CS/MATH 375, Fall 2021 — HOMEWORK # 5
Due : Sept. 28 at 6pm on UNM Learn

Instructions

- **Report:** In general, your report needs to read coherently. That is, start off by answering question 1. Fully answer the question, and provide all the information needed to understand your answer. If Matlab code or output is part of the question, include that code or output (e.g., screenshot) alongside your narrative answer. If discussion is required for a question, include that. *Your report should include your Matlab scripts, code output, and any figures.*
- **What to hand in:** Submission must be one **single PDF** document submitted on UNM Learn.

  Overall, your report is your narrative explanation of what was done, your answers to the specific questions, and how you arrived at your answers. If discussion is required for a question, include that. *Your report should include your Matlab scripts, code output, and any figures.*
- **Partners:** You are strongly encouraged to **work in pairs**. If you work with a partner, only one partner should submit a homework, but write both collaborators' names at the top. Groups of more than 2 students are not allowed.
- **Typesetting:** If you write your answers by hand, then make sure that your handwriting is readable. Otherwise, I cannot grade it.
- **Plots:** All plots/figures in the report must be generated in Matlab or Python and not hand drawn (unless otherwise specified in the homework question).

  In general, make sure to (1) title figures, (2) label both axes, and (3) include a legend for the plotted data sets. The font-size of all text in your figures must be large and easily readable.
- **Generating PDFs:** See the UNM Learn homework page for tips.

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1. Prove or provide a counter-example for each of the following statements. You can use Matlab to find counter-examples. Assume A is a matrix and c is a scalar.

   (a) $\|cA\| = |c|\,\|A\|$

   (b) $\kappa(cA) = \kappa(A)$ for any nonzero constant c

   (c) $\kappa(A)$ is the same for every matrix norm

2. Consider the matrix

$$A = \begin{bmatrix} 1 & k \\ 1 & 1 \end{bmatrix},$$

where $k \neq 1$.

(a) Find $\|A\|_1$ in terms of k.

(b) Find $\|A^{-1}\|_1$ in terms of k. Remember that

$$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$$

(c) Using the previous two parts, find $\kappa(A)$ in terms of k

(d) Explain geometrically why the condition number grows as $k \to 1$

(e) If you solve the linear system $Ax = b$ using Gaussian elimination without pivoting, for what k should you expect to have 10 digits of accuracy, when the computations are carried out using double precision? Recall that for double precision we have $\epsilon_m \approx 2.2 \times 10^{-16}$

3. Let

$$A = \begin{bmatrix} 1 & 1+\epsilon \\ 1-\epsilon & 1 \end{bmatrix}.$$

(a) Find the determinant of A

(b) Using double precision floating point arithmetic, for what values of ϵ will $\det(A)$ equal to 0?

(c) Find the LU factorization of A without pivoting

4. Let ϵ be some small positive number less than half machine epsilon and

$$A = \begin{bmatrix} \epsilon & 1 \\ 1 & 1 \end{bmatrix}.$$

(a) Compute $1 - \frac{1}{\epsilon}$ using floating-point arithmetic

(b) Without pivoting, find the LU factorization of this matrix in floating-point arithmetic (hint: you'll want to use the previous part)

(c) Compute LU for the L and U matrices found in the previous part

(d) Permute the rows of A, compute the LU factorization of the resulting matrix in floating-point arithmetic, and confirm the product LU is correct

5. Consider the symmetric positive definite matrix

$$E = \begin{bmatrix} A & \alpha \\ \alpha^t & c \end{bmatrix},$$

where c is a scalar, α is a $n \times 1$ vector, and A is an $n \times n$ matrix.

(a) Prove that c is positive and A is positive definite

(b) By the above, the Cholesky factorization $A = LL^t$ exists. Assuming $(L^{-1}\alpha)^t(L^{-1}\alpha) < c$, find the Cholesky factorization of E in terms of L, α, c.