1.-The linear congruential method produces a sequence of integers between zero and m-1 using the relationship:

$$X_{i+1} = (aX_i + c) \bmod m, \qquad i = 0, 1, 2, \ldots$$

- The initial value X_0 is called the seed;
- a is called the constant multiplier;
- c is the increment
- m is the modulus

When c != 0 the form is called the mixed congruential method; When c = 0, the form is known as the multiplicative congruential method.

2.-

$X_i + 1 = (a X_i + c) \bmod m \qquad i = 0, 1, 2 \ldots \qquad [0, m-1]$

$m > 0$

$X_0 = 1 \qquad a > m \qquad \therefore [0, 1] \qquad m = 2$

$X_1 = (3(1) + 1) \bmod 2 = 0 \qquad c < m \qquad a = 3$

$X_2 = (3(0) + 1) \bmod 2 = 1 \qquad X_0 < m \qquad c = 1$

$X_3 \ (3(1) + 1) \bmod 2 = 0 \qquad X_0 < m$

$\frac{1}{2}$

3.- What is the best way to choose the parameters for the generator? (The seed, the multiplier, the increment and the modulus)

The interval will be [0, m-1]

m > 0 and a < m, c < m, X < m

It has full period if and only if the following three conditions hold (Hull and Dobell, 1962):

1. The only positive integer that (exactly) divides both m and c is 1

2. If q is a prime number that divides m, then q divides a-1

3. If 4 divides m, then 4 divides a-1

•For m a power 2, m=2b, and c≠0
Longest possible period P=m=2b is achieved if c is relative prime to m and a=1+4k, where k is an integer

•For m a power 2, m=2b, and c=0
Longest possible period P=m/4=2b-2 is achieved if the seed X0 is odd and a=3+8k or a=5+8k, for k=0,1,...

•For m a prime and c=0
Longest possible period P=m-1 is achieved if the multiplier a has property that smallest integer k such that ak-1 is divisible by m is k = m-1

4.-

(4) $m=7$ $a=15$
$c=5$ $X_0=3$

1: Only int that divides m and C is 1
2: 7 divides m and a-1
3: Period should be 7

$X_0 = 3$
$X_1 = [15(3)+5] \bmod 7 = [50] \bmod 7 = 1$
$X_2 = 6$
$X_3 = 4$
$X_4 = 2$
$X_5 = 0$
$X_6 = 5$
$X = 3$
$X = 1$
$X = 6$

period = 7

M=7 period =7 (with the conditions established before meet)

$X_{i+1} = (aX_i + C) \bmod m$ $i = 0, 1, 2, ..$ $[0, m-1]$

$m > 0$
$a > m$
$c < m$
$X_0 < m$

$\therefore [0,1]$

$m=2$
$a=3$
$c=1$
$X_0 < m$
1

$X_0 = 1$
$X_1 = (3(1)+1) \bmod 2 = 0$
$X_2 = (3(0)+1) \bmod 2 = 1$
X_3 $(3(1)+1) \bmod 2 = 0$

M=2 period = 2

https://www.mi.fu-berlin.de/inf/groups/ag-tech/teaching/2012_SS/L_19540_Modeling_and_Performance_Analysis_with_Simulation/06.pdf