Juan Alejandro Ormaza

HW9 Nov 2^{nd} 2021

Problem 2.a) How many points are needed to ensure the composite trapezoid rule is accurate to 10^{-6} for

$$I = \int_0^{\pi} x \cdot \sin(x)\, dx$$

let $f(x) = x\sin(x)$

$$f'(x) = \sin(x) + x\cos(x)$$

$$f''(x) = \cos(x) + \cos(x) - x\sin(x)$$

$$= 2\cos(x) - x\cdot\sin(x)$$

$$f'''(x) = -2\sin(x) - \sin(x) - x\cos(x)$$

$$0 = -3\sin(x) - x\cos(x)$$

in 0 to π root at $x = 2.4556$ ~~thus~~ max

$$|f''(2.4556)| = 3.103$$

$$\frac{(b-a)h^2 f''(\eta)}{12} \leq 10^{-6}$$

$$\frac{(b-a)}{n} = h$$

$$\frac{\pi h^2 f''(\eta)}{12} \leq 10^{-6}$$

$$h^2 f''(\eta) \leq \frac{12\cdot 10^{-6}}{\pi}$$

$$h^2 \leq \frac{12\cdot 10^{-6}}{\pi(3.103)}$$

$$h \leq \sqrt{\frac{12}{\pi(3.103)}}\, 10^{-3}$$

$$\frac{\pi}{n} \leq \sqrt{\frac{12}{\pi(3.103)}}\cdot 10^{-3}$$

$$\frac{\pi\cdot 10^{3}}{1.109} \leq n$$

$$\boxed{2833 \leq n}$$

problem 2.d Approximate the above integral using the composite Simpson rule with 2 intervals of equal length (take $n=4$) How many evaluations of the function $f(x)$ are required?

$$\int_0^{\frac{\pi}{2}} f(x)\,dx + \int_{\pi/2}^{\pi} f(x)\,dx \qquad f(x) = x\sin(x)$$

$n=4$

$$[\underbrace{x_0, x_1}_{1}, \overbrace{x_1, x_2}^{2}, \underbrace{x_2, x_3}_{3}, \overbrace{x_3, x_4}^{4}]$$

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + f(b) + 4\sum_{i=1}^{n/2} f(a+(2i-1)h) + 2\sum_{i=1}^{(n/2)-1} f(a+2ih)\right]$$

Since $n=4$

$$\int_0^{\frac{\pi}{2}} f(x)dx = \frac{h}{3}\Big[\underbrace{f(0)}_{1\text{ eval}} + \underbrace{f(\pi/2)}_{2\text{ eval}} + 4\underbrace{\sum_{i=1}^{2} f(a+(2i-1)h)}_{4\text{ eval}} + 2\underbrace{\sum_{i=1}^{1} f(a+2ih)}_{5\text{ eval}}\Big]$$

Since there is 5 evaluations of $f(x)$ per interval and 2 intervals total, there will be 10 evaluations of $f(x)$ to approximate the integral