$$B(\phi) = \begin{pmatrix} B_1(\phi_1) & & 0 \\ & B_2(\phi_2) \ddots & \\ 0 & & B_N(\phi_N) \end{pmatrix} \quad \text{con}$$

$$B_m(\phi_m) = \begin{pmatrix} 0 & \gamma(v_m^2 + w_m^2) \\ -\gamma(v_m^2 + w_m^2) & 0 \end{pmatrix}$$

$$\phi^{n+1} - \phi^n + K\left(\vec{S} + B\left(\frac{\phi^{n+1} - \phi^n}{2}\right)\right)\left(\frac{\phi^{n+1} + \phi^n}{2}\right) = 0$$

Para $m = 1$ tenemos

$$v_1^{n+1} - v_1^n + \frac{\beta K}{2h^2}\left(\frac{2w_1^{n+1} + 2w_1^n - 2w_2^{n+1} - 2w_2^n}{2}\right)$$

$$+ \frac{K\gamma}{4}\left((v_1^{n+1} + v_1^n)^2 + (w_1^{n+1} + w_1^n)^2\right)\left(\frac{w_1^{n+1} + w_1^n}{2}\right) = 0$$

$$w_1^{n+1} - w_1^n + \frac{\beta K}{2h^2}\left(\frac{-2v_1^{n+1} - 2v_1^n + 2v_2^{n+1} + 2v_2^n}{2}\right)$$

$$- \frac{K\gamma}{4}\left((v_1^{n+1} + v_1^n)^2 + (w_1^{n+1} + w_1^n)^2\right)\left(\frac{v_1^{n+1} + v_1^n}{2}\right) = 0$$

Para $m = 2, \dots, N-1$

$$v_m^{n+1} - v_m^n + \frac{\beta K}{2h^2}\left(\frac{-w_{m-1}^{n+1} - w_{m-1}^n + 2w_m^{n+1} + 2w_m^n - w_{m+1}^{n+1} - w_{m+1}^n}{2}\right)$$

$$+ \frac{K\gamma}{4}\left((v_m^{n+1} + v_m^n)^2 + (w_m^{n+1} + w_m^n)^2\right)\left(\frac{w_m^{n+1} + w_m^n}{2}\right) = 0$$

$$\omega_m^{n+1} - \omega_m^n + \frac{\beta K}{2h^2}\left(\frac{v_{m-1}^{n+1} + v_{m-1}^n - 2v_m^{n+1} - 2v_m^n + v_{m+1}^{n+1} + v_{m+1}^n}{2}\right)$$

$$-\frac{K\gamma}{4}\left(\left(v_m^{n+1} + v_m^n\right)^2 + \left(\omega_m^{n+1} + \omega_m^n\right)^2\right)\left(\frac{v_m^{n+1} + v_m^n}{2}\right) = 0$$

Para $m = N$

$$v_N^{n+1} - v_N^n + \frac{\beta K}{2h^2}\left(\frac{-2\omega_{N-1}^{n+1} - 2\omega_{N-1}^n + 2\omega_N^{n+1} + 2\omega_N^n}{2}\right)$$

$$+\frac{K\gamma}{4}\left(\left(v_N^{n+1} + v_N^n\right)^2 + \left(\omega_N^{n+1} + \omega_N^n\right)^2\right)\left(\frac{\omega_N^{n+1} + \omega_N^n}{2}\right) = 0$$

$$\omega_N^{n+1} - \omega_N^n + \frac{\beta K}{2h^2}\left(\frac{2v_{N-1}^{n+1} + 2v_{N-1}^n - 2v_N^{n+1} - 2v_N^n}{2}\right)$$

$$-\frac{K\gamma}{4}\left(\left(v_N^{n+1} + v_N^n\right)^2 + \left(\omega_N^{n+1} + \omega_N^n\right)^2\right)\left(\frac{v_N^{n+1} + v_N^n}{2}\right) = 0$$

De manera que tenemos $\vec{F}(\phi^{n+1}) = 0$

con $\vec{F} = \left(\vec{f}_1, \vec{f}_2, \cdots, \vec{f}_N\right)^T$ con $\vec{f}_m = \left(f1_m, f2_m\right)^T$

$$J(\phi^{(j)})\,\vec{y} = \vec{F}(\phi^{(j)}) \qquad \phi^{(j+1)} = \phi^{(j)} - \vec{y}$$

con $$\vec{J}(\phi^{n+1}) = \begin{pmatrix} A_1 & C_1 & & 0 \\ B_2 & A_2 & \ddots & \\ & \ddots & \ddots & C_{N-1} \\ 0 & & B_N & A_N \end{pmatrix}$$

con $\quad A_i = \begin{pmatrix} \dfrac{\partial f1_i}{\partial v_i^{n+1}} & \dfrac{\partial f1_i}{\partial w_i^{n+1}} \\ \dfrac{\partial f2_i}{\partial v_i^{n+1}} & \dfrac{\partial f2_i}{\partial w_i^{n+1}} \end{pmatrix}$

$$B_i = \begin{pmatrix} \dfrac{\partial f1_i}{\partial v_{i-1}^{n+1}} & \dfrac{\partial f1_i}{\partial w_{i-1}^{n+1}} \\ \dfrac{\partial f2_i}{\partial v_{i-1}^{n+1}} & \dfrac{\partial f2_i}{\partial w_{i-1}^{n+1}} \end{pmatrix}$$

$$C_i = \begin{pmatrix} \dfrac{\partial f1_i}{\partial v_{i+1}^{n+1}} & \dfrac{\partial f1_i}{\partial w_{i+1}^{n+1}} \\ \dfrac{\partial f2_i}{\partial v_{i+1}^{n+1}} & \dfrac{\partial f2_i}{\partial w_{i+1}^{n+1}} \end{pmatrix}$$

Para $i=1$ tenemos

$$\frac{\partial f1_1}{\partial v_1^{n+1}} = 1 + \frac{K\gamma}{4} 2(v_1^{n+1} + v_1^n)\left(\frac{w_1^{n+1} + w_1^n}{2}\right)$$

$$\frac{\partial f1_1}{\partial v_2^{n+1}} = 0$$

$$\frac{\partial f1_1}{\partial w_1^{n+1}} = \frac{\beta K}{2h^2} + \frac{K\gamma}{4} 2(w_1^{n+1} + w_1^n)\left(\frac{w_1^{n+1} + w_1^n}{2}\right)$$

$$+ \frac{K\gamma}{4}\left((v_1^{n+1} + v_1^n)^2 + (w_1^{n+1} + w_1^n)^2\right)\frac{1}{2}$$

$$\frac{\partial f1_1}{\partial w_2^{n+1}} = \frac{-\beta K}{2h^2}$$

$$\frac{\partial f2_1}{\partial v_1^{n+1}} = \frac{-\beta K}{2h^2} - \frac{K\gamma}{4} 2(v_1^{n+1} + v_1^n)\left(\frac{v_1^{n+1} + v_1^n}{2}\right)$$

$$- \frac{K\gamma}{4}\left((v_1^{n+1} + v_1^n)^2 + (w_1^{n+1} + w_1^n)^2\right)\frac{1}{2}$$

$$\frac{\partial f2_1}{\partial w_1^{n+1}} = 1 - \frac{K\gamma}{4} 2(w_1^{n+1} + w_1^n)\left(\frac{v_1^{n+1} + v_1^n}{2}\right)$$

$$\frac{\partial f2_1}{\partial v_2^{n+1}} = \frac{\beta K}{2h^2}$$

$$\frac{\partial f2_1}{\partial w_2^{n+1}} = 0$$

Para $m = 2, \dots, N-1$

$$\frac{\partial f1_m}{\partial v_{m-1}^{n+1}} = 0$$

$$\frac{\partial f1_m}{\partial v_m^{n+1}} = 1 + \frac{K\gamma}{4} 2(v_m^{n+1} + v_m^n)\left(\frac{w_m^{n+1} + w_m^n}{2}\right)$$

$$\frac{\partial f1_m}{\partial v_{m+1}^{n+1}} = 0$$

$$\frac{\partial f1_m}{\partial w_{m-1}^{n+1}} = -\frac{\beta K}{4h^2}$$

$$\frac{\partial f1_m}{\partial \omega_m^{n+1}} = \frac{\beta K}{2h^2} + \frac{K\gamma}{4} 2(\omega_m^{n+1} + \omega_m^n)\left(\frac{\omega_m^{n+1} + \omega_m^n}{2}\right)$$

$$+ \frac{K\gamma}{4}\left((v_m^{n+1} + v_m^n)^2 + (\omega_m^{n+1} + \omega_m^n)^2\right) \cdot \frac{1}{2}$$

$$\frac{\partial f1_m}{\partial \omega_{m+1}^{n+1}} = -\frac{\beta K}{4h^2}$$

$$\frac{\partial f2_m}{\partial v_{m-1}^{n+1}} = \frac{\beta K}{4h^2}$$

$$\frac{\partial f2_m}{\partial v_m^{n+1}} = -\frac{\beta K}{2h^2} - \frac{K\gamma}{4} 2(v_m^{n+1} + v_m^n)\left(\frac{v_m^{n+1} + v_m^n}{2}\right)$$

$$-\frac{K\gamma}{4}\left((v_m^{n+1} + v_m^n)^2 + (\omega_m^{n+1} + \omega_m^n)\right)^2 \frac{1}{2}$$

$$\frac{\partial f2_m}{\partial v_{m+1}^{n+1}} = \frac{\beta K}{4h^2}$$

$$\frac{\partial f2_m}{\partial \omega_{m-1}^{n+1}} = 0$$

$$\frac{\partial f2_m}{\partial \omega_m^{n+1}} = 1 - \frac{K\gamma}{4} 2(\omega_m^{n+1} + \omega_m^n)\left(\frac{v_m^{n+1} + v_m^n}{2}\right)$$

$$\frac{\partial f2_m}{\partial \omega_{m+1}^{n+1}} = 0$$

Para $i = N$

$$\frac{\partial f1_N}{\partial v_{N-1}^{n+1}} = 0$$

$$\frac{\partial f1_N}{\partial v_N^{n+1}} = 1 + \frac{K\gamma}{4} 2(v_N^{n+1} + v_N^{n})\left(\frac{w_N^{n+1} + w_N^{n}}{2}\right)$$

$$\frac{\partial f1_N}{\partial w_{N-1}^{n+1}} = -\frac{\beta K}{2h^2}$$

$$\frac{\partial f1_N}{\partial w_N^{n+1}} = \frac{\beta K}{2h^2} + \frac{K\gamma}{4} 2(w_N^{n+1} + w_N^{n})\left(\frac{w_N^{n+1} + w_N^{n}}{2}\right)$$

$$+ \frac{K\gamma}{4}\left((v_N^{n+1} + v_N^{n})^2 + (w_N^{n+1} + w_N^{n})^2\right)\frac{1}{2}$$

$$\frac{\partial f2_N}{\partial v_{N-1}^{n+1}} = \frac{\beta K}{2h^2}$$

$$\frac{\partial f2_N}{\partial v_N^{n+1}} = \frac{-\beta K}{2h^2} - \frac{K\gamma}{4} 2(v_N^{n+1} + v_N^{n})\left(\frac{v_N^{n+1} + v_N^{n}}{2}\right)$$

$$-\frac{K\gamma}{4}\left((v_N^{n+1} + v_N^{n})^2 + (w_N^{n+1} + w_N^{n})^2\right)\frac{1}{2}$$

$$\frac{\partial f2_N}{\partial w_{N-1}^{n+1}} = 0$$

$$\frac{\partial f2_N}{\partial w_N^{n+1}} = 1 - \frac{K\gamma}{4} 2(w_N^{n+1} + w_N^{n})\left(\frac{v_N^{n+1} + v_N^{n}}{2}\right)$$