CS375 HW13 Dec 7 2021

Juan Alejandro Ormaza

1. $$\begin{cases} y'(t) = (y(t))^2 - y(t) & t \in [0,2] \\ y(0) = 0.5 \end{cases}$$

$h = 1.0$

t_i	y_i	$f(t_i, y_i)$	$y_{i+1} = y_i + h \cdot f(t_i, y_i)$
0	0.5	-0.25	0.25
1	0.25	-0.1875	0.0625
2	0.0625	-0.0586	0.0039

2. $$\begin{cases} y'(t) = y(t)^{1/3} \\ y(0) = 0 \end{cases}$$

$$\frac{dy}{dt} = y^{1/3}$$

$$\int \frac{dy}{y^{1/3}} = \int dt \Rightarrow \frac{3}{2} y^{2/3} = t + c$$

$$y(t) = \left(\frac{2}{3}(t+c)\right)^{3/2}$$

$$y_{i+1} = y_i + h \; f(t_i, y_i)$$

base case:

$$y_1 = 0 + h(0)^{1/3} = 0$$

then:

$y_2 = 0 + h(0)^{1/3} = 0$

$y_3 = 0$

$y_4 = ''$

$y_{i+1} = 0 \ldots$

Solution converges to 0

3. Matlab

4. a) $E[f(x)]$ on $[0,2]$

$$E[f(x)] = E[\sqrt{4-x^2}] = \int_0^2 \sqrt{4-x^2}\, p(x)\, dx$$

$$p(x) = \frac{1}{b-a} = \frac{1}{2} \longrightarrow = \int_0^2 \frac{1}{2}\sqrt{4-x^2}\, dx$$

$$= \frac{1}{2}\int_0^2 \sqrt{4-x^2} = \frac{\pi}{2}$$

the same as the average

b.

$$\sigma^2[f(x)] = \int_a^b \left(f(x) - E[f(x)]\right)^2 p(x)\, dx$$

$$\sigma^2[f(x)] = \int_0^2 \left(\sqrt{4-x^2} - \frac{\pi}{2}\right)^2 \frac{1}{2}\, dx$$

$$= \frac{1}{2}\int_0^2 4 - x^2 - \pi\sqrt{4-x^2} + \frac{\pi^2}{4}\, dx$$

$$= \frac{1}{2}\left[\cancel{8} - \cancel{8} - \pi^2 + \frac{\pi^2}{2}\right] = -\frac{1}{4}\pi^2$$

c) } Matlab
d) }

Contents

CS375 HW13

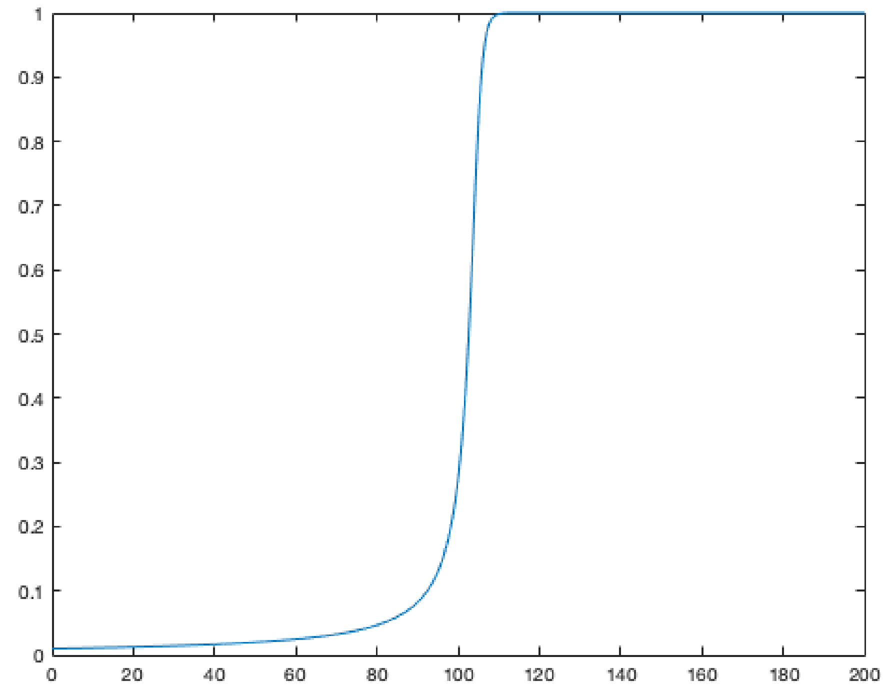
Juan Alejandro Ormaza DEC 7 2021

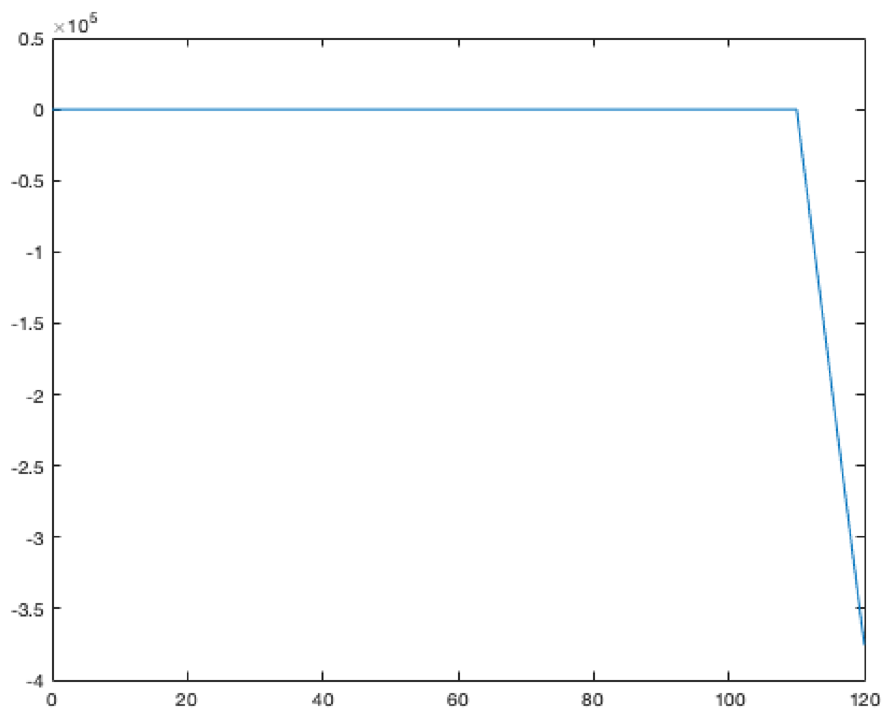
```
clear all;clc;close all;
```

Problem 3

```
f=@(t,y) y^2 - y^3;

[y,t]=RK4(f,0.01,0.1);

[y1,t1]=RK4(f,0.01,10);

figure(1);
plot(t,y)

figure(2)
plot(t1,y1)

fprintf("the plot for h=0.1 works well because h is small enough to not induce any error. \n")
fprintf("the plot for h=10 on the other hand has an oscillatory behavior that eventually overshoots \n")
fprintf("therefore, the solution with h=0.1 describes the behavior of the flame more precisely \n")
```

```
the plot for h=0.1 works well because h is small enough to not induce any error.
the plot for h=10 on the other hand has an oscillatory behavior that eventually overshoots
therefore, the solution with h=0.1 describes the behavior of the flame more precisely
```





4.C

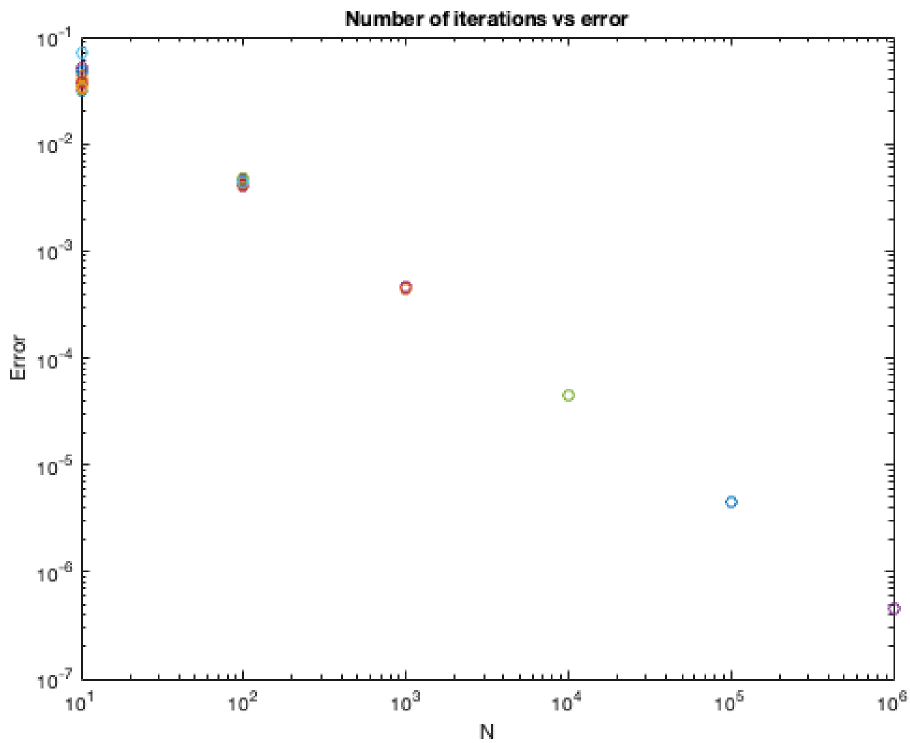
```
f_x=@(x) sqrt(4-x.^2);

% checking that the code works
% answer should be close to 1.57 or pi/2

monte_carlo(f_x,0,2,10000)
```

```
ans =

    1.5738
```

4.D

```
N = [10, 100, 1000, 10000, 100000, 1000000];

E=zeros(length(N),10);
V2=zeros(length(N),10);
error=zeros(length(N),10);

for i=1:length(N)
    for j=1:10
        E(i,j)=monte_carlo(f_x,0,2,N(i));
        f_v=@(x) (sqrt(4-x.^2)-E(i,j)).^2;
        V2(i,j)=monte_carlo(f_v,0,2,N(i));
        error(i,j) = sqrt(V2(i,j))/N(i);
    end
end

for i=1:length(N)
loglog(N(i),error(i,:),'o');
hold on
end
title('Number of iterations vs error');
xlabel('N');
ylabel('Error');


fprintf("the error becomes smaller as N increases (as expected.) Moreover, it is possible to see how \n");
fprintf("as N grows, the error starts to converge and cluster.\n")
```

```
the error becomes smaller as N increases (as expected.) Moreover, it is possible to see how
as N grows, the error starts to converge and cluster.
```



Published with MATLAB® R2021a

```
function [w,t] = RK4(f,delta,h)
% RK4
% Juan Alejandro Ormaza
% DEC 7 2021

a=0;
b=2/delta;
N=(b-a)/h;

w = zeros(N+1,1);
t = zeros(N+1,1);

w(1)=delta;

for i = 1:N

    K1 = h*f(t(i),w(i));
    K2 = h*f(t(i)+(h/2),w(i)+(K1/2));
    K3 = h*f(t(i)+(h/2),w(i)+(K2/2));
    K4 = h*f(t(i)+h,w(i)+K3);

    w(i+1) = w(i) + ((K1+(2*K2)+(2*K3)+K4)/6);
    t(i+1) = a + (i*h);
end

end
```

```
Not enough input arguments.

Error in RK4 (line 7)
b=2/delta;
```

Published with MATLAB® R2021a

```
function I = monte_carlo(f, a, b, N)
% Montecarlo
% Juan Alejandro Ormaza
% DEC 7 2021

x = a + (b-a).*rand(N,1);

A=f(x);

I=sum(A)/N;

end
```

```
Not enough input arguments.

Error in monte_carlo (line 6)
x = a + (b-a).*rand(N,1);
```