ISE 4623/5023: Deterministic Systems Models / Systems Optimization University of Oklahoma School of Industrial and Systems Engineering Fall 2024

Individual Assignment 1: Linear Algebra Basics and Mathematical Notation

1. (18 points) Let

$$A = \begin{bmatrix} 4 & 1 & 8 \\ 2 & -7 & 5 \end{bmatrix}, B = \begin{bmatrix} -9 & 7 \\ 1 & 5 \\ 8 & 3 \end{bmatrix}$$

   Solve:

   (a) (3 points) AB
   (b) (3 points) BA
   (c) (3 points) $-B$
   (d) (3 points) $B - A$
   (e) (3 points) A^T
   (f) (3 points) $-B^T B$

2. (21 points) If possible, solve for x and y. If they can't be computed, give a brief explanation of why:

   (a) (7 points)
$$3x + y = 5$$
$$8x + 5y = 2$$
   (b) (7 points)
$$15x + 5y = 40$$
$$3x - y = 8$$
   (c) (7 points)
$$2x - 24y = 42$$
$$-x + 12y = -20$$

3. (20 points) Let:

$$C = \begin{bmatrix} 3 & -2 \\ 12 & -8 \end{bmatrix}, D = \begin{bmatrix} -1 & 4 \\ 6 & -8 \end{bmatrix}, E = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} F = \begin{bmatrix} -3 & 5 & 9 \\ 8 & -2 & 5 \\ 1 & -2 & 14 \end{bmatrix}$$

   Solve:

   (a) (5 points) C^{-1}
   (b) (5 points) D^{-1}
   (c) (5 points) E^{-1}
   (d) (5 points) F^{-1}

4. (15 points) Translate into words:

   (a) (5 points) $\sum_{j \in \mathbb{R}} j$
   (b) (5 points) $\sum_{j \in \mathbb{Z}^+ | j \bmod 5 = 0} j$
   (c) (5 points) $\sum_{\substack{j=1 \\ j \in \mathbb{Z}}}^{40} j$

5. (12 points) Translate into mathematical expression:
   (a) (4 points) Summation of every element j over the set J such that the element is odd.
   (b) (4 points) Summation of every variable x_{ij} over j that belongs to the set J for all i that belongs to the set I.
   (c) (4 points) Summation of every variable x_{ij} over j that belongs to the set J and i that belongs to the set I, if the pair (i, j) belongs to the set of pairs A.
6. (14 points) Expand the expressions:
   (a) (7 points) $\sum_{j \in \{0,1,\ldots,12\}: j \bmod 2=0} j$
   (b) (7 points) Let

$$N = \{1, 2, 3, 4\},\ A = \{(1, 2), (2, 3), (2, 4), (3, 4), (2, 1)\},\ D = \{3\},\ S = \{1, 2\},\ T = \{4\}$$

$$b_i = \begin{cases} \alpha_i & i \in S \\ \beta_i & i \in D \\ 0 & i \in T \end{cases} \quad \forall\, i \in N$$

   Expand:

$$\sum_{j \in N:(i,j) \in A} x_{ij} - \sum_{j \in N:(j,i) \in A} x_{ji} = b_i \ \forall\, i \in N$$

The Greeks, just to keep in mind:

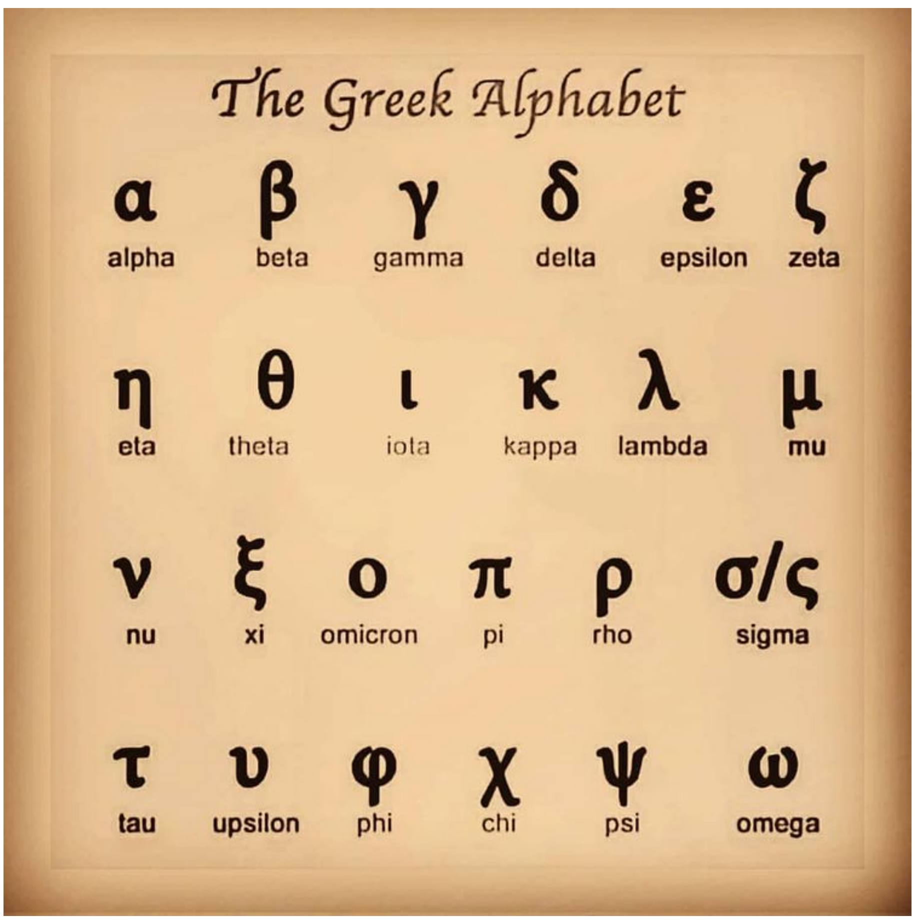

Figure 1: Retrieved from `https://www.instagram.com/joshi_physics_classes/`