Taller 7. Mínimos Cuadrados.

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Punto 7 (c)

Muestre que las derivadas parciales de la metrica son $\frac{\partial \chi^2(\vec{\theta})}{\partial \theta_i} = -2 \sum_{i=1}^{N} (y_i - M(x_i, \vec{\theta})) \frac{\partial M(x_i, \vec{\theta})}{\partial \theta_i}$

Se tiene que $\chi^2(\vec{\theta}) = \sum_{i=1}^{N} \left(\frac{y_i - M(x_i, \vec{\theta})}{\sigma_i} \right)^2$

con $\sigma_i = 1$ para todo i.

$$\chi^2(\vec{\theta}) = \sum_{i=1}^{N} \left(\frac{y_i - M(x_i, \vec{\theta})}{1} \right)^2 \rightarrow \chi^2(\vec{\theta}) = \sum_{i=1}^{N} \left(y_i - M(x_i, \vec{\theta}) \right)^2$$

- La sumatoria es lineal en todo el dominio
- Función derivable y suave (infinitamente diferenciable)

Primera derivada parcial respeto a θ_i

$$\frac{\partial \chi^2(\vec{\theta})}{\partial \theta_i} \left[\sum_{i=1}^{N} \left(y_i - M(x_i, \vec{\theta}) \right)^2 \right]$$

$$\frac{\partial \chi^2(\vec{\theta})}{\partial \theta_i} = \sum_{i=1}^{N} 2 \cdot \left(y_i - M(x_i, \vec{\theta}) \right) \cdot -\left(\frac{\partial M(x_i, \vec{\theta})}{\partial \theta_i} \right)$$

$$\frac{\partial \chi^2(\vec{\theta})}{\partial \theta_i} = -2 \sum_{i=1}^{N} \left(y_i - M(x_i, \vec{\theta}) \right) \cdot \left(\frac{\partial M(x_i, \vec{\theta})}{\partial \theta_i} \right)$$

R/.

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Punto 7 (d)

Muestre que el descenso del gradiente queda definido vectorialmente como

$$\vec{\theta}_{j+1} = \vec{\theta_j} - \gamma\left[-2\sum_{i=1}^{N}\left(y_i - M(x_i,\vec{\theta_j})\right)\cdot\nabla_\theta M(x_i,\vec{\theta_j})\right]$$

Se tiene que $\chi^2(\vec{\theta}) = \sum_{i=1}^{N}\left(\frac{y_i - M(x_i,\vec{\theta})}{\sigma_i}\right)^2$ con $\sigma_i = 1$ para todo i

siendo $M(x_i;\vec{\theta}) = \frac{\theta_0}{\theta_1 + e^{-\theta_2 x}}$ → Modelo de ajuste

De la definición del decenso de gradiente

$$\vec{x}^1 = \vec{x}^0 - \gamma\nabla\vec{F}(\vec{x_0})$$

Que por generalización $x_{j+1} = x_j - \gamma\nabla F(x_j)$ donde $\nabla F(x_j) = \frac{\partial\chi^2(\vec{\theta})}{\partial\theta_i}$

$$\frac{\partial\chi^2(\vec{\theta})}{\partial\theta_i} = -2\sum_{i=1}^{N}\left(y_i - M(x_i,\vec{\theta})\right)\cdot\left(\frac{\partial M(x_i,\vec{\theta})}{\partial\theta_i}\right)$$

Entonces se tiene $\vec{\theta}_{j+1} = \vec{\theta_j} - \gamma\left[-2\sum_{i=1}^{N}\left(y_i - M(x_i,\vec{\theta})\right)\cdot\left(\frac{\partial M(x_i,\vec{\theta})}{\partial\theta_i}\right)\right]$

Dado que se tienen 3 parametros $(\theta_1, \theta_2, \theta_3)$, se tienen 3 derivadas parciales:

$$\frac{\partial M(x_i,\vec{\theta})}{\partial\theta_i} = \nabla_\theta M(x_i,\vec{\theta})$$

$$\frac{\partial M(x_i,\vec{\theta})}{\partial\theta_i} = \left[\frac{\partial M(x_i,\vec{\theta})}{\partial\theta_0}, \frac{\partial M(x_i,\vec{\theta})}{\partial\theta_1}, \frac{\partial M(x_i,\vec{\theta})}{\partial\theta_2}\right]$$

$$\nabla_\theta M(x_i,\vec{\theta}) = \left[\frac{\partial M(x_i,\vec{\theta})}{\partial\theta_0}, \frac{\partial M(x_i,\vec{\theta})}{\partial\theta_1}, \frac{\partial M(x_i,\vec{\theta})}{\partial\theta_2}\right] \rightarrow 3 \text{ parametros } \theta_1, \theta_2, \theta_3$$

$$\vec{\theta}_{j+1} = \vec{\theta_j} - \gamma\left[-2\sum_{i=1}^{N}\left(y_i - M(x_i,\vec{\theta_j})\right)\cdot\nabla_\theta M(x_i,\vec{\theta_j})\right]$$ R/.