Contents

Homework 7

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```
clear all; clc; close all;
format long e
```

Problem 1

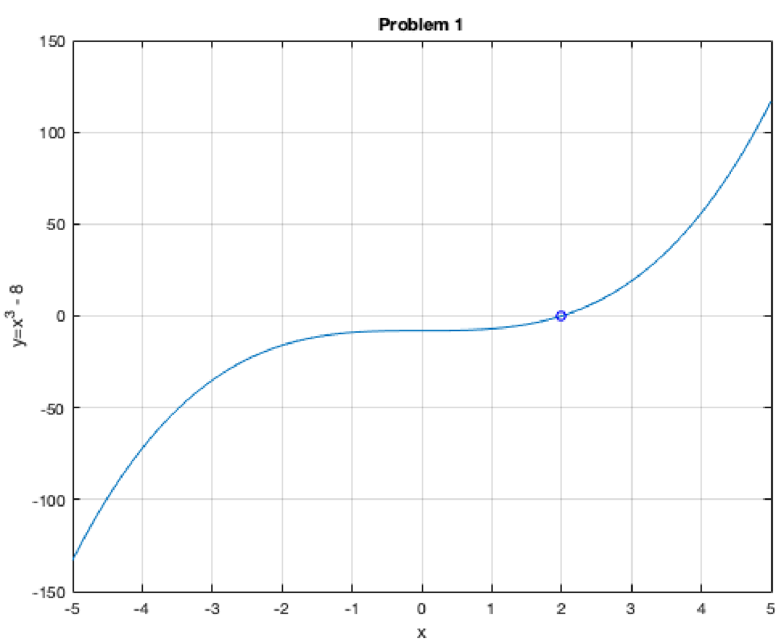
```
f_x=@(x) x.^3 - 8;
d_f=@(x) 3*x.^2;

x=linspace(-5,5,100);

figure()
plot(x,f_x(x))
hold on
plot(2,0,'bo')
title('Problem 1');
xlabel('x');
ylabel('y=x^3 - 8');
grid on

fprintf("there is only one zero at x=2\n\n")
```

```
there is only one zero at x=2
```



Problem 2 and 3

```
TOL=10^(-10);

x_bisection = my_bisection(f_x,1,4,TOL);

x_newton = my_newton(f_x,d_f,4,TOL);

x_secant = my_secant(f_x,1,4,TOL);

fprintf("the root of the bisection method is %2.14f and it took %3.0f iterations\n\n",x_bisection(end),length(x_bisection));

fprintf("the root of Newton's method is %2.14f and it took %3.0f iterations\n\n",x_newton(end),length(x_newton));

fprintf("the root of the Secant method is %2.14f and it took %3.0f iterations\n\n",x_secant(end),length(x_secant));
```

```
the root of the bisection method is 2.00000000002910 and it took  35 iterations

the root of Newton's method is 2.00000000000000 and it took   8 iterations

the root of the Secant method is 2.00000000000000 and it took  11 iterations
```

Problem 4

```
ek_bisection = abs(x_bisection-2);

ek_newton = abs(x_newton-2);

ek_secant = abs(x_secant-2);


ek1_bisection = circshift(ek_bisection,-1);

ek1_newton = circshift(ek_newton,-1);

ek1_secant = circshift(ek_secant,-1);


bisection_conv = ek1_bisection./ek_bisection;
bisection_conv(length(bisection_conv))=0;

newton_conv = ek1_newton./(ek_newton.^2);
%newton_conv(length(newton_conv))=0;

secant_conv = ek1_secant./(ek_secant.^(1.62));
%secant_conv(length(secant_conv))=0;


iter1=1:1:length(x_bisection);

iter2=1:1:length(x_newton);

iter3=1:1:length(x_secant);

fprintf("Bisection method:\n")
fprintf("iteration\t error\t\t convergence rate\n")
fprintf("%3.0f \t    %3.12f \t %3.12f\n",[iter1;ek_bisection;bisection_conv])
fprintf("\n")

fprintf("\nNewton's method:\n")
fprintf("iteration\t error\t\t convergence rate\n")
fprintf("%3.0f \t    %3.12f \t %3.12f\n",[iter2;ek_newton;newton_conv])
fprintf("\n")

fprintf("\nSecant method:\n")
fprintf("iteration\t error\t\t convergence rate\n")
fprintf("%3.0f \t    %3.12f \t %3.12f\n",[iter3;ek_secant;secant_conv])
fprintf("\n")
```

```
Bisection method:
iteration        error            convergence rate
  1         0.500000000000       0.500000000000
  2         0.250000000000       0.500000000000
  3         0.125000000000       0.500000000000
  4         0.062500000000       0.500000000000
  5         0.031250000000       0.500000000000
  6         0.015625000000       0.500000000000
  7         0.007812500000       0.500000000000
  8         0.003906250000       0.500000000000
  9         0.001953125000       0.500000000000
 10         0.000976562500       0.500000000000
 11         0.000488281250       0.500000000000
 12         0.000244140625       0.500000000000
 13         0.000122070312       0.500000000000
 14         0.000061035156       0.500000000000
 15         0.000030517578       0.500000000000
 16         0.000015258789       0.500000000000
 17         0.000007629395       0.500000000000
 18         0.000003814697       0.500000000000
 19         0.000001907349       0.500000000000
 20         0.000000953674       0.500000000000
 21         0.000000476837       0.500000000000
 22         0.000000238419       0.500000000000
 23         0.000000119209       0.500000000000
 24         0.000000059605       0.500000000000
 25         0.000000029802       0.500000000000
 26         0.000000014901       0.500000000000
 27         0.000000007451       0.500000000000
 28         0.000000003725       0.500000000000
 29         0.000000001863       0.500000000000
 30         0.000000000931       0.500000000000
 31         0.000000000466       0.500000000000
 32         0.000000000233       0.500000000000
 33         0.000000000116       0.500000000000
 34         0.000000000058       0.500000000000
 35         0.000000000029       0.000000000000


Newton's method:
iteration        error            convergence rate
  1         2.000000000000       0.208333333333
  2         0.833333333333       0.318339100346
  3         0.221068819685       0.435296037791
  4         0.021273536809       0.493001916400
  5         0.000223114608       0.499925636500
  6         0.000000024886       0.717046602291
  7         0.000000000000       0.000000000000
  8         0.000000000000       Inf


Secant method:
iteration        error            convergence rate
  1         1.000000000000       2.000000000000
  2         2.000000000000       0.216890309240
  3         0.666666666667       0.815994397306
  4         0.423076923077       0.876387440260
  5         0.217518135116       0.579055527207
  6         0.048917344679       0.667701420151
  7         0.005029353441       0.662748984007
  8         0.000125256065       0.661541563095
  9         0.000000315494       0.672011570641
 10         0.000000000020       0.000000000000
 11         0.000000000000       Inf
```

Problem 5

```
% BY HAND PROBLEM ATTACHED TO PDF
```

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CS375: HW7
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4. What is the rate of convergence that you observe for each method? Explain your reasoning for your observed convergence rate. Remember the numerical data may be a bit noisy.

- Bisection method

let $\delta_0 = b - a$

$\delta_1 = \frac{1}{2}\delta_0$

$\delta_2 = \frac{1}{2}\delta_1 = \frac{1}{4}\delta_0$

$\delta_n = \left(\frac{1}{2}\right)^n \delta_0$

$\frac{\delta_n}{\delta_0} = \left(\frac{1}{2}\right)^n$

$|x_* - c_n| \leq (b_n - a_n)/2 = \left(\frac{1}{2}\right)^{n+1} \delta_0$

$$\boxed{\frac{|e_{n+1}|}{|e_n|} = \frac{\cancel{\left(\frac{1}{2}\right)^{n+1}} \cancel{\delta_0}}{\cancel{\left(\frac{1}{2}\right)^{n}} \cancel{\delta_0}} = \left(\frac{1}{2}\right)}$$

Convergence rate agrees with results in matlab code.

- Newton's method

$$f(x_*) = f(x_k) + (x_* - x_k)f'(x_k) + \frac{1}{2}(x_* - x_k)^2 f''(\xi) = 0$$

$$\underbrace{\left(\frac{f(x_k)}{f'(x_k)} - x_k\right)}_{-x_{k+1}} + x_* + (x_* - x_k)^2 \left(\frac{1}{2}\right)\frac{f''(\xi)}{f'(x_k)} = 0$$

$$\frac{|x_* - x_{k+1}|}{|x_* - x_k|^2} = \left(\frac{1}{2}\right)\left|\frac{f''(\xi)}{f'(x_k)}\right|$$

when $f'(x_k) \to f'(r)$ convergence rate diverges because denominator $\to 0$

- Bisection Method

↳ Converges with $r = 1.62$
will also diverge as $x_k \to x_*$
because denominator becomes 0.

5.

Prove what the general convergence behavior of Newton's method is when used to find cube roots with

$$f(x) = x^3 - a$$

for any number $a \in \mathbb{R}$, $a \neq 0$. You may use any existing convergence theorems in the textbook or from the slides, and make assumptions like having a "suitable starting guess"

$$f'(x_k) = 3x_k^2$$

$$f''(\xi) = 6\xi$$

$$\left(\frac{1}{2}\right)\left|\frac{6\xi}{3x_k^2}\right|$$

as the interval $x_* \leq \xi \leq x_k$
gets smaller $x \approx x_k \approx \xi$
Where $x = \sqrt[3]{a}$ and $a \neq 0$

We thus find that $\lim \dfrac{|e_{k+1}|}{|e_k|^r} \leq C$

is always bounded.

Thus, in this case we find that there is always a constant C such that

$\left(\frac{1}{2}\right) \frac{6 \cdot x}{3x^2}$ is always bounded as long as $x = \sqrt[3]{a}$ and $a \neq 0$

for example with $a=8$ $x=2$

and $\boxed{\left(\frac{1}{2}\right) \cdot \frac{6(2)}{3(4)} = \frac{1}{2}}$ which (without the nase) was close to the magnitude of $\frac{|e_{k+1}|}{|e_k|^r}$ found in MatLab.

6) What is the convergence rate of Newton's method if $a=0$? Explain.

The convergence rate at $a=0$ is undefined and will cause the method to diverge because there will be a 0 at the denominator.

$\therefore$ $\boxed{\left(\frac{1}{2}\right) \frac{6}{3(0)} \therefore \text{ diverges}}$