Juan Mejia Homework 2

1. (70 points) Sooners Inc., a leading company in the house decorating and maintenance industry, is expanding its product line by launching a new branch dedicated to paint production. The company has decided to begin by manufacturing two paint colors: red and blue. Each gallon of red paint will be sold for \$10, while each gallon of blue paint will be sold for \$12.

   The production process for these paints requires specific resources. Each gallon of red paint requires 1 kilogram of pigments and 2 gallons of water. Similarly, each gallon of blue paint requires 2 kilograms of pigments and 2 gallons of water. However, the company faces resource constraints, having only 20 gallons of water and 25 kilograms of pigments available.

   Additionally, due to a partnership agreement with a local store, Sooners Inc. has committed to producing at least 2 gallons of each paint color.

   You have been hired to develop a production plan that will maximize the company's profit. To solve this, you first decide to formulate this problem as an LP model. In particular:

   (a) (10 points) Define the decision variables for this problem. What is its gradient?

   (b) (10 points) Plot the gradient and the feasible region (clearly indicating all the constraints and "shading" the feasible region). Solve this problem using the graphical method, indicating the values of all the variables and the objective function associated with the optimal solution.

   (c) (5 points) Explain the concept of isoclines. How do isoclines relate to the solution of the problem?

   (d) (10 points) Solve the problem using Excel Solver and indicate the values of all the variables and the objective function associated with the optimal solution. Compare your results with part b. Include a snapshot of your Excel model (Excel cells and the Solver window).

   (e) (10 points) Solve the problem using Gurobi/Python. Compare your results with parts b and c. Include a snapshot of your Gurobi/Python code and obtained results.

   (f) (25 points) After the successful launch of the new product line, Sooners Inc. has decided to invest more in it. Consequently, they have decided to expand their product lineup by adding more paint colors. For each paint color $p \in P$, they expect to charge r_p dollars per gallon, and each gallon of paint requires w_p gallons of water and i_p kilograms of pigments. The new resource limits are 1000 gallons of water and 600 kilograms of pigments. They have entered into new agreements with local stores, which require them to produce at least β_p gallons of each paint color p. Additionally, because the original paint colors have proven to be stellar products, a new rule has been added: they must produce at least 100 gallons more of the original colors than of the new ones. Formulate the updated mathematical model to maximize Sooners Inc.'s profit, considering the new constraints and expanded product line.

a) Decision variables:

Assuming a daily production for the information given:

x_1 : gallons of red paint manufactured daily

x_2 : gallons of blue paint manufactured daily

The gradient is defined as the vector of partial derivatives of the function. In this case, it's the function z, which we're trying to maximize

$$z = 10x_1 + 12x_2$$

The gradient is $\nabla z = [10, 12]$

b)

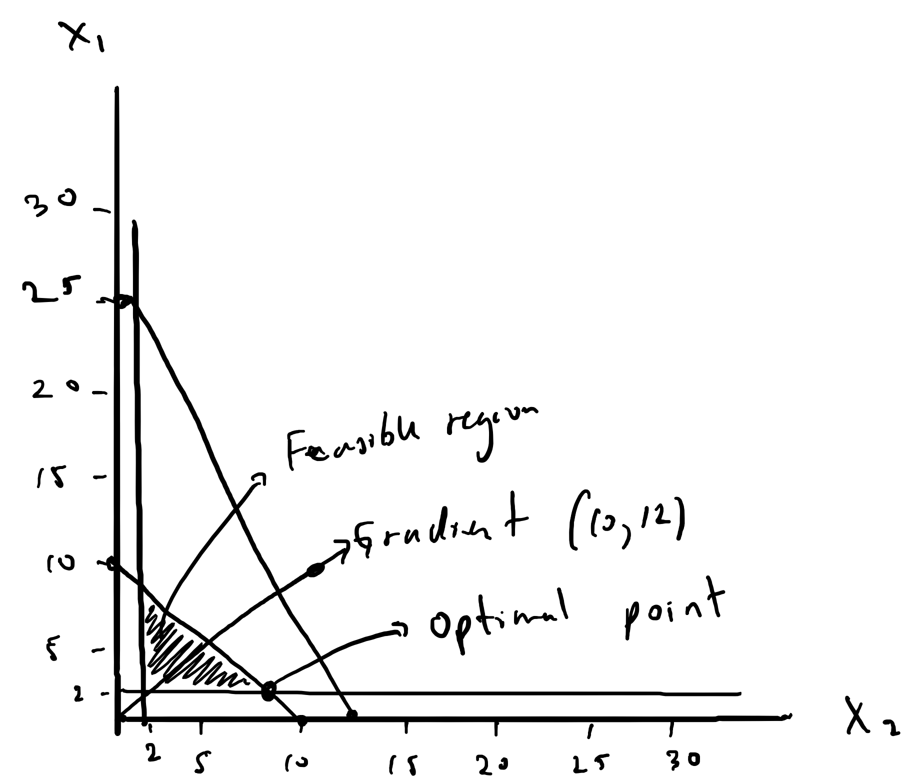
$$\text{Max} \quad z = 10x_1 + 12x_2$$

s.t

$$2x_1 + 2x_2 \leq 20 \quad (1)$$

$$x_1 + 2x_2 \leq 25 \quad (2)$$

$$x_1, x_2 \geq 2$$



The optimal point is guaranteed to be in a cornerpoint of the feasible region:

Intersection of $x_1 = 2$ and $2x_1 + 2x_2 = 20$

$2x_2 = 20 - 2(2) \Rightarrow x_2 = 8$ $\quad P_1(2, 8)$

Intersection of $x_2 = 2$ and $2x_1 + 2x_2 = 20$

$P_2(8, 2)$

Let's evaluate these two points in the objective function:

$z = 10(2) + 12(8) = 116$

$z = 10(8) + 12(2) = 104$

Hence, the optimal solution is $x_1 = 2$, $x_2 = 8$

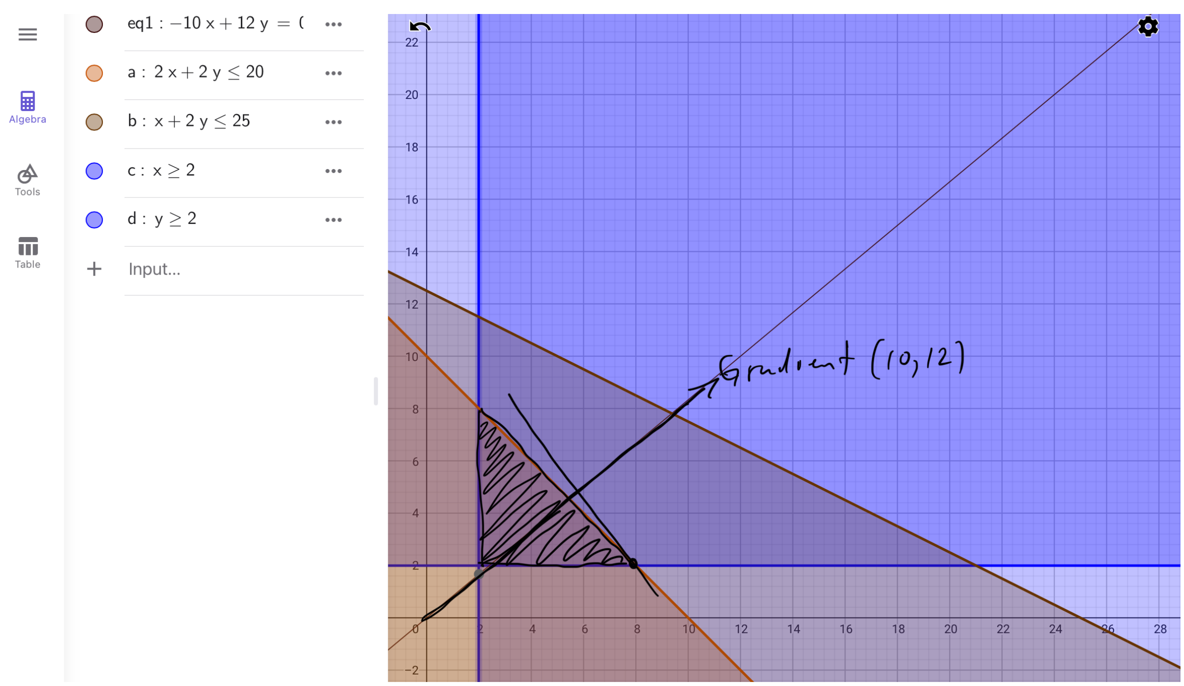


Figure 1. Model 1 graphical solution using geogebra.

c) Isoclines are lines where the objective function has the same value. The optimal solution occurs where the highest (in maximizing) or lowest (in minimizing) isocline touches the feasible region.

d)

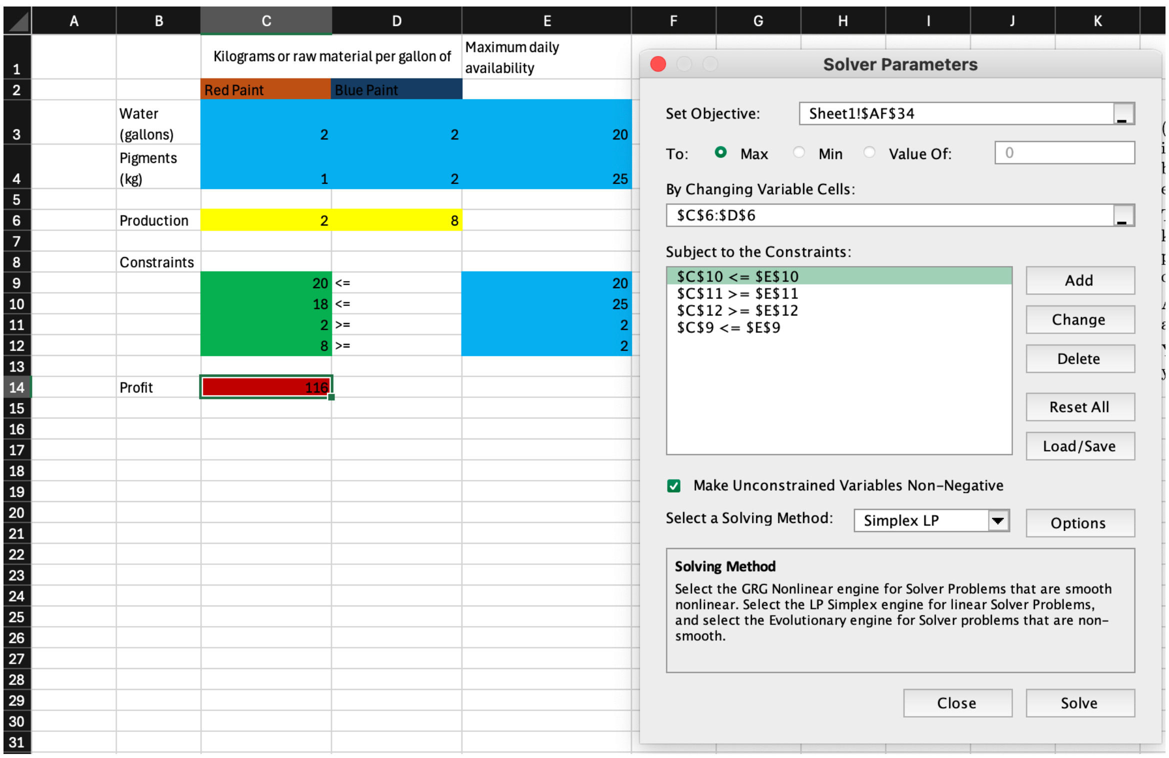


Figure 2. Model and solution for problem 1 using Excel solver

e)

```
import gurobipy as gp
from gurobipy import *
model = gp.Model("Sooner Inc Paints")
x1 = model.addVar(vtype= GRB.CONTINUOUS, lb= 0, ub= GRB.INFINITY, name="x1")
x2 = model.addVar(vtype= GRB.CONTINUOUS, lb= 0, ub= GRB.INFINITY, name="x2")

#Define the objective function
z = 10*x1 + 12*x2
model.setObjective(z)
model.modelSense = GRB.MAXIMIZE
model.update()

#Add Constraints
model.addConstr(2*x1 + 2*x2 <= 20)
model.addConstr(x1 + 2*x2 <= 25)
model.addConstr(x1 >= 2)
model.addConstr(x2 >= 2)
model.update()

model.optimize()

#Print output

if model.status == GRB.OPTIMAL:
    print("Optimal value:", model.objVal, "USD")
    print("Production quantity for red paint: ", x1.X)
    print("Production quantity for blue paint: ", x2.X)
```

Figure 3. Model 1 in gurobi

```
Gurobi Optimizer version 11.0.3 build v11.0.3rc0 (mac64[arm] - Darwin 23.6.0 23G93)

CPU model: Apple M2
Thread count: 8 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 4 rows, 2 columns and 6 nonzeros
Model fingerprint: 0x74497dc8
Coefficient statistics:
  Matrix range     [1e+00, 2e+00]
  Objective range  [1e+01, 1e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [2e+00, 2e+01]
Presolve removed 4 rows and 2 columns
Presolve time: 0.01s
Presolve: All rows and columns removed
Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    1.1600000e+02   0.000000e+00   0.000000e+00      0s

Solved in 0 iterations and 0.01 seconds (0.00 work units)
Optimal objective  1.160000000e+02
Optimal value: 116.0 USD
Production quantity for red paint:  2.0
Production quantity for blue paint:  8.0
```

Figure 4. Solution for model 1 in gurobi

F)

$$\text{Max} \quad \sum_{p \in P} x_p r_p$$

s.t

$$\sum_{p \in P} w_p x_p \leq 1000 \quad \forall p \in P \quad (1)$$

$$\sum_{p \in P} i_p x_p \leq 600 \quad \forall p \in P \quad (2)$$

$$x_p \leq \beta_p \quad \forall p \in P \quad (3)$$

$$x_1 + x_2 \geq \sum_{p \in P} x_p + 100 \quad (4)$$

where:

x_p = gallons produced for color p $\forall p \in P$

x_1, x_2 = Original colors (Red and Blue respectively)

2. (35 points) As a financial advisor, your role is to help clients achieve their savings goals while minimizing the cost of their total investment. Today, a couple has approached you to create an investment portfolio consisting of stocks from Company A and Company B. Their goal is to make at least \$20 in profit by the end of the year, in addition to recovering their initial investment.You know that for each dollar invested in Stock A, they will receive \$1.50 at the end of the year, while each dollar invested in Stock B will yield \$1.20. Due to regulatory constraints, they are allowed to invest at most 1.5 times more in Stock A than in Stock B. Additionally, the couple prefers Stock B for its lower risk, despite its lower return, and has requested that at least \$5 be invested in Stock B.

The total budget available for their investment is \$100. Your task is to determine the minimum-cost portfolio that meets all these requirements.

(a) (10 points) Define the decision variables for this problem. What is its gradient?

(b) (5 points) Plot the gradient and the feasible region (clearly indicating all the constraints and "shading" the feasible region). Solve this problem using the graphical method, indicating the values of all the variables and the objective function associated with the optimal solution.

(c) (10 points) Solve the problem using Excel Solver and indicate the values of all the variables and the objective function associated with the optimal solution. Compare your results with part c. Include a snapshot of your Excel model (Excel cells and the Solver window).

(d) (10 points) Solve the problem using Gurobi/Python. Compare your results with parts b and c. Include a snapshot of your Gurobi/Python code and obtained results.

a)

x_1 = dollars invested in stock A

x_2 dollars invested in stock B

$$\min \quad x_1 + x_2$$

s.t

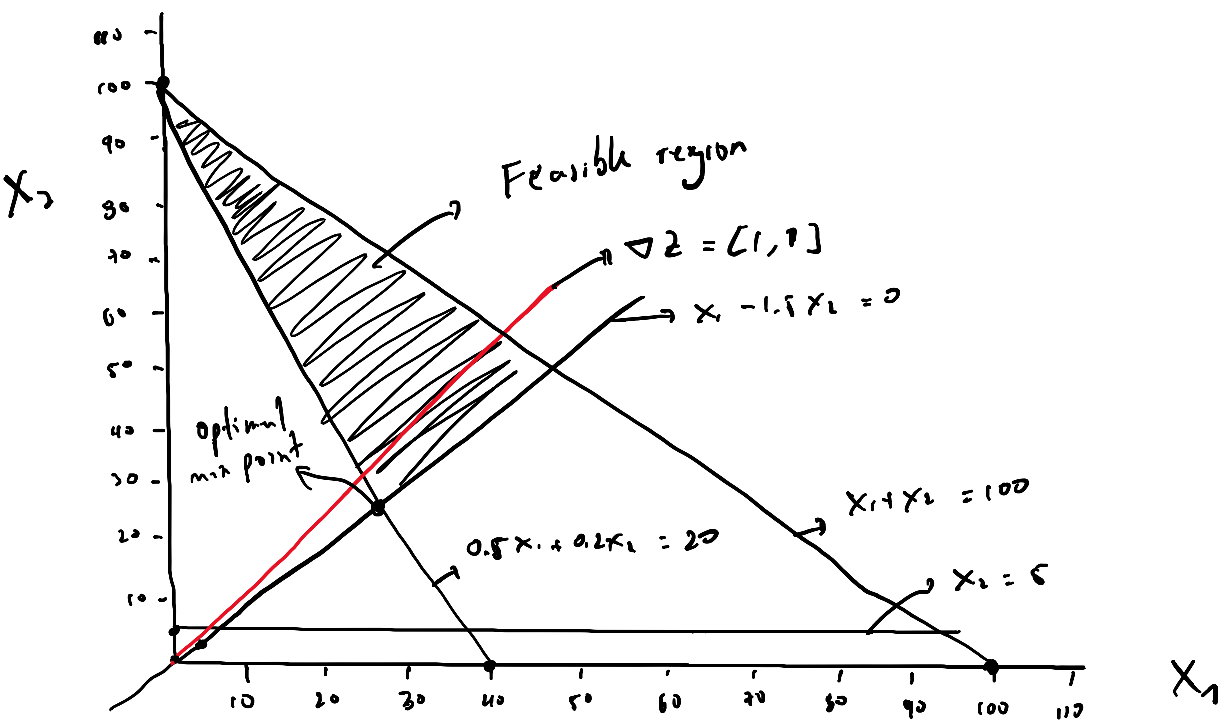
$$1.5x_1 + 1.2x_2 \geq 20 + (x_1 + x_2)$$
$$x_1 + x_2 \leq 100$$
$$x_1 \leq 1.5x_2$$
$$x_2 \geq 5$$
$$x_1, x_2 \geq 0$$

Hence, the gradient $\nabla z = [1, 1]$

$0.5x_1 + 0.2x_2 = 20 \rightarrow p_1(0,100)$, $p_2(40,0)$

$x_1 + x_2 = 100 \rightarrow p_1(100,0)$, $p_2(0,100)$

$x_1 - 1.5x_2 = 0 \rightarrow p_1(0,0)$, $p_2(3,2)$



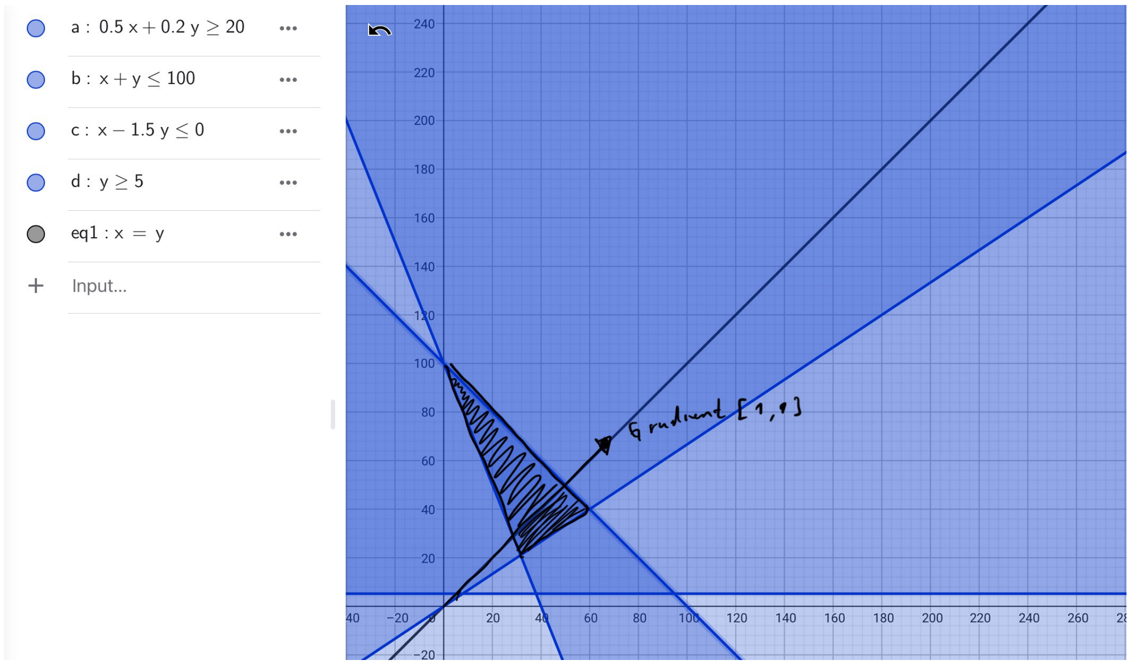


Figure 5. Model 2 graphical solution using geogebra.

c)

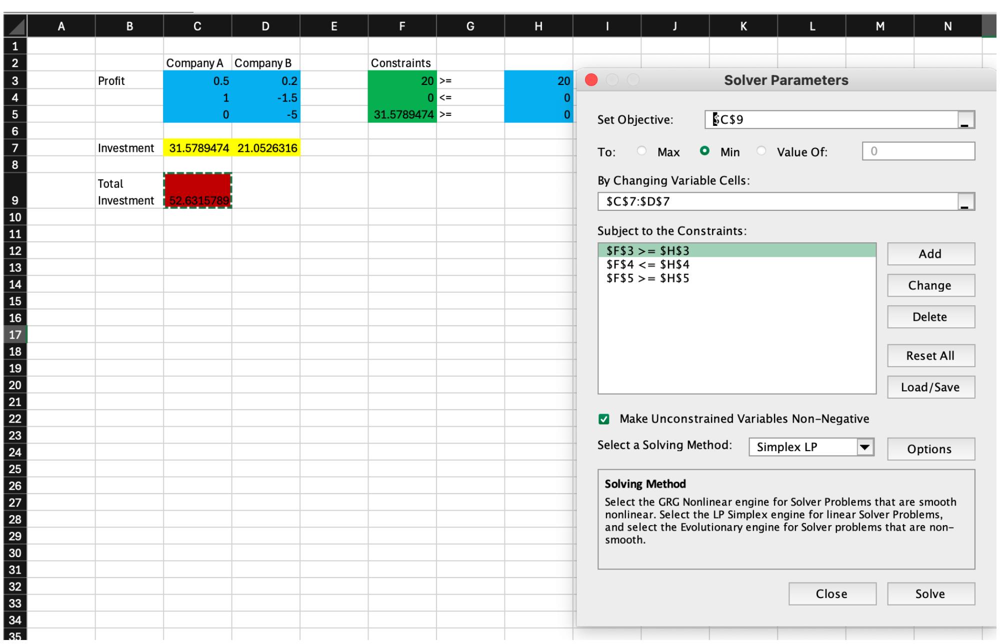


Figure 6. Model and solution for problem 2 using Excel solver

d)

```
import gurobipy as gp
from gurobipy import *
model = gp.Model("Stock Investment")
x1 = model.addVar(vtype= GRB.CONTINUOUS, lb= 0, ub= GRB.INFINITY, name="x1")
x2 = model.addVar(vtype= GRB.CONTINUOUS, lb= 0, ub= GRB.INFINITY, name="x2")

#Define the objective function
z = x1 + x2
model.setObjective(z)
model.modelSense = GRB.MINIMIZE
model.update()

#Add Constraints
model.addConstr(0.5*x1 + 0.2*x2 >= 20)
model.addConstr(x1 + x2 <= 100)
model.addConstr(x1 -1.5*x2 <= 2)
model.addConstr(x2 >= 5)
model.addConstr(x1 >= 0)
model.update()

model.optimize()

#Print output

if model.status == GRB.OPTIMAL:
    print("Optimal value:", model.objVal, "USD")
    print("Dollars invested in stock A: ", x1.X)
    print("Dollars invested in stock B: ", x2.X)
```

Figure 7. Model 2 in gurobi

```
Set parameter Username
Academic license - for non-commercial use only - expires 2025-08-19
Gurobi Optimizer version 11.0.3 build v11.0.3rc0 (mac64[arm] - Darwin 23.6.0 23G93)

CPU model: Apple M2
Thread count: 8 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 5 rows, 2 columns and 8 nonzeros
Model fingerprint: 0xf52ea53e
Coefficient statistics:
  Matrix range     [2e-01, 2e+00]
  Objective range  [1e+00, 1e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [2e+00, 1e+02]
Presolve removed 2 rows and 0 columns
Presolve time: 0.00s
Presolved: 3 rows, 2 columns, 6 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.3000000e+01   3.562500e+00   0.000000e+00      0s
       1    5.2000000e+01   0.000000e+00   0.000000e+00      0s

Solved in 1 iterations and 0.01 seconds (0.00 work units)
Optimal objective  5.200000000e+01
Optimal value: 52.0 USD
Production quantity for red paint:  32.0
Production quantity for blue paint:  20.0
```

Figure 8. Solution for model 1 in gurobi