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CS375: HW6
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1. Explain how the component-wise update formula for Jacobi given on the course slides is equivalent to the vector-update version. That is, for the Jacobi algorithm show that the vector update version

$$x^{(k)} = D^{-1}(C_L + C_U)\, x^{(k-1)} + D^{-1} b$$

is the same as the component-wise update,

$$x_i^{(k)} = -\sum_{j=1,\, j\neq i}^{n} \left(\frac{a_{ij}}{a_{ii}}\right) x_j^{(k-1)} + \frac{b_i}{a_{ii}}$$

Let's look at the i-th component of the vector update version of Jacobi

$$x^{(k)} = \underbrace{\begin{bmatrix} 0 \cdots \frac{1}{a_{ii}} \cdots 0 \end{bmatrix} \begin{bmatrix} 0 & \cdots & -a_{1n} \\ -a_{21} & & \vdots \\ -a_{i1} & 0 & -a_{in} \\ -a_{n1} & \cdots & 0 \end{bmatrix}} \cdot \begin{bmatrix} x_1^{(k-1)} \\ \vdots \\ x_n^{(k-1)} \end{bmatrix} + \begin{bmatrix} 0 \cdots \frac{1}{a_{ii}} \cdots 0 \end{bmatrix} \begin{bmatrix} b_1 \\ \vdots \\ b_i \\ \vdots \\ b_n \end{bmatrix}$$

$$x^{(k)} = \begin{bmatrix} \frac{-a_{i1}}{a_{ii}}, \frac{-a_{i2}}{a_{ii}}, \underbrace{0}_{a_{ii}=0}, \ldots, \frac{-a_{in}}{a_{ii}} \end{bmatrix} \begin{bmatrix} x_1^{(k-1)} \\ \vdots \\ x_n^{(k-1)} \end{bmatrix} + \frac{b_i}{a_{ii}}$$

$$\boxed{x^{(k)} = -\sum_{\substack{j=1 \\ j\neq i}}^{n} \left(\frac{a_{ij}}{a_{ii}}\right) x_j^{(k-1)} + \frac{b_i}{a_{ii}}}$$

It is the same

4.b

based on the answer of part a we know

$M=5$

thus Big-Oh is $O(m^2 n) = O(25n)$

additionally, we know that component-wise does not take advantage of sparsity because it goes through each element of the matrix.

Vector-update does take advantage of the speed that sparsity provides because it uses $Q|r$ to solve for each line/row

4.c $m = 2p = 2\sqrt{n}$

Since the solution for an m banded system Takes $O(m^2 n)$ to solve.

We know that the system with $m = 2\sqrt{n}$ will take: $O(4n^2)$ or essentially $O(n^2)$

with $n = \alpha = 2$

4.d

if $n=100$

let's assume i is the number of iterations

1) we want to find the moment when Jacobi becomes more expensive

$$\frac{i_1}{2500} \geq \frac{i_2}{10000} \qquad 4 i_1 \geq i_2$$

$i_1 \geq 0.25\, i_2$ at 4 times i_2 (number of iterations for c)

$i_1 \geq \frac{1}{4} i_2$ Jacobi will become more expensive.