For each of the following matrices determine:

a) If it represents a regular or non-regular Markov chain.
b) If it's an absorbing Markov chain.
c) The long trend or steady state of the matrix (if that's the case)

1.- (a) $\mathbf{P} = \begin{pmatrix} .5 & .5 \\ .5 & .5 \end{pmatrix}$. (n-1)^2+1 = 2

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}^2 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \qquad \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}^{10} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

a) Regular Markov Chain
b) Not absorbing markov chain
c) The long trend is (.5 .5)

2.- (b) $\mathbf{P} = \begin{pmatrix} .5 & .5 \\ 1 & 0 \end{pmatrix}$. (n-1)^2+1 = 2

$$\begin{pmatrix} 0.5 & 0.5 \\ 1 & 0 \end{pmatrix}^2 = \begin{pmatrix} 0.75 & 0.25 \\ 0.5 & 0.5 \end{pmatrix} \qquad \begin{pmatrix} 0.5 & 0.5 \\ 1 & 0 \end{pmatrix}^{10} = \begin{pmatrix} 0.667 & 0.333 \\ 0.666 & 0.334 \end{pmatrix} \qquad \begin{pmatrix} 0.5 & 0.5 \\ 1 & 0 \end{pmatrix}^{13} = \begin{pmatrix} 0.667 & 0.333 \\ 0.667 & 0.333 \end{pmatrix}$$

a)Regular markov chain
b) Not absorbing
c) The long trend is (.667 .333)

3.- (c) $\mathbf{P} = \begin{pmatrix} 1/3 & 0 & 2/3 \\ 0 & 1 & 0 \\ 0 & 1/5 & 4/5 \end{pmatrix}$ (n-1)^2+1 = 5

$$\begin{pmatrix} \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 1 & 0 \\ 0 & \frac{1}{5} & \frac{4}{5} \end{pmatrix}^5 = \begin{pmatrix} 0.00412 & 0.534 & 0.462 \\ 0 & 1 & 0 \\ 0 & 0.672 & 0.328 \end{pmatrix} \qquad \begin{pmatrix} \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 1 & 0 \\ 0 & \frac{1}{5} & \frac{4}{5} \end{pmatrix}^{100} = \begin{pmatrix} 1.94\cdot10^{-48} & 1.00 & 2.91\cdot10^{-10} \\ 0 & 1 & 0 \\ 0 & 1.00 & 2.04\cdot10^{-10} \end{pmatrix}$$

a) Not regular Markov Chain
b) Absorbing
c) Not long trend

4.- (d) $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. max = 2

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

a) Not regular
b) Not absorbing
c) Not Long Trend

5.- (e) $\mathbf{P} = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}.$ max = 5

a) $\begin{pmatrix} 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}^5 = \begin{pmatrix} 0.221 & 0.443 & 0.336 \\ 0.224 & 0.445 & 0.331 \\ 0.221 & 0.445 & 0.334 \end{pmatrix}$ $\begin{pmatrix} 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}^{100} = \begin{pmatrix} 0.222 & 0.444 & 0.333 \\ 0.222 & 0.444 & 0.333 \\ 0.222 & 0.444 & 0.333 \end{pmatrix}$

a)Regular Markov Chain
b)Not absorbing
c) The long Trend is (.222 .444 .333)

6.- (f) $\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 0 & 1 \end{pmatrix}$ max = 5

$\begin{pmatrix} 1 & 0 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0 & 0 & 1 \end{pmatrix}^5 = \begin{pmatrix} 1 & 0 & 0 \\ 0.484 & 0.0313 & 0.484 \\ 0 & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0 & 0 & 1 \end{pmatrix}^{100} = \begin{pmatrix} 1 & 0 & 0 \\ 0.500 & 7.89 \cdot 10^{-31} & 0.500 \\ 0 & 0 & 1 \end{pmatrix}$

a) Not Regular
b) Absorbing Chain
c) Not long trend

7.- **Example 11.1** According to Kemeny, Snell, and Thompson,[2] the Land of Oz is blessed by many things, but not by good weather. They never have two nice days in a row. If they have a nice day, they are just as likely to have snow as rain the next day. If they have snow or rain, they have an even chance of having the same the next day. If there is change from snow or rain, only half of the time is this a change to a nice day.

[2]J. G. Kemeny, J. L. Snell, G. L. Thompson, *Introduction to Finite Mathematics*, 3rd ed. (Englewood Cliffs, NJ: Prentice-Hall, 1974).

a)

	n	s	r
n	0	.5	.5
s	.25	.5	.25
r	.25	.25	.5

b) Tuesday is the initial matrix, Wednesday would me A^2 and Thursday A^3 so P = .406

$\begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.25 & 0.5 & 0.25 \\ 0.25 & 0.25 & 0.5 \end{pmatrix}^3 = \begin{pmatrix} 0.188 & 0.406 & 0.406 \\ 0.203 & 0.406 & 0.391 \\ 0.203 & 0.391 & 0.406 \end{pmatrix}$

c) $\begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.25 & 0.5 & 0.25 \\ 0.25 & 0.25 & 0.5 \end{pmatrix}^{100} = \begin{pmatrix} 0.200 & 0.400 & 0.400 \\ 0.200 & 0.400 & 0.400 \\ 0.200 & 0.400 & 0.400 \end{pmatrix}$ $\begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.25 & 0.5 & 0.25 \\ 0.25 & 0.25 & 0.5 \end{pmatrix}^{150} = \begin{pmatrix} 0.200 & 0.400 & 0.400 \\ 0.200 & 0.400 & 0.400 \\ 0.200 & 0.400 & 0.400 \end{pmatrix}$

The long trend is (.200 .400 .400)

8.- **Example 11.6** In the Dark Ages, Harvard, Dartmouth, and Yale admitted only male students. Assume that, at that time, 80 percent of the sons of Harvard men went to Harvard and the rest went to Yale, 40 percent of the sons of Yale men went to Yale, and the rest split evenly between Harvard and Dartmouth; and of the sons of Dartmouth men, 70 percent went to Dartmouth, 20 percent to Harvard, and 10 percent to Yale.

a)

	H	D	Y
H	.8	0	.2
D	.20	.70	.10
Y	.30	.30	.40

b) This matrix represents the probability of the next gen, grandson would be next next gen so M^3 would give me this. P = .217

$$\begin{pmatrix} 0.8 & 0 & 0.2 \\ 0.2 & 0.7 & 0.1 \\ 0.3 & 0.3 & 0.4 \end{pmatrix}^3 = \begin{pmatrix} 0.644 & 0.114 & 0.242 \\ 0.413 & 0.409 & 0.178 \\ 0.477 & 0.306 & 0.217 \end{pmatrix}$$

c) P = .114