Teórico:

Función de costo

$$\chi^2(a_0, a_1) = \sum_{i=1}^{n} [y_i - (a_0 + a_1 x_i)]^2$$

Minimizar χ^2: Parcial respecto a a_0 ^ a_1

$$\frac{\partial \chi^2}{\partial a_0} = -2 \sum_{i=1}^{n} [y_i - (a_0 + a_1 x_i)] = \emptyset$$

$$\frac{\partial \chi^2}{\partial a_1} = -2 \sum_{i=1}^{n} [y_i - (a_0 + a_1 x_i)] x_i = \emptyset$$

$$\sum_{i=1}^{n} [y_i - (a_0 + a_1 x_i)] = \emptyset$$

$$= \sum_{i=1}^{n} y_i - a_0 n - a_1 \sum_{i=1}^{n} x_i = \emptyset$$

$$\sum_{i=1}^{n} y_i x_i - a_0 \sum_{i=1}^{n} x_i - a_1 \sum_{i=1}^{n} x_i^2 = \emptyset$$

Simplificamos las Sumatorias para Comodidad:

$$S_x = \sum_{i=1}^{n} x_i \quad , \quad S_y = \sum_{i=1}^{n} y_i \ , \ S_{xx} = \sum_{i=1}^{n} x_i^2 \ , \ S_{xy} = \sum_{i=1}^{n} x_i y_i$$

Queda por tanto:

1. $a_0 n + a_1 S_x = S_y$
2. $a_0 S_x + a_1 S_{xx} = S_{xy}$

Para resolver el Sistema:

$$a_0 S_x n + a_1 S_x^2 = S_y S_x \quad (1.) \cdot S_x$$

$$[a_0 S_x n + a_1 S_x^2] - [a_0 S_x n + a_1 S_{xx} n] = S_y S_x - S_{xy} n$$

$$[1 - 2(n)]$$

$$a_1 (S_x^2 - S_{xx} n) = S_y S_x - S_{xy} n$$

$$a_1 = \frac{S_y S_x - S_{xy} n}{S_x^2 - S_{xx} n}$$

Calculando a_0

→ Desde la ecuación 1.

$$a_0 = \frac{S_y - a_1 S_x}{n} = \bar{y} - a_1 \bar{x}$$

La otra demostración da:

$$a_1 = \frac{\sum_{i=1}^{n} x_i y_i - \frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n}}{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}$$

Ajuste Cuadrático: Para nuestra función queda

$$\chi^2(a_0, a_1, a_2) = \sum_{i=1}^{n} \left[y_i - (a_0 + a_1 x_i + a_2 x_i^2)\right]^2$$

→ Para minimizarla: Parciales respecto a a_0, a_1 y a_2

$$\frac{\partial \chi^2}{\partial a_0} = -2 \sum_{i=1}^{n} \left[y_i - (a_0 + a_1 x_i + a_2 x_i^2)\right] = \emptyset$$

$$\frac{\partial \chi^2}{\partial a_1} = -2 \sum_{i=1}^{n} \left[y_i - (a_0 + a_1 x_i + a_2 x_i^2)\right] x_i = \emptyset$$

$$\frac{\partial \chi^2}{\partial a_2} = -2 \sum_{i=1}^{n} \left[y_i - (a_0 + a_1 x_i + a_2 x_i^2)\right] x_i^2 = \emptyset$$

Distribuimos las sumatorias de las parciales:

$$\sum_{i=1}^{n} y_i - a_0 n - a_1 \sum_{i=1}^{n} x_i - a_2 \sum_{i=1}^{n} x_i^2 = \emptyset \qquad \left(\frac{\partial \chi^2}{\partial a_0}\right)$$

$$\sum_{i=1}^{n} y_i x_i - a_0 \sum_{i=1}^{n} x_i - a_1 \sum_{i=1}^{n} x_i^2 - a_2 \sum_{i=1}^{n} x_i^3 = \emptyset \qquad \left(\frac{\partial \chi^2}{\partial a_1}\right)$$

$$\sum_{i=1}^{n} y_i x_i^2 - a_0 \sum_{i=1}^{n} x_i^2 - a_1 \sum_{i=1}^{n} x_i^3 - a_2 \sum_{i=1}^{n} x_i^4 = \emptyset \qquad \left(\frac{\partial \chi^2}{\partial a_2}\right)$$

Ahora Simplificando las ecuaciones:

1. $\sum_{i=1}^{n} y_i - a_0 n - a_1 \sum_{i=1}^{n} x_i - a_2 \sum_{i=1}^{n} x_i^2 = \emptyset$

2. $\sum_{i=1}^{n} y_i x_i - a_0 \sum_{i=1}^{n} x_i - a_1 \sum_{i=1}^{n} x_i^2 - a_2 \sum_{i=1}^{n} x_i^3 = \emptyset$

3. $\sum_{i=1}^{n} y_i x_i^2 - a_0 \sum_{i=1}^{n} x_i^2 - a_1 \sum_{i=1}^{n} x_i^3 - a_2 \sum_{i=1}^{n} x_i^4 = \emptyset$

Sumando a la notación que habíamos hecho anteriormente:

$S_x = \sum_{i=1}^{n} x_i$; $S_y = \sum_{i=1}^{n} y_i$; $S_{xx} = \sum_{i=1}^{n} x_i^2$; $S_{xy} = \sum_{i=1}^{n} x_i y_i$

$S_{x^3} = \sum_{i=1}^{n} x_i^3$; $S_{x^4} = \sum_{i=1}^{n} x_i^4$; $S_{x^2y} = \sum_{i=1}^{n} x_i^2 y_i$

→ Las ecuaciones normales forman un sistema lineal donde

x_i^k Con $0 \leq k \leq m$ se multiplica con un polinomio de grado m

Los coeficientes de los parámetros a_j son Sumas de x_i^{j+k}.

Las ecuaciones son simétricas y la matriz de coeficientes es una matriz de Vandermonde.