless building blocks dates back to more than 2000 years ago from Greek philosophers, where the word atom was defined[1]. The first time the atom was defined to have certain properties was by John Dalton in 1813, this is known as Dalton's atomic theory[2]. In 1827, botanist John Brown observed pollen suspended in water moving in random paths, just as if they were being pushed from all directions. At first, he thought the pollen was alive and moving on its own, but he repeated the experiment with other known inorganic particles and observed the same behavior. This was later explained by Einstein through statistical analysis as the collision of the pollen with the water molecules. John Perrin used Einstein's work to carry out an experiment, in which he calculated the mass and size of atoms, conclusively verifying Dalton's theory of atoms[3].

In this experiment, we applied an automated based particle tracking for video microscopy of 1 μm polystyrene spheres suspended in distilled water, measured their mean squared displacement, and from this calculated Boltzmann's constant k. Our results were obtained from tracking a total of 273 spheres at 15 frames per second for a total of 26.66 seconds.

2 Theoretical Background

The temperature of an object represents the kinetic energy its atoms contain, the higher the temperature, the faster the atoms are moving around. The collisions of these atoms or molecules with a bigger particle such as $1\mu m$ spheres can be calculated through statistical analysis, since we can assume these are all random. In one dimension, the average square displacement is given by

$$\frac{d < x^2 >}{dt} = \frac{2kT}{\mu}, \tag{1}$$

where k is Boltzmann's constant, T is the temperature of the sample, and μ is the linear drag coefficient. Because we are using spherical particles, through Stoke's law we obtain

$$\mu = 6\pi\eta a, \tag{2}$$

where a is the radius of the spheres, and η is the viscosity of the fluid. Since there is no preferred direction in our experiment, the mean squared displacement in both dimensions is equal to

$$< R^2 >= \frac{4kT}{6\pi\eta a}t. \tag{3}$$

In our experiment, we will calculate $< R^2 >$ as time progresses, and plot these two values, which means the slope will correspond to the value of

$$s = \frac{4kT}{6\pi\eta a}[4]. \tag{4}$$

The viscosity of water is given by the equation

$$\eta = A \cdot 10^{\frac{B}{T-C}}, \tag{5}$$

where A, B and C are constants with values of A = $2.414 \cdot 10^{-5} Pa \cdot s$, B = 247.8 K and C = 140 K, leaving only a dependence on the temperature of the water.

3 Experimental Procedure

We began our experiment by obtaining our sample. For this, we mixed one drop of a 1 μm solution with 65 $\pm$ .5mL of distilled water. We then placed a small amount on a slide with a cavity and placed the slide under a Olympus microscope with 10X magnification lens. Next, we used a bubble tool to level the microscope to avoid our particles from having a homogeneous drift due to gravity. We shook our sample and re-positioned it under the microscope until minimal sphere clumping was observed, as this would affect our results. Finally, we connected a camera that outputted the microscope's image into a computer running Thorcam, where we recorded the particles for 400 frames at 15 frames per second for a total of 26.666 seconds. Our setup is depicted in Figure 1.

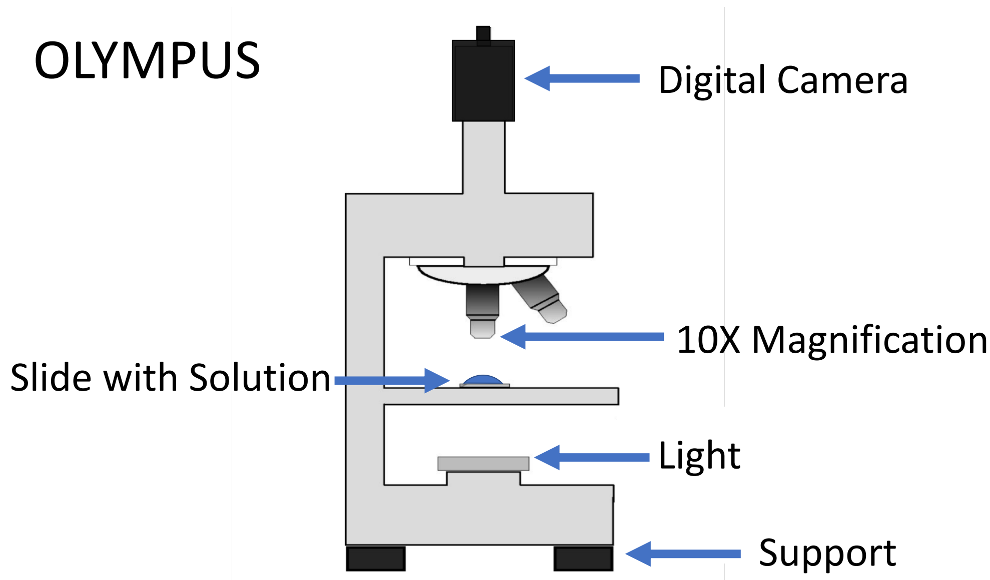

Figure 1: Sample containing microscopic spheres suspended in water, observed and recorded through an Olympus microscope

4 Data Analysis

In order to track the motion of our particles, we utilized the help of computer software. We used a python library called Trackpy, which is programmed to track the x and y coordinates of white dots on a black background. Our experiment had the opposite setup as our particles were black and the background white, so we inverted the colors of the images. The library also provided a drift function, which calculates the drift the whole ensemble of particles has caused by gravity or currents in the sample. This drift was subtracted from the path of our particles, leaving us only with the random motion from collision with water molecules. From this, we calculated the mean squared displacement of all tracked particles from all frames and plotted them against time, as depicted in Figure 2. This gave us a slope of $s = 1.8138 \pm .02 \mu m^2 s$.

We measured a temperature of our sample to be 296.0 K $\pm .3K$, by using this in equation 5 we obtain an $\eta = 936$ $\pm 15 \mu Pas$. The diameter of the spheres were given by the manufacturer of the sample as 1 μm, but no uncertainty was stated. So to be safe, we took the radius to be equal to $a = .50 \pm .01 \mu m$, which is mostly reported by other manufacturers [4]. We then took these calculated values for s, η, T, and a, and converted them into S.I. units, plugged them into equation 4, and solved for k. This gave us a value of $k = (1.35 \pm .05) * 10^{-27} \frac{J}{K}$, which agrees within error bounds to the accepted value of $1.38 * 10^{-23} \frac{J}{K}$.

The uncertainty of our slope comes from error in the fit, we utilized a python library called numpy. The fit returns a covariant matrix along with the results,

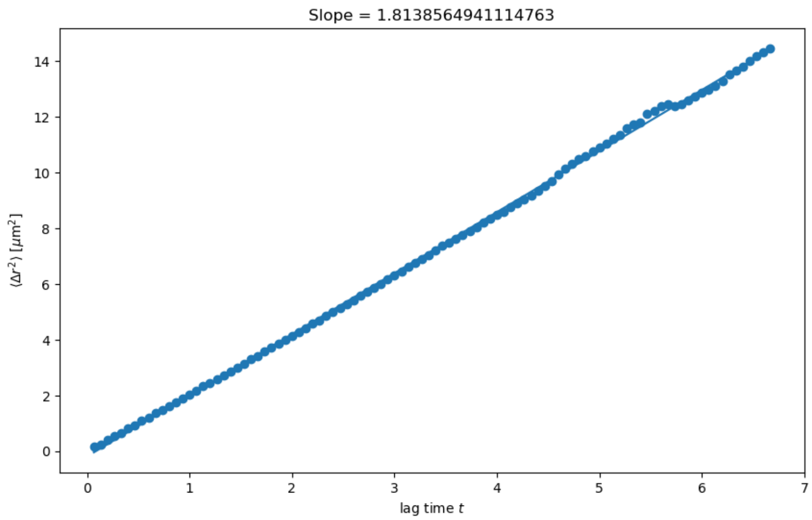

Figure 2: Plot of mean squared displacement of the particles against time, with a linear fit giving us a slope equal to $\frac{4kT}{6\pi\eta a}$.

which contains the error for our slope. To obtain the error for our k, we applied propagation of error as follows $\Delta k = \frac{6\pi \cdot \Delta\eta \cdot \Delta a \cdot \Delta s}{4 \cdot \Delta T}$

5 Conclusion

In this experiment, we measured the displacement of $1\mu m$ spheres suspended in water and obtained their mean squared displacement through computer software aid. From this, we were able to calculate Boltzmann's constant k to be $(1.35 \pm .05) * 10^{-27}\frac{J}{K}$, which agreed within error margins with the theoretical value of $1.38 * 10^{-23}\frac{J}{K}$. This verified Dalton's theoretical model of atoms, Einstein's calculations on the erratic movement of particles due to atom collisions, and invalidated Brown's thoughts on pollen being alive.

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