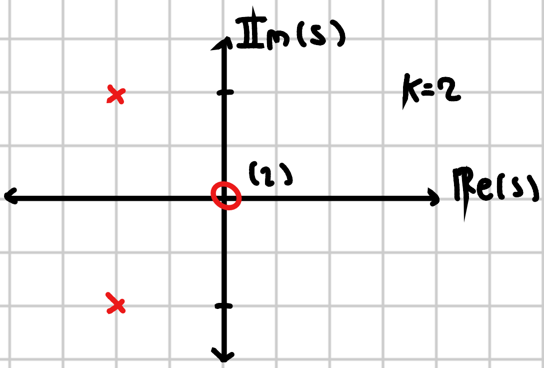
Taller 3 Laplace

1.1



x: polo
0: zero
n= p/2
$s = 1 + j$

$$X(s) = K \frac{s^2}{(s+1-j)(s+1+j)}$$

$$X(s) = K \frac{s^2}{(s+1)^2+1} = \frac{s^2}{s^2+2s+2} K$$

$$X(s) = 2\left(\frac{s^2}{s^2+2s+2}\right)$$

ROC causal $Re(s) > -1$
Anticausal $Re(s) < -1$

1.2 $\mathcal{L}\{\varepsilon(t)\ \text{sen}(w_0 t)\}$

$$= \int_{-\infty}^{\infty} x(t) e^{-st}\, dt$$ $\varepsilon(t)$ escalon por lo tanto es casual

$$= \int_{0}^{\infty} \text{sen}(w_0 t)\, e^{-st}\, dt = \int_0^\infty e^{-at} \text{sen}(bt)\, dt = \frac{b}{a^2+b^2} \quad Re(a) > 0$$

$$\mathcal{L}\{\varepsilon(t)\ \text{sen}(w_0 t)\} = \frac{w_0}{s^2 + w_0^2}$$

Polos $s = \pm j w_0$
ceros: NO Hay

Los ceros presentan simetria en el eje real, ya que se dan en pares conjugados

3.

$$L\{x(t-t_0)\} = e^{-st_0} X(s)$$

$$L\{x(t)\} = X(s) = \int_0^\infty x(t) e^{-st}\, dt$$

$$L\{x(t-t_0)\} = \int_0^\infty x(t-t_0)\, e^{-st}\, dt$$

$\tau = t - t_0$ $\quad t = \tau + t_0$ $\quad dt = d\tau$

$$= \int_0^{\infty} x(\tau) e^{-st(t_0+\tau)} \, \partial\tau$$

$$= e^{-st_0} \underbrace{\int_0^{\infty} x(\tau) e^{-st} \, \partial\tau}_{X(s)}$$

$$L\{x(t-t_0)\} = e^{-st_0} \, X(s)$$

ii) $L\{x(at)\} = \frac{1}{|a|} X\left(\frac{s}{a}\right)$

$$L\{x(at)\} = \int_0^{\infty} x(at) \, e^{-st} \, \partial t$$

$$\tau = at \qquad t = \frac{\tau}{a} \qquad \partial t = \frac{\partial\tau}{a}$$

$$\int_0^{\infty} x(\tau) e^{-s\tau/a} \, \frac{1}{a} \partial t$$

$$\frac{1}{|a|} \int_0^{\infty} x(\tau) \, e^{-s\tau/a} \, \partial t$$

$$\{x(at)\} = \frac{1}{|a|} X\left(\frac{s}{a}\right)$$

$$\mathcal{L}\left\{\frac{\partial x(t)}{\partial t}\right\} = sX(s)$$

$$\frac{\partial x(t)}{\partial t} = \int_0^{\infty} \frac{\partial x(t)}{\partial t} \, e^{-st} \, \partial t$$

$$\int_0^{\infty} e^{-st} \, \partial x(t) = e^{-st} x(t) \Big|_0^{\infty} = \int_0^{\infty} x(t) \, \partial e^{-st} = sX(s)$$

iv) $\mathcal{L}\{x(t) * y(t)\} = X(s) \, Y(s)$

$$x(t) * y(t) = \int_0^t x(\tau) \, y(t-\tau) \, \partial t$$

$$= \int_0^{\infty} \int_0^t x(\tau) \, y(t-\tau) \, \partial\tau \; e^{-st} \, \partial t$$

Reorganizando

$$= \int_0^{\infty} x(\tau) \int_0^{\infty} y(t-\tau) e^{-st} \, dt \, d\tau$$

$u = t - \tau \qquad t = \tau + u$

$$= \int_0^{\infty} x(\tau) \int_0^{\infty} y(u) e^{-s(u+\tau)} \, du \, d\tau$$

$$= \int_0^{\infty} x(\tau) e^{-s\tau} \int_0^{\infty} y(u) e^{-su} \, du \, dt$$

$= X(s) \; Y(s)$

7.4

i) $e^{-2t} u(t) + e^{-3t} u(t)$

$\mathcal{L}\{e^{-st}\} = \frac{1}{s+a}$ por lo tanto

$\mathcal{L}\{e^{-2t}\} = \frac{1}{s+2}$, $\mathcal{L}\{e^{-3t}\} = \frac{1}{s+3}$

ROC = $\mathrm{Re}(s) > -2$ ROC = $\mathrm{Re}(s) > -3$

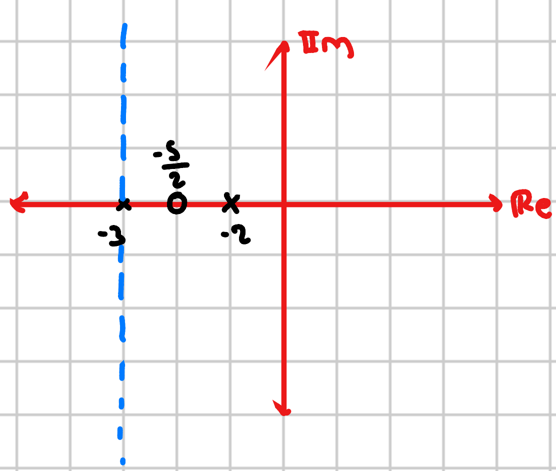
$$\mathcal{L}\{e^{-2t} u(t) + e^{-3t} u(t)\} = \frac{2s+5}{(s+2)(s+3)}$$

Ceros $= 2s+5=0$

$s = -5/2$

Polos: $s+2 \rightarrow s=-2$

$s+3 \rightarrow s=-3$



ii) $e^{2t}u(t) + e^{-3t}u(t)$

$\mathcal{L}\{e^{2t}u(t)\} = \frac{1}{s-2}$ ROC Re>2

Para $u(-t)$ es 1 cuando $t \leq 0$ y 0 cuando $t \geq 0$

$$\mathcal{L}\{e^{-3t}u(-t)\} = \int_{-\infty}^{0} e^{-3t}e^{-st}\,dt = \int_{-\infty}^{0} e^{-(s+3)t}\,dt = \frac{1}{s+3} \quad \text{ROC: Re} < -3$$

No hay interseccion comun, lo que nos indica que no hay ROC

$$\mathcal{L}\{e^{2t}u(t) + e^{-3t}u(t)\} = \frac{2s+1}{(s-2)(s+3)}$$

Polos = $s=2$ $s=-3$
Ceros = $s = -\frac{1}{2}$

iii) $e^{-a|t|}$

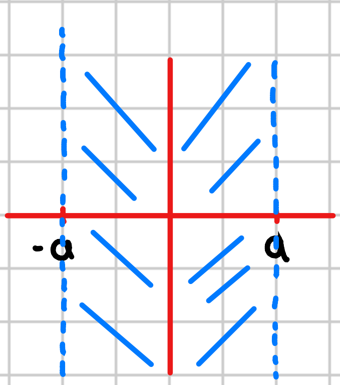
$$e^{-a|t|} = \begin{cases} e^{-a|t|} & \text{si } t \geq 0 \\ e^{a(t)} & \text{si } t < 0 \end{cases}$$

$$e^{-a|t|} = e^{-at}u(t) + e^{at}u(-t)$$

$\mathcal{L}\{e^{-at}u(t)\} = \frac{1}{s+a}$ ROC: Re(s) > -a

$\mathcal{L}\{e^{at}u(-t)\} = \frac{1}{-s+a}$ ROC: Re(s) < a

$$\mathcal{L}\{e^{-a|t|}\} = \frac{1}{s+a} + \frac{1}{-s+a} = \frac{-2a}{(s+a)(s-a)}$$



iv) $e^{-2t}[u(t) - u(t-5)]$

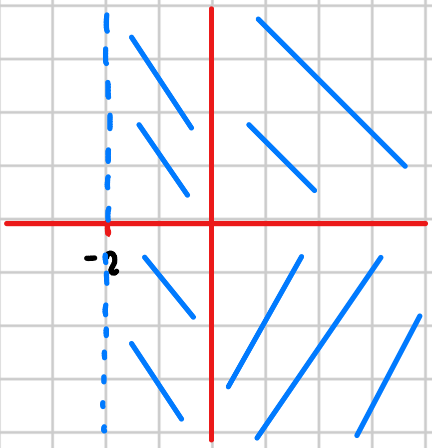
$\mathcal{L}\{e^{-2t}u(t)\} = \frac{1}{s+2}$ ROC : Re(s) > -2

Usar propiedad de desplazamiento en el tiempo

$\mathcal{L}\{e^{-3t} u(t-5)\} = e^{-5s}\,\mathcal{L}\{e^{-2t} u(t)\}$

$e^{-5s} \dfrac{1}{s+2}$

$\mathcal{L}\{e^{-2t}(u(t) - u(t-5))\} = \dfrac{1}{s+2}(1 - e^{-5s})$ Polos: $s=-2$



1.5

i) $X(s) = \dfrac{2s^2+14s+124}{s^3+8s^2+46s+68}$ $\Re(s) > -2$

Fracciones Parciales

$= \dfrac{A}{s+2} + \dfrac{Bs+C}{s^2+6s+34}$

$2s^2+14s+124 = A(s^2+6s+34)+(Bs+C)(s+2)$

$2s^2+14s+124 = s^2(A+B)+s(6A+2B+C)+(2C+34A)$

$A+B=2$ $A=4$

$6A+2B+C=14$ $B=-2$

$2C+34A=124$ $C=-6$

$\rightarrow X(s) = \dfrac{4}{s+2} - \dfrac{2s+6}{s^2+6s+34}$

$\mathcal{L}^{-1}[4/s+2] = 4e^{-2t}$

$s^2+6s+34 = (s+3)^2+25$

$(2s+6) = 2(s+3$

Utilizando transformada

$e^{at}\cos(bt) = \dfrac{s-a}{(s-a)^2+b^2}$

$$\frac{2(s+3)}{(s+3)^2+25} = 2e^{-3t}\cos(5t)$$

$$\mathcal{L}^{-1}\{X(s)\} = 4e^{-2t} - 2e^{3t}\cos(5t)$$

ii) $$\frac{1}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$1 = A(s+2)^2 + B(s+2)(s+1) + C(s+1)$$

$$1 = s^2(A+B) + s(4A+3B+C) + (4A+2B+C)$$

$A+B=0$ $\qquad$ $A=1$

$4A+3B+C=0$ $\qquad$ $B=-1$

$4A+2B+C=1$ $\qquad$ $C=-1$

$$X(s) = \frac{1}{s+1} - \frac{1}{s+2} - \frac{1}{(s+2)^2}$$

$$x(t) = e^{-t} - e^{-2t} - te^{2t}$$

1.6

i) $$a_0 y(t) + a_1 \frac{\partial y(t)}{\partial t} + a_2 \frac{\partial^2 y(t)}{\partial t^2} = x(t)$$

$$x(t) = e^{st}$$

Resolviendo la EDO

$$a_0 e^{st} + a_1 s e^{st} + a_2 s^2 e^{st} = e^{st}$$

$$e^{st}(a_0 + a_1 s + a_2 s^2) = e^{st}$$

$$a_0 + a_1 s + a_2 s^2 = 1$$

$$y(t) = \frac{e^{st}}{\lambda}$$

ii) EDO de orden n

$$a_n \frac{\partial^n y(t)}{\partial t^n} + a_{n-1} \frac{\partial^{n-1} y(t)}{\partial t} + \dots + a_1 \frac{\partial y(t)}{\partial t} + a_0 y(t) = x(t)$$

$(a_i \in \mathbb{R}e\ \forall i)$

Dado $x(t) = e^{st}$, $y(t) = \lambda e^{st}$

$$a_n s^n e^{st} + a_{n-1} s^{n-1} e^{st} + \dots + a_1 s e^{st} + a_0 e^{st} = e^{st}$$

$$a_n s^n e^{st} + a_{n-1} s^{n-1} e^{st} + \ldots + a_1 s e^{st} + a_0 e^{st} = 1$$

$$\lambda = \frac{1}{a_n s^n e^{st} + a_{n-1} s^{n-1} e^{st} + \ldots + a_1 s e^{st} + a_0 e^{st}}$$