ISE 4623/5023: Deterministic Systems Models / Systems Optimization University of Oklahoma School of Industrial and Systems Engineering Fall 2024

Instructions for Assignment Submission:

1. The assignment must be uploaded to Canvas by the indicated due date. The solution should be uploaded to Canvas as a PDF file.
2. Along with your main submission (in PDF format), please ensure you also attach any supplementary files (Excel, Python) that were instrumental in your work. If any Python code was used, upload it in HTML format.
3. If you've used Python or Excel, **kindly incorporate all relevant screenshots into your primary submission**. Phrases like *See Excel sheet* or *solved in Python* won't be accepted. While you're encouraged to use these tools, your primary submission (the PDF) should be comprehensive enough for grading.
4. Always label your answers with the corresponding problem number for clarity.

Individual Assignment 7: Knapsack and Production Modeling

1. (50 points) GreenTech Ventures is considering diversifying its investments into various renewable energy projects. They have a budget of $2 million USD. They are looking at several potential renewable energy projects, each with an associated cost (in thousands of USD) and an expected yearly return (in thousands of USD) based on government subsidies and market demand for green energy.

   The available projects vary in scale and technology (solar, wind, geothermal) for which the have different risk level. No more than 40% of the total budget can be allocated to high-risk projects.

   Furthermore, there are certain logical dependencies between projects:

   - If GreenTech Ventures selects Project A, they cannot select Project D.
   - If GreenTech Ventures selects Project B, they must also select Project E.

   To encourage strategic investment, GreenTech Ventures also recognizes synergies between certain projects. Specifically, if both Projects C and F are selected, there is a boost to their combined returns due to the complementary nature of the technologies. This synergy results in an additional $200,000 USD in annual returns if both projects are chosen.

   The goal is to determine how GreenTech Ventures can allocate its budget across different projects to maximize expected yearly returns while considering the constraints of the budget, risk considerations, and synergies.

   You are provided with a dataset (`GreenTech.xlsx`) that lists the cost, expected return, risk category (high or low).

   (a) (25 points) Formulate the optimization model, define the following components clearly:
      i. Sets
      ii. Parameters
      iii. Decision variables
      iv. Objective function
      v. Constraints

   (b) (25 points) Solve the problem using Gurobi: Solve the optimization model and find the optimal investment strategy. Present the results clearly, including:
      - Which projects where selected
      - The expected total return
      - Interpretation and conclusion in terms of the context of the problem.

2. (50 points) Gourmet Granola, a small artisan company specializing in high-quality granola, has reached out to you to help plan its production and inventory scheduling over the next 12 months. The company needs to decide how many units of granola should be produced and packed in-house or subcontracted to a third party to satisfy a proportion of the orders expected at the end of each month. Since 1 unit equals 1 order, fractional units cannot be ordered or produced.

   Gourmet Granola starts with an initial inventory of 50 units. Any orders not fulfilled at the end of the month will be carried forward as a backlog and must be satisfied in future months. Additionally, any unsold inventory will be stored for future demand.

   The company employs 10 workers who can be hired or laid off each month depending on demand. Each worker contributes a specific number of labor hours, and production requires a certain number of hours per unit. Employees can also work overtime, but overtime hours are capped each month.

   Tables 1 through 5 provide the necessary data for workforce dynamics, demand, costs, and production constraints.

Table 1: Monthly Demand (Units)

Month	1	2	3	4	5	6	7	8	9	10	11	12
Demand	150	200	250	180	220	300	280	260	230	240	190	210

Table 2: Production Costs

Type	Cost per Unit
In-house Production	\$10
Subcontracting	\$5

Table 3: Workforce Details

Parameter	Value
Initial Workforce	10 employees
Labor Hours per Employee (regular)	160 hours/month
Labor Hours per Employee (overtime)	40 hours/month (max)
Production Hours per Unit	1.5 hours/unit
Cost per Employee (regular hours)	\$20/hour
Cost per Employee (overtime hours)	\$30/hour
Hiring Cost per Employee	\$500/employee
Layoff Cost per Employee	\$750/employee

Table 4: Inventory and Backlog Costs

Parameter	Value
Initial Inventory	50 units
Holding Cost per Unit/month	\$2/unit/month
Backlog Cost per Unit/month	\$5/unit/month

Table 5: Hiring and Layoff Constraints

Parameter	Value
Maximum Hiring per Month	5 employees
Maximum Layoffs per Month	3 employees

   (a) (25 points) Write the associated mathematical model and solve it using Gurobi/Python. What is the optimal value of the objective function? Define the following clearly:

      i. Sets

ii. Parameters
iii. Decision variables
iv. Objective function (minimization of total costs)
v. Constraints

(b) (15 points) Having solved your optimization model, plot the following metrics as a function of the month and analyze the results:

i. Monthly demand, granola unit production, backlog of orders, and inventory at the end of each month.
ii. Monthly workforce size, monthly hiring, and monthly layoffs.
iii. Monthly number of overtime hours worked.

Discuss any trends or observations you find in the plots.

(c) (10 points) Assume the company can subcontract up to 600 units from a partner, but those units can only be used across a consecutive time window of three months (i.e., if they subcontract in January, they can get up to 600 units between January and March, and those units cannot be stored as part of the inventory).

i. Modify the mathematical model from part (a) to include this constraint and solve the new model using Gurobi/Python.
ii. How does this constraint change the original solution? Show the same 3 plots and interpret the results.
iii. Are there any similarities between this solution and the solution from part (c)?