Assignment for linear equations

Problem 1 Solve the following problems using matrix inversion. Check your solutions.

a.
$$\begin{aligned} 2x + y &= 5 \\ 3x - 9y &= 7 \end{aligned}$$

b.
$$\begin{aligned} -8x - 5y &= 4 \\ -2x + 7y &= 10 \end{aligned}$$

c.
$$\begin{aligned} 12x - 5y &= 11 \\ -3x + 4y + 7x_3 &= -3 \\ 6x + 2y + 3x_3 &= 22 \end{aligned}$$

d.
$$\begin{aligned} 6x - 3y + 4x_3 &= 41 \\ 12x + 5y - 7x_3 &= -26 \\ -5x + 2y + 6x_3 &= 16 \end{aligned}$$

Problem 2

a. Solve the following matrix equation for the matrix C

$$\mathbf{A}(\mathbf{BC} + \mathbf{A}) = \mathbf{B}$$

b. Evaluate the solution obtained in part a for this case

$$\mathbf{A} = \begin{bmatrix} 7 & 9 \\ -2 & 4 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 4 & -3 \\ 7 & 6 \end{bmatrix}$$

Problem 3 Use MATLAB to solve the following problems.

a.
$$\begin{aligned} -2x + y &= -5 \\ -2x + y &= 3 \end{aligned}$$

b.
$$\begin{aligned} -2x + y &= 3 \\ -8x + 4y &= 12 \end{aligned}$$

c.
$$\begin{aligned} -2x + y &= -5 \\ -2x + y &= -5.00001 \end{aligned}$$

d.
$$\begin{aligned} x_1 + 5x_2 - x_3 + 6x_4 &= 19 \\ 2x_1 - x_2 + x_3 - 2x_4 &= 7 \\ -x_1 + 4x_2 - x_3 + 3x_4 &= 30 \\ 3x_1 - 7x_2 - 2x_3 + x_4 &= -75 \end{aligned}$$

Problem 4

a. Use MATLAB to solve the following equations for x,y, and z as functions of the parameter c.

$$\begin{aligned} x - 5y - 2z &= 11c \\ 6x + 3y + z &= 13c \\ 7x + 3y - 5z &= 10c \end{aligned}$$

b. Plot the solutions for x,y, and z versus c on the same plot, for $-10 \leq c \leq 10$

Problem 5

Figure P7 illustrates a robot arm that has two "links" connected by two "joints"—a shoulder or base joint and an elbow joint. There is a motor at each joint. The joint angles are θ_1 and θ_2. The (x, y) coordinates of the hand at the end of the arm are given by

$$x = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$$

$$y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$$

where L_1 and L_2 are the lengths of the links.

Polynomials are used for controlling the motion of robots. If we start the arm from rest with zero velocity and acceleration, the following polynomials are used to generate commands to be sent to the joint motor controllers

$$\theta_1(t) = \theta_1(0) + a_1 t^3 + a_2 t^4 + a_3 t^5$$

$$\theta_2(t) = \theta_2(0) + b_1 t^3 + b_2 t^4 + b_3 t^5$$

where $\theta_1(0)$ and $\theta_2(0)$ are the starting values at time $t = 0$. The angles $\theta_1(t_f)$ and $\theta_2(t_f)$ are the joint angles corresponding to the desired destination of the arm at time t_f. The values of $\theta_1(0)$, $\theta_2(0)$, $\theta_1(t_f)$, and $\theta_2(t_f)$ can be found from trigonometry, if the starting and ending (x, y) coordinates of the hand are speci ed.

a. Set up a matrix equation to be solved for the coef cients a_1, a_2, and a_3, given values for $\theta_1(0)$, $\theta_1(t_f)$, and t_f. Obtain a similar equation for the coef cients b_1, b_2, and b_3.

b. Use MATLAB to solve for the polynomial coef cients given the values $t_f = 2$ sec, $\theta_1(0) = -19°$, $\theta_2(0) = 44°$, $\theta_1(t_f) = 43°$, and $\theta_2(t_f) = 151°$. (These values correspond to a starting hand location of $x = 6.5$, $y = 0$ ft and a destination location of $x = 0$, $y = 2$ ft for $L_1 = 4$ and $L_2 = 3$ ft.)

c. Use the results of part *b* to plot the path of the hand.

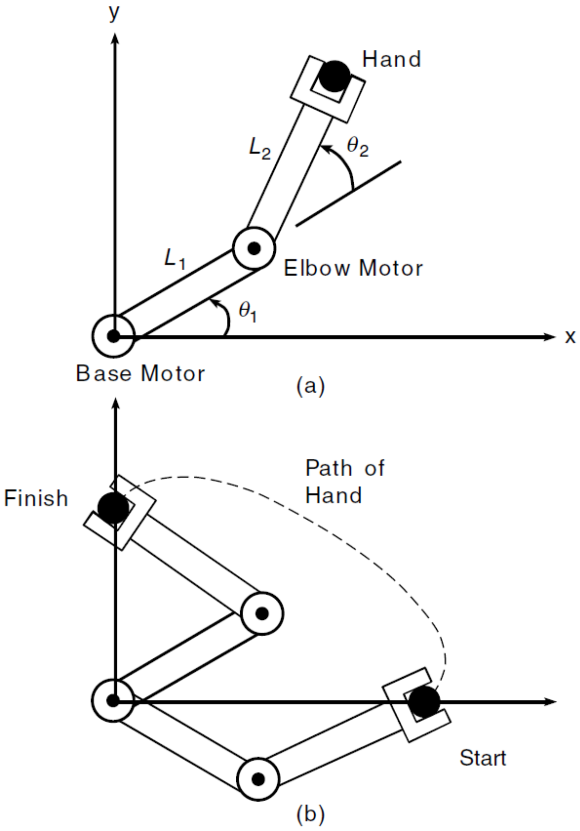
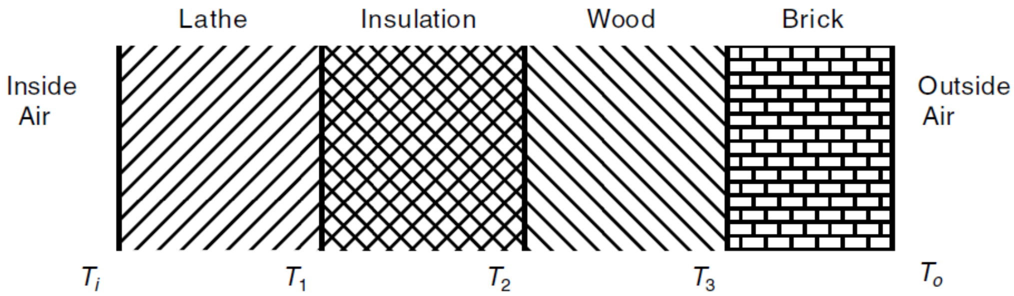

y
Hand
L_2
θ_2
L_1
Elbow Motor
θ_1
x
Base Motor
(a)
Path of
Hand
Finish
Start
(b)

Figure P7

Problem 6

Engineers must be able to predict the rate of heat loss through a building wall to determine the heating system requirements. They do this by using the concept of *thermal resistance* R, which relates the heat ow rate q through a material to the temperature difference ΔT across the material: $q = \Delta T/R$. This relation is like the voltage-current relation for an electric resistor: $i = v/R$. So the heat ow rate plays the role of electric current, and the temperature difference plays the role of the voltage difference. The SI unit for q is the *watt* (W), which is 1 joule/second (J/s).

The wall shown in Figure P8 consists of four layers: an inner layer of plaster/lathe 10 mm thick, a layer of ber glass insulation 125 mm thick, a



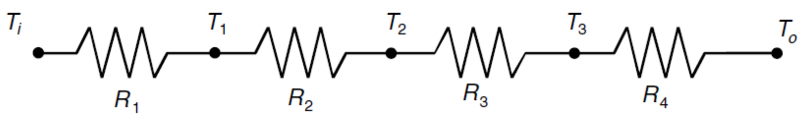
(a)

layer of wood 60 mm thick, and an outer layer of brick 50 mm thick. If we assume that the inner and outer temperatures T_i and T_o have remained constant for some time, then the heat energy stored in the layers is constant, and thus the heat ow rate through each layer is the same. Applying conservation of energy gives the following equations.

$$q = \frac{1}{R_1}(T_i - T_1) = \frac{1}{R_2}(T_1 - T_2) = \frac{1}{R_3}(T_2 - T_3) = \frac{1}{R_4}(T_3 - T_o)$$

The thermal resistance of a solid material is given by $R = D/k$, where D is the material thickness and k is the material's *thermal conductivity*. For the given materials, the resistances for a wall area of 1 m^2 are $R_1 = 0.036$, $R_2 = 4.01$, $R_3 = 0.408$, and $R_4 = 0.038$ K/W.

Suppose that $T_i = 20$°C and $T_o = -10$°C. Find the other three temperatures and the heat loss rate q, in watts. Compute the heat loss rate if the wall's area is 10 m^2.



(b)

Figure P8