ISE4623/5023: Deterministic Systems Models/Systems Optimization University of Oklahoma School of Industrial and Systems Engineering Fall 2024

Individual Assignment 5: Data Envelopment Analysis and General Formulation

Instructions for Assignment Submission:

1. The assignment must be uploaded to Canvas by the indicated due date. The solution should be uploaded to Canvas as a PDF file.
2. Along with your main submission (in PDF format), please ensure you also attach any supplementary files (Excel, Python) that were instrumental in your work. If any Python code was used, upload it in HTML format.
3. If you've used Python or Excel, **kindly incorporate all relevant screenshots into your primary submission**. Phrases like *See Excel sheet* or *solved in Python* won't be accepted. While you're encouraged to use these tools, your primary submission (the PDF) should be comprehensive enough for grading.
4. Always label your answers with the corresponding problem number for clarity.

Scoring: (100 points + 20 points extra credit/mandatory for graduate students + 10 points extra credit for all)

1. (60 points + 10 points extra credit/mandatory for graduate students) Sooners Inc., a retail company specializing in home decoration items, has recently faced challenges with several new product lines that failed to meet market expectations. As a result, the company has decided to halt the development of new products and focus instead on optimizing its existing expenditure structure. The management believes that by identifying and addressing inefficient spending, they can improve overall company performance. To achieve this, they have gathered data on 40 companies within the same sector, capturing information on their expenditure and performance metrics. Your task is to use Data Envelopment Analysis (DEA) in conjunction with the information provided in the Excel file **DEA DATA.xlsx** to assess the efficiency of these companies.

   (a) (15 points) Using the dataset provided, consider the revenue column as the output and the workers, publicity, and inventory management columns as the input variables. Your task is to analyze the efficiency of Sooners Inc (Company ID 18) by visualizing the relationship between the input and output variables.

      i. Create scatter plots of the output variable (revenue) against each input variable (workers, publicity, and inventory management). Ensure that each plot clearly shows how the input relates to the output.

      ii. Based on the visual trends in each plot, determine whether Sooners Inc (Company ID 18) is efficient in comparison to other companies, and justify your position.

   (b) (15 points) Based on your findings from Question 1, formulate an output-oriented DEA model to test whether your conclusion about Sooners Inc.'s efficiency (Company ID 18) was correct. Use the revenue column as the output and the workers, publicity, and inventory management columns as the inputs. Construct the mathematical formulation of the output-oriented DEA model. Clearly define the objective function and constraints, including the variables representing the $\lambda's$ and ϕ (growing factor). Use either Excel Solver or GurobiPy to solve the DEA model. What can you conclude from this exercise?

   (c) (15 points) Based on the results from part (b), analyze the slack variables obtained in the solution for the maximization of the growth factor for Sooners Inc (Company ID 18).

      i. Interpret the Slack Variables: Discuss what the slack variables represent in the context of the DEA solution. What do these values suggest about the inefficiencies in Sooners Inc's use of resources (workers, publicity, and inventory management)?

ii. Improving Resource Allocation: Using the growth factor obtained from the output-oriented DEA model as a parameter, propose a formulation to find the best resource allocation strategy that would allow Sooners Inc to achieve the same level of output (revenue). Solve the new resource allocation problem using either Excel Solver or GurobiPy. Present your solution, including the new input levels (workers, publicity, and inventory management) and any efficiency improvements achieved.

(d) (15 points) After reviewing your previous analysis, Sooners Inc. identified a new output variable that must be considered: the proportion of revenue that comes from cash transactions. This variable is important due to their efforts to reduce the reliance on cash-based revenue streams.

i. Please modify the output-oriented DEA model you designed in part (b) to incorporate a new output variable: the proportion of revenue from cash transactions. The objective of the model should now be the *minimization* of this proportion, while still maintaining the same efficient revenue you found earlier in (b). In addition, could you explain how incorporating this new output variable (the proportion of revenue from cash transactions) affects the structure of the DEA model?

ii. Solve the output-oriented problem using either Excel Solver or GurobiPy. Could you compare the results from the updated model with your original solution? Is Sooners Inc (Company ID 18) still efficient?

(e) (10 points - extra credit/mandatory for graduate students) Construct the Dual problem of the general formulation of the DEA output-oriented model.

2. (40 points + 10 points extra credit/mandatory for graduate students + 10 points extra credit for all) Oklahoma Brewery is planning its supply strategy for the next shipment to its customers. The brewery operates from four different factories, located in A, B, C, and D, each with the following stock of beer cans ready for delivery:

- Factory A: 100 cans
- Factory B: 200 cans
- Factory C: 150 cans
- Factory D: 175 cans

The demands of the 15 customers and the cost incurred for each undelivered unit of demand are shown in the table below:

Customer	**Demand**	**Cost of Undelivered Unit**
1	97	100
2	24	100
3	19	100
4	17	100
5	64	500
6	73	100
7	19	100
8	98	100
9	87	100
10	79	800
11	17	300
12	72	100
13	39	100
14	96	100
15	53	100

The table below shows the cost matrix for delivering beer from each factory (A, B, C, D) to each customer:

Customer	A	B	C	D
1	5	2	3	4
2	6	6	5	6
3	4	6	5	1
4	2	2	8	3
5	5	7	7	10
6	2	1	7	5
7	1	4	3	4
8	9	10	9	9
9	7	9	8	6
10	10	1	5	8
11	5	4	6	4
12	2	10	10	8
13	6	3	3	8
14	4	7	7	2
15	2	3	4	5

(a) (40 points) Formulate the minimum cost flow problem, assuming that each of the factories (A, B, C, D) has an unlimited number of beer cans available in storage. The goal is to minimize the total transportation cost of delivering beer from the factories to the customers, based on the cost matrix provided in the problem description.

i. Formulate the minimum cost flow problem. Define sets, parameters, decision variables, objective function, and constraints.

ii. Solve this minimum cost flow problem using either Python or Excel. In your solution, include a table that shows the number of units delivered from each factory to each customer.

(b) (10 points - Mandatory for graduate students/extra credit for undergraduate) In part (a), we assumed that each factory had unlimited supply. Now, consider that each factory has a distribution capacity limit as specified below:

- Factory A: 100 units
- Factory B: 200 units
- Factory C: 150 units
- Factory D: 175 units

Additionally, there is a cost associated with each undelivered unit of demand, as provided in the problem description.

i. Update your formulation from part (a) to include the distribution capacity constraints for each factory and the cost of undelivered units.

ii. Solve the updated model using either Python or Excel, ensuring the capacity constraints and undelivered unit costs are accounted for. How did your mathematical formulation change after adding the capacity constraints and undelivered unit costs? How did the delivery strategy change compared to part (a)?

(c) (10 points - extra credit for all) Construct the Dual problem of the general formulation of problem 2b. Solve it using Gurobi, show the results, and discuss their meaning in the context of this problem.