Universidad del Rosario | Escuela de Ingeniería, Ciencia y Tecnología | MACC

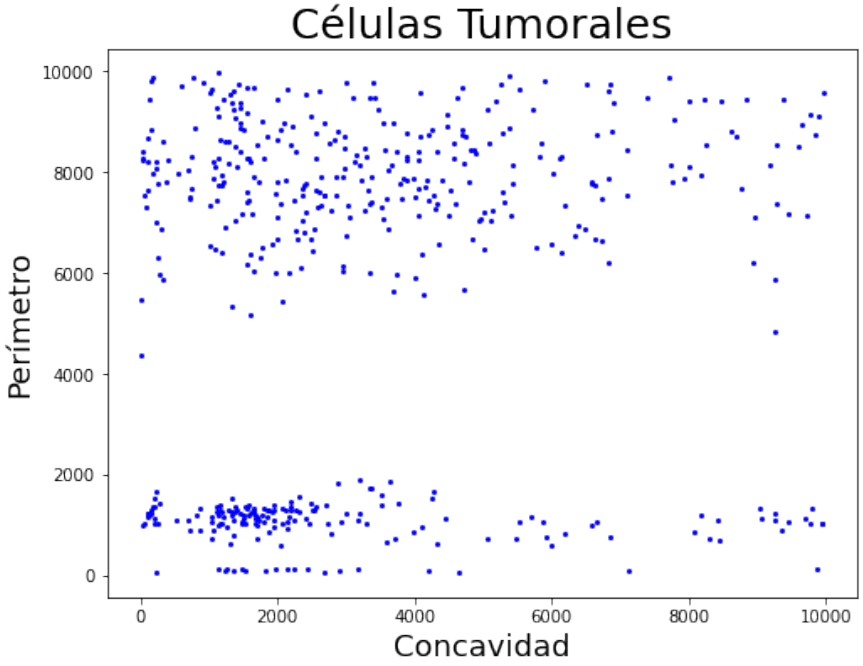
Computational and Differential Geometry
Homework 3

Professor: Nicolás Avilán Vargas, Ph.D.

Indicaciones

1. Fecha de entrega: 15 de noviembre de 2023 hasta las 11:55 pm.
2. Único medio de entrega e-aulas.
3. Formato de entrega: Un único archivo **.ipynb** con códigos en python, descripciones de códigos y procesos, y respuestas a las preguntas (No enviar archivos comprimidos).
4. Solo es permitido el uso de librerías "básicas" (numpy, matplotlib, plotly, seaborn, pandas, sympy, etc). En ningún caso será válida la solución lograda, total o parcialmente, por el uso de una librería especializada para resolver problemas de geometría computacional, con excepción del los diagramas de Voronoi del punto 2.
5. La **tarea** debe realizarse **individualmente**.
6. Cualquier tipo de fraude o plagio es causa de anulación directa de la evaluación y correspondiente proceso disciplinario.
7. Las entregas están sujetas a herramientas automatizadas de detección de plagio en códigos.
8. Las tareas no entregadas antes de la hora indicada tendrán calificación de 0.0.

Support each piece of code with a thorough explanation of its methods, techniques, functions, and tricks. Reference your search source (papers, books, tutorials, websites, etc.). Add any necessary bibliographical references or links.

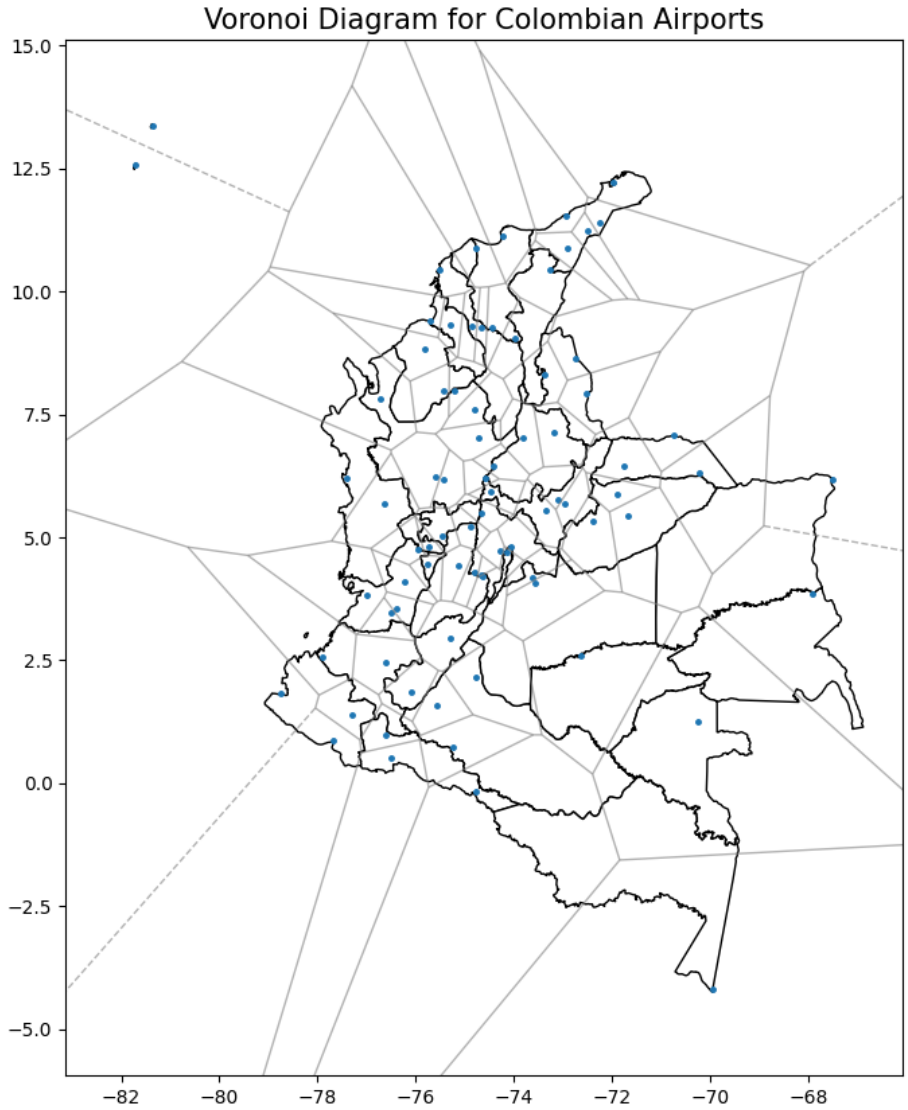


Read the data in the link cancer2.csv. This file has measurements of tumor cells; these are diagnosed as benign or malignant cells for each case. The scatter plot for the variables "Perimetro" and "Concavidad" is on the previous page.

1. (1 point) Implement the 2D Range Searching algorithm to run a query of the interval $[1000 : 3000] \times [500 : 1500]$ for the variables "Concavidad" and "Perimetro". Print the cell IDs for the results of the 2D query. You should implement the algorithm shown in class; the use of a common filter or a conditional is not the right answer.

   Do a scatterplot where points in the query have a different color from those which are out of the query.

Sometimes, when mechanics fail, flights have to follow an emergency protocol, which includes landing at the nearest airport. In such cases, knowledge about the Voronoi diagram associated with the airports is crucial, and according to that, the pilot could update the flight plan by searching for a faster landing.



Read the data in the link airports_CO.csv. This file has coordinates of airports in Colombia.

2. (0.8 points) Use the libraries Voronoi and voronoi_plot_2d and plot the Voronoi diagram associated with Colombian airports on the map as shown in the figure. It could be useful for you to explore the links: Voronoi_KMeans and Mapas.

Second part

All the calculations should be developed using the Sympy library in Python.

3. (0.8 points) Consider the curve

$$\alpha(t) = (\cos(3t)\cos(t), \cos(3t)\sin(t), 0), \qquad 0 < t < \pi$$

   - Plot the curve $\alpha(t)$ with the tangent vector when $t = 0$.
   - Plot the curveture $\kappa(t)$.
   - Evaluate the curve's length.
   - Evaluate the tosion index. Use numerical integration if necessary. Interpret your result.

4. (0.8 points) Consider the curve

$$\alpha(t) = \left(\frac{\cos(3t)}{1+\sin^2(t)}, \frac{\sin(t)\cos(t)}{1+\sin^2(t)}, 0\right), \qquad 0 < t < 2\pi$$

   - Plot the curve $\alpha(t)$ with the tangent vector when $t = 0$.
   - Plot the curveture $\kappa(t)$.
   - Evaluate the curve's length.
   - Evaluate the tosion index. Use numerical integration if necessary. Interpret your result.

5. (0.6 points) (1.3 points) Show that torsion $\tau(t)$ for a non-arc length parametrized curve $\alpha(t)$ is given by

$$\tau(t) = -\frac{(\alpha'(t) \times \alpha''(t)) \cdot \alpha'''(t)}{|\alpha'(t) \times \alpha''(t)|^2}$$

   Include the procedure in your notebook using LaTeX.

6. (1 points) Consider the toroidal spiral curve

$$\alpha(t) = ((a + \cos(\omega t))\cos(t), (a + \cos(\omega t))\sin(t), \sin(\omega t))$$

   - Evaluate the unit vectors $\vec{T}(t)$, $\vec{N}(t)$ and $\vec{B}(t)$ for $\alpha(t)$.
   - Evaluate and plot curvature $\kappa(t)$ and torsion $\tau(t)$ for $\alpha(t)$ when parameters are $a = 4$ and $\omega = 10$.
   - Add the vectors $\vec{T}(0)$, $\vec{N}(0)$ and $\vec{B}(0)$ to the previous plot.

Submit: Upload to the platform an **.ipynb** file with answers, codes, descriptions and plots.