HW 7 Juan Mejia

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1 Problem 1

1.1

Sets:

- P: Set of projects, $P = \{A, B, C, D, E, F, \ldots\}$

Parameters:

- c_p: Cost of project p, $\forall p \in P$
- r_p: Expected yearly return from project p, $\forall p \in P$
- risk_p: Binary indicator for whether project p is high risk (1 if high risk, 0 otherwise), $\forall p \in P$
- B: Total budget available ($2,000,000$ USD)
- α: Maximum allowable budget allocation for high-risk projects ($0.4B$)
- s_{CF}: Synergy bonus if both projects C and F are selected ($200,000$ USD)

Decision Variables:

- x_p: Binary variable indicating if project p is selected, $\forall p \in P$, $x_p \in \{0, 1\}$

Objective Function:

$$\text{Maximize} \sum_{p \in P} r_p \cdot x_p + s_{CF} \cdot x_C \cdot x_F$$

Constraints:

1. Budget Constraint:

$$\sum_{p \in P} c_p \cdot x_p \leq B$$

2. High-Risk Project Budget Constraint:

$$\sum_{p \in P, \text{risk}_p = 1} c_p \cdot x_p \leq \alpha \cdot B$$

3. Logical Dependencies:

- If selecting Project A excludes selecting Project D:

$$x_A + x_D \leq 1$$

- If selecting Project B requires selecting Project E:

$$x_B \leq x_E$$

4. Synergy Condition:

$$y_{CF} \leq x_C, \quad y_{CF} \leq x_F, \quad y_{CF} \geq x_C + x_F - 1$$

5. Non-Negativity and Binary Constraints:

$$x_p \in \{0, 1\} \quad \forall p \in P$$

1.2

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Optimal solution found (tolerance 1.00e-04)
Best objective 3.471329000000e+06, best bound 3.471329000000e+06, gap 0.0000%
Selected projects: ['A', 'C', 'F', 'G', 'H', 'I', 'J']
Expected total return: 3471329.0
Total cost: 1994342
High-risk projects cost: 277106
```

Figure 1

Results Summary:

- **Selected Projects**: The optimal set of projects selected includes Projects A, C, F, G, H, I, and J.
- **Expected Total Return**: The expected total return from the selected projects is \$3,471,329.
- **Total Cost**: The total cost of the selected projects is \$1,994,342.
- **High-Risk Projects Cost**: The cost of the selected high-risk projects is \$277,106.

Interpretation and Conclusion:

The results show that GreenTech Ventures can get the highest return by choosing Projects A, C, F, G, H, I, and J. This plan stays within the total budget of \$2,000,000 and keeps the spending on high-risk projects at \$277,106, which is within the allowed 40% of the budget for high-risk investments. The combination of Projects C and F added an extra \$200,000 to the total return because of their synergy.

This plan is the best solution because it gives the highest possible return while meeting all the rules and limits, such as the budget and the allowed spending on high-risk projects. GreenTech Ventures should follow this plan to get the most yearly profit while staying within their budget and following investment rules.

2 Problem 2

2.1

Sets:

- T: Set of time periods (months), $T = \{1, 2, \ldots, 12\}$

Parameters:

- D_t: Demand in month t, $\forall t \in T$
- c_{in}: Cost per unit for in-house production (10/unit)
- h_{reg}: Regular labor hours per employee per month (160 hours)
- h_{ot}: Maximum overtime hours per employee per month (40 hours)
- p_h: Production hours required per unit (1.5 hours/unit)
- c_{reg}: Cost per regular labor hour (20/hour)
- c_{ot}: Cost per overtime labor hour (30/hour)
- c_{hire}: Hiring cost per employee (500/employee)
- c_{layoff}: Layoff cost per employee (750/employee)
- h_c: Holding cost per unit per month (2/unit/month)
- b_c: Backlog cost per unit per month (5/unit/month)

- I_0: Initial inventory (50 units)
- W_0: Initial workforce (10 employees)
- $H_{\max}$: Maximum number of employees that can be hired per month (5 employees)
- $L_{\max}$: Maximum number of employees that can be laid off per month (3 employees)

Decision Variables:

- W_t: Number of employees at the end of month t, $\forall t \in T$, $W_t \in Z_+$
- H_t: Number of employees hired in month t, $\forall t \in T$, $H_t \in Z_+$
- F_t: Number of employees laid off in month t, $\forall t \in T$, $F_t \in Z_+$
- $P_{\text{in},t}$: Number of units produced in-house in month t, $\forall t \in T$, $P_{\text{in},t} \in Z_+$
- I_t: Inventory level at the end of month t, $\forall t \in T$, $I_t \in Z_+$
- B_t: Backlog at the end of month t, $\forall t \in T$, $B_t \in Z_+$
- O_t: Overtime hours used in month t, $\forall t \in T$, $O_t \in R_+$

Objective Function:

$$\text{Minimize} \sum_{t \in T} (c_{\text{in}} P_{\text{in},t} + h_c I_t + b_c B_t + c_{\text{hire}} H_t + c_{\text{layoff}} F_t + c_{\text{reg}} \cdot h_{\text{reg}} \cdot W_t + c_{\text{ot}} O_t)$$

Constraints:

1. Initial Conditions:

$$W_1 = W_0, \quad I_1 = I_0, \quad B_{12} = 0$$

2. Workforce Balance:

$$W_t = W_{t-1} + H_t - F_t \quad \forall t \in \{2, \ldots, 12\}$$

3. Inventory Balance:

$$I_t = I_{t-1} - B_{t-1} + P_{\text{in},t} - D_t + B_t \quad \forall t \in T$$

4. Production Capacity:

$$P_{\text{in},t} \cdot p_h \leq W_t \cdot h_{\text{reg}} + O_t \quad \forall t \in T$$

5. Overtime Limit:

$$O_t \leq W_t \cdot h_{\text{ot}} \quad \forall t \in T$$

6. Hiring and Layoff Limits:

$$H_t \leq H_{\max}, \quad F_t \leq L_{\max} \quad \forall t \in T$$

7. Non-Negativity and Integer Constraints:

$$W_t, H_t, F_t, P_{\text{in},t}, I_t, B_t \geq 0 \quad \forall t \in T$$

2.2

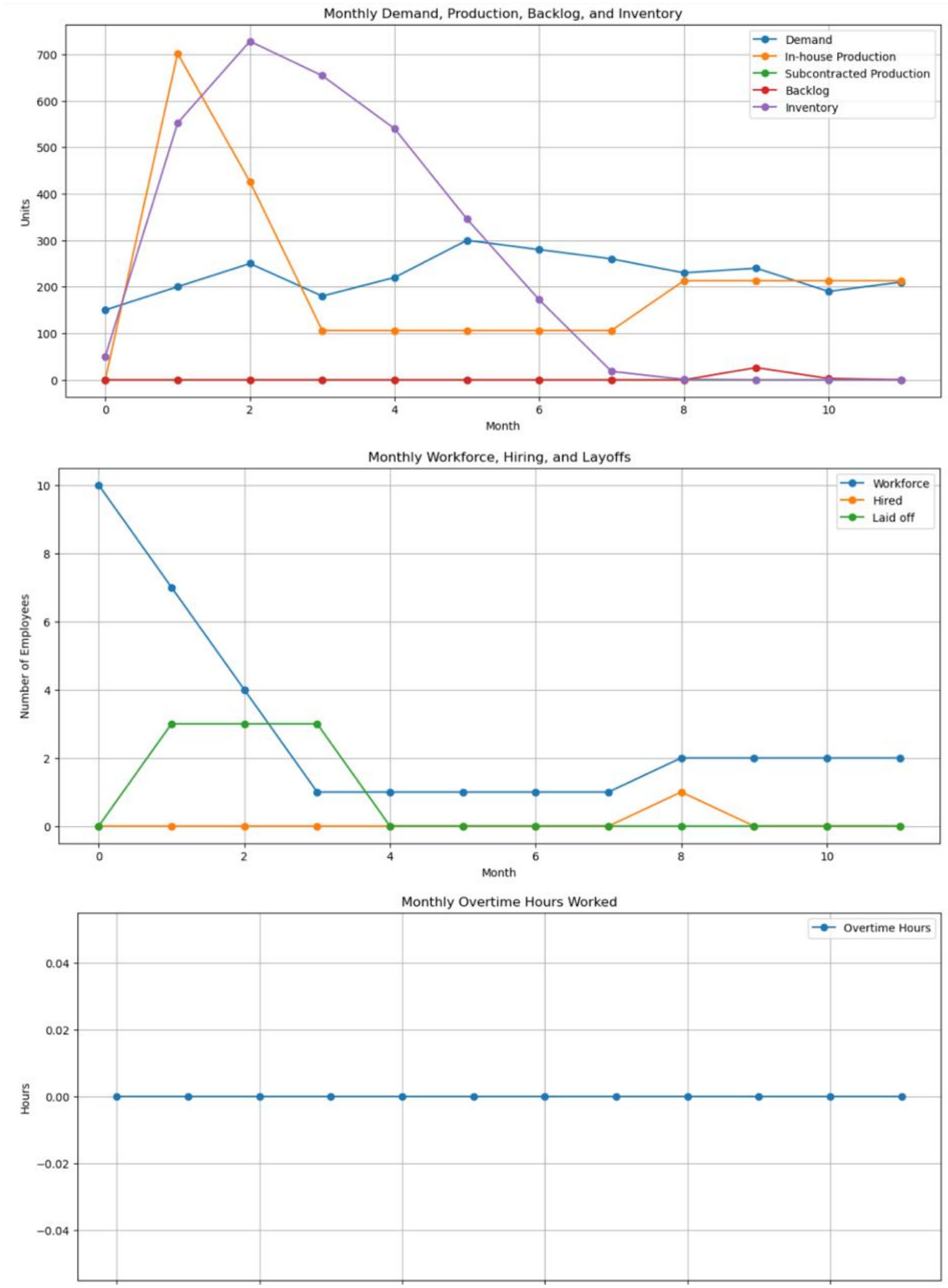

Figure 2: Plot of aggregate planning without subcontracting

2.3

Sets:

- T: Set of time periods (months), $T = \{1, 2, \ldots, 12\}$

Parameters:

- D_t: Demand in month t, $\forall t \in T$
- c_{in}: Cost per unit for in-house production (10/unit)
- c_{sub}: Cost per unit for subcontracting (5/unit)
- h_{reg}: Regular labor hours per employee per month (160 hours)
- h_{ot}: Maximum overtime hours per employee per month (40 hours)
- p_h: Production hours required per unit (1.5 hours/unit)
- c_{reg}: Cost per regular labor hour (20/hour)
- c_{ot}: Cost per overtime labor hour (30/hour)
- c_{hire}: Hiring cost per employee (500/employee)
- c_{layoff}: Layoff cost per employee (750/employee)
- h_c: Holding cost per unit per month (2/unit/month)
- b_c: Backlog cost per unit per month (5/unit/month)
- I_0: Initial inventory (50 units)
- W_0: Initial workforce (10 employees)
- $H_{\max}$: Maximum number of employees that can be hired per month (5 employees)
- $L_{\max}$: Maximum number of employees that can be laid off per month (3 employees)
- $U_{\max}$: Maximum total subcontracted units (600 units)

Decision Variables:

- W_t: Number of employees at the end of month t, $\forall t \in T$, $W_t \in Z_+$
- H_t: Number of employees hired in month t, $\forall t \in T$, $H_t \in Z_+$
- F_t: Number of employees laid off in month t, $\forall t \in T$, $F_t \in Z_+$
- $P_{\text{in},t}$: Number of units produced in-house in month t, $\forall t \in T$, $P_{\text{in},t} \in Z_+$
- $P_{\text{sub},t}$: Number of units subcontracted in month t, $\forall t \in T$, $P_{\text{sub},t} \in Z_+$
- I_t: Inventory level at the end of month t, $\forall t \in T$, $I_t \in Z_+$
- B_t: Backlog at the end of month t, $\forall t \in T$, $B_t \in Z_+$
- O_t: Overtime hours used in month t, $\forall t \in T$, $O_t \in R_+$
- Z_t: Binary variable indicating if subcontracting starts at month t, $\forall t \in \{1, \ldots, 10\}$, $Z_t \in \{0, 1\}$

Objective Function:

$$\text{Minimize} \sum_{t\in T}(c_{\text{in}}P_{\text{in},t} + c_{\text{sub}}P_{\text{sub},t} + h_c I_t + b_c B_t + c_{\text{hire}}H_t + c_{\text{layoff}}F_t + c_{\text{reg}} \cdot h_{\text{reg}} \cdot W_t + c_{\text{ot}}O_t)$$

Constraints:

1. Initial Conditions:

$$W_1 = W_0, \quad I_1 = I_0, \quad B_{12} = 0$$

2. Workforce Balance:

$$W_t = W_{t-1} + H_t - F_t \quad \forall t \in \{2, \ldots, 12\}$$

3. Inventory Balance:

$$I_t = I_{t-1} - B_{t-1} + P_{\text{in},t} - D_t + B_t \quad \forall t \in T$$

4. Production Capacity:

$$P_{\text{in},t} \cdot p_h \leq W_t \cdot h_{\text{reg}} + O_t \quad \forall t \in T$$

5. Overtime Limit:

$$O_t \leq W_t \cdot h_{\text{ot}} \quad \forall t \in T$$

6. Hiring and Layoff Limits:

$$H_t \leq H_{\max}, \quad F_t \leq L_{\max} \quad \forall t \in T$$

7. Total Subcontracting Limit:

$$\sum_{t\in T} P_{\text{sub},t} \leq U_{\max}$$

8. Subcontracting Activation:

$$\sum_{t=1}^{10} Z_t = 1$$

9. Three-Month Subcontracting Sum:

$$P_{\text{sub},t} + P_{\text{sub},t+1} + P_{\text{sub},t+2} \leq U_{\max} \cdot Z_t \quad \forall t \in \{1, \ldots, 10\}$$

10. Non-Negativity and Integer Constraints:

$$W_t, H_t, F_t, P_{\text{in},t}, P_{\text{sub},t}, I_t, B_t \geq 0, \quad Z_t \in \{0, 1\} \quad \forall t \in T$$

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Optimal Solution Found!
Total Cost: $147,417.00
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	Month	Workforce	Hired	Laid off	In-house Production	Inventory	Backlog	Overtime Hours	Demand
0	0	10.000000	-0.000000	-0.000000	-0.000000	50.000000	0.000000	0.000000	150
1	1	7.000000	-0.000000	3.000000	702.000000	552.000000	-0.000000	0.000000	200
2	2	4.000000	-0.000000	3.000000	426.000000	728.000000	-0.000000	0.000000	250
3	3	1.000000	-0.000000	3.000000	106.000000	654.000000	-0.000000	0.000000	180
4	4	1.000000	-0.000000	0.000000	106.000000	540.000000	-0.000000	0.000000	220
5	5	1.000000	-0.000000	0.000000	106.000000	346.000000	-0.000000	0.000000	300
6	6	1.000000	0.000000	-0.000000	106.000000	172.000000	-0.000000	0.000000	280
7	7	1.000000	-0.000000	-0.000000	106.000000	18.000000	-0.000000	0.000000	260
8	8	2.000000	1.000000	-0.000000	213.000000	1.000000	-0.000000	0.000000	230
9	9	2.000000	-0.000000	-0.000000	213.000000	-0.000000	26.000000	0.000000	240
10	10	2.000000	0.000000	-0.000000	213.000000	-0.000000	3.000000	0.000000	190
11	11	2.000000	-0.000000	-0.000000	213.000000	0.000000	0.000000	0.000000	210

Figure 3: Aggregate planning without subcontracting

2.4

Total Cost: $90,550.00

	Month	Workforce	Hired	Laid off	In-house Production	Subcontracted Production	Inventory	Backlog	Overtime Hours	Demand
0	0	10.000000	-0.000000	-0.000000	-0.000000	-0.000000	50.000000	0.000000	0.000000	150
1	1	7.000000	-0.000000	3.000000	-0.000000	150.000000	0.000000	0.000000	0.000000	200
2	2	4.000000	-0.000000	3.000000	-0.000000	250.000000	0.000000	0.000000	0.000000	250
3	3	1.000000	-0.000000	3.000000	-0.000000	180.000000	0.000000	0.000000	0.000000	180
4	4	-0.000000	-0.000000	1.000000	-0.000000	220.000000	0.000000	0.000000	0.000000	220
5	5	-0.000000	-0.000000	-0.000000	-0.000000	300.000000	0.000000	0.000000	0.000000	300
6	6	-0.000000	-0.000000	-0.000000	-0.000000	280.000000	0.000000	0.000000	0.000000	280
7	7	-0.000000	-0.000000	-0.000000	-0.000000	260.000000	0.000000	0.000000	0.000000	260
8	8	-0.000000	-0.000000	-0.000000	-0.000000	230.000000	0.000000	0.000000	0.000000	230
9	9	-0.000000	-0.000000	-0.000000	-0.000000	240.000000	0.000000	0.000000	0.000000	240
10	10	0.000000	-0.000000	0.000000	-0.000000	190.000000	0.000000	0.000000	0.000000	190
11	11	0.000000	-0.000000	-0.000000	-0.000000	210.000000	0.000000	0.000000	0.000000	210

Figure 4: Aggregate planning allowing subcontracting without restrictions

Total Cost: $129,160.00

	Month	Workforce	Hired	Laid off	In-house Production	Subcontracted Production	Inventory	Backlog	Overtime Hours	Demand
0	0	10.000000	-0.000000	-0.000000	-0.000000	-0.000000	50.000000	0.000000	0.000000	150
1	1	7.000000	0.000000	3.000000	742.000000	-0.000000	592.000000	-0.000000	0.000000	200
2	2	4.000000	0.000000	3.000000	426.000000	-0.000000	768.000000	-0.000000	0.000000	250
3	3	1.000000	0.000000	3.000000	106.000000	0.000000	694.000000	-0.000000	0.000000	180
4	4	1.000000	0.000000	0.000000	106.000000	0.000000	580.000000	-0.000000	0.000000	220
5	5	1.000000	0.000000	0.000000	106.000000	0.000000	386.000000	-0.000000	0.000000	300
6	6	1.000000	0.000000	0.000000	106.000000	0.000000	212.000000	-0.000000	0.000000	280
7	7	1.000000	0.000000	0.000000	106.000000	0.000000	58.000000	0.000000	0.000000	260
8	8	1.000000	0.000000	0.000000	106.000000	0.000000	-0.000000	66.000000	0.000000	230
9	9	1.000000	0.000000	0.000000	106.000000	0.000000	0.000000	200.000000	0.000000	240
10	10	0.000000	0.000000	1.000000	0.000000	0.000000	-0.000000	390.000000	0.000000	190
11	11	0.000000	0.000000	0.000000	0.000000	600.000000	0.000000	0.000000	0.000000	210

Figure 5: Aggregate planning allowing subcontracting only with restriction of 600 units

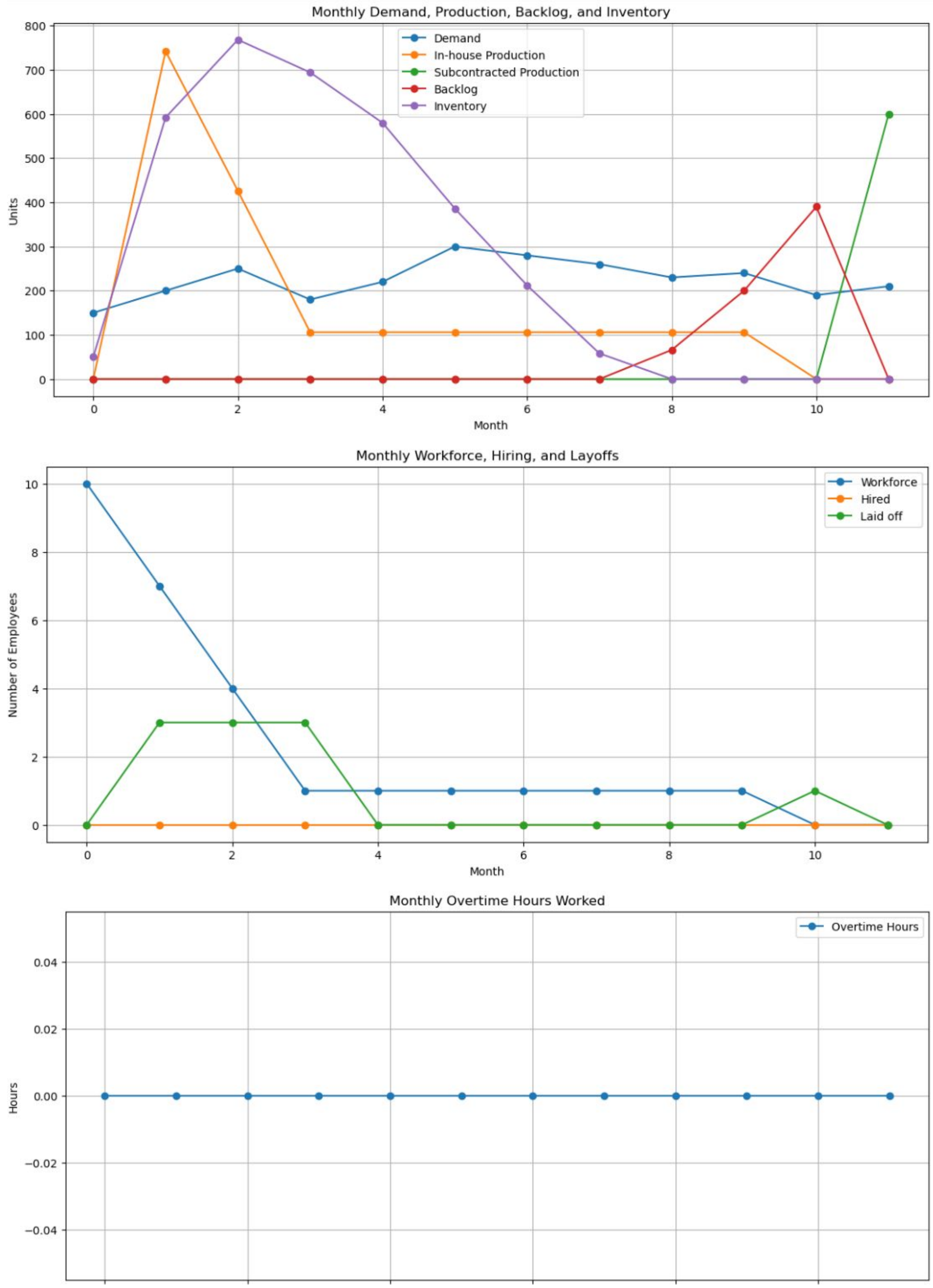

Figure 6: Plot of aggregate planning allowing subcontracting only with restriction of 600 units

Conclusion:

Both scenarios are very similar, with the latter having subcontracting units in the last month, lowering the cost and thus being preferable.