Punto 1.

El sistema masa, resorte y amortiguadores se puede modelar a partir de la conservación de fuerzas.

$$F_S(t) + F_F(t) + F_I(t) = F_E(t)$$

donde

$$F_S(t) = k y(t) \quad , \quad F_F(t) = c \frac{dy(t)}{dt} \quad y \quad F_I = m \frac{d^2 y(t)}{dt^2}.$$

Por consiguiente:

$$m \frac{d^2 y(t)}{dt^2} + c \frac{dy(t)}{dt} + k y(t) = F_E(t) = x(t)$$

se Aplica tranformada Laplace

$$\mathcal{L}\left\{ \frac{d^n x(t)}{dt} \right\} = \int^n x(s) \quad \text{se tiene que}$$

$$m s^2 y(s) + c s y(s) + k y(s) = x(s)$$

$$y \quad H(s) = \frac{y(s)}{x(s)} = \frac{1}{m s^2 + cs + k}$$

Función de transferencia sistema masa, resorte, amortiguador

Ahora, función de transferencia para el cto.

LVK malla interna .

$$-V_i(t) + L \frac{d}{dt} i_1(t) + \frac{1}{c} \int_0^t (i_1(t) - i_2(t)) dt = 0$$

$$V_i(s) = Ls \; I_1(s) + (I_1(s) - I_2(s)) \frac{1}{cs} \qquad (1)$$

Ahora LVK en malla $i_2(t)$

$$i_2(t) R + \frac{1}{C} \int_0^t i_2(t) - i_1(t) \, dt = 0$$

donde $V_0(t) = i_2(t) R$

se utilizan las impedancias transformadas

$$I^2(s) R + (I_2(s) - I_1(s)) \frac{1}{cs} = 0$$

se despeja $I_1(s)$

$$\frac{I_1(s)}{cs} + \frac{I_2(s)}{cs} + I_2(s) R$$

$$I_1(s) = \frac{I_2(s)}{Cs} Cs + I_2(s) RCs$$

$$I_1(s) = I_2(s)(1 + CRs) \quad (2)$$

se reemplazar (2) en (1)

$$Vi(s) = Ls\, I_2(s)(1+CRs) + (I_2(s)(1+CRs) - I_2(s))\frac{1}{Cs}$$

$$Vi(s) = Ls\, I_2(s) + (RLs^2 I_2(s) + R\, I_2(s))$$

$$Vi(s) = I_2(s)\,[CRLs^2 + Ls + R]$$

$$\frac{I_2(s)}{Vi(s)} = \frac{1}{CRLs^2 + Ls + R}$$

Se reemplaza $I_2(s) = \frac{Vo(s)}{R}$

$$\frac{Vo(s)}{R\,Vi(s)} = \frac{1}{CRLs^2 + Ls + R}$$

$$\frac{Vo(s)}{Vi(s)} = \frac{R}{CRLs^2 + Ls + R} \cdot \left(\frac{1/R}{1/R}\right)$$

$$\frac{Vo(s)}{Vi(s)} = \frac{1}{CLs^2 + \frac{L}{R}s + 1}$$ → Funcion transferencia cto eléctrico

equivalencia del cto eléctrico en péndulo elástico

$$CL = m$$
$$L/R = C$$
$$1 = K$$

Entonces $H(s) = \frac{1}{LCs^2 + \frac{L}{R}s + 1}$

En péndulo es:

$$H(s) = \frac{1}{ms^2 + Cs + K} = \frac{1/m}{\left(s^2 + \frac{Cs}{m} + \frac{K}{m}\right)}$$

Comparando

$$s^2 + s\xi\, w_n s + w_n^2 = s^2 + \frac{cs}{m} + \frac{k}{m}$$

$1 = 1 \rightarrow$ coe F s^2

$2\xi w_n = \frac{c}{m} \rightarrow$ coe F s

$w_n^2 = \frac{k}{m} \rightarrow$ coef indep.

Freq natural no amortiguada

$$w_n = \sqrt{\frac{k}{m}}$$

Factor amortiguamiento

$$2\xi\sqrt{\frac{k}{m}} = \frac{c}{m} \Rightarrow \xi = \frac{c}{2m\sqrt{\frac{k}{m}}}$$

$$K w_n^2 = \frac{1}{m} \Rightarrow K = \frac{1}{m w_n^2} = \frac{1}{\cancel{m}\frac{k}{\cancel{m}}} \Rightarrow K = \frac{1}{k}$$

Forma canónica de segundo orden

$$H(s) = K \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2}$$

$$H(s) = \frac{1}{k} \cdot \frac{k/m}{s^2 + 2\left(\frac{c}{2m\sqrt{\frac{k}{m}}}\right)\sqrt{k/m}\, s + \frac{k}{m}}$$

$$H(s) = \frac{1}{m\left(s^2 + \frac{c}{m} + \frac{k}{m}\right)}$$

Se halla la freq natural amortiguada.

$$w_d = w_n\sqrt{1-\xi^2}$$

$$w_d = \left(\sqrt{\frac{k}{m}}\right)\left(\sqrt{1-\left(\frac{c}{2m\sqrt{k/m}}\right)^2}\right)$$

$$w_d = \frac{\sqrt{\frac{k}{m}}\sqrt{4km - c^2}}{2\sqrt{km}}$$

tiempo de establecimiento

$$t_s = \frac{3}{\xi w_n} \Rightarrow t_s = \frac{3}{\left(\frac{c}{2m\sqrt{\frac{k}{m}}}\right)\sqrt{\frac{k}{m}}} = \frac{6m}{c}$$

$$HLC = \frac{H(s)}{1 + A(s)\,H(s)}$$

se reemplaza a

$$H(s) = \frac{1}{ms^2 + cs + k}$$ → Función trans lazo abierto

$$A(s) = 1$$

En $HLC(s)$

$$HLC = \frac{\frac{1}{ms^2 + cs + k}}{1 + \frac{1}{ms^2 + cs + k}} = \frac{\frac{1}{ms^2 + cs + k}}{\frac{ms^2 + cs + k + 1}{ms^2 + cs + k}} = \frac{\cancel{ms^2 + cs + k}}{\cancel{(ms^2 + cs + k)}(ms^2 + cs + k)}$$

$$HLC = \frac{1}{ms^2 + cs + k} \quad ó \quad \frac{1/m}{s^2 + \frac{c}{m}s + \left(\frac{k+1}{m}\right)}$$

forma canónica segundo orden.

$$s^2 + s\,\xi\,\omega_n s + \omega_n^2 = s^2 + \frac{c}{m}s + \left(\frac{k+1}{m}\right)$$

$$1 = 1 \rightarrow \text{coeF } s^2$$

$$2\xi\omega_n = \frac{c}{m} \rightarrow \text{coeF } s$$

$$\omega_n^2 = \frac{k+1}{m} \rightarrow \text{coeF ind.}$$

Freq natural no amortiguada

$$\omega_n = \sqrt{\frac{k+1}{m}}$$

factor amortiguado

$$\xi = \frac{c}{2m\omega_n} = \frac{c}{2m\sqrt{\frac{k+1}{m}}}$$

La ganancia:

$$K\omega_n^2 = \frac{1}{m} \Rightarrow K = \frac{1}{m\omega_n^2} = \frac{1}{m\left(\sqrt{\frac{k+2}{m}}\right)^2}$$

$$K = \frac{1}{k+1}$$

Forma canónica segundo orden:

$$H_{LC}(S) = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}$$

$$H_{LC}(S) = \frac{(k+1)/m}{S^2 + 2\left(\frac{c}{2m\left(\sqrt{\frac{k+1}{m}}\right)}\sqrt{k+1/m} \cdot S \;\; \frac{k+1}{m}\right)}$$

$$H_{LC}(S) = \frac{1}{m\left(S^2 + \frac{c}{m}S + \frac{k+1}{m}\right.}$$

Referencia amortiguada

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\omega_d = \sqrt{\frac{k+1}{m}}\sqrt{1 - \left(\frac{c}{2m\sqrt{(k+1)/m}}\right)^2}$$

$$\omega_d = \frac{\sqrt{\frac{k+1}{m}}\sqrt{4km + 4m - c^2}}{2\sqrt{m(k+1)}}$$

tiempo de establecimiento:

$$t_s = \frac{3}{\xi\omega_n} = \frac{3}{\frac{c}{2m\sqrt{\frac{(k+1)}{m}}} \cdot \sqrt{\frac{k+1}{m}}} = \frac{6m}{c}$$