Assignment 1: Phasor-field Non-Line-Of-Sight Imaging - Lab session 1 -

1 Introduction

This assignment involves the implementation and analysis of efficient phasor-based non-line-of-sight (NLOS) imaging methods using pico-second time-resolved measurements acquired at a relay wall. Your main task will be implementing different NLOS imaging modalities based on phasor-field virtual wave optics [1], testing them on different datasets under different SNR levels and parameter values, and analyzing their ability to provide images of hidden scenes.To evaluate your implementation, you can find several datasets in the folder linked in Moodle.

2 Single-frequency NLOS imaging (6.5 points)

In this first task you will implement an efficient single-frequency phasor-field NLOS imaging model based on plane-to-plane Rayleigh-Sommerfeld propagation. You will evaluate it in planar scenes under different scene and imaging parameters.

2.1 Efficient single-frequency RSD propagation (4 points)

The RSD operator to propagate a single-frequency phasor field $\mathcal{P}_{\Omega'}(\mathbf{x}_a)$ at a domain $\mathbf{x}_a \in A$ to another phasor field $\mathcal{P}_{\Omega'}(\mathbf{x}_b)$ at a domain $\mathbf{x}_b \in B$ can be expressed as:

$$\mathcal{P}_{\Omega'}(\mathbf{x}_b) = \int_A \frac{e^{2\pi i \Omega' t_{ab}}}{c t_{ab}} \mathcal{P}_{\Omega'}(\mathbf{x}_a) \mathrm{d}\mathbf{x}_a := \mathcal{R}_A(\mathcal{P}_{\Omega'}(\mathbf{x}_a), \mathbf{x}_b). \tag{1}$$

When $\mathbf{x}_a \in A$ and $\mathbf{x}_b \in B$ are two different co-planar domains at a distance z_b from each other, RSD propagation of a phasor field $\mathcal{P}_{\Omega'}$ with frequency Ω' from A (at $z = 0$) to B (at $z = z_b$) can be expressed as a 2D convolution

$$\begin{aligned}
\mathcal{P}_{\Omega'}(x_b, y_b, z_b) &= \iint_{-\infty}^{\infty} \frac{e^{2\pi i \Omega' \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2 + z_b^2}/c}}{\sqrt{(x_b - x_a)^2 + (y_b - y_a)^2 + z_b^2}} \mathcal{P}_{\Omega'}(x_a, y_a, 0) dx_a dy_a, && (2)\\
&= G(x_a, y_a, z_b) *_{x_a, y_a} \mathcal{P}_{\Omega'}(x_a, y_a, 0) && (3)\\
&= \hat{G}(\Omega_{x_a}, \Omega_{y_a}, z_b) \hat{\mathcal{P}_{\Omega'}}(\Omega_{x_a}, \Omega_{y_a}, 0). && (4)
\end{aligned}$$

Use this expression to implement an efficient RSD-based single-frequency solver for the conventional virtual camera model. This model involves a single RSD propagation from the sensor aperture $\mathbf{x}_s \in S$ to another plane $\mathbf{x}_v \in V$ within the hidden scene:

$$f_{\Omega'}(\mathbf{x}_v) = \int_S \frac{e^{2\pi i \Omega' t_{sv}}}{c t_{sv}} \int_L \mathcal{P}_{\Omega'}(\mathbf{x}_l) H_{\Omega'}(\mathbf{x}_l, \mathbf{x}_s) \mathrm{d}\mathbf{x}_l \mathrm{d}\mathbf{x}_s$$

Implement this operator as a spatial-domain convolution using the `conv2(..., 'same')` Matlab function (Equation (3)), and as a multiplication in the domain of $x - y$ spatial frequencies

(Equation (4)). To use all the laser positions of any dataset as light sources, simply set $\mathcal{P}_{\Omega'}(\mathbf{x}_l) \equiv 1$. **TIP:** Define your convolution kernel G with the same lateral resolution as the sensor grid. When computing RSD propagation using a frequency-domain multiplication, you will need to compute the FFT of the convolution kernel G using `fft2`. Before this, apply a `fftshift` operation over the convolution kernel to ensure the results are not shifted half the size of the kernel size.

$H_{\Omega'}$ represents the temporal frequency component of the impulse response function $H(\mathbf{x}_l, \mathbf{x}_s, t)$ with frequency Ω':

$$H_{\Omega'}(\mathbf{x}_l, \mathbf{x}_s) = \int_{-\infty}^{\infty} e^{2\pi i \Omega' t} H(\mathbf{x}_l, \mathbf{x}_s, t) \mathrm{d}t.$$

Evaluate your implementation in a non-confocal dataset with a planar hidden scene (e.g. the Z or the USAF datasets), placing the plane $\mathbf{x}_v \in V$ at exactly the distance z_v at which the hidden object is w.r.t. the relay wall, so that the object will become in perfect focus in the resulting image. Choose $\Omega' = 1/\lambda_c'$ with $\lambda_c = 2\Delta x_s$, where Δx_s is the distance between adjacent sampled points on the regular sensor grid on the relay wall. The leftmost image of Figure 1 shows a result of the USAF dataset at $z = 0.5$ m using $\lambda_c = 2$ cm. Do you observe any performance improvements when using the spatial convolution implementation compared to the the frequency-domain multiplication implementation?

2.2 Effect of λ_c (0.5 points)

Evaluate your implementation under different λ_c values, below and above the $2\Delta x_s$ threshold. What is the effect of going below this value? And above?

2.3 Sensitivity to signal degradation (1 points)

The performance of NLOS imaging methods is strongly dependent on the quality of the impulse response function H. One factor that determines the quality of such data is the number of photons collected by the sensor. A lower number of photons will decrease the signal-to-noise ratio (SNR) of the signal, which may have a strong impact on the quality of the output reconstruction.

In this task you need to evaluate your implementation in the chosen dataset under different SNR levels of the impulse response function. To vary their SNR, you can artificially add shot noise (which mimics photon counting in optical devices) using a Poisson process via `poissrnd` Matlab function. This function can be invoked with a single parameter as input (i.e. the impulse response function H) to simulate Poisson noise at every spatio-temporal bin of H. The Poisson

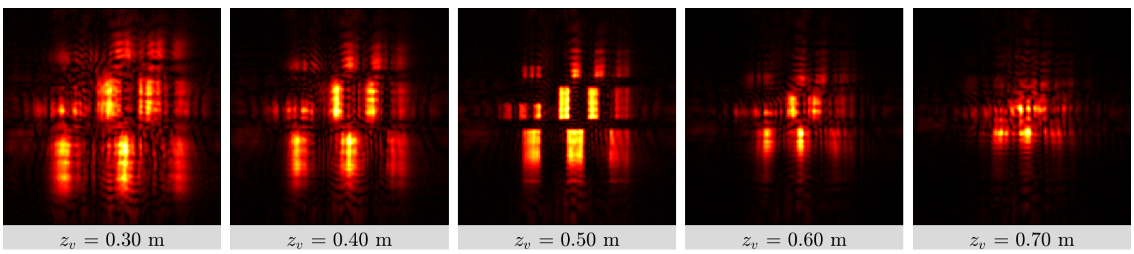

Figure 1: Reconstructions of the USAF hidden scene placed at 0.5 meters from the relay wall. From left to right: Single-frequency reconstructions with $\lambda_c = 2$cm at focal distances $z_v = [0.3, 0.4, 0.5, 0.6, 0.7]$ meters, respectively.

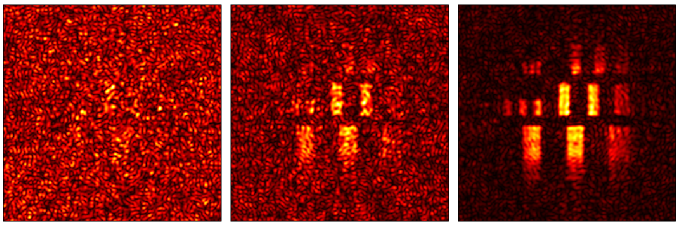

Figure 2: Reconstructions of the USAF hidden scene placed at 0.5 meters from the relay wall. From left to right: Single-frequency reconstructions with $\lambda_c = 2$cm at a focal distance $z_v = 0.5$ meters, under 100k, 1M, and 10M photons, respectively.

process interprets the value of each spatio-temporal bin as an estimated number of photons collected by that bin. When called, it will simulate a photon counting process at each bin using that value.

While you cannot determine how many photons arrive at every bin individually, you can estimate a total number of photons collected by the entire impulse response function based on the sum of all its bins. For example, calling `poissrnd(`$H_{(1)}$`)` for

$$H_{(1)} = \frac{H}{\sum\limits_{x,y,t} H} \tag{5}$$

will simulate an impulse response function that only captured *one* photon. To simulate an impulse response function that collected p photons, you can simply call `poissrnd(`$H_{(p)}$`)` where $H_{(p)} = p \cdot H_{(1)}$.

Evaluate your single-frequency implementation on the dataset from the previous task under different photon counts. Vary the photon counts by an order of magnitude on each step, starting at e.g. 1k photons, then 10k, then 100k, then 1M, until you cannot notice the degradation in the quality of the output reconstruction. You can see examples of this effect on the USAF dataset in Figure 2

2.4 Multiple focal planes (1 points)

Evaluate your implementation of single-frequency NLOS imaging at multiple planes of the hidden scene to observe how the planar object becomes out of focus as you move the planes further or closer to the relay wall. Create a 3D volume from multiple equidistant reconstructed planes. Display the resulting volume using any volumetric visualization tool (e.g. `volshow`) using maximum intensity projection and the colomap `hot`. Can you distinguish the planar object in the 3D reconstruction? Why?

3 Multi-frequency RSD propagation

In this task you need to leverage your single-frequency implementation from the previous task to implement and evaluate the multi-frequency virtual confocal camera model for NLOS imaging:

$$f(\mathbf{x}_v, \Omega) = \int\limits_S \frac{e^{2\pi i \Omega t_{sv}}}{c t_{sv}} \int\limits_L \frac{e^{2\pi i \Omega t_{lv}}}{c t_{lv}} \mathcal{P}(\mathbf{x}_l, \Omega) H(\mathbf{x}_l, \mathbf{x}_s, \Omega) \mathrm{d}\mathbf{x}_l \mathrm{d}\mathbf{x}_s \tag{6}$$

A reconstruction of the geometry of the hidden scene can be obtained by evaluating

$$f(\mathbf{x}_v) = \int_{-\infty}^{\infty} f(\mathbf{x}_v, \Omega) \mathrm{d}\Omega$$

. This model is akin to a regular filtered backprojection method.

3.1 Efficient multi-frequency RSD propagation (3.5 points)

Implement an efficient solver for Equation (6) based on RSD propagation using a simple illumination function $\mathcal{P}(\mathbf{x}_l, \Omega) \equiv 1$ which weighs all temporal frequencies of H equally. This corresponds to a delta function in the temporal domain $\mathcal{P}(\mathbf{x}_l, t) \equiv \delta(t)$.

Note that the model in Equation (6) has now two different RSD propagations, one corresponding to the sensor aperture $\mathbf{x}_s \in S$, and another corresponding to the laser aperture $\mathbf{x}_l \in L$. You will need to perform propagations over all the involved frequencies of H.

For this you will need to retrieve all the Fourier components of H and $\mathcal{P}$ using fast Fourier transforms over the temporal domain (check the `fftn` function in Matlab). For each Fourier component, you need to calculate the value of the corresponding frequency Ω to perform RSD propagation. Check the Matlab documentation of the `fft`-related functions to understand the meaning of the output.

TIP: Frequencies vary linearly in the output of the FFT, ranging from $[0..\Omega_{\max}, -\Omega_{\max}..0]$. Following the Nyquist-Shannon sampling theorem, the maximum temporal frequency $\Omega_{\max}$ captured by the imaging system is half the inverse of the temporal bin width $\Omega_{\max} = 1/(2\Delta t)$.

Evaluate your implementation in the planar scene you used in the previous task, and compare the multi-frequency results with the single-frequency results, both in the plane containing the geometry, and in the volumetric result with multiple planes. Evaluate your implementation also in a non-confocal dataset of non-planar scene (e.g. the Bunny). Discuss the characteristics and differences that you observe on the results.

3.2 Narrow-band NLOS imaging using virtual pulsed illumination (2 points)

Using a constant virtual illumination function $\mathcal{P}(\mathbf{x}_l, \Omega) = 1$ is quite inefficient as it preserves all frequencies of H and requires computing. Implement a virtual pulsed illumination function as

$$\mathcal{P}(\mathbf{x}_l, t) = e^{2i\pi\Omega_c t} e^{\frac{-t^2}{2\sigma^2}}$$

where $\Omega_c = 1/\lambda_c$ and $\sigma \in [\frac{\lambda_c}{2\log 2}, 2\lambda_c]$. Follow the same criteria as in Section 2 to define λ_c based on the sensor and laser grid spacing. Once you implement this function, you can calculate $\mathcal{P}(\mathbf{x}_l, \Omega)$ as a simple Fourier transform on the temporal domain.

If you plot the absolute value of the frequency spectrum of $\mathcal{P}(\mathbf{x}_l, \Omega)$, you will notice only a small subset of the Fourier components are non-zero. Exploit this characteristic to increase efficiency when evaluating Equation (6), performing RSD propagation on the Fourier components of the integrand $\mathcal{P}(\mathbf{x}_l, \Omega) H(\mathbf{x}_l, \mathbf{x}_s, \Omega)$ that are non-zero. Choose a reasonable threshold to discard frequencies (e.g. all those components of $\mathcal{P}(\mathbf{x}_l, \Omega)$ whose absolute value is greater than 1e−3).

Evaluate your implementation in the non-confocal dataset of a non-planar scene you chose for the previous task, and compare the execution times.

4 Submission and evaluation

You will have to submit a report of up-to 3 pages maximum, plus figures. The report should include the results required for every task, and a brief discussion of what is shown on every result. The maximum score possible for this assignment is 10 points, but note the scores of all tasks add up to 12, so you can still get the maximum grade without completing all the tasks:

Single-frequency NLOS imaging (Section 2)	**6.5**
→ Efficient RSD propagation	4
→ Effect of λ_c	0.5
→ Signal degradation	1
→ Multiple focal planes	1
Multi-frequency NLOS imaging (Section 3)	**5.5**
→ Efficient RSD propagation	3.5
→ Narrow-band RSD using virtual pulsed illumination	2

References

[1] Xiaochun Liu et al. "Non-Line-of-Sight Imaging using Phasor Fields Virtual Wave Optics". In: Nature (2019).