HW 6 Juan Mejia

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1 Question 1

1.1

Sets:

- S: Set of suppliers ($S = \{1, 2, 3\}$)
- P: Set of production plants ($P = \{1, 2, 3\}$)

Parameters:

- c_{sp}: Cost per ton of transporting from supplier s to plant p
- A_s: Availability of raw ingredients (in tons) at supplier s
- D_p: Demand for raw ingredients (in tons) at plant p

Decision Variables:

$$x_{sp} : \text{Tons of raw ingredients shipped from supplier } s \text{ to plant } p$$

Objective Function:

$$\text{Minimize } Z = \sum_{s \in S} \sum_{p \in P} c_{sp} \cdot x_{sp}$$

The objective is to minimize the total transportation cost.

Constraints:

1. Supply Constraints (Supplier Availability):

$$\sum_{p \in P} x_{sp} \leq A_s \quad \forall s \in S$$

This ensures that the total amount shipped from each supplier does not exceed its availability.

2. Demand Constraints (Plant Requirements):

$$\sum_{s \in S} x_{sp} \geq D_p \quad \forall p \in P$$

This ensures that each plant receives enough raw ingredients to meet its demand.

3. Non-Negativity Constraints:

$$x_{sp} \geq 0 \quad \forall s \in S, p \in P$$

All the decision variables are non-negative.

Data:

- Transportation costs (c_{sp}):

From/To	Plant 1	Plant 2	Plant 3
Supplier 1	320	215	400
Supplier 2	410	245	360
Supplier 3	290	460	190

- Supplier availability (A_s):

$$A_1 = 50, \quad A_2 = 120, \quad A_3 = 180$$

- Plant demand (D_p):

$$D_1 = 80, \quad D_2 = 90, \quad D_3 = 60$$

1.2

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Solved in 1 iterations and 0.01 seconds (0.00 work units)
Optimal objective  5.515000000e+04
Optimal transportation strategy:
Ship 50.00 tons from Supplier 1 to Plant 2 at cost 215 per ton
Ship 40.00 tons from Supplier 2 to Plant 2 at cost 245 per ton
Ship 80.00 tons from Supplier 3 to Plant 1 at cost 290 per ton
Ship 60.00 tons from Supplier 3 to Plant 3 at cost 190 per ton

Total transportation cost: 55150.00
```

Figure 1

1.3

The supply disruptions in 1.c lead to an infeasible model because the reduced availability from suppliers (40, 60, and 25 percent losses) makes it impossible to meet the total demand at the three plants. In contrast, the model in 1.b found a feasible and cost-efficient solution. This highlights the impact of supply chain disruptions on the ability to meet demand and the importance of contingency planning.

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Solved in 1 iterations and 0.00 seconds (0.00 work units)
Infeasible model
No optimal solution found.
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Figure 2

1.4

Sets and Parameters

- S: Set of suppliers.
- P: Set of production plants.
- c_{sp}: Transportation cost per ton from supplier s to plant p.
- c_{pq}: Internal transportation cost per ton from plant p to plant q.
- a_s: Availability of raw ingredients at supplier s (in tons).
- d_p: Demand for raw ingredients at plant p (in tons).
- M: A large number, used to link the continuous and binary decision variables.

Decision Variables

- x_{sp}: Tons of raw ingredients shipped from supplier s to plant p.
- y_{pq}: Tons of raw ingredients shipped from plant p to plant q.
- z_p: Binary variable; 1 if plant p ships to another plant, 0 otherwise.
- w_{pq}: Binary variable; 1 if plant p ships to plant q, 0 otherwise.

$$\text{Minimize } Z = \sum_{s\in S}\sum_{p\in P} c_{sp}x_{sp} + \sum_{p\in P}\sum_{q\in P, p\neq q} c_{pq}y_{pq}$$

Subject to:

$$\sum_{p\in P} x_{sp} \leq a_s \quad \forall s \in S$$

$$\sum_{s\in S} x_{sp} + \sum_{q\in P, q\neq p} y_{qp} = d_p \quad \forall p \in P$$

$$\sum_{p\in P} z_p = 1$$

$$y_{pq} \leq M \cdot w_{pq} \quad \forall p, q \in P, p \neq q$$

$$w_{pq} \leq z_p \quad \forall p, q \in P, p \neq q$$

$$\sum_{q\in P, q\neq p} w_{pq} \leq 1 \quad \forall p \in P$$

$$x_{sp} \geq 0, \quad y_{pq} \geq 0, \quad z_p \in \{0,1\}, \quad w_{pq} \in \{0,1\}$$

2

2.1

Sets:

- H: Set of hubs
- W: Set of warehouses
- C: Set of customers
- T: Set of sauce types, where r stands for regular and s stands for special

Parameters:

- c_{hw}: Cost of shipping per order from hub h to warehouse w
- c_{wc}: Cost of shipping per order from warehouse w to customer c
- u_cost_{ct}: Cost per unit of unmet demand for customer c and sauce type t
- M_{hw}: Maximum number of orders that can be shipped from hub h to warehouse w
- M_{wc}: Maximum number of orders that can be shipped from warehouse w to customer c
- d_{ct}: Demand for customer c and sauce type t

Decision Variables:

$$x_{hwt} : \text{Number of orders of type } t \text{ shipped from hub } h \text{ to warehouse } w$$

$$y_{wct} : \text{Number of orders of type } t \text{ shipped from warehouse } w \text{ to customer } c$$

$$u_{ct} : \text{Unmet demand for customer } c \text{ and sauce type } t$$

Objective Function:

$$\text{Minimize } Z = \sum_{h\in H}\sum_{w\in W}\sum_{t\in T}(c_{hw} \cdot x_{hwt}) + \sum_{w\in W}\sum_{c\in C}\sum_{t\in T}(c_{wc} \cdot y_{wct}) + \sum_{c\in C}\sum_{t\in T}(u\_cost_{ct} \cdot u_{ct})$$

Constraints:

1. Flow from hubs to warehouses (Capacity Constraint):

$$\sum_{t \in T} x_{hwt} \leq M_{hw} \quad \forall h \in H, w \in W$$

2. Flow from warehouses to customers (Capacity Constraint):

$$\sum_{t \in T} y_{wct} \leq M_{wc} \quad \forall w \in W, c \in C$$

3. Demand Satisfaction for Sauce Types:

$$\sum_{w \in W} y_{wct} + u_{ct} = d_{ct} \quad \forall c \in C, t \in T$$

4. Flow Conservation:

$$\sum_{h \in H} x_{hwt} = \sum_{c \in C} y_{wct} \quad \forall w \in W, t \in T$$

5. Non-Negativity Constraints:

$$x_{hwt}, y_{wct}, u_{ct} \geq 0 \quad \forall h \in H, w \in W, c \in C, t \in T$$

2.2

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Optimal solution found (tolerance 1.00e-04)
Best objective 7.672250000000e+05, best bound 7.672250000000e+05, gap 0.0000%
Orders shipped from hubs to warehouses:
Orders shipped from hub 1 to warehouse 1 for sauce s: 50.0
Orders shipped from hub 1 to warehouse 2 for sauce s: 30.0
Orders shipped from hub 1 to warehouse 3 for sauce r: 20.0
Orders shipped from hub 1 to warehouse 3 for sauce s: 40.0
Orders shipped from hub 2 to warehouse 1 for sauce r: 20.0
Orders shipped from hub 2 to warehouse 1 for sauce s: 20.0
Orders shipped from hub 2 to warehouse 2 for sauce r: 35.0
Orders shipped from hub 2 to warehouse 3 for sauce r: 85.0
Orders shipped from hub 3 to warehouse 1 for sauce r: 45.0
Orders shipped from hub 3 to warehouse 2 for sauce r: 20.0
Orders shipped from hub 3 to warehouse 2 for sauce s: 30.0
Orders shipped from hub 3 to warehouse 3 for sauce s: 55.0

Orders shipped from warehouses to customers:
Orders shipped from warehouse 1 to customer 1 for sauce r: 40.0
Orders shipped from warehouse 1 to customer 3 for sauce r: 25.0
Orders shipped from warehouse 1 to customer 4 for sauce s: 45.0
Orders shipped from warehouse 1 to customer 5 for sauce s: 25.0
Orders shipped from warehouse 2 to customer 1 for sauce r: 30.0
Orders shipped from warehouse 2 to customer 3 for sauce r: 25.0
Orders shipped from warehouse 2 to customer 5 for sauce s: 60.0
Orders shipped from warehouse 3 to customer 1 for sauce r: 65.0
Orders shipped from warehouse 3 to customer 2 for sauce s: 25.0
Orders shipped from warehouse 3 to customer 3 for sauce r: 40.0
Orders shipped from warehouse 3 to customer 4 for sauce s: 20.0
Orders shipped from warehouse 3 to customer 5 for sauce s: 50.0

Unmet demand for customers:
Unmet demand for customer 1 for sauce r: 65.0
Unmet demand for customer 1 for sauce s: 100.0
Unmet demand for customer 2 for sauce r: 250.0
Unmet demand for customer 2 for sauce s: 25.0
Unmet demand for customer 3 for sauce r: 60.0
Unmet demand for customer 3 for sauce s: 100.0
Unmet demand for customer 4 for sauce r: 100.0
Unmet demand for customer 4 for sauce s: 135.0
Unmet demand for customer 5 for sauce r: 200.0
Unmet demand for customer 5 for sauce s: 215.0

Total cost: 767225.0
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Figure 3