Parcial 2: Señales y Sistemas 2024-1

1) Encuentre la expresion del espectro de fourier (forma exponencial y trigonometrica) para la señal $x(t) = |A\sin(2\pi F_0 t)|^2$

$T = \frac{1}{2F_0} - \left(-\frac{1}{2F_0}\right) = \frac{1}{F_0}$, $T_0 = \frac{1}{F_0}$

$t \in \left[-\frac{T_0}{2}, \frac{T_0}{2}\right]$

$x(t) = A^2 \operatorname{sen}^2(2\pi F_0 t)$

$x(t) = A^2\left[\frac{1}{2} - \frac{1}{2}\cos(2.2\pi F_0 t)\right]$

$x(t) = \frac{A^2}{2} - \frac{A^2}{2}\cos(2.2\pi F_0 t)$

$a_0 = \frac{A^2}{2}$

Cuenta con simetria par por lo que su espectro es Re

$\frac{A^2}{2} - \frac{A^2}{2}\cos(2.2\pi.F_0 t) = a_0 + \sum_{n=0}^{N} a_n \cos(n w_0 t)$

$a_0 = \frac{A^2}{2}$, $C_0 = \frac{A^2}{2}$

cuando $n=2$, $a_2 = -\frac{A^2}{2}$

Su espectro en forma trigonometrica

$x(t) = a_0 + \sum_{n=1}^{N} a_n \cos(n w_0 t)$

$x(t) = a_0 + a_2 \cos(2.2\pi F_0 t)$

$$a_n = \begin{cases} 0 & \forall\ n\ [0,2] \\ -\frac{A^2}{2} ; & n=2 \end{cases}$$

$x(t) = \frac{A^2}{2} - \frac{A^2}{2}\cos(2.2\pi F_0 t)$

Para la exponencial

$$C_0 = a_0 = \frac{A^2}{2} \quad , \quad C_n = \frac{a_n - jb_n}{2}$$

$$X(t) = \sum_{n=-N}^{N} C_n e^{jn\omega_0 t}$$

$$X(t) = C_{-2}\, e^{j2\cdot 2\pi f_0 t} + C_0 e^{0} + C_2 e^{j2\cdot 2\pi}$$

$$C_n = \begin{cases} -\frac{A^2}{4} \; ; & n=-2 \\ \frac{A^2}{2} \; ; & n=0 \\ -\frac{A^2}{4} \; ; & n=2 \\ 0 & \forall\, n\,[-2,0,2] \end{cases}$$

$$\rightarrow X(t) = \frac{A^2}{2}\left[\cos(2\cdot 2\pi f_0 t) - j\sin(2\cdot 2\pi f_0 t)\right] + \ldots + \frac{A^2}{2} + \left(-\frac{A^2}{4}\right)\left[\cos 2\cdot 2\pi f_0 t\right] + j\sin(2\cdot 2\pi f_0 t)$$

Para el error relativo

$$E_r[\%] \cdot \frac{\vec{P_e}}{\vec{P_x}}\, 100\,[\%] = \left(1 - \frac{\sum_{n=-N}^{N} |C_n|^2 P_n}{\vec{P_x}}\right) \times 100$$

$$P_x = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2\, dt = \frac{1}{T_0}\int \left|\frac{A^2}{2} - \frac{A^2}{2}\cos(2\cdot 2\pi f_0 t)\right|^2 dt$$

$$= \frac{1}{T_0}\left[\frac{A^2}{2}\int dt - \frac{A^4}{2}\int \cos(2\cdot 2\pi f_0 t)\, dt + \frac{A^4}{4}\int \cos^2(2\cdot 2\pi f_0 t)\, dt\right]$$

$$= \frac{1}{T_0}\left[\frac{A^4}{4} T_0 - \frac{A^4}{2}\left[\sin(\cancel{2\cdot 2\pi f_0 \tfrac{T_0}{2}})^{0} - \sin(\cancel{2\cdot 2\pi f_0 (-\tfrac{T_0}{2})})^{0}\right] + \frac{A^4}{4}\left[\frac{1}{2}\int dt + \frac{1}{2}\int \cos(4\cdot 2\pi f_0 t)\, dt\right]\right]$$

$$= \frac{1}{T_0}\left[\frac{A^4}{4} T_0 + \frac{A^4}{4}\left[\frac{T_0}{2}\right]\right] = \frac{1}{\cancel{T_0}}\left[\cancel{T_0}\left[\frac{A^4}{4} + \frac{A^4}{8}\right]\right]$$

$$\vec{P_x} = \frac{3A^4}{8} \quad , \quad \vec{P_n} = 1$$

$$E_r[\%] = 1 - \left(\frac{1}{P_x}\sum_{n=-N}^{N}|c_n|^2\right)$$

3)

$$y(t) = \left(1 + \frac{m(t)}{A_c}\right) \cdot c(t) \quad , \quad c(t) = A_c \sin(2\pi f_c t$$

La transformada de fourier a la señal moduldda

$$Y(\omega) = F\{y(t)\} = F\left\{\left(1 + \frac{m(t)}{A_c}\right)\right\} = F\{c(t)\} + \frac{1}{A_c} F\{m(t) \cdot c(t)\}$$

Usando la tabla de transformadas de fourier

$$C(\omega) = F\{c(t)\} = F\{A_c \sin(2\pi f_c t)\}$$

$$C(\omega) = A_c F\left\{\frac{e^{j2\pi f_c t} - e^{j2\pi f_c t}}{2j}\right\}$$

$$C(\omega) = \frac{A_c}{2j}\left[F\{e^{j2\pi f_c t}\} - F\{e^{-j2\pi f_c t}\}\right]$$

$$C(\omega) = \frac{A_c}{j}\pi\left[\delta(\omega - 2\pi f_c) - \delta(\omega + 2\pi f_c)\right]$$

Usando otra vez las tablas de transformadas de fourier

$$m(t)\,c(t) = m(t)\left[A_c \sin(2\pi f_c t)\right]$$

$$m(t)\,c(t) = m(t)\,A_c\left[\frac{e^{j2\pi f_c t} - e^{-j2\pi f_c t}}{2j}\right]$$

$$\rightarrow F\{\cdot\}$$

$$= F\left\{\frac{A_c}{2j} m(t)\, e^{j2\pi f_c t}\right\} - F\left\{\frac{A_c}{2j} m(t)\, e^{-j2\pi f_c t}\right\}$$

$$F\{f(t)\, e^{j\omega_0 t}\} = F(\omega - \omega_0)$$

$$\frac{A_c}{2j}\left[M(\omega - 2\pi f_c) - M(\omega + 2\pi f_c)\right]$$

Finalmente la transformada de fourier

$$y(\omega) = \frac{A_c}{j}\pi\left[\delta(\omega-2\pi f_c) - \delta(\omega+2\pi f_c)\right] + \frac{1}{2j}\left[M(\omega-2\pi f_c) - M(\omega-2\pi f_c)\right]$$