Homework 3

1.

(a) (15 points) Based on the solution of Assignment 2, Problem 1:
Let x_R be the amount red paint gallons produced, and x_B be the amount blue paint gallons produced
.

$$\text{Maximize} \quad 10x_R + 12x_B$$

st:

$$x_R + 2x_B \leq 25$$
$$2x_R + 2x_B \leq 20$$
$$x_R \geq 2$$
$$x_B \geq 2$$

$$x_R \geq 0, \quad x_B \geq 0$$

Write this problem in its standard form.

$$\text{Max } z = 10x_R + 12x_B$$

$$\text{s.t.: } x_R + 2x_B + S_1 = 25$$
$$2x_R + 2x_B + S_2 = 20$$
$$x_R - S_3 = 2$$
$$x_B - S_4 = 2$$

$$x_R, x_B, S_1, S_2, S_3, S_4 \geq 0$$

(b) (20 points) Assume that they decide to produce 2 gallons of each paint color. Is this solution optimal? why or why not? if this solution is not optimal, find the basic solution associated with these variables' values and use it (as an initial feasible solution) in the Simplex Algorithm to find the optimal solution to the problem. Use Excel and indicate the values of all the variables and the objective function associated with the initial basic solution and the optimal solution in case the initial basic solution is not optimal. Include a snapshot of your procedure in Excel.

The (2,2) solution is not optimal, as seen in the graphical method (figure1), the optimal solution is not (2,2). It can be seen as well in the simplex tableau formulation (figure 2) because we are trying to maximize the objective function and we have negative values in the Z row, this means we have not reached the optimal point. In fact, in the last iteration of the simplex tableau we can observe that the optimal values are 2 for red paint and 8 gallons for blue paint.

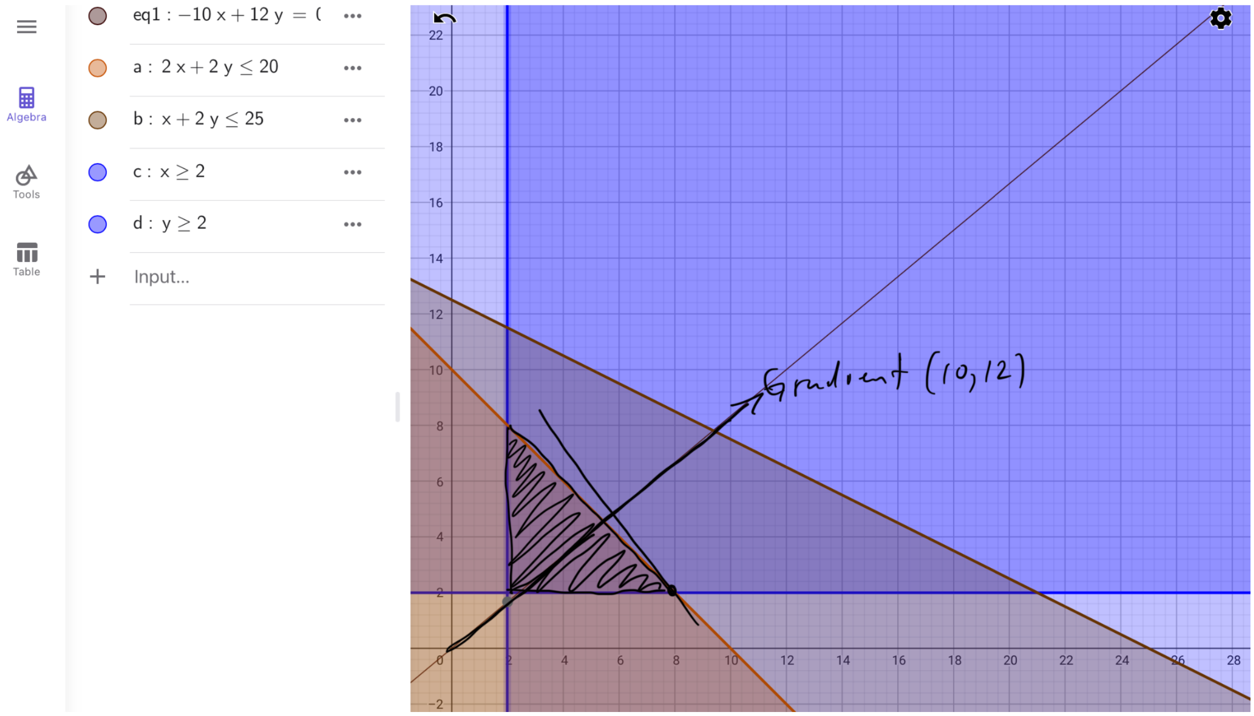

Figure 1. Model 1 graphical solution using geogebra

xr=2	xb=2							
Basic	z	xr	xb	s1	s2	s3	s4	Solution
z	1	-10	-12	0	0	0	0	0
s1	0	1	2	1	0	0	0	25
s2	0	2	2	0	1	0	0	20
xr	0	1	0	0	0	-1	0	2
xb	0	0	1	0	0	0	-1	2

make xr identity

Basic	z	xr	xb	s1	s2	s3	s4	Solution
z	1	0	-12	0	0	-10	0	20
s1	0	0	2	1	0	1	0	23
s2	0	0	2	0	1	2	0	16
xr	0	1	0	0	0	-1	0	2
xb	0	0	1	0	0	0	-1	2

make xb identiy

Basic	z	xr	xb	s1	s2	s3	s4	Solution	Min Ratio
z	1	0	0	0	0	-10	-12	44	
s1	0	0	0	1	0	1	2	19	9.5
s2	0	0	0	0	1	2	2	12	6
xr	0	1	0	0	0	-1	0	2	#DIV/0!
xb	0	0	1	0	0	0	-1	2	-2

make s4 basic

Basic	z	xr	xb	s1	s2	s3	s4	Solution
z	1	0	0	0	6	2	0	116
s1	0	0	0	1	-1	-1	0	7
s4	0	0	0	0	0.5	1	1	6
xr	0	1	0	0	0	-1	0	2
xb	0	0	1	0	0.5	1	0	8

xr	2
xb	8

Figure 2. Model 1 simplex formulation and solution

In figure 2 we see the optimal values for xr and xb in blue, the value for the objective function (z) is also in blue.

2.

(a) (9 points) Based on the solution of Assignment 2, Problem 2:
Let x_A be the amount of dollars invested in Stock A, and x_B be the amount of dollars invested in Stock B.

$$\text{Minimize} \quad x_A + x_B$$

st:

$$0.5x_A + 0.2x_B \geq 20$$

$$x_A \leq 1.5x_B$$

$$x_B \geq 5$$

$$x_A + x_B \leq 100$$

$$x_A \geq 0, \quad x_B \geq 0$$

Write this problem in its standard form.

Min $\quad X_a + X_b$

S.t

$$0.5\,X_a + 0.2\,X_b - S_1 = 20$$

$$X_a - 1.5\,X_b + S_2 = 0$$

$$X_b - S_3 = 5$$

$$X_a + X_b + S_4 = 100$$

$$X_a, X_b, S_1, S_2, S_3, S_4 \geq 0$$

(b) (9 points) Find an initial basic solution using the big M initialization method. Show the problem's formulation under this initialization method. Use Excel and include snapshots of your procedure.

(c) (9 points) Find an initial basic solution using the two-phase initialization method. Show the problem's formulation under this initialization method. Use Excel and include snapshots of your procedure.

(d) (8 points) Starting from one of the initial basic solutions found in either (b) or (c), solve this problem using the Simplex algorithm. Indicate the values of all the variables and the objective function associated with the optimal solution. Use Excel and include snapshots of your procedure.

b.

$$\text{Min} \quad z = x_a + x_b + M(r_1 + r_3)$$

$$z - x_a - x_b - Mr_1 - Mr_3 = 0$$

Let's assume $r_1, r_2 = 10000$

Basic	z	xa	xb	s1	s2	s3	s4	r1	r3	Solution
z	1	-1	-1	0	0	0	0	-10000	-10000	0
r1	0	0.5	0.2	-1	0	0	0	1	0	20
s2	0	1	-1.5	0	1	0	0	0	0	0
r3	0	0	1	0	0	-1	0	0	1	5
s4	0	1	1	0	0	0	1	0	0	100

Basic	z	xa	xb	s1	s2	s3	s4	r1	r3	Solution
z	1	4999	1999	-10000	0	0	0	0	-10000	200000
r1	0	0.5	0.2	-1	0	0	0	1	0	20
s2	0	1	-1.5	0	1	0	0	0	0	0
r3	0	0	1	0	0	-1	0	0	1	5
s4	0	1	1	0	0	0	1	0	0	100

Basic	z	xa	xb	s1	s2	s3	s4	r1	r3	Solution	Min Ratio
z	1	4999	11999	-10000	0	-10000	0	0	0	250000	
r1	0	0.5	0.2	-1	0	0	0	1	0	20	100
s2	0	1	-1.5	0	1	0	0	0	0	0	0
r3	0	0	1	0	0	-1	0	0	1	5	5
s4	0	1	1	0	0	0	1	0	0	100	100

Figure 3. Model 2 simplex tableau formulation and solution using the big M method.

The initial basis for the big M methos is shown in figure 3 highlighted in yellow (r1,r3,s2,s4) c.

For the two Phase we have :

$$\text{Min} \sum r_i$$

Basic	z	xa	xb	s1	s2	s3	s4	r1	r3	Solution
z	1	0	0	0	0	0	0	-1	-1	
	0	0.5	0.2	-1	0	0	0	1	0	20
	0	1	-1.5	0	1	0	0	0	0	0
	0	0	1	0	0	-1	0	0	1	5
	0	1	1	0	0	0	1	0	0	100

Basic	z	xa	xb	s1	s2	s3	s4	r1	r3	Solution
z	1	0.5	0.2	-1	0	0	0	0	-1	20
	0	0.5	0.2	-1	0	0	0	1	0	20
	0	1	-1.5	0	1	0	0	0	0	0
	0	0	1	0	0	-1	0	0	1	5
	0	1	1	0	0	0	1	0	0	100

Basic	z	xa	xb	s1	s2	s3	s4	r1	r3	Solution	Min Ratio
z	1	0.5	1.2	-1	0	-1	0	0	0	25	
r1	0	0.5	0.2	-1	0	0	0	1	0	20	100
s2	0	1	-1.5	0	1	0	0	0	0	0	0
xb	0	0	1	0	0	-1	0	0	1	5	5
s4	0	1	1	0	0	0	1	0	0	100	100

Basic	z	xa	xb	s1	s2	s3	s4	r1	r3	Solution	Min Ratio
z	1	0.5	0	-1	0	0.2	0	0	-1.2	19	
r1	0	0.5	0	-1	0	0.2	0	1	-0.2	19	38
xa	0	1	0	0	1	-1.5	0	0	1.5	7.5	7.5
xb	0	0	1	0	0	-1	0	0	1	5	#DIV/0!
s4	0	1	0	0	0	1	1	0	-1	95	95

Basic	z	xa	xb	s1	s2	s3	s4	r1	r3	Solution	Min Ratio
z	1	0	0	-1	-0.5	0.95	0	0	-1.95	15.25	
s3	0	0	0	-1	-0.5	0.95	0	1	-0.95	15.25	16.052632
xa	0	1	0	0	1	-1.5	0	0	1.5	7.5	-5
xb	0	0	1	0	0	-1	0	0	1	5	-5
s4	0	0	0	0	-1	2.5	1	0	-2.5	87.5	35

Basic	z	xa	xb	s1	s2	s3	s4	r1	r3	Solution
z	1	0	0	0	0	0	0	-1	-1	0
s3	0	0	0	-1.052632	-0.526316	1	0	1.0526316	-1	16.052632
xa	0	1	0	-1.578947	0.2105263	0	0	1.5789474	0	31.578947
xb	0	0	1	-1.052632	-0.526316	0	0	1.0526316	0	21.052632
s4	0	0	0	2.6315789	0.3157895	0	1	-2.631579	0	47.368421

Figure 4. Two phase model 2 formulation and base solution

The initial basis using the two phase method is shown in figure 4 highlighted in yellow (xa,xb,s3,s4) d.

Basic	z	xa	xb	s1	s2	s3	s4	r1	r3	Solution
z	1	-1	-1	0	0	0	0	-10000	-10000	0
r1	0	0.5	0.2	-1	0	0	0	1	0	20
s2	0	1	-1.5	0	1	0	0	0	0	0
r3	0	0	1	0	0	-1	0	0	1	5
s4	0	1	1	0	0	0	1	0	0	100

Basic	z	xa	xb	s1	s2	s3	s4	r1	r3	Solution
z	1	4999	1999	-10000	0	0	0	0	-10000	200000
r1	0	0.5	0.2	-1	0	0	0	1	0	20
s2	0	1	-1.5	0	1	0	0	0	0	0
r3	0	0	1	0	0	-1	0	0	1	5
s4	0	1	1	0	0	0	1	0	0	100

Basic	z	xa	xb	s1	s2	s3	s4	r1	r3	Solution	Min Ratio
z	1	4999	11999	-10000	0	-10000	0	0	0	250000	
r1	0	0.5	0.2	-1	0	0	0	1	0	20	100
s2	0	1	-1.5	0	1	0	0	0	0	0	0
r3	0	0	1	0	0	-1	0	0	1	5	5
s4	0	1	1	0	0	0	1	0	0	100	100

Basic	z	xa	xb	s1	s2	s3	s4	r1	r3	Solution	Min Ratio
z	1	4999	0	-10000	0	1999	0	0	-11999	190005	
r1	0	0.5	0	-1	0	0.2	0	1	-0.2	19	38
s2	0	1	0	0	1	-1.5	0	0	1.5	7.5	7.5
xb	0	0	1	0	0	-1	0	0	1	5	#DIV/0!
s4	0	1	0	0	0	1	1	0	-1	95	95

Basic	z	xa	xb	s1	s2	s3	s4	r1	r3	Solution	Min Ratio
z	1	0	0	-10000	-4999	9497.5	0	0	-19497.5	152512.5	
r1	0	0	0	-1	-0.5	0.95	0	1	-0.95	15.25	16.05263
xa	0	1	0	0	1	-1.5	0	0	1.5	7.5	-5
xb	0	0	1	0	0	-1	0	0	1	5	-5
s4	0	0	0	0	-1	2.5	1	0	-2.5	87.5	35

Basic	z	xa	xb	s1	s2	s3	s4	r1	r3	Solution
z	1	0	0	-2.631579	-0.315789	0	0	-9997.368	-10000	52.63158
s3	0	0	0	-1.052632	-0.526316	1	0	1.052632	-1	16.05263
xa	0	1	0	-1.578947	0.210526	0	0	1.578947	0	31.57895
xb	0	0	1	-1.052632	-0.526316	0	0	1.052632	0	21.05263
s4	0	0	0	2.631579	0.315789	0	1	-2.631579	0	47.36842

xa	31.6
xb	21.05

Figure 5. Model 2 solution using the big M method.

In figure 5 we see the optimal values for xa and xb in blue, the value for the objective function (z) is also in blue.

3.

(30 points) Given the context from Problem 2. After reviewing your initial investment recommendation, the couple became concerned about the risks associated with their portfolio. To mitigate this risk, they have decided to diversify their investments further. They have requested that you create a new investment portfolio, this time including all stocks from a broader set S.The couple now asks you to achieve an aggregate profit amounting to at least P.

To manage risk, they have imposed additional constraints:

- You may not invest more than 30% more in any one stock than in any other stock.
- Among the available stocks, there are certain "safe" stocks with minimal risk, and you are required to invest at least m dollars in each of these safe stocks.
- The total new budget available for investment is B dollars.

(a) (15 points) Formulate the problem mathematically in a general manner.

(b) (15 points) Provide the standard form of the general mathematical formulation for the problem.

a.

x_i : Amount of dollars invested in stock i where i belongs to the set S.

r_i : return per dollar of stock i

$S_{safe} \subseteq S$

$$\text{Min} \quad \sum_{i \in S} x_i$$

s.t

$$\sum_{i \in S} r_i x_i \geq P$$

$$x_i \leq 1.3\, x_j \quad \forall\ i,j \text{ in } S,\ j \neq i$$

$$x_i \geq m \quad \forall\ i \in S_{safe}$$

$$\sum_{i \in S} x_i \leq B$$

$$x_i \geq 0 \quad \forall\ i \in S$$

b.

standard form

$$\text{Min} \sum_{i \in S} x_i$$

s.t

$$\sum_{i \in S} r_i x_i + S_1 = P$$

$$x_i - 1.3 x_j + S_2 = 0 \qquad \forall\ i,j \in S,\ i \neq j$$

$$x_i - m - S_3 = 0 \qquad \forall\ i \in S$$

$$\sum_{i \in S} x_i + S_4 = B$$

$$x_i,\ S_1,\ S_2,\ S_3,\ S_4 \geq 0$$