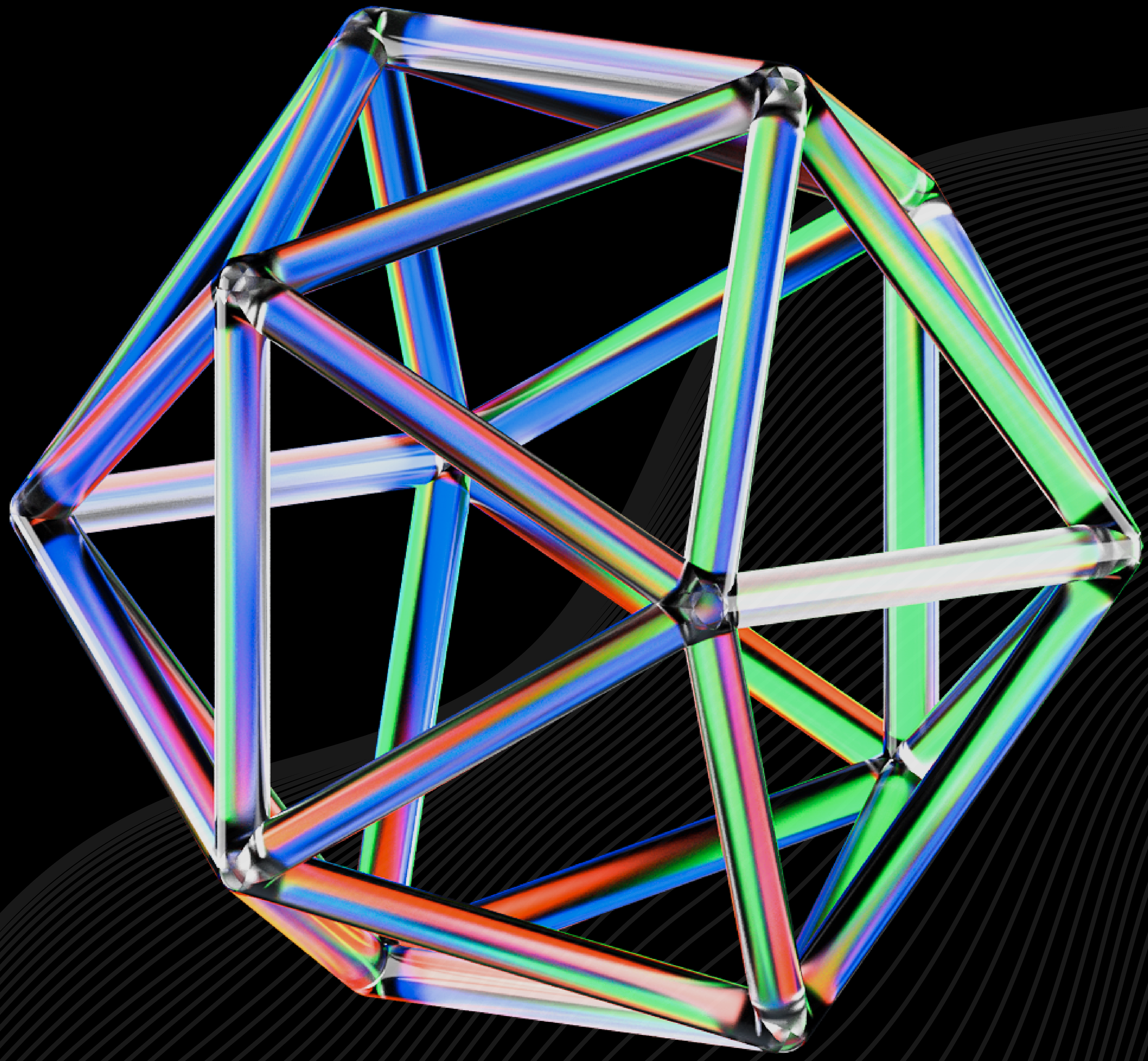
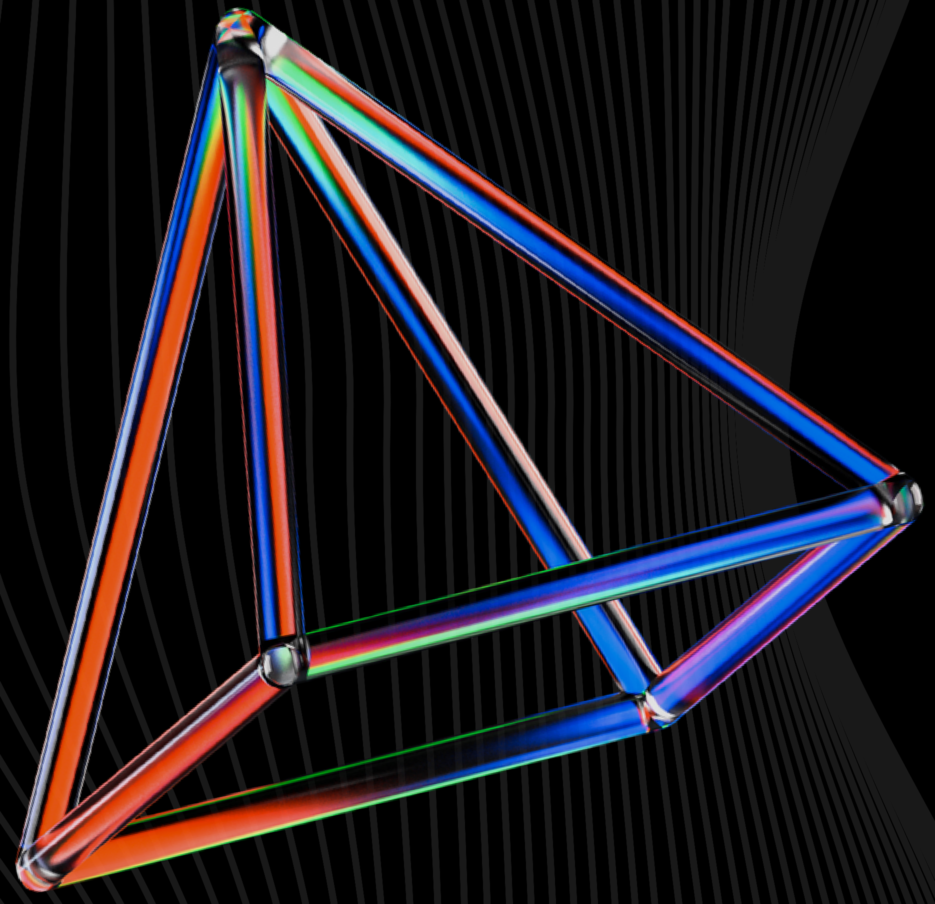
PARCIAL III

SOLUCIÓN DE ECUACIONES DIFERENCIALES POR EL MÉTODO SHOOTING



JOSUÉ JARAMILLO
ESNEIDER VELEZ

CURSO FÍSICA COMPUTACIONAL II



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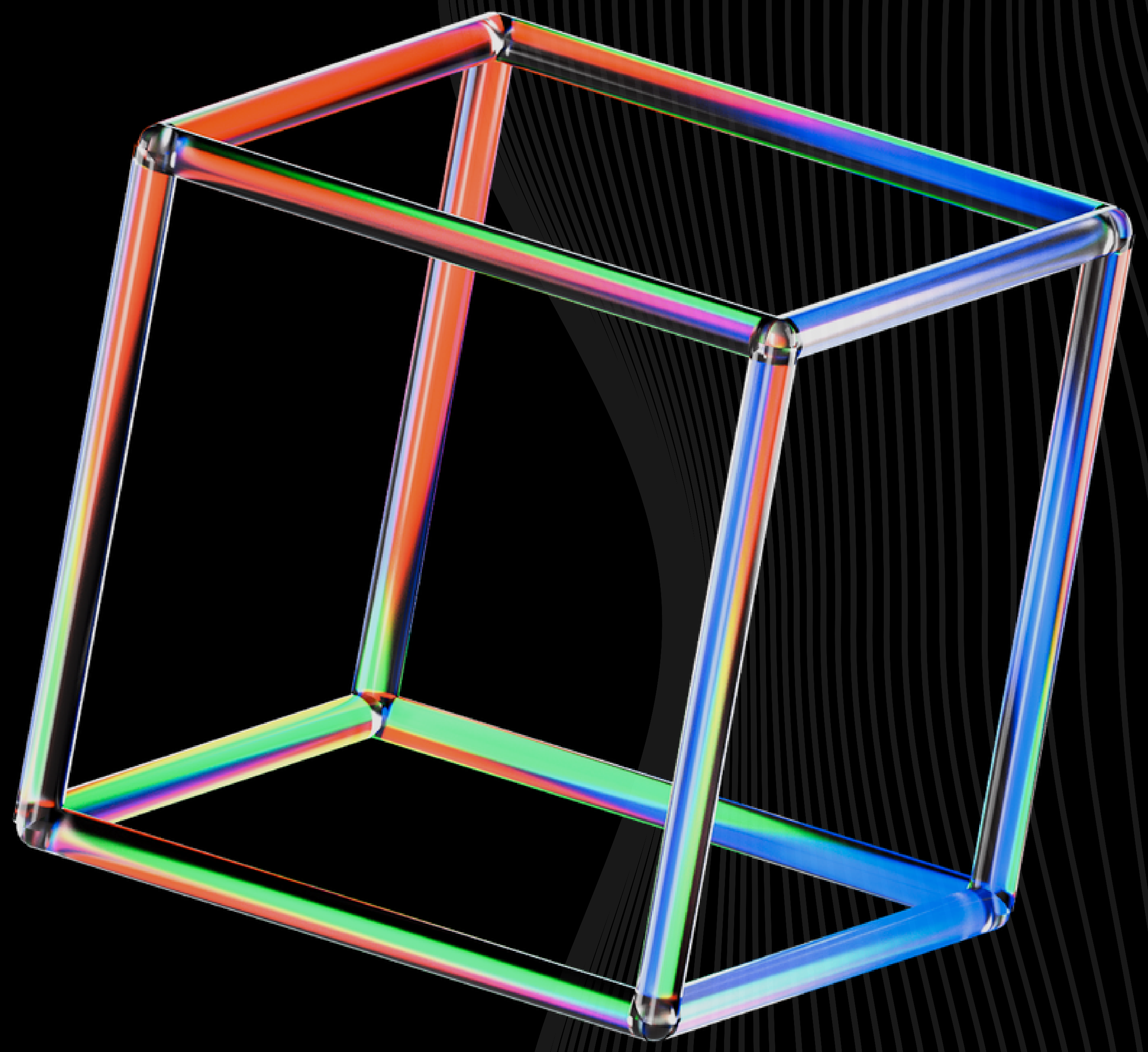
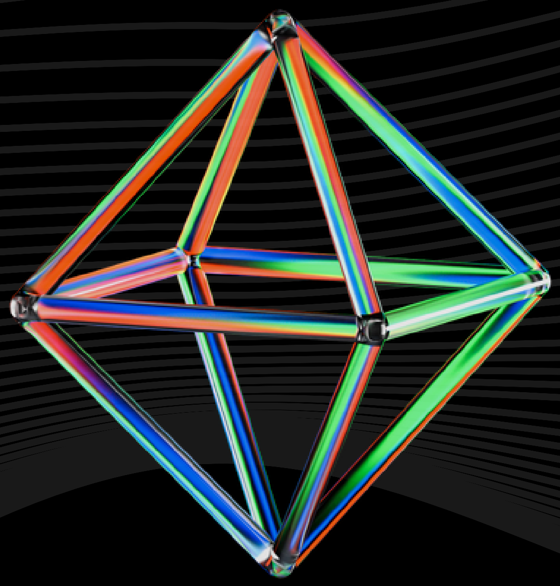
Introducción

Las ecuaciones diferenciales son una herramienta fundamental en la modelización de sistemas físicos, biológicos, económicos y sociales. Sin embargo, en muchos casos, estas ecuaciones no pueden resolverse de manera analítica, y se requiere de métodos numéricos para encontrar una solución aproximada.





Método shooting

$$y'' = f(x, y, y'), \quad \text{for } a \le x \le b$$

$$y(a) = \alpha \quad \text{and} \quad y(b) = \beta$$

Teorema de unicidad de la solución

Theorem 11.1 Suppose the function f in the boundary-value problem

$$y'' = f(x, y, y'), \quad \text{for } a \le x \le b, \text{ with } y(a) = \alpha \text{ and } y(b) = \beta,$$

is continuous on the set

$$D = \{\, (x, y, y') \mid \text{for } a \le x \le b, \text{ with } -\infty < y < \infty \text{ and } -\infty < y' < \infty \,\},$$

and that the partial derivatives f_y and $f_{y'}$ are also continuous on D. If

(i) $f_y(x, y, y') > 0$, for all $(x, y, y') \in D$, and

(ii) a constant M exists, with

$$\left| f_{y'}(x, y, y') \right| \le M, \quad \text{for all } (x, y, y') \in D,$$

then the boundary-value problem has a unique solution.

Método Shooting Lineal

Problema de valor en la frontera

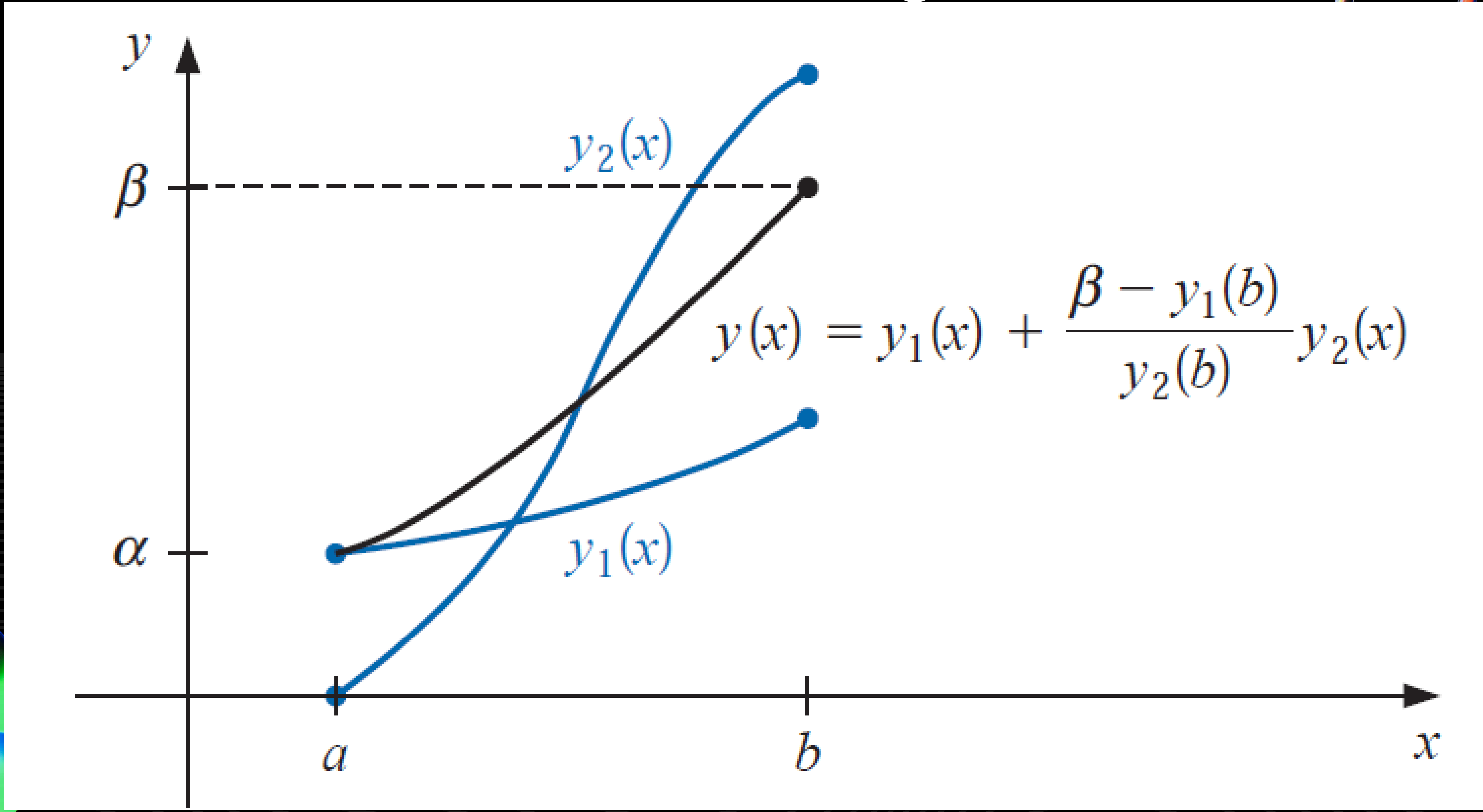
$$y'' = p(x)y' + q(x)y + r(x), \quad \text{for } a \le x \le b, \text{ with } y(a) = \alpha \text{ and } y(b) = \beta$$

Dos problemas de valor inicial

$$y'' = p(x)y' + q(x)y + r(x), \text{ with } \quad a \le x \le b, \quad y(a) = \alpha, \text{ and } \quad y'(a) = 0$$

$$y'' = p(x)y' + q(x)y, \text{ with } \quad a \le x \le b, \quad y(a) = 0, \text{ and } \quad y'(a) = 1$$

Método Shooting Lineal



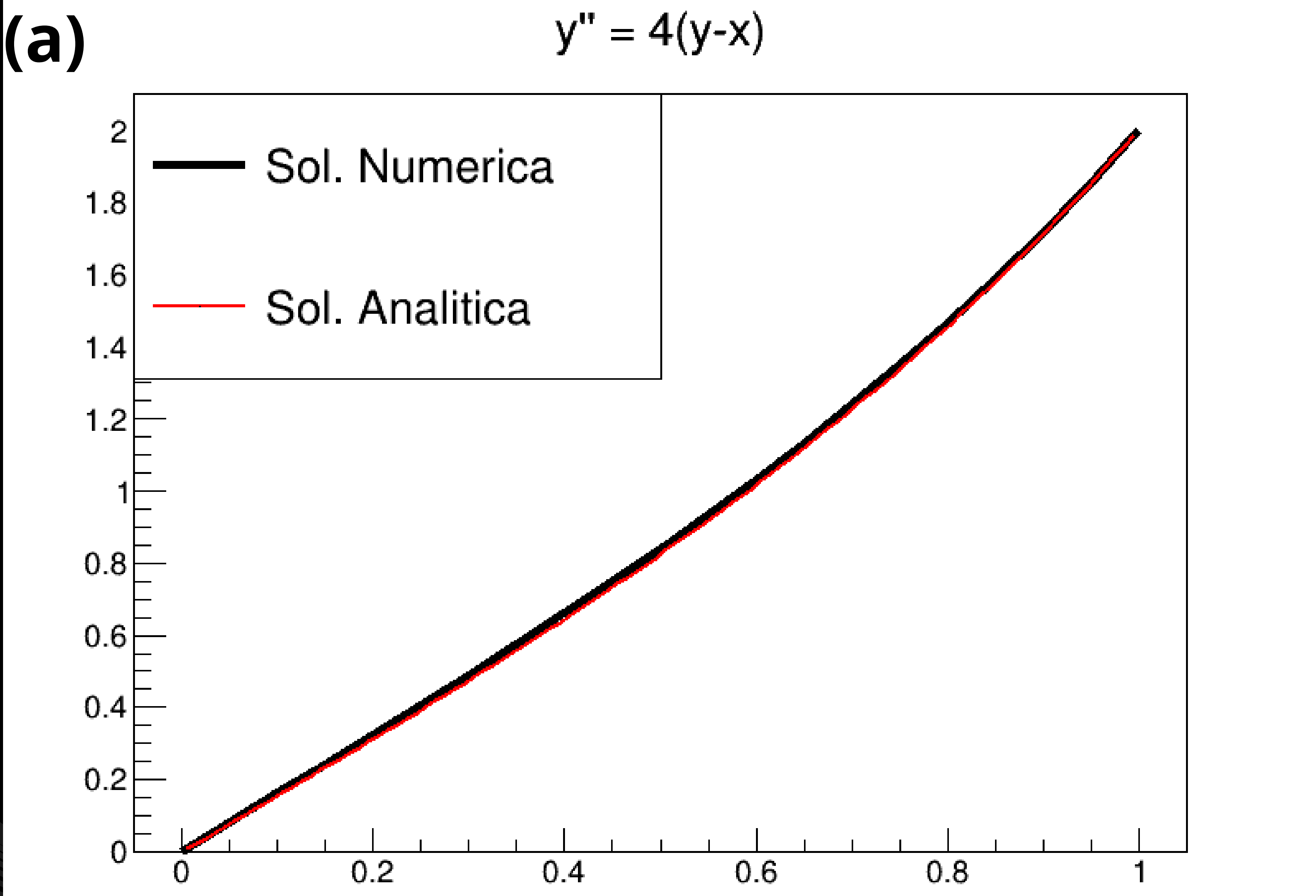
Evaluación en los puntos extremos

$$y(a) = y_1(a) + \frac{\beta - y_1(b)}{y_2(b)} y_2(a) = \alpha + \frac{\beta - y_1(b)}{y_2(b)} \cdot 0 = \alpha$$

$$y(b) = y_1(b) + \frac{\beta - y_1(b)}{y_2(b)} y_2(b) = y_1(b) + \beta - y_1(b) = \beta.$$

Ejemplos de aplicación

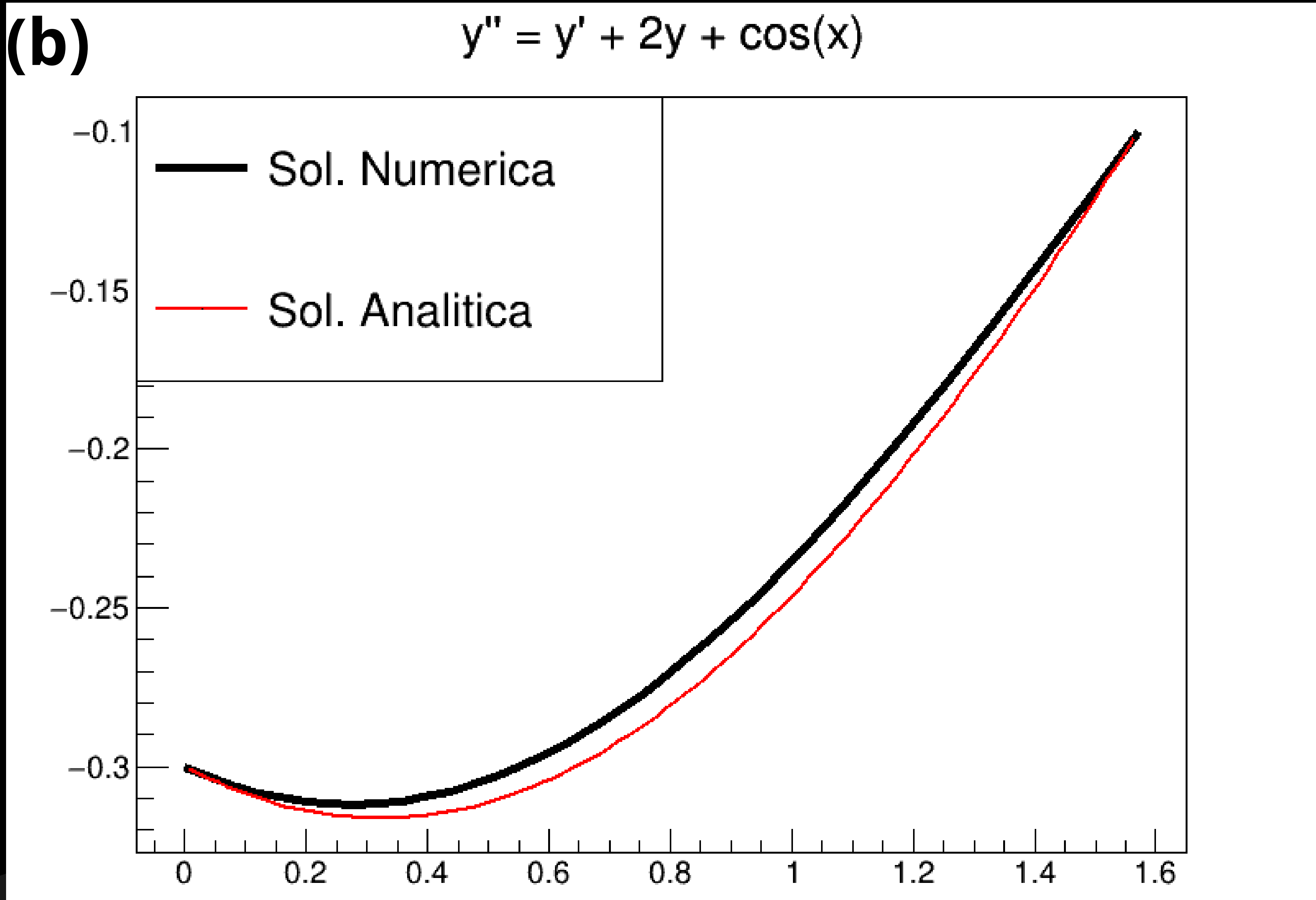
Ejercicio 1 (Burden)



$$y'' = 4(y - x), \quad 0 \le x \le 1, \quad y(0) = 0, \quad y(1) = 2,$$

$$y(x) = e^2(e^4 - 1)^{-1}(e^{2x} - e^{-2x}) + x.$$

Ejercicio 2 (Burden)



$$y'' = y' + 2y + \cos x, \quad 0 \le x \le \tfrac{\pi}{2}, \quad y(0) = -0.3, \quad y\left(\tfrac{\pi}{2}\right) = -0.1$$

$$y(x) = -\frac{1}{10}(\sin x + 3\cos x).$$

Ejemplo 2, pag 676 (Burden)

Example 2 Apply the Linear Shooting technique with $N = 10$ to the boundary-value problem

$$y'' = -\frac{2}{x}y' + \frac{2}{x^2}y + \frac{\sin(\ln x)}{x^2}, \quad \text{for } 1 \le x \le 2, \text{ with } y(1) = 1 \text{ and } y(2) = 2,$$

and compare the results to those of the exact solution

$$y = c_1 x + \frac{c_2}{x^2} - \frac{3}{10}\sin(\ln x) - \frac{1}{10}\cos(\ln x),$$

where

$$c_2 = \frac{1}{70}[8 - 12\sin(\ln 2) - 4\cos(\ln 2)] \approx -0.03920701320$$

and

$$c_1 = \frac{11}{10} - c_2 \approx 1.1392070132.$$

Método Shooting No Lineal

Problema de valor en la frontera

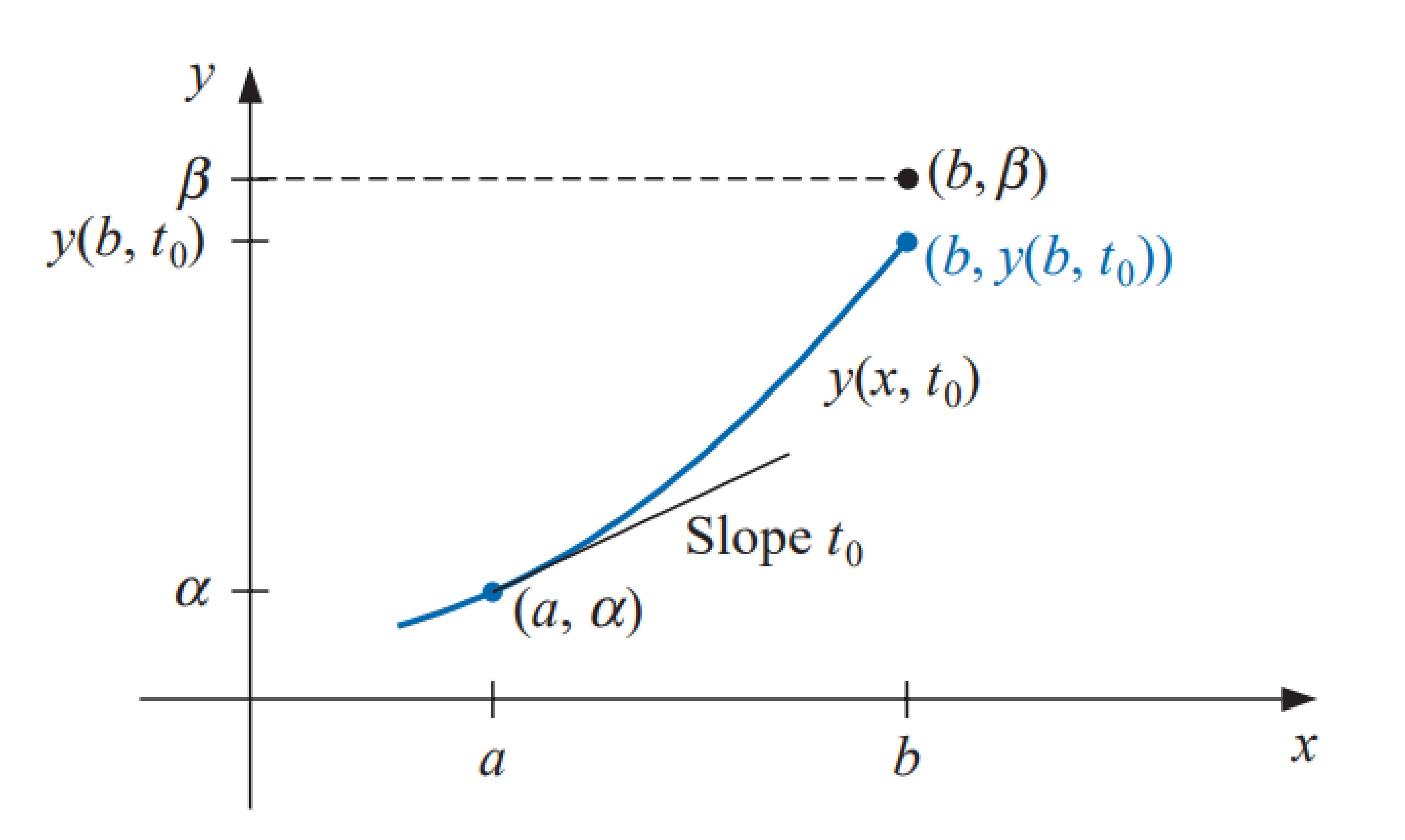
$$y'' = f(x, y, y'), \quad \text{for } a \leq x \leq b,$$

$$y(a) = \alpha \quad \text{and} \quad y(b) = \beta$$

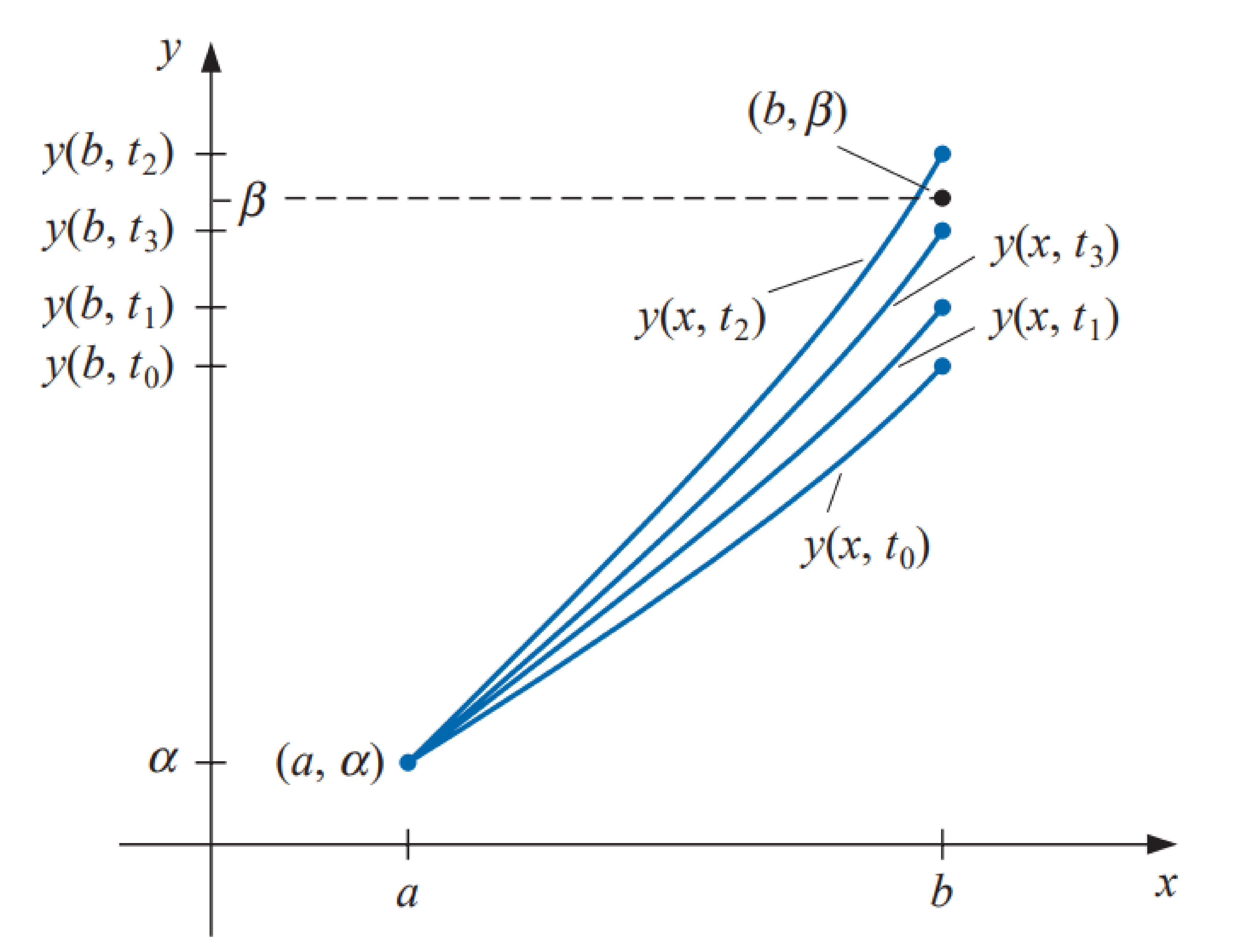
Se plantea el problema de valor inicial

for $a \leq x \leq b$, with $y(a) = \alpha$ and $y'(a) = t$

Método Shooting No Lineal



Método Shooting No Lineal



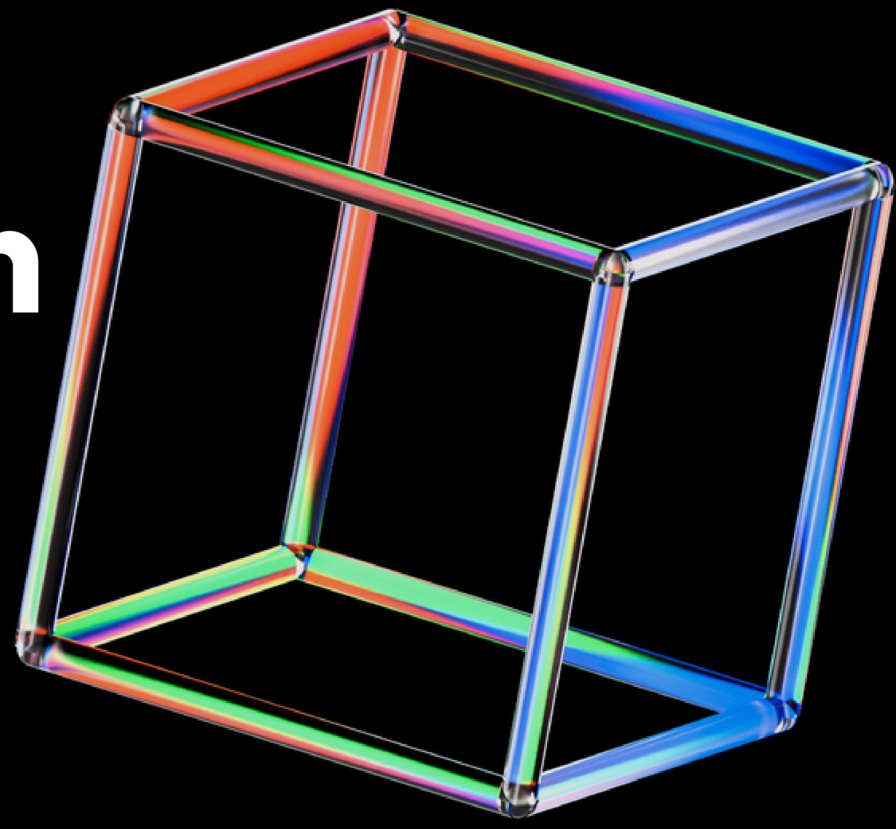
Método Shooting No Lineal

$$\lim_{k\to\infty} y(b, t_k) = y(b) = \beta,$$

$$\downarrow$$

$$y(b, t) - \beta = 0.$$

Método Shooting No Lineal con Newton Rhapson



$$t_k = t_{k-1} - \frac{y(b, t_{k-1}) - \beta}{\frac{dy}{dt}(b, t_{k-1})},$$

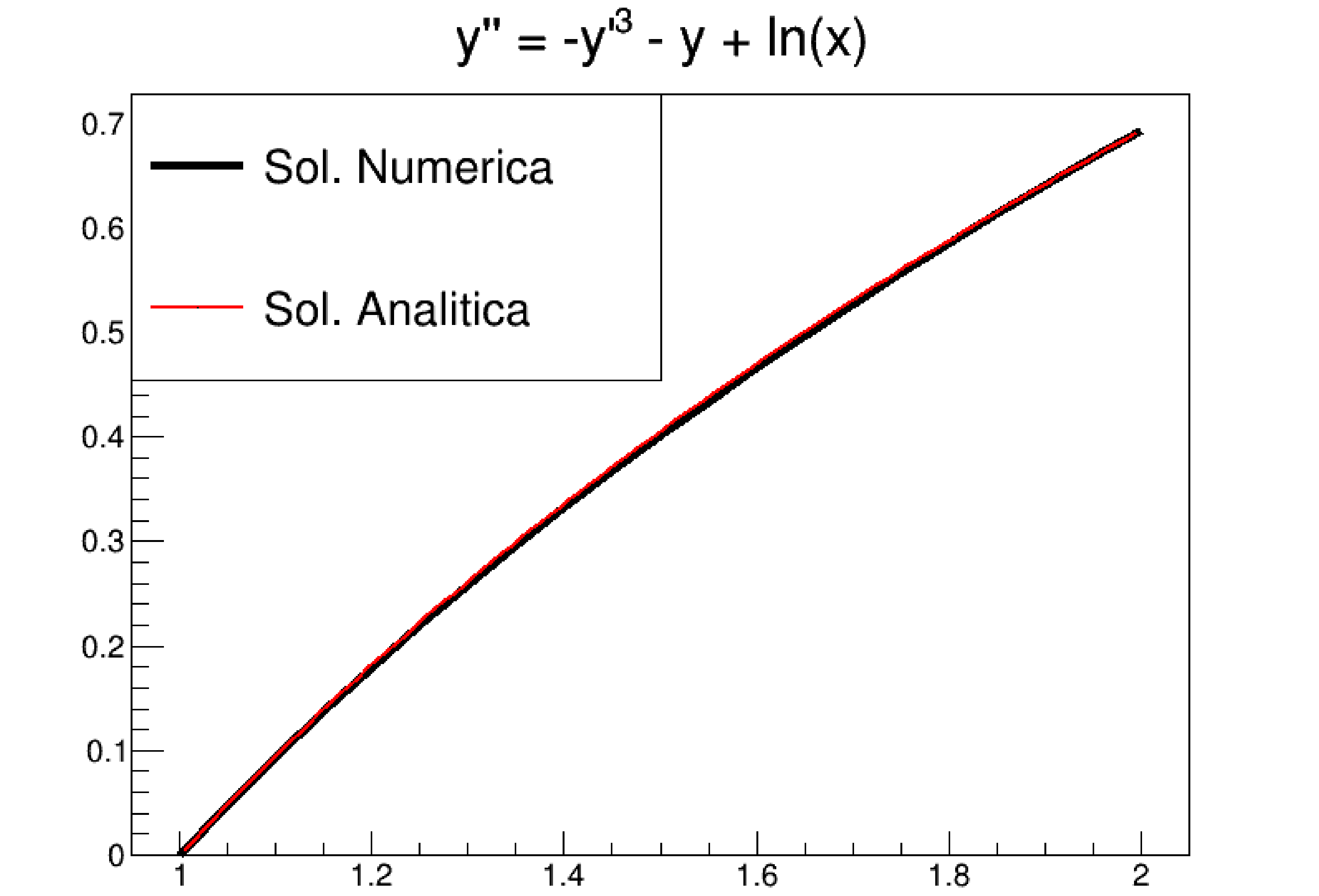
Método Shooting No Lineal con Newton Rhapson

$$z''(x,t) = \frac{\partial f}{\partial y}(x,y,y')z(x,t) + \frac{\partial f}{\partial y'}(x,y,y')z'(x,t), \quad \text{for } a \le x \le b,$$

$$z(a,t) = 0 \text{ and } z(a,t) = 1$$

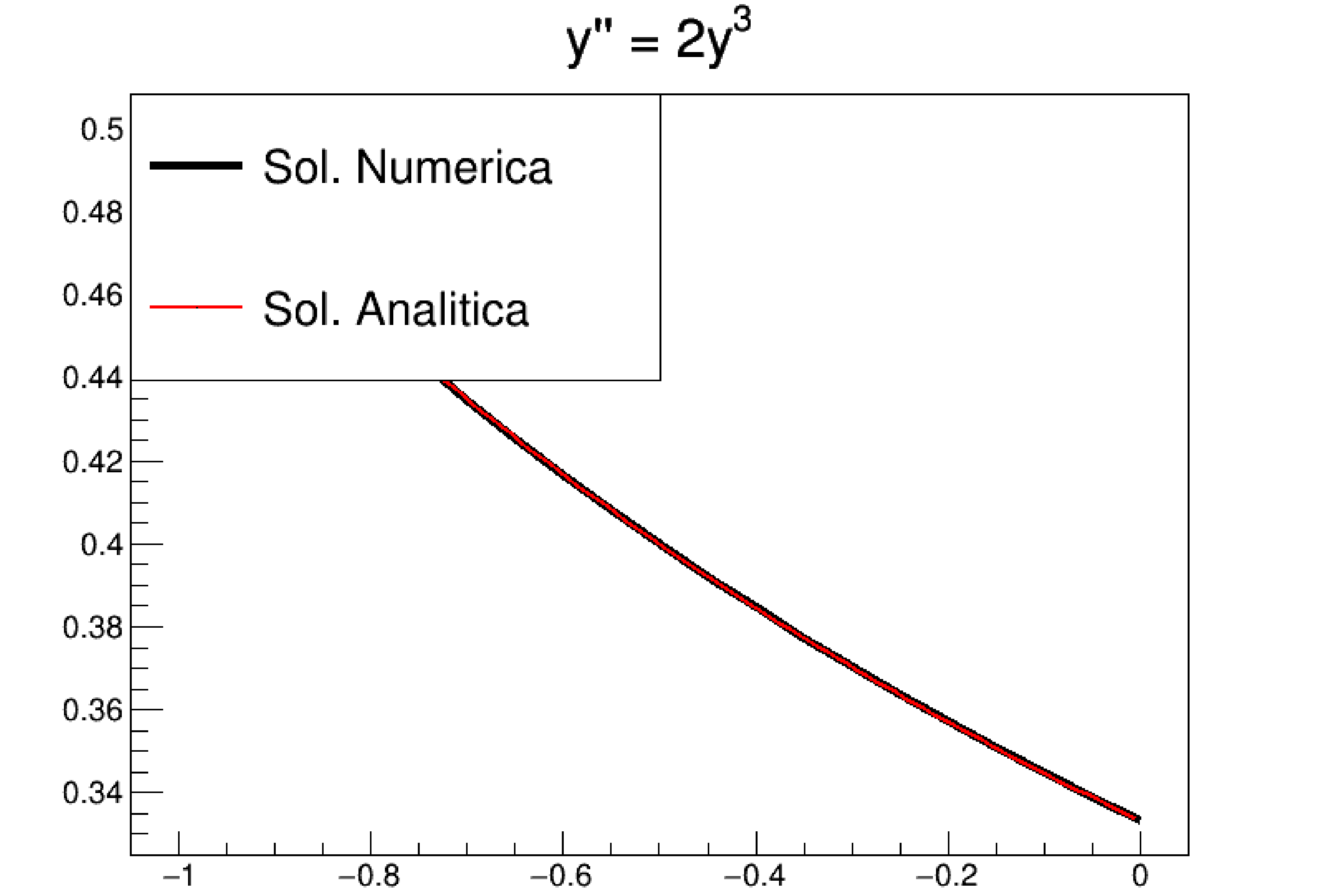
$$t_k = t_{k-1} - \frac{y(b,t_{k-1}) - \beta}{z(b,t_{k-1})}.$$

Ejemplos de aplicación



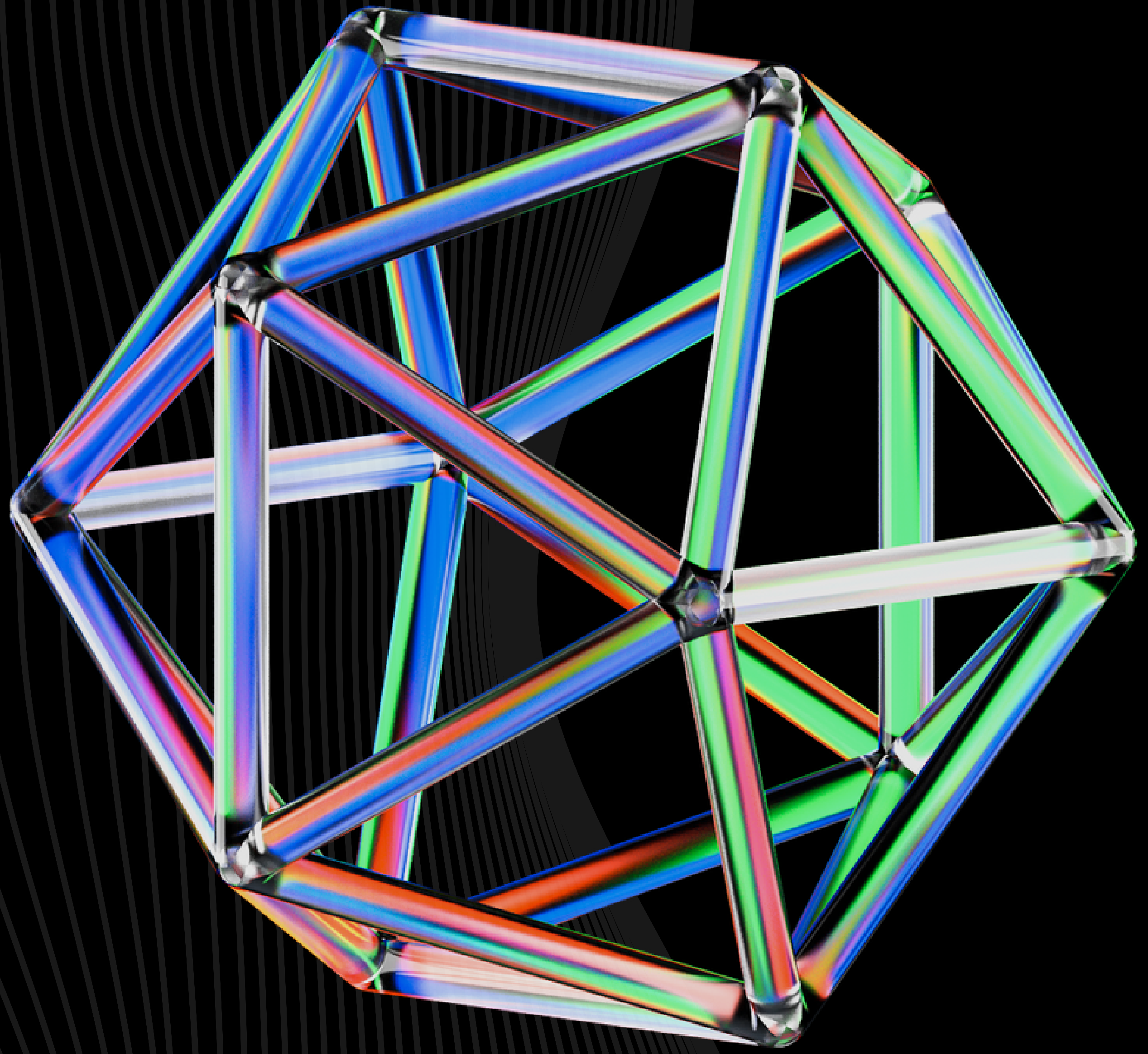
$$y'' = -(y')^2 - y + \ln x, \quad 1 \le x \le 2, \quad y(1) = 0, \quad y(2) = \ln 2.$$

$$y = \ln x.$$



$$y'' = 2y^3, \quad -1 \le x \le 0, \quad y(-1) = \frac{1}{2}, \quad y(0) = \frac{1}{3}.$$

$$y(x) = 1/(x+3).$$



¡GRACIAS!