ISE 4623/5023: Deterministic Systems Models / Systems Optimization

University of Oklahoma

School of Industrial and Systems Engineering
Fall 2024

Final Exam (100 points – undergraduate students; 120 points –graduate students)

To solve this Exam, you can use your notes, the book, and any material uploaded in Canvas. You may not look online (outside Canvas) for codes or any help of any kind. Also, you cannot receive assistance of any type from anyone else.

You need to write your name and sign on each page. You need to upload a copy of the document with the solutions in Canvas (PDF format) along with the Gurobi/Excel files used to solve the exam and any other relevant support files. Problems without a proper support will not receive any points. If you are solving the exam with Excel, I suggest using only a single Excel file for the entire exam (using one sheet per problem).

Last name: Mejia

First name: Juan

Student ID: 113688069

Section (mark with X): ___ ISE 4623

X ISE 5023

Pledge: "On my honor, I have neither given nor received inappropriate assistance in the completion of this Exam."

Student signature: Juan Mejia

Student name: Juan Mejia
Student signature: Juan Mejia

Problem 1 (40 points): Knapsack problem

Imagine that you are planning your next vacations. For your trip, first you want to determine which items to pack. To take this decision, you make a list of 30 items from which you want to pick the ones to be packed. For each of these 30 items, you know its weight (in pounds) and its associated intrinsic value (the more you want to take the item, the higher its "value"), as indicated below (Table 1).

Table 1. Weight and value of each item

ITEM	Weight (pounds)	Value
Item 1	4	3
Item 2	5	2
Item 3	5	5
Item 4	9	4
Item 5	9	4
Item 6	4	4
Item 7	7	4
Item 8	7	9
Item 9	6	7
Item 10	3	9
Item 11	7	8
Item 12	8	9
Item 13	3	8
Item 14	8	3
Item 15	4	6
Item 16	3	5
Item 17	7	7
Item 18	9	8
Item 19	6	2
Item 20	7	2
Item 21	6	7
Item 22	8	2
Item 23	7	5
Item 24	9	3
Item 25	3	7
Item 26	4	4
Item 27	5	7
Item 28	4	10
Item 29	3	4
Item 30	8	6

PART I. (20 points)

Assume that for your trip you can only use a single bag, with a maximum capacity of 15 pounds. Considering this weight constraint, calculate the optimal packing strategy (the one that maximizes the total value of the items packed)

1.1. (2 points) In the optimal solution, is item 8 packed?
A. Yes
B. No

1.2. (2 points) In the optimal solution, is item 10 packed?
A. Yes
B. No

1.3. (2 points) In the optimal solution, is item 13 packed?
A. Yes
B. No

1.4. (2 points) In the optimal solution, is item 18 packed?
A. Yes
B. No

1.5. (2 points) In the optimal solution, is item 27 packed?
A. Yes
B. No

1.6. (5 points) In the optimal packing strategy (the one that maximizes the total value packed) what is the total value associated with the items that you can pack in your bag?
A. 95
B. 98
C. 34
D. 15
E. 42
F. 79

1.7. (5 points) In the optimal packing strategy (the one that maximizes the total value packed) what is the total weight associated with the items that you can pack in your bag?
A. 10
B. 11
C. 12
D. 13
E. 14
F. 15

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PART II. (20 points)

Now, assume that for your trip you can use two bags. The first bag has a maximum capacity of 15 pounds and the second one has a maximum capacity of 50 pounds.

1.8. (2 points) In the new optimal solution, is item 8 packed?
A. Yes
B. No

1.9. (2 points) In the new optimal solution, is item 10 packed?
A. Yes
B. No

1.10. (2 points) In the new optimal solution, is item 13 packed?
A. Yes
B. No

1.11. (2 points) In the new optimal solution, is item 18 packed?
A. Yes
B. No

1.12. (2 points) In the new optimal solution, is item 27 packed?
A. Yes
B. No

1.13. (5 points) In the optimal packing strategy (the one that maximizes the total value packed) what is the total value associated with the items that you can pack in your bag?
A. 95
B. 98
C. 34
D. 15
E. 42
F. 79

1.14. (5 points) In the optimal packing strategy (the one that maximizes the total value packed) what is the total weight associated with the items that you can pack in the first bag?
A. 10
B. 11
C. 12
D. 13
E. 14
F. 15

Student name: Juan Mejia
Student signature: Juan Mejia

Problem 2. (40 points) (additional 20 points for Graduate Students or extra credit for undergraduate): TSP

You are still planning your next vacations. For your vacations, you have decided to visit six cities (labeled as city 1, 2, ..., 6). <u>Your trip will start and finish in your hometown (labeled as city 7)</u>. To visit each of the cities, you can either take a bus or a plane. The travel costs (in USD) and times (in hours) are shown in the tables below (Tables 2 and 3). Note that, as expected, the bus is often much cheaper, but usually takes longer. Also, not that these costs and travel times are not symmetric.

Table 2. Travel times and travel costs for bus trips

Travel time - bus (hours)

Departure city \ Destination city	1	2	3	4	5	6	7
1	0	4.7	5.4	4.5	10.1	7	9.4
2	4.5	0	7.1	6.7	10.8	6.9	10.2
3	3.5	6.1	0	5.1	6.4	7.4	7.6
4	2.8	6.6	5.9	0	10.7	10.7	10.7
5	9.3	11.4	7.6	10.5	0	7.4	5
6	6.9	7	7.6	9.5	9	0	5.4
7	8.9	9.3	6.7	11.3	6.2	4.2	0

Travel cost - bus (USD)

Departure city \ Destination city	1	2	3	4	5	6	7
1	0	21	21	25	38	27	25
2	13	0	30	32	31	17	25
3	24	16	0	12	19	32	21
4	25	24	19	0	21	36	38
5	22	42	16	39	0	17	11
6	23	27	16	34	25	0	13
7	19	19	25	37	28	19	0

Table 3. Travel times and travel costs for airplane trips

Travel time - airplane (hours)

Departure city \ Destination city	1	2	3	4	5	6	7
1	0	0.9	1.1	1	2.8	1.8	2.5
2	0.8	0	1.7	1.8	3.1	1.8	2.4
3	1.1	1.6	0	1.4	1.7	1.7	1.7
4	0.9	1.9	1.3	0	2.7	2.8	2.9
5	2.8	3	1.6	2.8	0	2	1.3
6	1.9	1.7	1.8	2.8	2.2	0	1.2
7	2.6	2.5	1.7	2.9	1.3	1.2	0

Travel cost - airplane (USD)

Departure city \ Destination city	1	2	3	4	5	6	7
1	0	85	160	99	280	194	302
2	123	0	223	232	385	234	312
3	162	240	0	202	175	213	209
4	150	248	197	0	291	295	349
5	344	327	193	314	0	258	132
6	202	176	201	281	246	0	181
7	326	271	230	318	130	174	0

PART I. (20 points)

Assume that you want to minimize your total travel time. Help: for this particular problem, this means that you will travel using only airplanes (not buses).

2.1 (5 points) What is the total travel time (in hours) for the optimal trip (the one that minimizes the total travel time)?

A. 10.3
B. 9.0
C. 8.7
D. 7.6
E. 9.4
F. 11.3

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Student signature: Juan Magin

2.2 (5 points) What is the total cost (in USD) associated with the optimal trip (the one that minimizes the total travel time)?

A. 895
B. 1283
C. 978
D. 1076
E. 756
F. 852

2.3 (2 points) For the trip that minimizes the total travel time, which was the first city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.4 (2 points) For the trip that minimizes the total travel time, which was the second city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.5 (2 points) For the trip that minimizes the total travel time, which was the third city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.6 (2 points) For the trip that minimizes the total travel time, which was the fourth city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.7 (2 points) For the trip that minimizes the total travel time, which was the fifth city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

PART II. (20 points)

Assume that now you want to minimize your total travel cost. Help: for this particular problem, this means that you will travel using only buses (not airplanes).

2.8 (5 points) What is the total travel time (in hours) for the optimal trip (the one that minimizes the total travel cost)?

A. 50.4
B. 22.6
C. 47.8
D. 39.7
E. 63.5
F. 57.4

2.9 (5 points) What is the total cost (in USD) associated with the optimal trip (the one that minimizes the total travel cost)?

A. 116
B. 231
C. 158
D. 287
E. 129
F. 174

2.10 (2 points) For the trip that minimizes the total travel cost, which was the first city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.11(2 points) For the trip that minimizes the total travel cost, which was the second city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.12(2 points) For the trip that minimizes the total travel cost, which was the third city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.13(2 points) For the trip that minimizes the total travel cost, which was the fourth city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.14(2 points) For the trip that minimizes the total travel cost, which was the fifth city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

PART III. (20 points) Graduate students only (or Extra Credit for Undergraduate Students)

Assume that now you want to minimize your total travel cost, but while guaranteeing that the total travel time is less than or equal to 20 hours. In this case, it is necessary to use both buses and airplanes.

2.15 (5 points) What is the total travel time (in hours) for the optimal trip (the one that minimizes the total travel cost using less than or equal to 20 hours)?

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A. 19.9
B. 18.6
C. 19.5
D. 19.8
E. 18.9
F. 20.0

2.16 (5 points) What is the total cost (in USD) associated with the optimal trip (the one that minimizes the total travel cost using less than or equal to 20 hours)?

A. 295
B. 643
C. 435
D. 821
E. 542
F. 718

2.17 (2 points) For the trip that minimizes the total travel cost using less than or equal to 20 hours, which was the first city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.18 (2 points) For the trip that minimizes the total travel cost using less than or equal to 20 hours, which was the second city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.19(2 points) For the trip that minimizes the total travel cost using less than or equal to 20 hours, which was the third city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.20(2 points) For the trip that minimizes the total travel cost using less than or equal to 20 hours, which was the fourth city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.21(2 points) For the trip that minimizes the total travel cost using less than or equal to 20 hours, which was the fifth city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

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Student signature: Juan Mejia

Problem 1)

1) Part I

We want to maximize value while not surpassing the max weight in the bag

→ Items packed can't be more than 15 pounds

Sets

I = set of items $(1, 2, \dots n)$

Parameters

w_i = weight of item $i \in I$

v_i = Value of the object $i \in I$

W = max allowable weight (capacity of the Knapsack)

Variables:

x_i = binary variable that indicates if item $i \in I$ is packed

Objective function:

$$\max \sum_{i \in I} v_i x_i$$

Constraints:

$$\sum_{i \in I} v_i x_i \leq 15 \text{ (max allowable weight)}, \quad x_i, v_i \geq 0$$

Part 2

For part 2 we add another bag of capacity = 50 and the model is as follows

Sets

I = items $(1, 2, n)$

parameters

w_i = weight of item $i \in I$

v_i = value of item $i \in I$

W_1 = Max capacity of bag 1

W_2 = Max capacity of bag 2

Variables:

$x_{i,1}$ = binary variable indicating if item i in bag 1

$x_{i,2}$ = binary variable indicating if item i in bag 2

Objective function

$$\max \sum_{i \in I} v_i \cdot (x_{i,1} + x_{i,2})$$

Constraints

$$\sum_{i \in I} v_i x_{i,1} \leq 15, \quad \sum v_i x_{i,2} \leq 50$$

$$x_{i,1} + x_{i,2} \leq 1 \text{ (assigned to at most one bag)}$$

$$x_{i,1} ; x_{i,2} ; v_i \geq 0$$

Problem 1 part 1

```
Optimal solution found:
Item 10: Included (Weight = 3, Value = 9)
Item 13: Included (Weight = 3, Value = 8)
Item 25: Included (Weight = 3, Value = 7)
Item 28: Included (Weight = 4, Value = 10)
Total Value: 34.0
```

Part 2

```
Optimal solution found:
Item 3: Included in Bag 2 (Weight = 5, Value = 5)
Item 6: Included in Bag 2 (Weight = 4, Value = 4)
Item 8: Included in Bag 2 (Weight = 7, Value = 9)
Item 9: Included in Bag 2 (Weight = 6, Value = 7)
Item 10: Included in Bag 2 (Weight = 3, Value = 9)
Item 11: Included in Bag 2 (Weight = 7, Value = 8)
Item 12: Included in Bag 2 (Weight = 8, Value = 9)
Item 13: Included in Bag 2 (Weight = 3, Value = 8)
Item 15: Included in Bag 2 (Weight = 4, Value = 6)
Item 16: Included in Bag 2 (Weight = 3, Value = 5)
Item 25: Included in Bag 1 (Weight = 3, Value = 7)
Item 27: Included in Bag 1 (Weight = 5, Value = 7)
Item 28: Included in Bag 1 (Weight = 4, Value = 10)
Item 29: Included in Bag 1 (Weight = 3, Value = 4)
Total Weight in Bag 1: 15
Total Weight in Bag 2: 50
Total Value: 98.0
```

Problem 2)

This is a typical TSP with the added novelty of having the constraint of the initial and final point

Sets

N: set of nodes (1, 2, .., n)

A: set of Arcs

Parameters:

c_{ij}: cost of traveling through arc $(i,j) \in A$ (travel time in hrs)

Variables:

x_{ij}: Binary variable indicating if salesman travels through arc $(i,j) \in A$

u_i: Label / order of visit of node $i \in N \setminus \{1\}$

Constraints

- $\sum_{i:(i,j) \in A} x_{ij} = 1 \quad \forall i \in N$ (each city exactly one outgoing route)
- $\sum_{j:(i,j) \in A} x_{ij} = 1 \quad \forall i \in N$ (each city exactly one incoming route)
- $u_i \leq u_j - 1 + n(1 - x_{ij}) \quad \forall (i,j) \in A : j \neq 1$ (subtour elimination)
- $\sum_{i:(i,1) \in A,\ i \neq 1} x_{i1} = 1$ (traveler starts at hometown city 1)
- $\sum_{j:(1,j) \in A : j \neq 1} x_{1j} = 1$ (traveler ends at hometown city 1)

$x_{ij} \in \{0,1\} \quad \forall (i,j) \in A, \quad u_i \geq 0 \quad \forall i \in N \setminus \{1\}$

Objective function

$$\text{Min} \sum_{(i,j) \in A} c_{ij} \cdot x_{ij}$$

This model works for both part I and II, it just changes the definition of c_{ij}, that goes from cost or travel time using airplanes to the monetary cost using buses.

Part III

Now, we use both airplanes and buses while having a max time of 20 hours in our travel time. The model now needs to decide which arcs should be connected by bus or airplane. hence, we need to define two decision variables instead of just one. The updated model is as follows:

Sets
N = set of nodes (cities) $(1, 2 .. n)$

A = set of arcs

Parameters
c_{ij}^{bus} = travel time cost by bus from i to j through arc $(i,j) \in A$

c_{ij}^{plane} = travel time cost by plane from i to j through arc $(i,j) \in A$

T_{max} = Max allowed travel time

Decision variables
x_{ij}^{bus} = Binary variable indicating if salesman travels through arc $(i,j) \in A$ by bus

x_{ij}^{plane} = Binary variable indicating if salesman travels through arc $(i,j) \in A$ by plane

u_i = label / order of visit of city $i \in N \setminus \{1\}$

Objective function

$$\text{Min} \sum_{(i,j) \in A} c_{ij}^{bus} \cdot x_{ij}^{bus} + \sum_{(i,j) \in A} c_{ij}^{plane} \cdot x_{ij}^{plane}$$

Constraints

- $\sum_{(i,j) \in A} \left(x_{ij}^{bus} + x_{ij}^{plane} \right) = 1 \quad \forall i \in N$ (each city exactly one outgoing route)
- $\sum_{(i,j) \in A} x_{ij}^{bus} + x_{ij}^{plane} = 1 \quad \forall j \in N$ (each city exactly one incoming route)
- $u_i - u_j + n \left(x_{ij}^{bus} + x_{ij}^{plane} \right) \leq n-1 \quad \forall (i,j) \in A : j \neq 1$ (subtour elimination)
- $x_{ij}^{bus} + x_{ij}^{plane} \leq 1 \quad \forall (i,j) \in A$ (only one mode of transportation chosen)
- $\sum_{(i,j) \in A} c_{ij}^{bus} \cdot x_{ij}^{bus} + \sum_{(i,j) \in A} c_{ij}^{plane} \cdot x_{ij}^{plane} \leq T_{max}$ (Not exceed max time allowed)

- $\sum_{(i,j)\in A,\ j\neq 7} \left(X_{7j}^{bus} + X_{7j}^{plane} \right) = 1$ (traveler starts at hometown)

- $\sum_{(i,j)\in A} \left(X_{i7}^{bus} + X_{i7}^{plane} \right) = 1$ (traveler ends at hometown)

$X_{ij}^{bus}, X_{ij}^{plane} \in \{0,1\} \quad \forall\ i,j \in A$

$u_i \geq 0 \quad \forall\ i \in N$

For 2.16 only changes the meaning of the cost parameters, now it would mean monetary cost instead of time cost, the rest is the same.

Problem 3)

Objective: minimize transportation costs.
The cost depends on the amount shipped. so we need a variable to represent shipments (X_{ij}). Also, the production capacity is not given, we have to determine it (y_i). The model is as follows:

Sets

$P = \{P_1, P_2\}$ set of production plants
$C = \{C_1, C_2, C_3\}$ sets of cities (demand nodes)
A = set of arcs between i and j

Parameters

C_{ij} = Transportation cost $\forall\ i \in P,\ j \in C$
D_j = demand of city j $\forall\ j \in C$
F_{ij} = flow capacity of arc (i,j) $\forall\ i \in P,\ j \in C$

Variables

X_{ij} = Amount of paint transported from plant i to city j $\forall\ i \in P,\ j \in C$
y_i Total production capacity of plant i $\forall\ i \in P$

objective function

$$\text{Min} \sum_{i\in P} \sum_{j \in C} C_{ij} \cdot X_{ij}$$

Problem 2 part 1

```
Optimal solution found:
Total time (hours): 9.0
Total Cost (USD): 1076.00
Routes:
Plane: 1 -> 4
Plane: 2 -> 1
Plane: 3 -> 5
Plane: 4 -> 3
Plane: 5 -> 7
Plane: 6 -> 2
Plane: 7 -> 6
```

Part 2

```
Optimal solution found:
Total travel cost: 116.0
Total travel time: 47.80 hours
Routes:
Bus: 1 -> 3
Bus: 2 -> 1
Bus: 3 -> 4
Bus: 4 -> 5
Bus: 5 -> 6
Bus: 6 -> 7
Bus: 7 -> 2
```

Part 3

```
Optimal solution found:
Total travel cost: 643.00 USD
Total travel time: 19.80 hours
Routes:
Plane: 1 -> 2
Bus: 2 -> 6
Bus: 3 -> 4
Bus: 4 -> 1
Plane: 5 -> 3
Plane: 6 -> 7
Plane: 7 -> 5
```

Constraints

- $\sum_{i \in P} x_{ij} = D_j \quad ; \forall \; j \in C$ (each city's demand must be met)

- $x_{ij} \leq f_{ij} \quad \forall \; i,j$ (shipment on each arc can't exceed its capacity)

- $\sum_{j \in C} x_{ij} \leq y_i \quad \forall \; i \in P$ (shipments can't exceed production capacity)

- $x_{ij}, y_i \geq 0 \quad \forall \; i \in P, j \in C$

Problem 3

```
Optimal solution found:
Total cost: 1085.00
P1 production capacity: 15.00 tons
P2 production capacity: 65.00 tons
Transport 5.00 tons from P1 to C1
Transport 10.00 tons from P1 to C3
Transport 20.00 tons from P2 to C1
Transport 15.00 tons from P2 to C2
Transport 30.00 tons from P2 to C3
```