Constrained Wasserstein Barycenter

Juan Nicolas Mendoza Roncancio, ~~Welington De Oliveira~~

Mines Paris PSL-CMA

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Problem and state of the art

~~How can we find the "mean" of a set of images?~~

$$\min_{\mu \in \mathcal{P}(\mathbb{R}^d)} \sum_{m=1}^{M} \frac{1}{M} W_2^2(\mu, \nu^m)$$

~~On images we got a **Huge** LP. We can even add restrictions on μ:~~

$$\begin{cases} \min\limits_{\cancel{p \in X}, \pi \geq 0} \sum_{m=1}^{M} \frac{1}{M} \sum_{r=1}^{R} \sum_{s=1}^{S^m} \|\xi_r - \zeta_s^m\|^2 \pi_{rs}^m \\ \textbf{s.t} \ \sum_{r=1}^{R} \pi_{rs}^{(m)} = q_s^{(m)}, \quad \sum_{s=1}^{S^{(m)}} \pi_{rs}^{(m)} = p_r \end{cases}$$

In has various aplications: **Bias reduction, distribution clustering, Data augmentation**.
~~To solve this problem we have various algorithms:~~

- ~~IBP~~
- ~~MAM~~
- ~~CMAM~~
- ~~Bundle Methods for DTS penalties~~

Research Question

~~What happens if we have the following:~~

$$
\begin{cases}
\min\limits_{p \geq 0, \pi \in Y} \sum_{m=1}^{M} \frac{1}{M} \sum_{r=1}^{R} \sum_{s=1}^{S^m} \|\xi_r - \zeta_s^m\|^2 \pi_{rs}^m \\
\textbf{s.t} \quad \sum_{r=1}^{R} \pi_{rs}^{(m)} = q_s^{(m)}, \quad \sum_{s=1}^{S^{(m)}} \pi_{rs}^{(m)} = p_r
\end{cases}
$$

~~For instance: $Y = \{\pi \mid \|\pi^{(m)}\|_F \leq \tau\}$ or $\sum_s \pi_{rs} \leq c_r$ This leads to an extension of (C)MAM, IBP or even to a new algorithm with applications on finance:~~

- Prices cannot be pushed consistently upward or downward.
- The transport plan cannot promise payoffs incompatible with observed market prices. [Yan Dolinsky, 2018]
- The transport plan must respect temporal information.
- Price trajectories cannot oscillate in an excessively violent manner. [A. Galichon1, 2014]

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