

## Load all the relevant packages

```
In [17]: using IntervalArithmetic
```

```
In [18]: using IntervalRootFinding
```

```
In [19]: using StaticArrays
```

```
In [20]: using DifferentialEquations
```

```
In [21]: Pkg.add("ForwardDiff")
using ForwardDiff
```

```
unknown package ForwardDiff
macro expansion at .\pkg\entry.jl:53 [inlined]
(::Base.Pkg.Entry.##1#3{String,Base.Pkg.Types.VersionSet})() at .\task.jl:335
```

Stacktrace:

```
[1] sync_end() at .\task.jl:287
[2] macro expansion at .\task.jl:303 [inlined]
[3] add(::String, ::Base.Pkg.Types.VersionSet) at .\pkg\entry.jl:51
[4] (::Base.Pkg.Dir.##4#7{Array{Any,1},Base.Pkg.Entry.##add,Tuple{String}})()
at .\pkg\dir.jl:36
[5] cd(::Base.Pkg.Dir.##4#7{Array{Any,1},Base.Pkg.Entry.##add,Tuple{String}},
::String) at .\file.jl:59
[6] #cd#1(::Array{Any,1}, ::Function, ::Function, ::String, ::Vararg{String,N}
) where N at .\pkg\dir.jl:36
[7] add(::String) at .\pkg\pkg.jl:117
```

```
In [22]: using Plots
```

## Define the ODE to be analysed

```
In [23]: Angeli2(u, p) = SVector(
    p[1]*(100-u[1])-p[2]*u[1]*(p[3]*u[2])^p[4]/(p[5]+(p[3]*u[2])^p[4]),
    p[6]*(100-u[2])-p[7]*u[2]*u[1]^p[8]/(p[9]+u[1]^p[8])
)
```

```
Out[23]: Angeli2 (generic function with 1 method)
```

```
In [24]: # set the parameters
alpha1=1
alpha2=1
beta1=200
beta2=10
gamma1=4
gamma2=4
K1=30
K2=1
v=1

p=[alpha1, beta1, v, gamma1, K1, alpha2, beta2, gamma2, K2];
```

**For each variable, create a matrix to put all the solutions (preallocate). Since we don't know the number of solutions we will get, make the matrix big enough**

```
In [25]: x1_matrix=NaN*ones(5,2001); #create a matrix to put at most 5 solutions of
        y1_matrix=NaN*ones(5,2001); #create a matrix to put at most 5 solutions of
```

## Let's start!!!

```
In [26]: counter=1; # initialize the counter

        for v in 0:0.005:1 #this is our bifurcation parameter.

            p=[alpha1, beta1, v, gamma1, K1, alpha2, beta2, gamma2, K2]; #update parameter vector

            X = IntervalBox(0..100, 2) # create the interval box in the phase plane. in this case, it is a 2d box.
            # should be as big as state space to make sure we are not missing any solutions

            aa=roots(u->Angeli2(u, p), X) #compute all the roots MAGIC!
            #output of this fantastic code are small boxes (intervals) with contain the solutions.

            #Since we want to get not an interval, but a real number, we need to take the middle value:

            mids = mid.([rt.interval for rt in aa]) #take the middle value for the roots in the boxes

            count_dif_sols=1
            # now, it is possible that we get several boxes. this can be because (1) we have several solutions (which we expect,
            # at least for some parameter values), or because the boxes spit out by the roots function are too small, i.e. they are finding
            # effectively the same root just with different "names" so to say. we shall filter out this second option in the following:
            #and fill our matrices
            for sol in 1:length(mids)

                if (sol==1)
                    x1_matrix[count_dif_sols,counter]= mids[sol][1]
                    y1_matrix[count_dif_sols,counter]= mids[sol][2]
                    count_dif_sols=count_dif_sols+1
                else
                    if (abs(mids[sol][1]-mids[sol-1][1])>0.001)
                        x1_matrix[count_dif_sols,counter]= mids[sol][1]
                        y1_matrix[count_dif_sols,counter]= mids[sol][2]
                        count_dif_sols=count_dif_sols+1
                    end
                end
            end
            counter=counter+1
        end
```

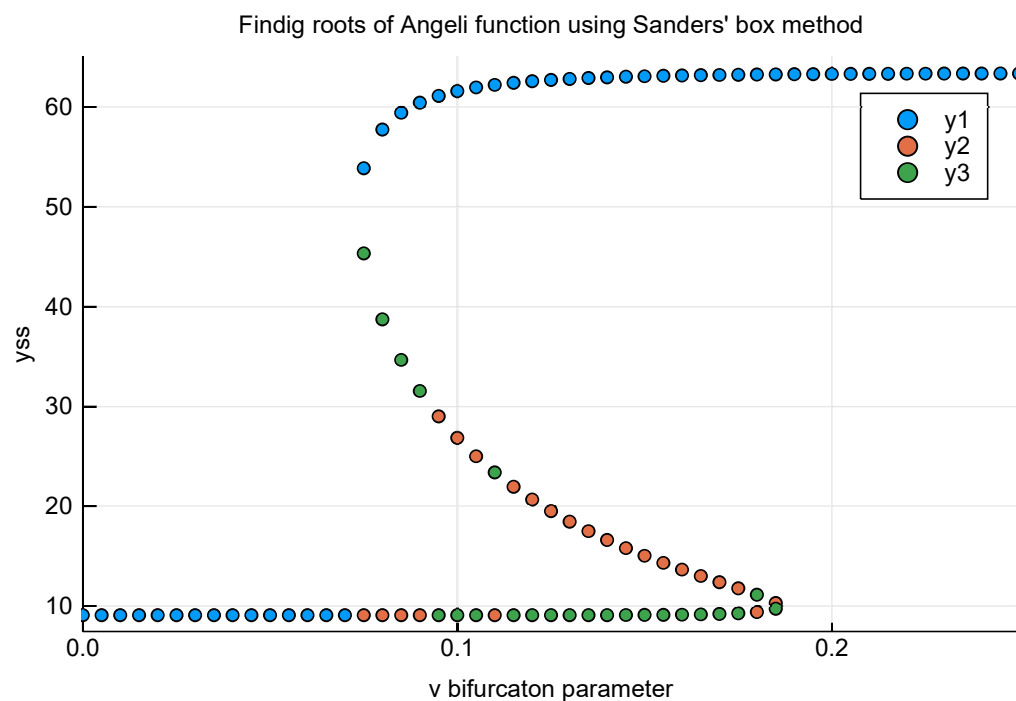
yeiii! we are ready. let's plot our results.

```
In [27]: # first, create a vector with the bifurcation parameter values tested in the pre
        #vious step
        vvect=collect(0:0.005:1);

        # we are ready to plot now:
        scatter(vvect, y1_matrix[1,:])
        scatter!(vvect, y1_matrix[2,:])
        scatter!(vvect, y1_matrix[3,:])

        #just fixing the labels and stuff
        plot!(xlabel="v bifurcaton parameter")
        plot!(ylabel="yss")
        plot!(title="Findig roots of Angeli function using Sanders' box method")
        plot!(xlims=(0,.25),xticks=0:.1:.25)
```

Out[27]:



In [ ]:

In [ ]: