Homework1

Saturday, August 24, 2024 6:40 PM

1)

$A = \begin{bmatrix} 4 & 1 & 8 \\ 2 & -7 & 5 \end{bmatrix}_{2\times 3}$ $B = \begin{bmatrix} -9 & 7 \\ 1 & 5 \\ 8 & 3 \end{bmatrix}_{3\times 2}$

a) AB

$$\begin{bmatrix} (4\times-9)+(1\times 1)+(8\times 8) & (4\times 7)+(1\times 5)+(8\times 3) \\ (2\times 9)+(-7\times 1)+(5\times 8) & (2\times 7)+(-7\times 5)+(5\times 3) \end{bmatrix} = \begin{bmatrix} 29 & 57 \\ 15 & -6 \end{bmatrix}$$

b) BA

$$\begin{bmatrix} -9 & 7 \\ 1 & 5 \\ 8 & 3 \end{bmatrix} \times \begin{bmatrix} 4 & 1 & 8 \\ 2 & -7 & 5 \end{bmatrix} = \begin{bmatrix} -22 & -58 & -37 \\ 14 & -34 & 33 \\ 38 & -13 & 79 \end{bmatrix}$$

c) $-B = -1 \cdot B = \begin{bmatrix} 9 & -7 \\ -1 & -5 \\ -8 & -3 \end{bmatrix}$

d) To sum or subtract matrices, both have to have the same dimensions, in this case A(2x3) and B(3x2) don't

e) $A^T = \begin{bmatrix} 4 & 2 \\ 1 & -7 \\ 8 & 5 \end{bmatrix}$

F) $-B^T \cdot B$

$$\begin{bmatrix} 9 & -1 & -8 \\ -7 & -5 & -3 \end{bmatrix} \begin{bmatrix} -9 & 7 \\ 1 & 5 \\ 8 & 3 \end{bmatrix}$$

$$\begin{bmatrix} (9\times-9)+(-1\times 1)+(-8\times 8) & (9\times 7)+(-1\times 5)+(-8\times 3) \\ (-7\times-9)+(-5\times 1)+(-3\times 8) & (-7\times 7)+(-5\times 5)+(-3\times 3) \end{bmatrix}$$

$$= \begin{bmatrix} 148 & -34 \\ -34 & 83 \end{bmatrix}$$

2)

a)

$15x + 5y = 25$ ①
$8x + 5y = 2$ ②
$\rightarrow$ ① - ② = $7x = 23$

$\boxed{x = \frac{23}{7}}$

$8(23/7) + 5y = 2 \rightarrow \boxed{y = -34/7}$

b)

$3x + y = 8$ ①
$3x - y = 8$ ②

① + ② = $6x = 16 \rightarrow x = \frac{16}{6}$

$3(\frac{16}{6}) - y = 8 \rightarrow \boxed{y = 0}$

c)

$2x - 24y = 42$ ①
$-2x + 24y = -40$ ②

① + ② = $0 = 2$

contradiction

the contradiction is because the equations are parallel lines that never intersect

3)

a) $C = \begin{bmatrix} 3 & -2 \\ 12 & -8 \end{bmatrix}$ $C^{-1} = \frac{1}{\det C} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$C^{-1} = \frac{1}{0}$ since the determinant is zero, the matrix C doesn't have an inverse

b)

$D = \begin{bmatrix} -1 & 4 \\ 6 & -8 \end{bmatrix}$ $D^{-1} = \frac{1}{-16} \begin{bmatrix} -8 & -4 \\ -6 & -1 \end{bmatrix} = \begin{bmatrix} 8/16 & 4/16 \\ 6/16 & 1/16 \end{bmatrix}$

c)

$E = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ $\det E = 1\cdot(5\times 9 - 6\times 8) - 2(4\times 9 - 6\times 7) + 3(4\times 8 - 5\times 7)$

$\det E = 0$ hence matrix E does not have an inverse

d)

$F = \begin{bmatrix} -3 & 5 & 9 \\ 8 & -2 & 5 \\ 1 & -2 & 14 \end{bmatrix}$ $\det F = -3(-2\times 14 + 5\times 2) - 5(8\cdot 14 - 5\cdot 1) + 9(8\cdot -2 + 2\cdot 1) = -607$

$F^{-1} = \frac{1}{-607} \cdot adj(F)$

$\begin{vmatrix} -2 & 5 \\ -2 & 14 \end{vmatrix} = -18$ $\begin{vmatrix} 8 & 5 \\ 1 & 14 \end{vmatrix} = 107$ $\begin{vmatrix} 8 & -2 \\ 1 & -2 \end{vmatrix} = -14$

$\begin{vmatrix} 5 & 9 \\ -2 & 14 \end{vmatrix} = 88$ $\begin{vmatrix} -3 & 9 \\ 1 & 14 \end{vmatrix} = -51$ $\begin{vmatrix} -3 & 5 \\ 1 & -2 \end{vmatrix} = 1$

$\begin{vmatrix} 5 & 9 \\ -2 & 5 \end{vmatrix} = 43$ $\begin{vmatrix} -3 & 9 \\ 8 & 5 \end{vmatrix} = -87$ $\begin{vmatrix} -3 & 5 \\ 8 & -2 \end{vmatrix} = -34$

$$adj(F) = \begin{vmatrix} -18 & -107 & -14 \\ -88 & -51 & -1 \\ 43 & 87 & -34 \end{vmatrix}$$

$$F^{-1} = \begin{bmatrix} \frac{18}{607} & \frac{107}{607} & \frac{14}{607} \\ \frac{88}{607} & \frac{51}{607} & \frac{1}{607} \\ \frac{-43}{607} & \frac{-87}{607} & \frac{34}{607} \end{bmatrix}$$

4)

a) $\sum_{j \in \mathbb{R}} j$ = "Summation of all j where j belongs to the real numbers"

b) $\sum_{j \in \mathbb{Z}^+ | j \bmod 5 = 0} j$ = "Sum of all j where j are all positive integers that are divisible by 5"

c) $\sum_{\substack{j=1 \\ j \in \mathbb{Z}}}^{40} j$ = "Sum of elements j from j equal 1 to j equal 40 where j are integers"

5)

a) $\sum_{j \in J | j \bmod 2 \neq 0} j$

b) $\sum_{i \in I} \sum_{j \in J} x_{ij}$

c) $\sum_{i \in J} \sum_{j \in J | (i,j) \in A} x_{ij}$

6)

a) $\sum_{j \in \{0,1,2,\dots,12\} : j \bmod 2 = 0} j$ = this means that from the set we will only select the even numbers

so the sum would be $= 0+2+4+6+8+10+12 = 42$

b)

$N = \{1,2,3,4\},\ A = \{(1,2),(2,3),(2,4),(3,4),(2,1)\},\ D = \{3\},\ S = \{1,2\},\ T = \{4\}$

$$b_i = \begin{cases} \alpha_i & i \in S \\ \beta_i & i \in D \\ 0 & i \in T \end{cases} \quad \forall i \in N$$

Expand:

$$\sum_{j \in N:(i,j) \in A} x_{ij} - \sum_{j \in N:(j,i) \in A} x_{ji} = b_i \ \forall i \in N$$

for $i = 1$ $(i \in S)$:
outgoing edges: $(1,2)$
Incoming edges = $(2,1)$

$x_{12} - x_{21} = \alpha_1$

for $i = 2$ $(i \in S)$
outgoing edges: $(2,3), (2,4), (2,1)$
incoming edges: $(1,2)$

$x_{21} + x_{23} + x_{24} - x_{12} = \alpha_1$

for $i = 3$ $(i \in D)$
outgoing edges: $(3,4)$
incoming edges: $(2,3)$

$x_{34} - x_{23} = \beta_i$

for $i = 4$ $(i \in T)$
outgoing: None (No $(4,j)$ pairs in A)
Incoming: $(2,4), (3,4)$

$-x_{24} - x_{34} = 0$

Set of equations:

$x_{12} - x_{21} = \alpha_1$

$-x_{12} + x_{21} + x_{23} + x_{24} = \alpha_1$

$x_{34} - x_{23} = \beta_i$

$-x_{24} - x_{34} = 0$