2-11 Encontrar la expresion del espectro de fourier

$$X(t) = |6 Sen(3t + \pi/4)|^2 = 6^2 sen^2(3t + \pi/4)$$

Por identidad:

$\rightarrow$ $Sen^2(\theta) = \frac{1}{2} - \frac{\cos(2\theta)}{2}$

$$X(t) = 36\left(\frac{1}{2} - \frac{Cos(6t + \pi/2)}{2}\right) = \frac{36}{2} - 18 Cos(6t + \pi/2)$$

$$X(t) = 18 - 18 Cos(6t + \pi/2)$$

$\Rightarrow$ $Cos(\theta + \pi/2) = -Sen(\theta)$

$$X(t) = 18 + 18 Sen(6t)$$

Forma trigonometrica:

$$X(t) = a_0 + \sum_{n=-N}^{N} a_n Cos(nwot) + b_n Sen(nwot)$$

Ya que $x(t)$ corresponde a una funcion seno, el seno presenta simetria impar, entonces $X(t) = -X(-t)$

Entonces; $a_n = 0$

$$X(t) = 18 + 18 sen(6t) = a_0 + \sum_{n=-N}^{n} b_n sen(nwot)$$

$$a_0 = C_0 = \frac{1}{tf - t_1}\int_{t_1}^{tf} x(t)dt$$

$$a_0 = \frac{1}{\pi - (-\pi)}\int_{-\pi}^{\pi} 18 + 18 Sen(6t) dt$$

$$= \underbrace{\frac{18}{2\pi}\int_{-\pi}^{\pi} dt}_{1} + \underbrace{\frac{18}{2\pi}\int_{-\pi}^{\pi} Sen(6t) dt}_{2}$$

• 1) $\frac{18}{2\pi}\int_{-\pi}^{\pi} dt = \frac{18}{2\pi}(\pi) - \frac{18}{2\pi} = 9 - (-9) = 18$

• 2) $\frac{18}{2\pi}\int_{-\pi}^{\pi} sen(6t = \frac{18}{12\pi} - Cos(6t)\Big|_{-\pi}^{\pi} = \frac{18}{12\pi}[-\overset{0}{\cancel{cos(6\pi)}} + Cos(6(-\pi))]$

$= \frac{18}{12\pi}[0] = 0$

$a_0 = 18$

Para bn

$$b_n = \frac{2}{t_f - t_i}\int_{t_i}^{t_f} x(t)\,Sen(n w_0 t)\,dt$$

$$b_n = \frac{2}{\pi-(-\pi)}\int_{-\pi}^{\pi} 18 + 18\,sen(6t)\,Sen(w_0 t)\,dt$$

$$b_n = \frac{2}{2\pi}\left[\int_{-\pi}^{\pi} 18\,Sen(n w_0 t)\,dt + \int_{-\pi}^{\pi} 18\,Sen(6t)\,Sen(n w_0 t)\,dt\right.$$

Identidad

$$Sen(\theta)\,Sen(\alpha) = \frac{Cos(\theta-\alpha) - Cos(\theta+\alpha)}{2}$$

$Sen\,\theta = Sen\,6t \qquad Sen\,\alpha = Sen(n w_0 t)$

$$= \frac{Cos(6t - n w_0 t) - Cos(6t + n w_0 t)}{2} = \frac{Cos((6-n w_0)t) - Cos((6+n w_0)t)}{2}$$

$w_0 = \frac{2\pi}{T} \qquad T = 2\pi \qquad w_0 = \frac{2\pi}{2\pi} = 1\ rad/s$

$$b_n = \frac{2}{2\pi}\left[\underbrace{\int_{-\pi}^{\pi} 18\,Sen(nt)\,dt}_{①} + \underbrace{\int_{-\pi}^{\pi} \frac{18\,Cos((6-n)t) - Cos((6+n)t)}{2}\,dt}_{②}\right]$$

$$① \; \frac{18}{\pi}\int_{-\pi}^{\pi} Sen(nt)\,dt = \frac{-18}{\pi}\cos(nt)\Big|_{-\pi}^{\pi} = -\frac{18}{\pi}\left[\cancel{Cos(n\pi)} - \cancel{Cos(-n\pi)}\right] = 0$$

$$② \; \frac{18}{2\pi}\int_{-\pi}^{\pi} Cos((6-n)t) - Cos((6+n)t)\,dt = \frac{18}{2\pi}\left[\int_{-\pi}^{\pi} Cos((6-n)t)\,dt - \int_{-\pi}^{\pi} Cos((6+n)t)\,dt\right]$$

$$\frac{18}{2\pi}\left[\left[\frac{sen((6-n)\pi)}{6-n} - \frac{Sen((6-n)(-\pi))}{6-n}\right] - \left[\frac{Sen((6+n)\pi)}{6+n} - \frac{Sen((6+n)-\pi)}{6+n}\right]\right]$$

$$= \frac{18}{2\pi}\left[\frac{\text{Sen}((6-n)\pi) - \text{Sen}((6-n)-\pi)}{6-n} - \frac{\text{Sen}((6+n)\pi) - \text{Sen}((6+n)-\pi)}{6+n}\right]$$

$$= \frac{18\,\text{Sen}((6-n)\pi) - \text{Sen}((6-n)-\pi)}{2\pi(6-n)} - \frac{18\,\text{Sen}((6+n)\pi) - \text{Sen}((6+n)-\pi)}{2\pi(6+n)}$$

Para $n \neq 6$, $b = n$. No obstante, para $n=6$ calcular el limite

$$b_6 = 18 \lim_{n\to 6} \frac{\frac{d}{dn}\,\text{Sen}((6-n)\pi) - \text{Sen}((6-n)-\pi)}{\frac{d}{dn}\,2\pi(6-n)}$$

$$b_6 = 18 \lim_{n\to 6} \frac{\cos((6-n)\pi)(-\pi) - \cos(-(6-n)\pi)\pi}{-2\pi}$$

$$b_6 = 18\,\frac{\overset{1}{\cos(0)}(-\pi) - \overset{1}{\cos(0)}\,\pi}{-2\pi} = \frac{18(-2\pi)}{(-2\pi)} = 18$$

$$b_6 = 18 \quad y \quad b_{-6} = -18$$

Por tanto,

$$a_n = \begin{cases} 18 & n=0 \\ 0 & \forall n \setminus \{0\} \end{cases}$$

$$b_n = \begin{cases} 18 & n=6 \\ -18 & n=-6 \\ 0 & \forall n \setminus \{6,-6\} \end{cases}$$

Forma exponencial: $c_0 = a_0 = 18$

$$c_n = \frac{a_n - jb_n}{2} \qquad c_6 = \frac{0 - j18}{2} = \frac{-j18}{2} = -j9$$

$$c_{-6} = j9$$

$$c_n = \begin{cases} 18 & n=0 \\ -9j & n=6 \\ 9j & n=-6 \\ 0 & \forall n \setminus \{0,6,-6\} \end{cases}$$

$$X(t) = \sum_{n=-N}^{N} C_n e^{j\theta}$$

$$X(t) = C_6 e^{-j6t} + C_0 e^{0} + C_6 e^{j6t}$$

El error relativo se calcula:

$$E_x[\%] = \left[1 - \frac{1}{P_x} \sum_{n=-N}^{N} |C_n|^2\right] 100\%$$

Para $P_x =$

$$P_x = \frac{1}{T}\int_{t_1}^{t_1+T} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\pi}^{\pi} |18 + 18\,\mathrm{Sen}(6t)|^2 dt$$

$$P_x = \frac{1}{2\pi}\left[\int_{-\pi}^{\pi} 18^2 - 2(18)18\,\mathrm{Sen}(6t) + 18^2 \mathrm{Sen}^2(6t)\, dt\right]$$

$$P_x = \frac{1}{2\pi}\left[\int_{-\pi}^{\pi} 18^2 dt - \int 648\,\mathrm{Sen}(6t) + 18^2 \int \left(\frac{1}{2} - \frac{\cos(12t)}{2}\right) dt\right]$$

$$P_x = \frac{1}{2\pi}\left[324t\Big|_{-\pi}^{\pi} + \frac{648}{6}\cos(6t)\Big|_{-\pi}^{\pi} + 324\left[\frac{1}{2}t\Big|_{-\pi}^{\pi} - \frac{\mathrm{Sen}(12t)}{24}\Big|_{-\pi}^{\pi}\right]\right]$$

$$P_x = \frac{1}{2\pi}\left[\left[324\pi - 324(-\pi)\right] + 108\left[\cos(6(\pi)) - \cos(6(-\pi))\right] + 324\left[\left[\frac{1}{2}\pi - \frac{1}{2}(-\pi)\right] - \left[\frac{\mathrm{Sen}\,12\pi}{24} - \frac{\mathrm{Sen}\,12(-\pi)}{24}\right]\right]\right]$$

$$P_x = \frac{1}{2\pi}\left[648\pi + 0 + 324\pi\right] = \frac{972\pi}{2\pi} = 486$$

$$P_x = 486$$

$$E_x = \left[\frac{1 - (-9)^2 + (18)^2 + (9)^2}{486}\right] \times 100\% = 0\%$$

2-21) $C(t) = A_c \cos(2\pi F_c t)$, $A_c, F_c \in R$ $\quad y(t) = \left(1 + \frac{m(t)}{A_c}\right) C(t)$

$$F\{C(t)\} + F\{m(t)C(t)\}$$

$$F\{A_c \cos(2\pi F_c t)\} = A_c \cdot F\left\{\frac{e^{j2\pi F_c t} + e^{-j2\pi F_c t}}{2}\right\}$$

$$A_c\left[F\left\{\frac{e^{j2\pi F_c t}}{2}\right\} + F\left\{\frac{e^{-j2\pi F_c t}}{2}\right\}\right]$$

$$\frac{A_c}{\cancel{2}}\left[\cancel{2}\pi\delta(\omega - 2\pi F_c) + \cancel{2}\pi\delta(\omega + 2\pi F_c)\right]$$

$$A_c\pi\delta(\omega - 2\pi F_c) + A_c\pi\delta(\omega + 2\pi F_c) \rightarrow C(\omega) = A_c\pi\delta\left[(\omega - 2\pi F_c) + (\omega + 2\pi F_c)\right]$$

$$F\left\{\frac{m(t)(\cancel{A_c}\cos(2\pi F_c t))}{\cancel{A_c}}\right\} = F\{\cos(2\pi F_c t)m(t)\} =$$

$$F\left\{\frac{m(t)e^{j2\pi F_c t}}{2}\right\} + F\left\{\frac{m(t)e^{-j2\pi F_c t}}{2}\right\}$$

$$\frac{m(\omega - 2\pi F_c t)}{2} + \frac{m(\omega + 2\pi F_c t)}{2} = \frac{1}{2}m\left[(\omega - 2\pi F_c t) + (\omega + 2\pi F_c t)\right]$$

$$\rightarrow y(\omega) = A_c\pi\delta\left[(\omega - 2\pi F_c t) + (\omega + 2\pi F_c t)\right] + \frac{1}{2}m\left[(\omega - 2\pi F_c t) + (\omega + 2\pi F_c t)\right]$$