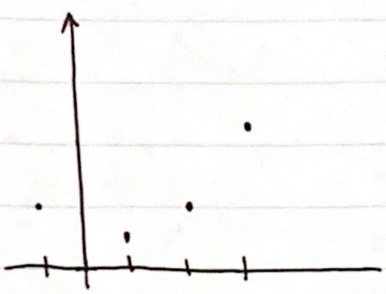
Juan Alejandro Ormaza October 25^{th} 2021

HW8

1. Polynomial interpolation

How many degree d polynomials pass through $(-1,3), (1,1), (2,3), (3,7)$, for $d=2,3,6$? If at least one polynomial passes through these points, write it down. If no such polynomial exists, explain why.



per theorem:

if points $x_0, \dots, x_n$ are distinct, then for arbitrary $y_0, \dots y_n$, there is a unique polynomial $p(x)$ of degree at most n such that $p(x_i) = y_i$ for $i=0,\dots,n$

$\therefore$ for $n+1$ points $=4$ $n=3$ and $p(x)$ is of degree at most 3.

Therefore $d=2$ and $d=3$ are possible polynomials.

we can use the Vandermonde Matrix to visualize this.

$$\underbrace{\begin{bmatrix} 1 & x_1 & x_1^2 & x_1^3 \\ 1 & x_2 & x_2^2 & x_2^3 \\ 1 & x_3 & x_3^2 & x_3^3 \\ 1 & x_4 & x_4^2 & x_4^3 \end{bmatrix}}_{\text{Degree 3 Polynomial}} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \left.\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}\right\} \text{4 elements}$$

2. a By hand problem Assume (x_j, y_j), $j=0,\dots,n$ are given. Derive the system $Va=y$ that determines the coefficients $a=[a_0,\dots a_n]^T$ (here $y=[y_0,y_1,\dots,y_n]^T$), that is, find the structure (i.e., general form) of V.

for each point

y_j we can assume a polynomial $p(x)$ that when evaluated at x_j yields y_j

$\therefore$

$$\underbrace{\begin{aligned} a_0 + a_1 x_0 + a_2 x_0^2 + \dots + a_n x_0^n &= y_0 \\ a_0 + a_1 x_1 + a_2 x_1^2 + \dots + a_n x_1^n &= y_1 \\ &\vdots \\ a_0 + a_1 x_n + a_2 x_n^2 + \dots + a_n x_n^n &= y_n \end{aligned}}$$

$$\underbrace{\begin{bmatrix} 1 & x_0 & x_0^2 & \cdots & x_0^n \\ 1 & x_1 & x_1^2 & \cdots & x_1^n \\ 1 & x_2 & x_2^2 & \cdots & x_2^n \\ \vdots & & & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^n \end{bmatrix}}_{V} \underbrace{\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}}_{a} = \underbrace{\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}}_{y}$$