Assignment for linear equations

Problem 1

Integrate the following function analytically and using the trapezoidal rule, with $n = 1, 2, 3$, and 4:

$$\int_1^2 (x + 1/x)^2 \, dx$$

Use the analytical solution to compute true percent relative errors to evaluate the accuracy of the trapezoidal approximations.

Problem 2

Integrate the following function both analytically and using Simpson's rules, with $n = 4$ and 5. Discuss the results.

$$\int_{-3}^5 (4x - 3)^3 \, dx$$

Problem 3

Integrate the following function both analytically and numerically. Use both the trapezoidal and Simpson's $1/3$ rules to numerically integrate the function. For both cases, use the multiple-application version, with $n = 4$. Compute percent relative errors for the numerical results.

$$\int_0^3 x^2 e^x \, dx$$

Problem 4

The following data was collected for a cross-section of a river (y = distance from bank, H = depth and U = velocity):

y, m	0	1	3	5	7	8	9	10
H, m	0	1	1.5	3	3.5	3.2	2	0
U, m/s	0	0.1	0.12	0.2	0.25	0.3	0.15	0

Use numerical integration to compute the (*a*) average depth, (*b*) cross-sectional area, (*c*) average velocity, and (*d*) the flow rate. Note that the cross-sectional area (A_c) and the flow rate (Q) can be computed as

$$A_c = \int_0^y H(y)\,dy \qquad\qquad Q = \int_0^y H(y)U(y)\,dy$$

Problem 5

The outflow concentration from a reactor is measured at a number of times over a 24-hr period:

t, hr	0	1	5.5	10	12	14	16	18	20	24
c, mg/L	1	1.5	2.3	2.1	4	5	5.5	5	3	1.2

The flow rate for the outflow in m^3/s can be computed with the following equation:

$$Q(t) = 20 + 10\sin\left(\frac{2\pi}{24}(t-10)\right)$$

Use the best numerical integration method to determine the flow-weighted average concentration leaving the reactor over the 24-hr period,

$$\bar{c} = \frac{\int_0^t Q(t)c(t)dt}{\int_0^t Q(t)dt}$$

Problem 6

A transportation engineering study requires that you determine the number of cars that pass through an intersection traveling during morning rush hour. You stand at the side of the road and count the number of cars that pass every 4 minutes at several times

as tabulated below. Use the best numerical method to determine **(a)** the total number of cars that pass between 7:30 and 9:15, and **(b)** the rate of cars going through the intersection per minute. (*Hint:* Be careful with units.)

Time (hr)	7:30	7:45	8:00	8:15	8:45	9:15
Rate (cars per 4 min)	18	24	26	20	18	9