In Class Exercise for ODE with I.C.

Problem 1

- The outbreak of an insect population can be modeled with the equation below.
- R=growth rate
- C=carrying capacity
- N=# of insects
- N_c=critical population
- Second term is due to bird predation

$$\frac{dN}{dt} = RN\left(1 - \frac{N}{C}\right) - \frac{rN^2}{N_c^2 + N^2}$$

Parameters

- $0<t<50$ days
- R=0.55 /day
- N(0)=10,000
- C=10,000
- N_c=10,000
- r=10,000 /day

- What is steady state population?
- How long does it take to get there?

$$\frac{dN}{dt} = RN\left(1 - \frac{N}{C}\right) - \frac{rN^2}{N_c^2 + N^2}$$

Problem 2

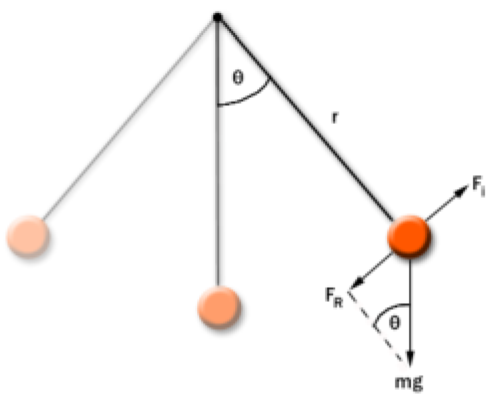
- A rocket's mass decreases as it burns fuel
- Find the final velocity of a rocket if:
- T=48000 N; m_0=2200 kg
- R=0.8; g=9.81 m/s^2; b=40 s

$$\boxed{\begin{aligned} m\frac{dv}{dt} &= T - mg \\ m &= m_0\left(1 - \frac{rt}{b}\right) \end{aligned}}$$

Problem 3 Nonlinear pendulum

- r=1 m; g=9.81 m/s^2
- Initial angle =π/8, π/2, π-0.1

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{r}\sin(\theta)}$$



Problem 4

- Consider an ecosystem of rabbits r and foxes f. Rabbits are fox food.
- Start with 300 rabbits and 150 foxes
- α=0.01

$$\boxed{\begin{aligned}\frac{dr}{dt} &= 2r - \alpha rf \\ \frac{df}{dt} &= -f + \alpha rf\end{aligned}}$$