

ISE 4623/5023: Deterministic Systems Models / Systems Optimization
University of Oklahoma School of Industrial and Systems Engineering Fall 2024

Individual Assignment 3: Simplex and Initialization Methods (100 points)

1. (35 points) Sooners Inc., a leading company in the house decorating and maintenance industry, is expanding its product line by launching a new branch dedicated to paint production. The company has decided to begin by manufacturing two paint colors: red and blue. Each gallon of red paint will be sold for \$10, while each gallon of blue paint will be sold for \$12.

The production process for these paints requires specific resources. Each gallon of red paint requires 1 kilogram of pigments and 2 gallons of water. Similarly, each gallon of blue paint requires 2 kilograms of pigments and 2 gallons of water. However, the company faces resource constraints, having only 20 gallons of water and 25 kilograms of pigments available.

Additionally, due to a partnership agreement with a local store, Sooners Inc. has committed to producing at least 2 gallons of each paint color.

You have been hired to develop a production plan that will maximize the company's profit. To solve this, you first decide to formulate this problem as an LP model. In particular:

- (a) (15 points) Based on the solution of Assignment 2, Problem 1:

Let x_R be the amount red paint gallons produced, and x_B be the amount blue paint gallons produced

$$\text{Maximize} \quad 10x_R + 12x_B$$

st:

$$x_R + 2x_B \leq 25$$

$$2x_R + 2x_B \leq 20$$

$$x_R \geq 2$$

$$x_B \geq 2$$

$$x_R \geq 0, \quad x_B \geq 0$$

Write this problem in its standard form.

- (b) (20 points) Assume that they decide to produce 2 gallons of each paint color. Is this solution optimal? why or why not? if this solution is not optimal, find the basic solution associated with these variables' values and use it (as an initial feasible solution) in the Simplex Algorithm to find the optimal solution to the problem. Use Excel and indicate the values of all the variables and the objective function associated with the initial basic solution and the optimal solution in case the initial basic solution is not optimal. Include a snapshot of your procedure in Excel.

2. (35 points) As a financial advisor, your role is to help clients achieve their savings goals while minimizing the cost of their total investment. Today, a couple has approached you to create an investment portfolio consisting of stocks from Company A and Company B. Their goal is to make at least \$20 in profit by the end of the year, in addition to recovering their initial investment. You know that for each dollar invested in Stock A, they will receive \$1.50 at the end of the year, while each dollar invested in Stock B will yield \$1.20. Due to regulatory constraints, they are allowed to invest at most 1.5 times more in Stock A than in Stock B. Additionally, the couple prefers Stock B for its lower risk, despite its lower return, and has requested that at least \$5 be invested in Stock B.

The total budget available for their investment is \$100. Your task is to determine the minimum-cost portfolio that meets all these requirements.

- (a) (9 points) Based on the solution of Assignment 2, Problem 2:

Let x_A be the amount of dollars invested in Stock A, and x_B be the amount of dollars invested in Stock B.

$$\text{Minimize } x_A + x_B$$

st:

$$0.5x_A + 0.2x_B \geq 20$$

$$x_A \leq 1.5x_B$$

$$x_B \geq 5$$

$$x_A + x_B \leq 100$$

$$x_A \geq 0, \quad x_B \geq 0$$

Write this problem in its standard form.

- (b) (9 points) Find an initial basic solution using the big M initialization method. Show the problem's formulation under this initialization method. Use Excel and include snapshots of your procedure.
- (c) (9 points) Find an initial basic solution using the two-phase initialization method. Show the problem's formulation under this initialization method. Use Excel and include snapshots of your procedure.
- (d) (8 points) Starting from one of the initial basic solutions found in either (b) or (c), solve this problem using the Simplex algorithm. Indicate the values of all the variables and the objective function associated with the optimal solution. Use Excel and include snapshots of your procedure.
3. (30 points) Given the context from Problem 2. After reviewing your initial investment recommendation, the couple became concerned about the risks associated with their portfolio. To mitigate this risk, they have decided to diversify their investments further. They have requested that you create a new investment portfolio, this time including all stocks from a broader set S . The couple now asks you to achieve an aggregate profit amounting to at least P .

To manage risk, they have imposed additional constraints:

- You may not invest more than 30% more in any one stock than in any other stock.
- Among the available stocks, there are certain "safe" stocks with minimal risk, and you are required to invest at least m dollars in each of these safe stocks.
- The total new budget available for investment is B dollars.

- (a) (15 points) Formulate the problem mathematically in a general manner.

- (b) (15 points) Provide the standard form of the general mathematical formulation for the problem.