Statistics Kingdom

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Linear Regression Calculator

Linear regression calculator and prediction interval calculator with step-by-step solution.

Simple Linear regression   Multiple Linear regression   Logistic regression   Multinomial logistic regression

How to do with R?

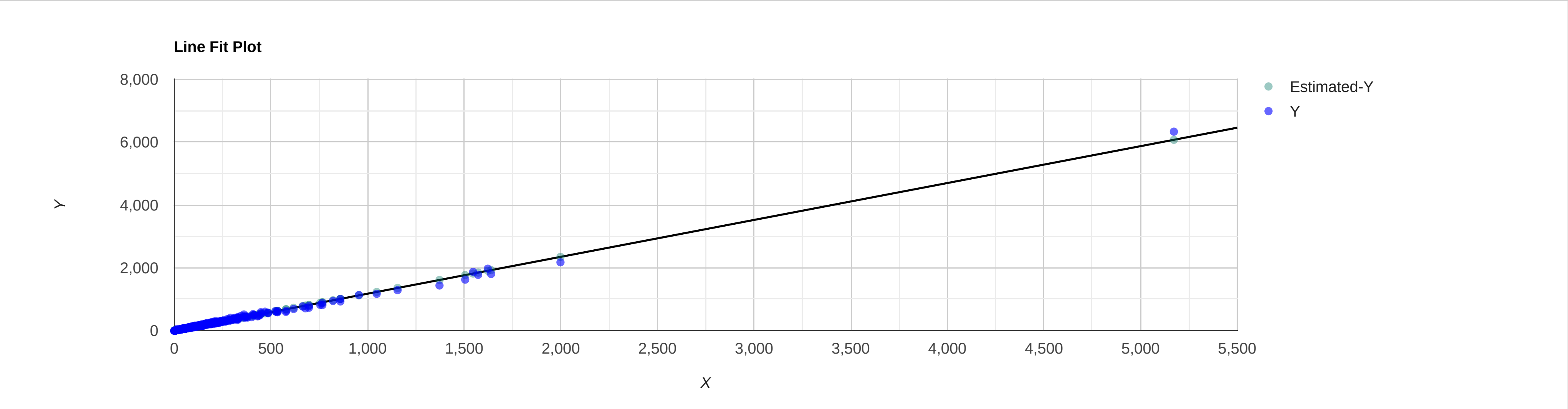
Regression line equation

Ŷ = 3.6532 + 1.1739X

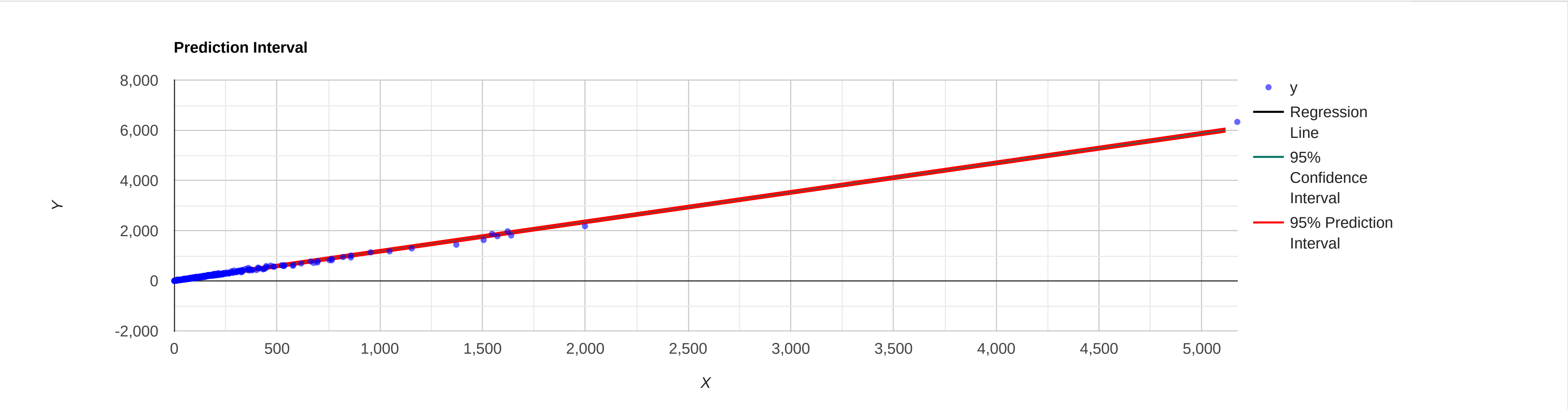
Reporting linear regression in APA style

X predicted Y, R^2 = , F(1,498) = 108193.83, p < .001.
β = 1.17, p < .001, α = 3.65, p = .006.

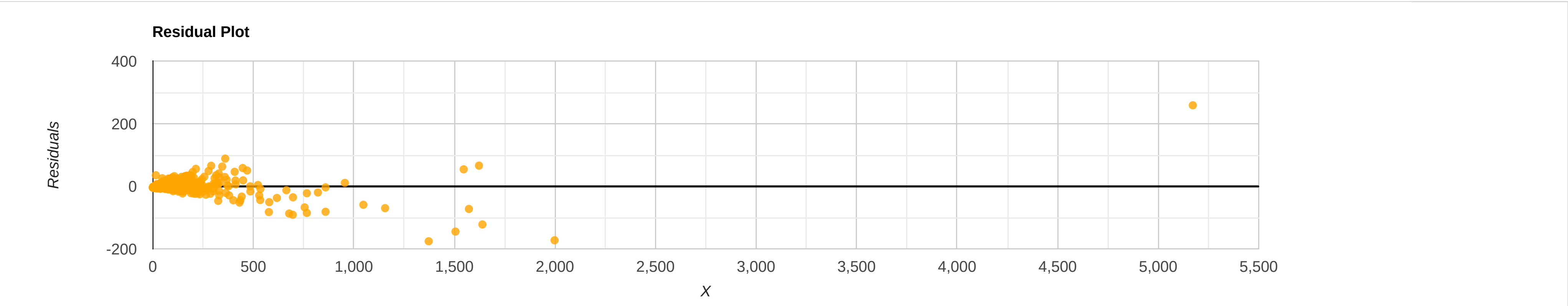
Line Fit Plot



Prediction online

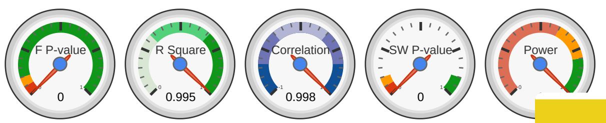


Residual Plot



Prediction

Interpretation of the results



Regression ANOVA

Hover over the cells to see the formulas.

Source	DF	Sum of Square	Mean Square	F Statistic (df_1,df_2)	P-value
Regression (between $\hat{y}_i$ and $\bar{y}$)	1	71813488.0628	71813488.0628	108193.8287 (1,498)	0
Residual (between y_i and $\hat{y}_i$)	498	330546.7372	663.7485		
Total (between y_i and $\bar{y}$)	499	72144034.8	144577.224		

1. Y and X relationship

R-Squared (R^2) equals **0.9954.** This means that 99.5% of the variability of Y is explained by X.
Correlation (R) equals **0.9977**. This means that there is a **very strong direct relationship** between X and Y.
The Standard deviation of the residuals (S_{res}) equals **25.7633**.
The slope: b_1=**1.1739** CI[1.1669, 1.1809] means that when you increase X by 1, the value of Y increases by 1.1739.
The y-intercept: b_0=**3.6532** CI[1.0697, 6.2368] means that when X equals 0, the prediction of Y's value is 3.6532.
The x-intercept equals -3.1121.

2. Goodness of fit

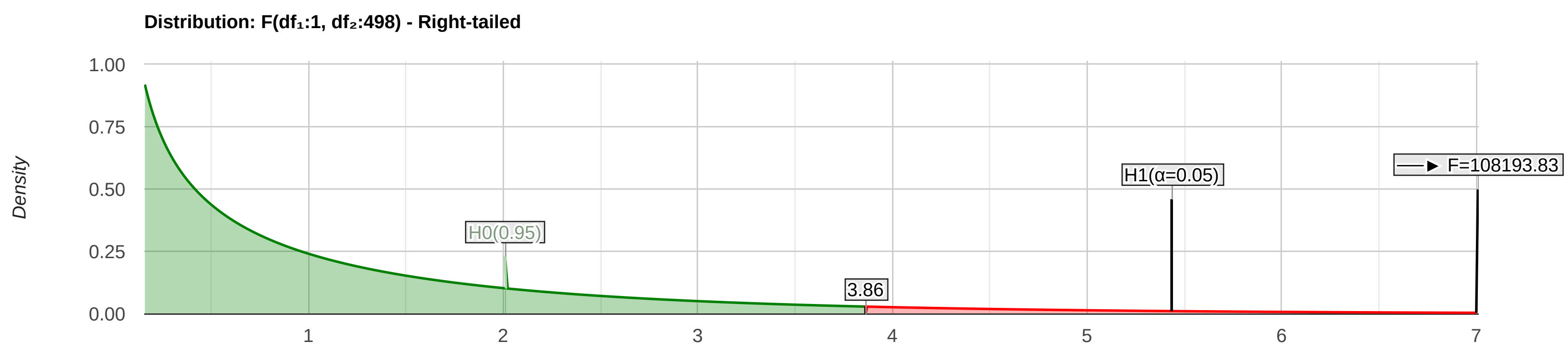
Overall regression: right-tailed, F(1,498) = **108193.8287**, p-value = **0**. Since p-value < α (0.05), we reject H_0.
The linear regression model, $\mathbf{Y = b_0 + b_1X + \varepsilon}$, provides a better fit than the model without the independent variable resulting in $\mathbf{Y = b_0 + \varepsilon}$.
The slope (b_1): two-tailed, T(498)=**328.9283**, p-value = **0**. For one predictor it is the same as the p-value for the overall model.
The y-intercept (b_0): two-tailed, T(498) = **2.7782**, p-value = **0.005673**. Hence, b_0 is significantly different from zero.

3. Residual normality

The linear regression model assumes normality for residual errors. The Shapiro-Wilk p-value equals **0**. It is assumed that the data is not normally distributed, But since the sample size is large, it should not adversely affect the regression model.

4. Outliers

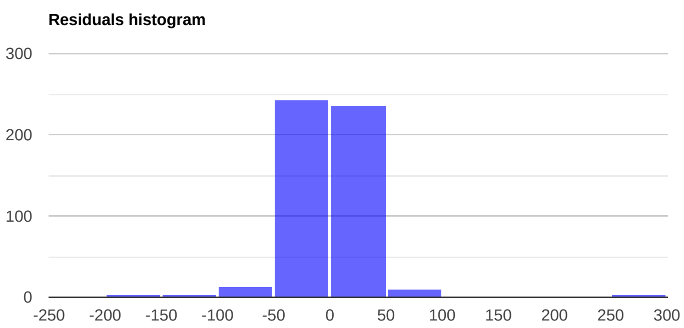
Outliers may affect the regression line.
In this case, the distribution of the residuals is normal. Therefore, the probability of detecting 11 valid outliers or more is 1, (outliers: 88.7552,-81.9736,-86.534,-90.6636,-84.8341,-81.0036,-175.1966,-144.3207,-121.6186,-172.0363,259.1149).
You should only remove outliers if you identify them as errors!

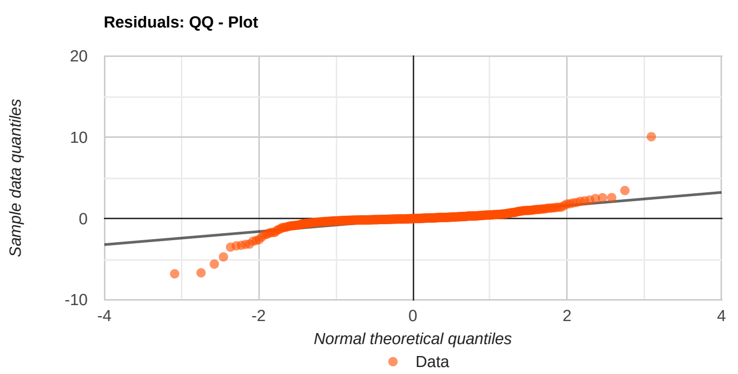


Significance level (α)

Annotations

Residuals normality





Calculation

Step-by-step solution

$\hat{Y} = b_0 + b_1X$

$b_1 = \frac{SP_{xy}}{SS_x} = \frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{\Sigma(x_i-\bar{x})^2}$

$b_1 = \frac{61176935.6}{52115800.95} = 1.1739$

$b_0 = \bar{y} - b_1\bar{x}$
$\bar{x} = 177.59$
$\bar{y} = 212.12$
$b_0 = 212.12-1.1739*177.59 = 3.6532$

$R^2 = \frac{SS_{Regression}}{SS_{total}} = \frac{\Sigma(\hat{y}_i - \bar{y})^2}{\Sigma(y_i - \bar{y})^2} = \frac{71813488.0628}{72144034.8} = 0.9954$

The standard deviation of the residuals is:

$MS_{residual} = S^2_{res} = \frac{\Sigma(y_i - \hat{y})^2}{n - 2}$

Residual outliers

$S_{res} = \sqrt{MSE} = \sqrt{663.7485} = 25.7633$.

The average of the residuals is always zero.

The thresholds used to calculate the outliers are: $\pm k*S_{res}$.

In this case, the thresholds are ±3*25.7633 = ±77.2899.

We tagged the outliers with an arrow (⇐) at the 'Residual' column.

SS_x and SP_{xy}

$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})^2$	$(x-\bar{x})(y-\bar{y})$

-177.59	-212.12	31538.2081	37670.3908
-177.59	-212.12	31538.2081	37670.3908
-177.59	-212.12	31538.2081	37670.3908
-177.59	-212.12	31538.2081	37670.3908
-177.59	-212.12	31538.2081	37670.3908
-177.59	-212.12	31538.2081	37670.3908
-175.59	-210.12	30831.8481	36894.9708
-175.59	-210.12	30831.8481	36894.9708
-175.59	-210.12	30831.8481	36894.9708
-174.59	-209.12	30481.6681	36510.2608
-174.59	-209.12	30481.6681	36510.2608
-176.59	-208.12	31184.0281	36751.9108
-173.59	-208.12	30133.4881	36127.5508
-173.59	-207.12	30133.4881	35953.9608
-173.59	-207.12	30133.4881	35953.9608
-173.59	-206.12	30133.4881	35780.3708
-174.59	-206.12	30481.6681	35986.4908
-171.59	-206.12	29443.1281	35368.1308
-170.59	-205.12	29100.9481	34991.4208
-170.59	-205.12	29100.9481	34991.4208
-171.59	-205.12	29443.1281	35196.5408
-172.59	-205.12	29787.3081	35401.6608
-171.59	-204.12	29443.1281	35024.9508
-170.59	-203.12	29100.9481	34650.2408
-170.59	-203.12	29100.9481	34650.2408
-168.59	-202.12	28422.5881	34075.4108
-169.59	-202.12	28760.7681	34277.5308
-170.59	-202.12	29100.9481	34479.6508
-170.59	-202.12	29100.9481	34479.6508
-166.59	-200.12	27752.2281	33337.9908
-165.59	-198.12	27420.0481	32806.6908
-165.59	-198.12	27420.0481	32806.6908
-166.59	-197.12	27752.2281	32838.2208
-163.59	-197.12	26761.6881	32246.8608
-164.59	-195.12	27089.8681	32114.8008
-162.59	-195.12	26435.5081	31724.5608
-164.59	-195.12	27089.8681	32114.8008
-162.59	-195.12	26435.5081	31724.5608
-161.59	-194.12	26111.3281	31367.8508
-159.59	-194.12	25468.9681	30979.6108
-162.59	-194.12	26435.5081	31561.9708
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-160.59	-184.12	25789.1481	29567.8308
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-147.59	-173.12	21782.8081	25550.7808
-145.59	-173.12	21196.4481	25204.5408
-143.59	-173.12	20618.0881	24858.3008
-150.59	-173.12	22677.3481	26070.1408
-145.59	-173.12	21196.4481	25204.5408
-142.59	-172.12	20331.9081	24542.5908
-141.59	-171.12	20047.7281	24228.8808
-143.59	-170.12	20618.0881	24427.5308
-138.59	-170.12	19207.1881	23576.9308
-140.59	-168.12	19765.5481	23635.9908
-141.59	-168.12	20047.7281	23804.1108
-137.59	-167.12	18931.0081	22994.0408
-144.59	-167.12	20906.2681	24163.8808
-148.59	-166.12	22078.9881	24683.7708
-145.59	-164.12	21196.4481	23894.2308
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-91.59	-104.12	8388.7281	9536.3508
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-82.59	-98.12	6821.1081	8103.7308
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-81.59	-97.12	6656.9281	7924.0208
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-84.59	-95.12	7155.4681	8046.2008
-79.59	-94.12	6334.5681	7491.0108
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-68.59	-86.12	4704.5881	5906.9708
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-76.59	-84.12	5866.0281	6442.7508
-79.59	-84.12	6334.5681	6695.1108
-66.59	-83.12	4434.2281	5534.9608
-75.59	-82.12	5713.8481	6207.4508
-63.59	-81.12	4043.6881	5158.4208
-64.59	-80.12	4171.8681	5174.9508
-63.59	-80.12	4043.6881	5094.8308
-64.59	-79.12	4171.8681	5110.3608
-70.59	-78.12	4982.9481	5514.4908
-64.59	-77.12	4171.8681	4981.1808
-70.59	-77.12	4982.9481	5443.9008
-76.59	-76.12	5866.0281	5830.0308
-86.59	-76.12	7497.8281	6591.2308
-62.59	-75.12	3917.5081	4701.7608
-49.59	-75.12	2459.1681	3725.2008
-63.59	-74.12	4043.6881	4713.2908
-60.59	-73.12	3671.1481	4430.3408
-65.59	-73.12	4302.0481	4795.9408
-64.59	-70.12	4171.8681	4529.0508
-62.59	-70.12	3917.5081	4388.8108
-49.59	-69.12	2459.1681	3427.6608
-50.59	-69.12	2559.3481	3496.7808
-59.59	-68.12	3550.9681	4059.2708

-53.59	-64.12	2871.8881	3436.1908
-52.59	-64.12	2765.7081	3372.0708
-79.59	-64.12	6334.5681	5103.3108
-49.59	-63.12	2459.1681	3130.1208
-41.59	-63.12	1729.7281	2625.1608
-40.59	-60.12	1647.5481	2440.2708
-48.59	-60.12	2360.9881	2921.2308
-72.59	-60.12	5269.3081	4364.1108
-48.59	-58.12	2360.9881	2824.0508
-29.59	-57.12	875.5681	1690.1808
-53.59	-55.12	2871.8881	2953.8808
-37.59	-54.12	1413.0081	2034.3708
-38.59	-52.12	1489.1881	2011.3108
-51.59	-52.12	2661.5281	2688.8708
-70.59	-50.12	4982.9481	3537.9708
-33.59	-50.12	1128.2881	1683.5308
-31.59	-50.12	997.9281	1583.2908
-40.59	-50.12	1647.5481	2034.3708
-47.59	-49.12	2264.8081	2337.6208
-60.59	-49.12	3671.1481	2976.1808
-35.59	-49.12	1266.6481	1748.1808
-48.59	-49.12	2360.9881	2386.7408
-47.59	-48.12	2264.8081	2290.0308
-31.59	-48.12	997.9281	1520.1108
-25.59	-47.12	654.8481	1205.8008
-49.59	-47.12	2459.1681	2336.6808
-38.59	-46.12	1489.1881	1779.7708
-48.59	-46.12	2360.9881	2240.9708
-29.59	-45.12	875.5681	1335.1008
-39.59	-44.12	1567.3681	1746.7108
-49.59	-42.12	2459.1681	2088.7308
-34.59	-41.12	1196.4681	1422.3408
-36.59	-40.12	1338.8281	1467.9908
-29.59	-40.12	875.5681	1187.1508
-35.59	-39.12	1266.6481	1392.2808
-34.59	-38.12	1196.4681	1318.5708
-33.59	-38.12	1128.2881	1280.4508
-43.59	-36.12	1900.0881	1574.4708
-52.59	-36.12	2765.7081	1899.5508
-23.59	-34.12	556.4881	804.8908
-29.59	-33.12	875.5681	980.0208
-21.59	-33.12	466.1281	715.0608
-38.59	-32.12	1489.1881	1239.5108
-35.59	-32.12	1266.6481	1143.1508
-41.59	-30.12	1729.7281	1252.6908
-37.59	-29.12	1413.0081	1094.6208
-23.59	-28.12	556.4881	663.3508
-24.59	-27.12	604.6681	666.8808
-25.59	-26.12	654.8481	668.4108
-32.59	-26.12	1062.1081	851.2508
-23.59	-24.12	556.4881	568.9908
-29.59	-24.12	875.5681	713.7108
-39.59	-24.12	1567.3681	954.9108
-12.59	-23.12	158.5081	291.0808
-39.59	-23.12	1567.3681	915.3208
-22.59	-22.12	510.3081	499.6908
-22.59	-22.12	510.3081	499.6908
-16.59	-20.12	275.2281	333.7908
-16.59	-19.12	275.2281	317.2008
-23.59	-18.12	556.4881	427.4508
-29.59	-18.12	875.5681	536.1708
-15.59	-17.12	243.0481	266.9008
-5.59	-13.12	31.2481	73.3408
-27.59	-13.12	761.2081	361.9808
-11.59	-12.12	134.3281	140.4708
-14.59	-12.12	212.8681	176.8308
1.41	-10.12	1.9881	-14.2692
-34.59	-10.12	1196.4681	350.0508
-25.59	-10.12	654.8481	258.9708
-10.59	-9.12	112.1481	96.5808
-11.59	-9.12	134.3281	105.7008
-22.59	-8.12	510.3081	183.4308
11.41	-8.12	130.1881	-92.6492
-1.59	-4.12	2.5281	6.5508
1.41	-2.12	1.9881	-2.9892
9.41	-1.12	88.5481	-10.5392
1.41	-0.12	1.9881	-0.1692
8.41	-0.12	70.7281	-1.0092
-3.59	1.88	12.8881	-6.7492
10.41	4.88	108.3681	50.8008
10.41	4.88	108.3681	50.8008
-18.59	4.88	345.5881	-90.7192
24.41	5.88	595.8481	143.5308
8.41	5.88	70.7281	49.4508
-13.59	7.88	184.6881	-107.0892
3.41	9.88	11.6281	33.6908
-17.59	11.88	309.4081	-208.9692
1.41	12.88	1.9881	18.1608
-13.59	12.88	184.6881	-175.0392
2.41	13.88	5.8081	33.4508
9.41	13.88	88.5481	130.6108
-14.59	15.88	212.8681	-231.6892
19.41	15.88	376.7481	308.2308
35.41	17.88	1253.8681	633.1308

8.41	19.88	70.7281	167.1908
16.41	19.88	269.2881	326.2308
-5.59	20.88	31.2481	-116.7192
16.41	21.88	269.2881	359.0508
19.41	21.88	376.7481	424.6908
-9.59	22.88	91.9681	-219.4192
16.41	23.88	269.2881	391.8708
33.41	24.88	1116.2281	831.2408
15.41	25.88	237.4681	398.8108
30.41	25.88	924.7681	787.0108
22.41	25.88	502.2081	579.9708
24.41	29.88	595.8481	729.3708
20.41	29.88	416.5681	609.8508
26.41	30.88	697.4881	815.5408
42.41	30.88	1798.6081	1309.6208
41.41	32.88	1714.7881	1361.5608
49.41	36.88	2441.3481	1822.2408
21.41	38.88	458.3881	832.4208
55.41	39.88	3070.2681	2209.7508
12.41	39.88	154.0081	494.9108
9.41	39.88	88.5481	375.2708
34.41	41.88	1184.0481	1441.0908
32.41	42.88	1050.4081	1389.7408
7.41	44.88	54.9081	332.5608
38.41	49.88	1475.3281	1915.8908
45.41	50.88	2062.0681	2310.4608
15.41	51.88	237.4681	799.4708
49.41	52.88	2441.3481	2612.8008
56.41	53.88	3182.0881	3039.3708
49.41	56.88	2441.3481	2810.4408
37.41	56.88	1399.5081	2127.8808
39.41	58.88	1553.1481	2320.4608
48.41	58.88	2343.5281	2850.3808
49.41	58.88	2441.3481	2909.2608
26.41	58.88	697.4881	1555.0208
48.41	60.88	2343.5281	2947.2008
48.41	61.88	2343.5281	2995.6108
56.41	63.88	3182.0881	3603.4708
71.41	66.88	5099.3881	4775.9008
57.41	67.88	3295.9081	3896.9908
66.41	69.88	4410.2881	4640.7308
20.41	69.88	416.5681	1426.2508
53.41	72.88	2852.6281	3892.5208
86.41	74.88	7466.6881	6470.3808
65.41	75.88	4278.4681	4963.3108
56.41	77.88	3182.0881	4393.2108
59.41	81.88	3529.5481	4864.4908
58.41	82.88	3411.7281	4841.0208
72.41	86.88	5243.2081	6290.9808
60.41	87.88	3649.3681	5308.8308
86.41	92.88	7466.6881	8025.7608
89.41	96.88	7994.1481	8662.0408
92.41	98.88	8539.6081	9137.5008
36.41	98.88	1325.6881	3600.2208
66.41	100.88	4410.2881	6699.4408
90.41	100.88	8173.9681	9120.5608
108.41	103.88	11752.7281	11261.6308
100.41	117.88	10082.1681	11836.3308
78.41	122.88	6148.1281	9635.0208
107.41	124.88	11536.9081	13413.3608
147.41	126.88	21729.7081	18703.3808
123.41	129.88	15230.0281	16028.4908
126.41	142.88	15979.4881	18061.4608
152.41	150.88	23228.8081	22995.6208
124.41	154.88	15477.8481	19268.6208
151.41	163.88	22924.9881	24813.0708
132.41	165.88	17532.4081	21964.1708
99.41	165.88	9882.3481	16490.1308
141.41	174.88	19996.7881	24729.7808
129.41	177.88	16746.9481	23019.4508
147.41	180.88	21729.7081	26663.5208
154.41	191.88	23842.4481	29628.1908
185.41	196.88	34376.8681	36503.5208
137.41	196.88	18881.5081	27053.2808
112.41	197.88	12636.0081	22243.6908
201.41	207.88	40565.9881	41869.1108
153.41	208.88	23534.6281	32044.2808
222.41	216.88	49466.2081	48236.2808
150.41	217.88	22623.1681	32771.3308
192.41	228.88	37021.6081	44038.8008
199.41	234.88	39764.3481	46837.4208
180.41	241.88	32547.7681	43637.5708
189.41	243.88	35876.1481	46193.3108
253.41	245.88	64216.6281	62308.4508
257.41	257.88	66259.9081	66380.8908
167.41	259.88	28026.1081	43506.5108
264.41	277.88	69912.6481	73474.2508
234.41	281.88	54948.0481	66075.4908
233.41	292.88	54480.2281	68361.1208
182.41	302.88	33273.4081	55248.3408
229.41	315.88	52628.9481	72466.0308
271.41	337.88	73663.3881	91704.0108
307.41	344.88	94500.9081	106019.5608
306.41	359.88	93887.0881	110270.8308

356.41	374.88	127028.0881	133610.9808
269.41	374.88	72581.7481	100996.4208
351.41	383.88	123488.9881	134899.2708
399.41	386.88	159528.3481	154523.7408
291.41	392.88	84919.7881	114489.1608
345.41	409.88	119308.0681	141576.6508
357.41	411.88	127741.9081	147210.0308
401.41	420.88	161129.9881	168945.4408
439.41	478.88	193081.1481	210424.6608
500.41	500.88	250410.1681	250645.3608
518.41	517.88	268748.9281	268474.1708
486.41	558.88	236594.6881	271844.8208
519.41	574.88	269786.7481	298598.4208
588.41	605.88	346226.3281	356505.8508
577.41	610.88	333402.3081	352728.2208
588.41	668.88	346226.3281	393575.6808
681.41	718.88	464319.5881	489852.0208
643.41	735.88	413976.4281	473472.5508
681.41	796.88	464319.5881	543002.0008
777.41	923.88	604366.3081	718233.5508
869.41	961.88	755873.7481	836268.0908
977.41	1077.88	955330.3081	1053530.6908
1194.41	1226.88	1426615.2481	1465397.7408
1327.41	1413.88	1762017.3081	1876798.4508
1394.41	1564.88	1944379.2481	2182084.3208
1461.41	1593.88	2135719.1881	2329312.1708
1368.41	1660.88	1872545.9281	2272764.8008
1444.41	1761.88	2086320.2481	2544877.0908
1820.41	1964.88	3313892.5681	3576887.2008
4994.41	6121.88	24944131.2481	30575178.6908
0	0	**52115800.95** (SS_x)	**61176935.6** (SP_{xy})

Linear regression calculator

The linear regression calculator generates the linear regression equation. It also draws: a linear regression line, a histogram, a residuals QQ-plot, a residuals x-plot, and a distribution chart.
It calculates the R-squared, the R, and the outliers, then testing the fit of the linear model to the data and checking the residuals' normality assumption and the priori power.

What is linear regression?

The linear regression is the linear equation that best fits the points.
There is no one way to choose the best fit ting line, the most common one is the ordinary least squares (OLS). The linear regression describes the relationship between the dependent variable (Y) and the independent variables (X).
The linear regression model calculates the dependent variable (DV) based on the independent variables (IV, predictors).

What is "ordinary least squares"?

The ordinary least squares method chooses the line parameters that minimize the sum of squares of the differences between the observed dependent variables (Y) and the estimated value by the linear regression ($\hat{Y}$).

Why do you need linear regression?

We may use linear regression when we want to do one of the following

- Predict the dependent variable ($\hat{Y}$).
- Estimate the effect of each independent variable (X) on the dependent variable (Y).
- Calculate the correlation between the dependent variable and the independent variables.
- Test the linear model significance level.

How to calculate linear regression?

Following the linear regression formula:

$$\hat{Y} = b_0 + b_1 x$$

b_0 - the y-intercept, where the line crosses the y-axis.
b_1 - the slope, describes the line's direction and incline.

$$b_1 = \frac{SP_{xy}}{SS_x} = \frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{\Sigma(x_i-\bar{x})^2}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

linear regression prediction

The prediction calculator uses the linear regrssion to predict the depdendent variable based on the independent value. The calculator also creates the confidence interval, and the prediction interval.

Confidence interval of the prediction

The prediction interval for the **mean value** of the dependent variable.
This is the interval for the equation line, the true value equation will be in this interval. If we would know the true equation then the width of this interval would be zero.
If you would calculate the confidence interval over an infinite number of regressions with the same sample size, 95% (confidence level) of the calculated confidence intervals will contain the mean's true value.
Since this interval is for the mean, the standard error is smaller and the the range is narrower than the range of the prediction interval.

$$MS_{residual} = S^2_{residual} = \frac{\Sigma(y_i - \hat{y})^2}{n - 2}$$

$$S.E^2_{ci} = S^2_{residual}\left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{SS_x}\right)$$

$$\hat{Y} \pm T_{1-\alpha/2}(n-2) * S.E_{ci}$$

Prediction Interval

The prediction interval for a **particular observation** of the dependent variable.
This is the interval for any single value.
The prediction inteval takes into consideration the fact that you don't know the true equatio, and the fact the the liner regression explaned only part of the variance (the part is R-squared). Even if we would know the true equation then the width of this interval would be greater than zero.
Since this interval is for a single observation, the standard error is larger and the range is wider than the range of the confidence interval

$S.E^2_{prediction} = S^2_{residual}\left(1+\frac{1}{n}+\frac{(x_0 - \bar{x})^2}{SS_x}\right)$

$\hat{Y} \pm T_{1-\alpha/2}(n-2)*S.E_{prediction}$

How to calculate R squares?

R squares is the percentage of the variance explain by the regression ($SS_{Regression}$) from the overall variance (SS_{Total}).

$R^2 = \frac{SS_{Regression}}{SS_{Total}}$

Linear regression in calculator

This online calculator supports all the basic functionality and more.

The **right-tailed** F test checks if the entire regression model is statistically significant. Why only right tail?

For **Multiple regression calculator** with the stepwise method and assumptions validations: **multiple regression calculator**

The following statistic checks if the linear regression model supports better results than the average of Y.

Hypotheses

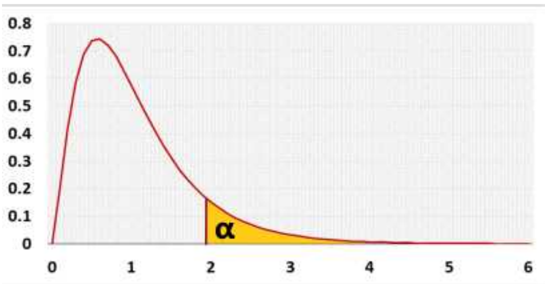
H_0: $Y = b_0$

H_1: $Y = b_0+b_1X$

Test statistic

$F = \frac{MS(regression)}{MS\ (residual)}$

F distribution



R Code

The following R code should produce similar results

```
rm(list = ls())
if(!"car" %in% installed.packages()){install.packages("car")}
library(car)
x10 <-
c(0,0,0,0,0,0,2,2,2,3,3,1,4,4,4,4,3,6,7,7,6,5,6,7,7,9,8,7,7,11,12,12,11,14,13,15,13,15,16,18,15,14,19,23,22,17,24,22,25,18,30,28,28,25,24,26,28,29,30,32,27,30,32,34,27,32,35,36,34,39,37,36,40,33,29,32,40,32,36,4(
x11 <-
c(130,117,142,129,130,146,152,128,139,129,148,138,128,143,141,148,142,143,144,134,125,154,148,156,139,142,136,140,154,153,152,145,154,148,138,165,138,155,155,161,161,154,148,162,172,150,166,163,17
x1 <- c(x10,x11)
y10 <-
c(0,0,0,0,0,0,2,2,2,3,3,4,4,5,5,6,6,6,7,7,7,7,8,9,9,10,10,10,10,12,14,14,15,15,17,17,17,17,18,18,18,21,24,25,25,28,28,29,30,32,32,34,34,34,35,36,36,38,38,38,39,39,39,39,39,39,40,41,42,42,44,44,45,45,46,48,49,49
y11 <-
c(163,163,163,163,164,164,165,165,166,166,167,168,170,171,172,172,173,174,174,176,176,178,179,179,180,180,182,183,184,185,186,186,188,188,188,189,189,190,190,192,193,194,194,195,199,199,200,200,2(
y1 <- c(y10,y11)
model1 = lm(y1~x1)
summary(model1)
```

What is linear regression?

Tutorial

Calculators

Multiple regression
Correlation
Regression sample size