Statistics Kingdom



# Linear Regression Calculator

Linear regression calculator and prediction interval calculator with step-by-step solution.



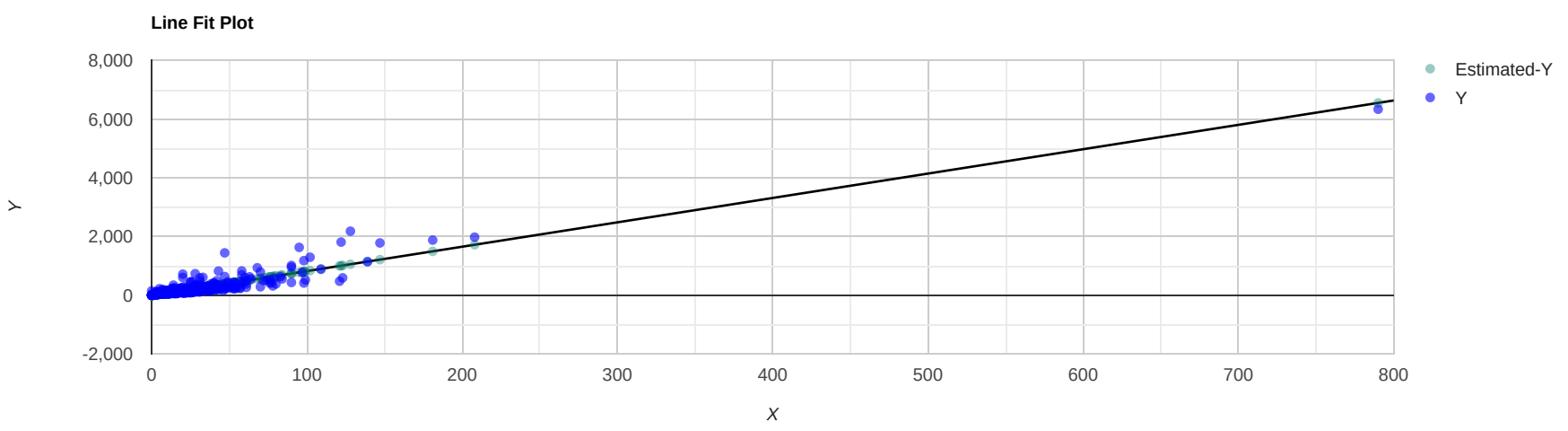
## Regression line equation

$$\hat{Y} = -12.6294 + 8.3074X$$

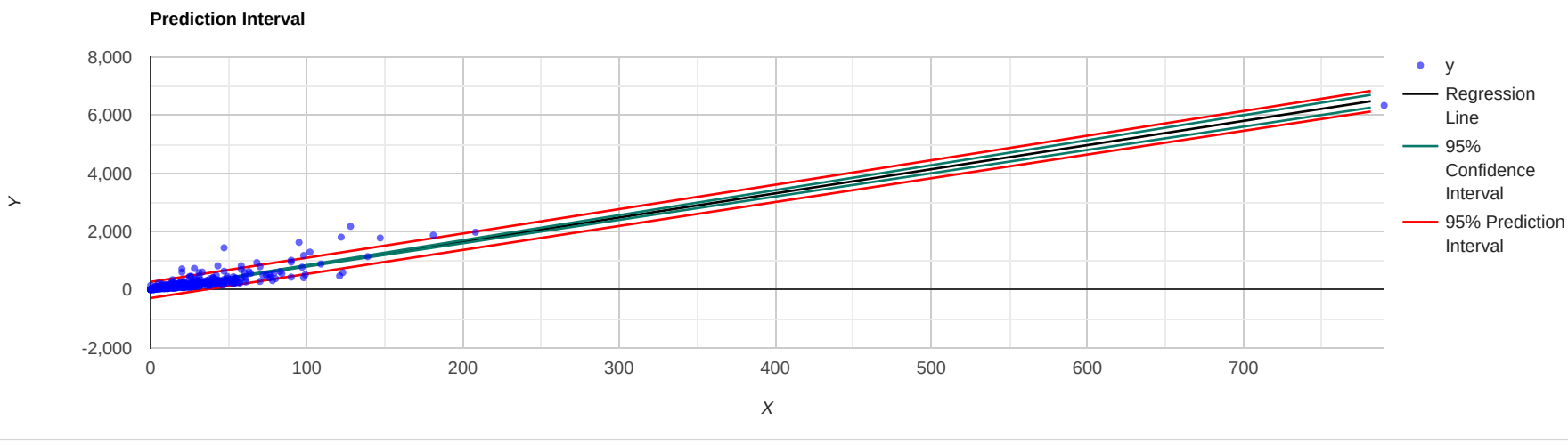
## Reporting linear regression in APA style

X predicted Y, $R^2$ = .86, $F(1,498) = 3124.72$, $p < .001$.
β = 8.31, $p < .001$, α = -12.63, $p = .092$.

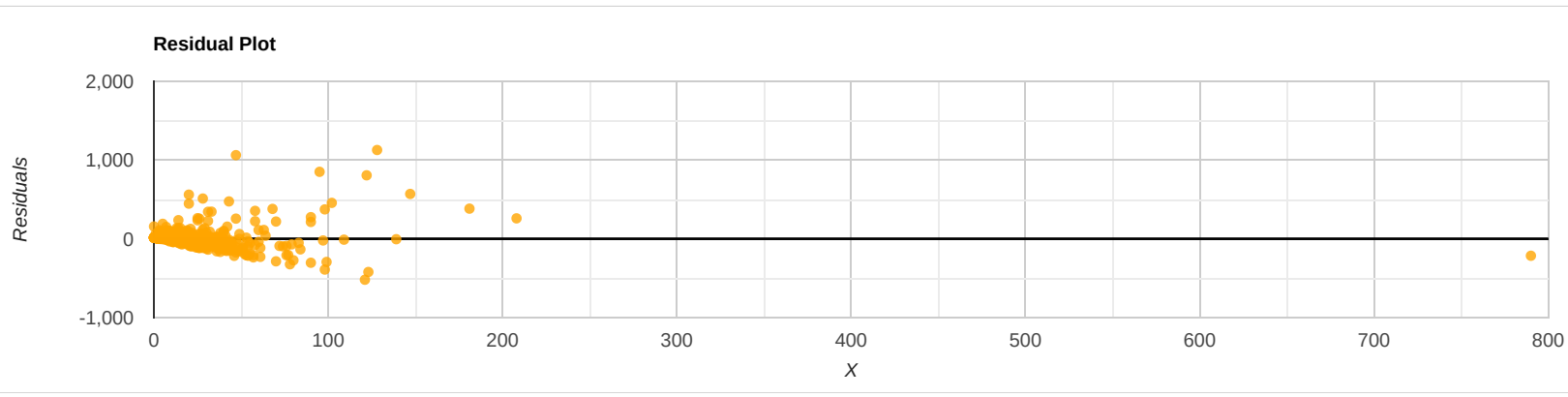
## Line Fit Plot




## Prediction online

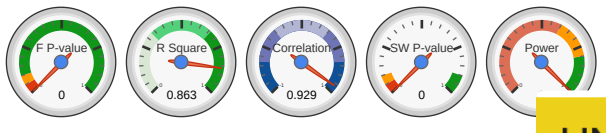



## Residual Plot




## Prediction

## Interpretation of the results




## Regression ANOVA

Hover over the cells to see the formulas.

| Source | DF | Sum of Square | Mean Square | F Statistic ($df_1$,$df_2$) | P-value |
|---|---|---|---|---|---|
| **Regression** (between $\hat{y}_i$ and $\bar{y}$ ) | 1 | 62226688.0363 | 62226688.0363 | 3124.7159 (1,498) | 0 |
| **Residual** (between $y_i$ and $\hat{y}_i$) | 498 | 9917346.7637 | 19914.3509 | | |
| **Total** (between $y_i$ and $\bar{y}$) | 499 | 72144034.8 | 144577.224 | | |

## 1. Y and X relationship

R-Squared ($R^2$) equals **0.8625.** This means that 86.3% of the variability of Y is explained by X.
Correlation (R) equals **0.9287**. This means that there is a **very strong direct relationship** between X and Y.
The Standard deviation of the residuals ($S_{res}$) equals **141.1182**.
The slope: $b_1$=**8.3074** CI[8.0154, 8.5994] means that when you increase X by 1, the value of Y increases by 8.3074.
The y-intercept: $b_0$=**-12.6294** CI[-27.3314, 2.0726] means that when X equals 0, the prediction of Y's value is -12.6294.
The x-intercept equals 1.5203.

## 2. Goodness of fit

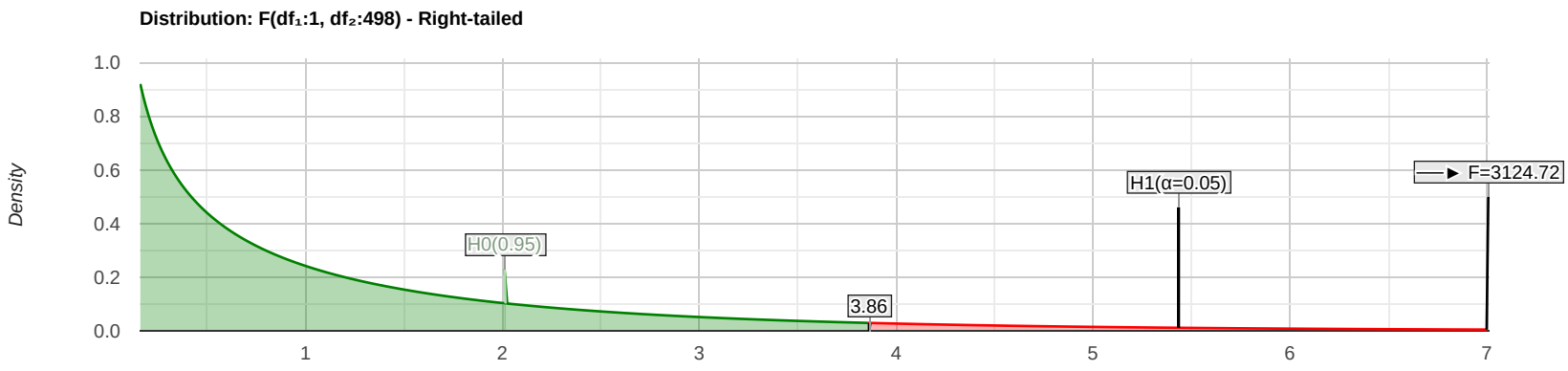
Overall regression: right-tailed, F(1,498) = **3124.7159**, p-value = **0**. Since p-value < α (0.05), we reject $H_0$.
The linear regression model, $\mathbf{Y = b_0 + b_1X + \varepsilon}$, provides a better fit than the model without the independent variable resulting in $\mathbf{Y = b_0 + \varepsilon}$.
The slope ($b_1$): two-tailed, T(498)=**55.8992**, p-value = **0**. For one predictor it is the same as the p-value for the overall model.
The y-intercept ($b_0$): two-tailed, T(498) = **-1.6878**, p-value = **0.09208**. Hence, $b_0$ is not significantly different from zero. It is still most likely recommended not to force $b_0$ to be zero.

## 3. Residual normality

The linear regression model assumes normality for residual errors. The Shapiro-Wilk p-value equals **0**. It is assumed that the data is not normally distributed, But since the sample size is large, it should not adversely affect the regression model.

## 4. Outliers

Outliers may affect the regression line.
In this case, the distribution of the residuals is normal. Therefore, the probability of detecting 11 valid outliers or more is 1, (outliers: -520.5705,445.4807,559.4807,510.0212,473.4096,455.2708,1061.1799,849.4229,568.4362,805.1221,1126.2775).
You should only remove outliers if you identify them as errors!

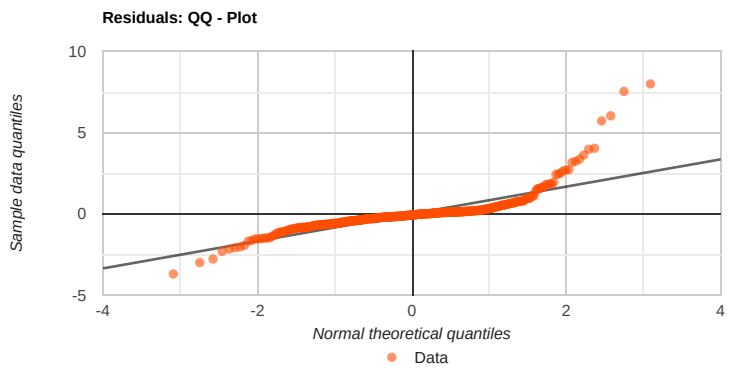



Significance level (α)

Annotations

## Residuals normality




## Calculation

### Step-by-step solution

$\hat{Y} = b_0 + b_1X$

$b_1 = \frac{SP_{xy}}{SS_x} = \frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{\Sigma(x_i-\bar{x})^2}$

$b_1 = \frac{7490479.76}{901659.542} = 8.3074$

$b_0 = \bar{y} - b_1\bar{x}$

$\bar{x} = 27.054$

$\bar{y} = 212.12$

$b_0 = 212.12-8.3074*27.054 = -12.6294$

$R^2 = \frac{SS_{Regression}}{SS_{total}} = \frac{\Sigma(\hat{y}_i - \bar{y})^2}{\Sigma(y_i - \bar{y})^2} = \frac{62226688.0363}{72144034.8} = 0.8625$

The standard deviation of the residuals is:

$MS_{residual} = S^2_{res} = \frac{\Sigma(y_i - \hat{y})^2}{n - 2}$

**Residual outliers**

$S_{res} = \sqrt{MSE} = \sqrt{19914.3509} = 141.1182$.

The average of the residuals is always zero.

The thresholds used to calculate the outliers are: $\pm k*S_{res}$.

In this case, the thresholds are $\pm 3*141.1182 = \pm 423.3546$.

We tagged the outliers with an arrow (⇐) at the 'Residual' column.

**$SS_x$ and $SP_{xy}$**

| $x-\bar{x}$ | $y-\bar{y}$ | $(x-\bar{x})^2$ | $(x-\bar{x})(y-\bar{y})$ |
|---|---|---|---|

| | | | |
|---|---|---|---|
| -27.054 | -212.12 | 731.9189 | 5738.6945 |
| -27.054 | -212.12 | 731.9189 | 5738.6945 |
| -27.054 | -212.12 | 731.9189 | 5738.6945 |
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| -27.054 | -210.12 | 731.9189 | 5684.5865 |
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| -27.054 | -206.12 | 731.9189 | 5576.3705 |
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| -25.054 | -202.12 | 627.7029 | 5063.9145 |
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| | | | |
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| 0.946 | -90.12 | 0.8949 | -85.2535 |
| -1.054 | -90.12 | 1.1109 | 94.9865 |
| -13.054 | -89.12 | 170.4069 | 1163.3725 |
| -2.054 | -88.12 | 4.2189 | 180.9985 |
| -4.054 | -87.12 | 16.4349 | 353.1845 |
| -18.054 | -87.12 | 325.9469 | 1572.8645 |
| 1.946 | -87.12 | 3.7869 | -169.5355 |
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| -17.054 | -86.12 | 290.8389 | 1468.6905 |
| -16.054 | -86.12 | 257.7309 | 1382.5705 |
| -13.054 | -86.12 | 170.4069 | 1124.2105 |
| 8.946 | -85.12 | 80.0309 | -761.4835 |
| -2.054 | -84.12 | 4.2189 | 172.7825 |
| -0.054 | -84.12 | 0.002916 | 4.5425 |
| -11.054 | -83.12 | 122.1909 | 918.8085 |
| -8.054 | -82.12 | 64.8669 | 661.3945 |
| -14.054 | -81.12 | 197.5149 | 1140.0605 |
| -8.054 | -80.12 | 64.8669 | 645.2865 |
| -11.054 | -80.12 | 122.1909 | 885.6465 |
| -10.054 | -79.12 | 101.0829 | 795.4725 |
| -1.054 | -78.12 | 1.1109 | 82.3385 |
| -7.054 | -77.12 | 49.7589 | 544.0045 |
| -7.054 | -77.12 | 49.7589 | 544.0045 |
| 1.946 | -76.12 | 3.7869 | -148.1295 |
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| -18.054 | -75.12 | 325.9469 | 1356.2165 |
| -6.054 | -74.12 | 36.6509 | 448.7225 |
| -9.054 | -73.12 | 81.9749 | 662.0285 |
| -4.054 | -73.12 | 16.4349 | 296.4285 |
| -1.054 | -70.12 | 1.1109 | 73.9065 |
| -1.054 | -70.12 | 1.1109 | 73.9065 |
| -12.054 | -69.12 | | |
| -27.054 | -69.12 | | |
| -5.054 | -68.12 | | |

| | | | |
|---|---|---|---|
| -5.054 | -64.12 | 25.5429 | 324.0625 |
| -11.054 | -64.12 | 122.1909 | 708.7825 |
| -13.054 | -64.12 | 170.4069 | 837.0225 |
| -10.054 | -63.12 | 101.0829 | 634.6085 |
| -14.054 | -63.12 | 197.5149 | 887.0885 |
| -17.054 | -60.12 | 290.8389 | 1025.2865 |
| -15.054 | -60.12 | 226.6229 | 905.0465 |
| 18.946 | -60.12 | 358.9509 | -1139.0335 |
| -2.054 | -58.12 | 4.2189 | 119.3785 |
| -20.054 | -57.12 | 402.1629 | 1145.4845 |
| 4.946 | -55.12 | 24.4629 | -272.6235 |
| -19.054 | -54.12 | 363.0549 | 1031.2025 |
| -10.054 | -52.12 | 101.0829 | 524.0145 |
| 4.946 | -52.12 | 24.4629 | -257.7855 |
| -5.054 | -50.12 | 25.5429 | 253.3065 |
| -17.054 | -50.12 | 290.8389 | 854.7465 |
| -18.054 | -50.12 | 325.9469 | 904.8665 |
| -7.054 | -50.12 | 49.7589 | 353.5465 |
| 3.946 | -49.12 | 15.5709 | -193.8275 |
| 2.946 | -49.12 | 8.6789 | -144.7075 |
| -9.054 | -49.12 | 81.9749 | 444.7325 |
| 4.946 | -49.12 | 24.4629 | -242.9475 |
| 1.946 | -48.12 | 3.7869 | -93.6415 |
| -12.054 | -48.12 | 145.2989 | 580.0385 |
| -16.054 | -47.12 | 257.7309 | 756.4645 |
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| -2.054 | -46.12 | 4.2189 | 94.7305 |
| 8.946 | -46.12 | 80.0309 | -412.5895 |
| -11.054 | -45.12 | 122.1909 | 498.7565 |
| -1.054 | -44.12 | 1.1109 | 46.5025 |
| 10.946 | -42.12 | 119.8149 | -461.0455 |
| -0.054 | -41.12 | 0.002916 | 2.2205 |
| -1.054 | -40.12 | 1.1109 | 42.2865 |
| -6.054 | -40.12 | 36.6509 | 242.8865 |
| -3.054 | -39.12 | 9.3269 | 119.4725 |
| -14.054 | -38.12 | 197.5149 | 535.7385 |
| 1.946 | -38.12 | 3.7869 | -74.1815 |
| 6.946 | -36.12 | 48.2469 | -250.8895 |
| 13.946 | -36.12 | 194.4909 | -503.7295 |
| -9.054 | -34.12 | 81.9749 | 308.9225 |
| 0.946 | -33.12 | 0.8949 | -31.3315 |
| -7.054 | -33.12 | 49.7589 | 233.6285 |
| 6.946 | -32.12 | 48.2469 | -223.1055 |
| 9.946 | -32.12 | 98.9229 | -319.4655 |
| 14.946 | -30.12 | 223.3829 | -450.1735 |
| 10.946 | -29.12 | 119.8149 | -318.7475 |
| -1.054 | -28.12 | 1.1109 | 29.6385 |
| -0.054 | -27.12 | 0.002916 | 1.4645 |
| 5.946 | -26.12 | 35.3549 | -155.3095 |
| 3.946 | -26.12 | 15.5709 | -103.0695 |
| 2.946 | -24.12 | 8.6789 | -71.0575 |
| 0.946 | -24.12 | 0.8949 | -22.8175 |
| 10.946 | -24.12 | 119.8149 | -264.0175 |
| -5.054 | -23.12 | 25.5429 | 116.8485 |
| 11.946 | -23.12 | 142.7069 | -276.1915 |
| -4.054 | -22.12 | 16.4349 | 89.6745 |
| 4.946 | -22.12 | 24.4629 | -109.4055 |
| 3.946 | -20.12 | 15.5709 | -79.3935 |
| -2.054 | -19.12 | 4.2189 | 39.2725 |
| 8.946 | -18.12 | 80.0309 | -162.1015 |
| 11.946 | -18.12 | 142.7069 | -216.4615 |
| -20.054 | -17.12 | 402.1629 | 343.3245 |
| -6.054 | -13.12 | 36.6509 | 79.4285 |
| 1.946 | -13.12 | 3.7869 | -25.5315 |
| 4.946 | -12.12 | 24.4629 | -59.9455 |
| 7.946 | -12.12 | 63.1389 | -96.3055 |
| -14.054 | -10.12 | 197.5149 | 142.2265 |
| 16.946 | -10.12 | 287.1669 | -171.4935 |
| 13.946 | -10.12 | 194.4909 | -141.1335 |
| -1.054 | -9.12 | 1.1109 | 9.6125 |
| 5.946 | -9.12 | 35.3549 | -54.2275 |
| 19.946 | -8.12 | 397.8429 | -161.9615 |
| -12.054 | -8.12 | 145.2989 | 97.8785 |
| 1.946 | -4.12 | 3.7869 | -8.0175 |
| 2.946 | -2.12 | 8.6789 | -6.2455 |
| -9.054 | -1.12 | 81.9749 | 10.1405 |
| -0.054 | -0.12 | 0.002916 | 0.00648 |
| -9.054 | -0.12 | 81.9749 | 1.0865 |
| 8.946 | 1.88 | 80.0309 | 16.8185 |
| -1.054 | 4.88 | 1.1109 | -5.1435 |
| -1.054 | 4.88 | 1.1109 | -5.1435 |
| 25.946 | 4.88 | 673.1949 | 126.6165 |
| -22.054 | 5.88 | 486.3789 | -129.6775 |
| -4.054 | 5.88 | 16.4349 | -23.8375 |
| 26.946 | 7.88 | 726.0869 | 212.3345 |
| 4.946 | 9.88 | 24.4629 | 48.8665 |
| 29.946 | 11.88 | 896.7629 | 355.7585 |
| 12.946 | 12.88 | 167.5989 | 166.7445 |
| 24.946 | 12.88 | 622.3029 | 321.3045 |
| 16.946 | 13.88 | 287.1669 | 235.2105 |
| 8.946 | 13.88 | 80.0309 | 124.1705 |
| -0.054 | 15.88 | | |
| -6.054 | 15.88 | | |
| -14.054 | 17.88 | | |

| | | | |
|---|---|---|---|
| 12.946 | 19.88 | 167.5989 | 257.3665 |
| 5.946 | 19.88 | 35.3549 | 118.2065 |
| 19.946 | 20.88 | 397.8429 | 416.4725 |
| 6.946 | 21.88 | 48.2469 | 151.9785 |
| 0.946 | 21.88 | 0.8949 | 20.6985 |
| 21.946 | 22.88 | 481.6269 | 502.1245 |
| 5.946 | 23.88 | 35.3549 | 141.9905 |
| -13.054 | 24.88 | 170.4069 | -324.7835 |
| 0.946 | 25.88 | 0.8949 | 24.4825 |
| 0.946 | 25.88 | 0.8949 | 24.4825 |
| 9.946 | 25.88 | 98.9229 | 257.4025 |
| 3.946 | 29.88 | 15.5709 | 117.9065 |
| 13.946 | 29.88 | 194.4909 | 416.7065 |
| 6.946 | 30.88 | 48.2469 | 214.4925 |
| -8.054 | 30.88 | 64.8669 | -248.7075 |
| -5.054 | 32.88 | 25.5429 | -166.1755 |
| -12.054 | 36.88 | 145.2989 | -444.5515 |
| 4.946 | 38.88 | 24.4629 | 192.3005 |
| -8.054 | 39.88 | 64.8669 | -321.1935 |
| 29.946 | 39.88 | 896.7629 | 1194.2465 |
| 7.946 | 39.88 | 63.1389 | 316.8865 |
| 8.946 | 41.88 | 80.0309 | 374.6585 |
| 11.946 | 42.88 | 142.7069 | 512.2445 |
| 27.946 | 44.88 | 780.9789 | 1254.2165 |
| 11.946 | 49.88 | 142.7069 | 595.8665 |
| 7.946 | 50.88 | 63.1389 | 404.2925 |
| 33.946 | 51.88 | 1152.3309 | 1761.1185 |
| 8.946 | 52.88 | 80.0309 | 473.0645 |
| -1.054 | 53.88 | 1.1109 | -56.7895 |
| 3.946 | 56.88 | 15.5709 | 224.4485 |
| 19.946 | 56.88 | 397.8429 | 1134.5285 |
| 22.946 | 58.88 | 526.5189 | 1351.0605 |
| 16.946 | 58.88 | 287.1669 | 997.7805 |
| 0.946 | 58.88 | 0.8949 | 55.7005 |
| 16.946 | 58.88 | 287.1669 | 997.7805 |
| 16.946 | 60.88 | 287.1669 | 1031.6725 |
| 14.946 | 61.88 | 223.3829 | 924.8585 |
| 12.946 | 63.88 | 167.5989 | 826.9905 |
| -0.054 | 66.88 | 0.002916 | -3.6115 |
| 6.946 | 67.88 | 48.2469 | 471.4945 |
| 4.946 | 69.88 | 24.4629 | 345.6265 |
| 42.946 | 69.88 | 1844.3589 | 3001.0665 |
| 15.946 | 72.88 | 254.2749 | 1162.1445 |
| -6.054 | 74.88 | 36.6509 | -453.3235 |
| 13.946 | 75.88 | 194.4909 | 1058.2225 |
| 15.946 | 77.88 | 254.2749 | 1241.8745 |
| 17.946 | 81.88 | 322.0589 | 1469.4185 |
| 23.946 | 82.88 | 573.4109 | 1984.6445 |
| 14.946 | 86.88 | 223.3829 | 1298.5085 |
| 23.946 | 87.88 | 573.4109 | 2104.3745 |
| 8.946 | 92.88 | 80.0309 | 830.9045 |
| 12.946 | 96.88 | 167.5989 | 1254.2085 |
| 10.946 | 98.88 | 119.8149 | 1082.3405 |
| 50.946 | 98.88 | 2595.4949 | 5037.5405 |
| 16.946 | 100.88 | 287.1669 | 1709.5125 |
| 8.946 | 100.88 | 80.0309 | 902.4725 |
| 2.946 | 103.88 | 8.6789 | 306.0305 |
| 15.946 | 117.88 | 254.2749 | 1879.7145 |
| 26.946 | 122.88 | 726.0869 | 3311.1245 |
| 0.946 | 124.88 | 0.8949 | 118.1365 |
| -13.054 | 126.88 | 170.4069 | -1656.2915 |
| 4.946 | 129.88 | 24.4629 | 642.3865 |
| 19.946 | 142.88 | 397.8429 | 2849.8845 |
| 1.946 | 150.88 | 3.7869 | 293.6125 |
| 13.946 | 154.88 | 194.4909 | 2159.9565 |
| 10.946 | 163.88 | 119.8149 | 1793.8305 |
| 33.946 | 165.88 | 1152.3309 | 5630.9625 |
| 52.946 | 165.88 | 2803.2789 | 8782.6825 |
| 27.946 | 174.88 | 780.9789 | 4887.1965 |
| 30.946 | 177.88 | 957.6549 | 5504.6745 |
| 21.946 | 180.88 | 481.6269 | 3969.5925 |
| 26.946 | 191.88 | 726.0869 | 5170.3985 |
| 12.946 | 196.88 | 167.5989 | 2548.8085 |
| 48.946 | 196.88 | 2395.7109 | 9636.4885 |
| 70.946 | 197.88 | 5033.3349 | 14038.7945 |
| 12.946 | 207.88 | 167.5989 | 2691.2145 |
| 49.946 | 208.88 | 2494.6029 | 10432.7205 |
| -2.054 | 216.88 | 4.2189 | -445.4715 |
| 62.946 | 217.88 | 3962.1989 | 13714.6745 |
| 25.946 | 228.88 | 673.1949 | 5938.5205 |
| 32.946 | 234.88 | 1085.4389 | 7738.3565 |
| 21.946 | 241.88 | 481.6269 | 5308.2985 |
| -2.054 | 243.88 | 4.2189 | -500.9295 |
| -1.054 | 245.88 | 1.1109 | -259.1575 |
| 3.946 | 257.88 | 15.5709 | 1017.5945 |
| 93.946 | 259.88 | 8825.8509 | 24414.6865 |
| 14.946 | 277.88 | 223.3829 | 4153.1945 |
| 44.946 | 281.88 | 2020.1429 | 12669.3785 |
| 46.946 | 292.88 | 2203.9269 | 13749.5445 |
| 71.946 | 302.88 | 5176.2269 | 21791.0045 |
| 48.946 | 315.88 | 2395.7109 | 15461.0625 |
| 56.946 | 337.88 | | |
| 36.946 | 344.88 | | |
| 51.946 | 359.88 | | |

| | | | |
|---|---|---|---|
| 3.946 | 374.88 | 15.5709 | 1479.2765 |
| 95.946 | 374.88 | 9205.6349 | 35968.2365 |
| 32.946 | 383.88 | 1085.4389 | 12647.3105 |
| -7.054 | 386.88 | 49.7589 | -2729.0515 |
| 5.946 | 392.88 | 35.3549 | 2336.0645 |
| 35.946 | 409.88 | 1292.1149 | 14733.5465 |
| 55.946 | 411.88 | 3129.9549 | 23043.0385 |
| 19.946 | 420.88 | 397.8429 | 8394.8725 |
| 30.946 | 478.88 | 957.6549 | 14819.4205 |
| -7.054 | 500.88 | 49.7589 | -3533.2075 |
| 0.946 | 517.88 | 0.8949 | 489.9145 |
| 69.946 | 558.88 | 4892.4429 | 39091.4205 |
| 42.946 | 574.88 | 1844.3589 | 24688.7965 |
| 15.946 | 605.88 | 254.2749 | 9661.3625 |
| 30.946 | 610.88 | 957.6549 | 18904.2925 |
| 81.946 | 668.88 | 6715.1469 | 54812.0405 |
| 40.946 | 718.88 | 1676.5749 | 29435.2605 |
| 62.946 | 735.88 | 3962.1989 | 46320.7025 |
| 62.946 | 796.88 | 3962.1989 | 50160.4085 |
| 111.946 | 923.88 | 12531.9069 | 103424.6705 |
| 70.946 | 961.88 | 5033.3349 | 68241.5385 |
| 74.946 | 1077.88 | 5616.9029 | 80782.7945 |
| 19.946 | 1226.88 | 397.8429 | 24471.3485 |
| 67.946 | 1413.88 | 4616.6589 | 96067.4905 |
| 119.946 | 1564.88 | 14387.0429 | 187701.0965 |
| 94.946 | 1593.88 | 9014.7429 | 151332.5305 |
| 153.946 | 1660.88 | 23699.3709 | 255685.8325 |
| 180.946 | 1761.88 | 32741.4549 | 318805.1385 |
| 100.946 | 1964.88 | 10190.0949 | 198346.7765 |
| 762.946 | 6121.88 | 582086.5989 | 4670663.8585 |
| 0 | 0 | **901659.542** ($SS_x$) | **7490479.76** ($SP_{xy}$) |

# Linear regression calculator

The linear regression calculator generates the linear regression equation. It also draws: a linear regression line, a histogram, a residuals QQ-plot, a residuals x-plot, and a distribution chart.
It calculates the R-squared, the R, and the outliers, then testing the fit of the linear model to the data and checking the residuals' normality assumption and the priori power.

## What is linear regression?

The linear regression is the linear equation that best fits the points.
There is no one way to choose the best fit ting line, the most common one is the ordinary least squares (OLS). The linear regression describes the relationship between the dependent variable (Y) and the independent variables (X).
The linear regression model calculates the dependent variable (DV) based on the independent variables (IV, predictors).

## What is "ordinary least squares"?

The ordinary least squares method chooses the line parameters that minimize the sum of squares of the differences between the observed dependent variables (Y) and the estimated value by the linear regression ($\hat{Y}$).

## Why do you need linear regression?

We may use linear regression when we want to do one of the following

- Predict the dependent variable ($\hat{Y}$).
- Estimate the effect of each independent variable (X) on the dependent variable (Y).
- Calculate the correlation between the dependent variable and the independent variables.
- Test the linear model significance level.

## How to calculate linear regression?

Following the linear regression formula:

$$\hat{Y} = b_0 + b_1 x$$

$b_0$ - the y-intercept, where the line crosses the y-axis.
$b_1$ - the slope, describes the line's direction and incline.

$$b_1 = \frac{SP_{xy}}{SS_x} = \frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{\Sigma(x_i-\bar{x})^2}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

## linear regression prediction

The prediction calculator uses the linear regrssion to predict the depdendent variable based on the independent value. The calculator also creates the confidence interval, and the prediction interval.

### Confidence interval of the prediction

The prediction interval for the **mean value** of the dependent variable.
This is the interval for the equation line, the true value equation will be in this interval. If we would know the true equation then the width of this interval would be zero.
If you would calculate the confidence interval over an infinite number of regressions with the same sample size, 95% (confidence level) of the calculated confidence intervals will contain the mean's true value.
Since this interval is for the mean, the standard error is smaller and the the range is narrower than the range of the prediction interval.

$$MS_{residual} = S^2_{residual} = \frac{\Sigma(y_i - \hat{y})^2}{n - 2}$$

$$S.E^2_{ci} = S^2_{residual}\left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{SS_x}\right)$$

$$\hat{Y} \pm T_{1-\alpha/2}(n-2) * S.E_{ci}$$

### Prediction Interval

The prediction interval for a **particular observation** of the dependent variable.
This is the interval for any single value.
The prediction inteval takes into consideration the fact that you don't know the true equatio, and the fact the the liner regression explaned only part of the variance (the part is R-squared). Even if we would know the true equation then the width of this interval would be gre
Since this interval is for a single observation, the standa

$S.E^2_{prediction} = S^2_{residual}\left(1+\frac{1}{n}+\frac{(x_0 - \bar{x})^2}{SS_x}\right)$

$\hat{Y} \pm T_{1-\alpha/2}(n-2)*S.E_{prediction}$

## How to calculate R squares?

R squares is the percentage of the variance explain by the regression ($SS_{Regression}$) from the overall variance ($SS_{Total}$).

$R^2 = \frac{SS_{Regression}}{SS_{Total}}$

## Linear regression in calculator

This online calculator supports all the basic functionality and more.

The **right-tailed** F test checks if the entire regression model is statistically significant. Why only right tail?

For **Multiple regression calculator** with the stepwise method and assumptions validations: **multiple regression calculator**

The following statistic checks if the linear regression model supports better results than the average of Y.

### Hypotheses

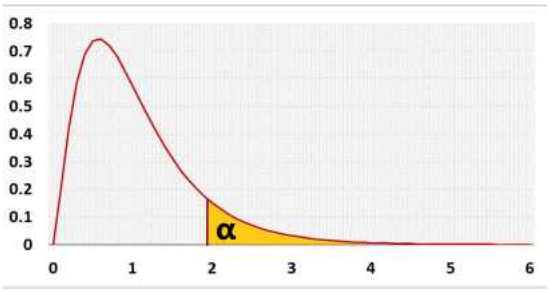
$H_0$: $Y = b_0$

$H_1$: $Y = b_0+b_1X$

### Test statistic

$F = \frac{MS(regression)}{MS\ (residual)}$

### F distribution




## R Code

The following R code should produce similar results

```
rm(list = ls())
if(!"car" %in% installed.packages()){install.packages("car")}
library(car)
x10 <-
c(0,0,0,0,0,0,0,0,0,0,2,0,1,1,2,2,0,0,1,1,2,2,2,2,1,2,3,2,1,1,2,3,1,4,2,1,1,2,2,3,1,5,2,3,9,4,6,5,8,2,4,4,6,11,3,7,7,6,6,11,6,3,4,11,7,5,5,8,3,3,7,5,10,10,16,8,15,11,13,9,4,14,12,10,9,7,10,11,8,1,11,9,6,14,6,7,10,15,14,14
x11 <-
c(31,30,18,32,29,15,11,32,25,36,16,26,38,27,26,21,24,13,29,34,41,18,28,20,34,37,42,38,26,27,33,31,30,28,38,22,39,23,32,31,25,36,39,7,21,29,32,35,13,44,41,26,33,47,15,29,30,18,27,18,36,26,26,53,5,23,54,32,57
x1 <- c(x10,x11)
y10 <-
c(0,0,0,0,0,0,2,2,2,3,3,4,4,5,5,6,6,6,7,7,7,7,8,9,9,10,10,10,10,12,14,14,15,15,17,17,17,17,18,18,18,21,24,25,25,28,28,29,30,32,32,34,34,34,35,36,36,38,38,38,39,39,39,39,39,39,40,41,42,42,44,44,45,45,46,48,49,49
y11 <-
c(163,163,163,163,164,164,165,165,166,166,167,168,170,171,172,172,173,174,174,176,176,178,179,179,180,180,182,183,184,185,186,186,188,188,188,189,189,190,190,192,193,194,194,195,199,199,200,200,20
y1 <- c(y10,y11)
model1 = lm(y1~x1)
summary(model1)
```

What is linear regression?

Tutorial

Calculators

Multiple regression
Correlation
Regression sample size