

# ISE 4623/5023: Deterministic Systems Models / Systems Optimization University of Oklahoma School of Industrial and Systems Engineering Fall 2024

## Individual Assignment 2: Graphical Method (100 points)

1. (70 points) Sooners Inc., a leading company in the house decorating and maintenance industry, is expanding its product line by launching a new branch dedicated to paint production. The company has decided to begin by manufacturing two paint colors: red and blue. Each gallon of red paint will be sold for \$10, while each gallon of blue paint will be sold for \$12.

The production process for these paints requires specific resources. Each gallon of red paint requires 1 kilogram of pigments and 2 gallons of water. Similarly, each gallon of blue paint requires 2 kilograms of pigments and 2 gallons of water. However, the company faces resource constraints, having only 20 gallons of water and 25 kilograms of pigments available.

Additionally, due to a partnership agreement with a local store, Sooners Inc. has committed to producing at least 2 gallons of each paint color.

You have been hired to develop a production plan that will maximize the company's profit. To solve this, you first decide to formulate this problem as an LP model. In particular:

- (a) (10 points) Define the decision variables for this problem. What is its gradient?
  - (b) (10 points) Plot the gradient and the feasible region (clearly indicating all the constraints and "shading" the feasible region). Solve this problem using the graphical method, indicating the values of all the variables and the objective function associated with the optimal solution.
  - (c) (5 points) Explain the concept of isoclines. How do isoclines relate to the solution of the problem?
  - (d) (10 points) Solve the problem using Excel Solver and indicate the values of all the variables and the objective function associated with the optimal solution. Compare your results with part b. Include a snapshot of your Excel model (Excel cells and the Solver window).
  - (e) (10 points) Solve the problem using Gurobi/Python. Compare your results with parts b and c. Include a snapshot of your Gurobi/Python code and obtained results.
  - (f) (25 points) After the successful launch of the new product line, Sooners Inc. has decided to invest more in it. Consequently, they have decided to expand their product lineup by adding more paint colors. For each paint color  $p \in P$ , they expect to charge  $r_p$  dollars per gallon, and each gallon of paint requires  $w_p$  gallons of water and  $i_p$  kilograms of pigments. The new resource limits are 1000 gallons of water and 600 kilograms of pigments. They have entered into new agreements with local stores, which require them to produce at least  $\beta_p$  gallons of each paint color  $p$ . Additionally, because the original paint colors have proven to be stellar products, a new rule has been added: they must produce at least 100 gallons more of the original colors than of the new ones. Formulate the updated mathematical model to maximize Sooners Inc.'s profit, considering the new constraints and expanded product line.
2. (35 points) As a financial advisor, your role is to help clients achieve their savings goals while minimizing the cost of their total investment. Today, a couple has approached you to create an investment portfolio consisting of stocks from Company A and Company B. Their goal is to make at least \$20 in profit by the end of the year, in addition to recovering their initial investment. You know that for each dollar invested in Stock A, they will receive \$1.50 at the end of the year, while each dollar invested in Stock B will yield \$1.20. Due to regulatory constraints, they are allowed to invest at most 1.5 times more in Stock A than in Stock B. Additionally, the couple prefers Stock B for its lower risk, despite its lower return, and has requested that at least \$5 be invested in Stock B.

The total budget available for their investment is \$100. Your task is to determine the minimum-cost portfolio that meets all these requirements.

- (a) (10 points) Define the decision variables for this problem. What is its gradient?

- (b) (5 points) Plot the gradient and the feasible region (clearly indicating all the constraints and “shading” the feasible region). Solve this problem using the graphical method, indicating the values of all the variables and the objective function associated with the optimal solution.
- (c) (10 points) Solve the problem using Excel Solver and indicate the values of all the variables and the objective function associated with the optimal solution. Compare your results with part c. Include a snapshot of your Excel model (Excel cells and the Solver window).
- (d) (10 points) Solve the problem using Gurobi/Python. Compare your results with parts b and c. Include a snapshot of your Gurobi/Python code and obtained results.