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HW9

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Problem 2.a) How many points are needed to ensure the composite trapezoid rule is accurate to 10^{-6} for

$$I = \int_0^{\pi} x \cdot \sin(x) dx$$

let $f(x) = x \sin(x)$

$$f'(x) = \sin(x) + x \cos(x)$$

$$f''(x) = \cos(x) + \cos(x) - x \sin(x)$$

$$= 2 \cos(x) - x \sin(x)$$

$$f'''(x) = -2 \sin(x) - \sin(x) - x \cos(x)$$

$$0 = -3 \sin(x) - x \cos(x)$$

in 0 to π root at $x = 2.4556$ this max

$$|f''(2.4556)| = 3.103$$

$$\frac{(b-a)h^2 f''(\eta)}{12} \leq 10^{-6}$$

$$\frac{(b-a)}{n} = h$$

$$\frac{\pi h^2 f''(\eta)}{12} \leq 10^{-6}$$

$$\frac{\pi}{n} \leq \sqrt{\frac{12}{\pi(3.103)}} \cdot 10^{-3}$$

$$h^2 f''(\eta) \leq \frac{12 \cdot 10^{-6}}{\pi}$$

$$\frac{\pi \cdot 10^3}{1.109} \leq n$$

$$h^2 \leq \frac{12 \cdot 10^{-6}}{\pi(3.103)}$$

$$2833 \leq n$$

$$h \leq \sqrt{\frac{12}{\pi(3.103)}} \cdot 10^{-3}$$

problem 2.d Approximate the above integral using the composite Simpson rule with 2 intervals of equal length (take $n=4$)
How many evaluations of the function $f(x)$ are required?

$$\int_0^{\frac{\pi}{2}} f(x) dx + \int_{\frac{\pi}{2}}^{\pi} f(x) dx \quad f(x) = x \sin(x)$$

$n=4$

$$[x_0, x_1, x_2, x_3, x_4]$$

Diagram showing nodes x_0, x_1, x_2, x_3, x_4 with brackets indicating intervals of length h . Brackets are labeled 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100.

$$\int_a^b f(x) dx \approx \frac{h}{3} \left[f(a) + f(b) + 4 \sum_{i=1}^{n/2} f(a + (2i-1)h) + 2 \sum_{i=1}^{(n/2)-1} f(a + 2ih) \right]$$

Since $n=4$

$$\int_0^{\frac{\pi}{2}} f(x) dx = \frac{h}{3} \left[f(0) + f(\frac{\pi}{2}) + 4 \sum_{i=1}^2 f(a + (2i-1)h) + 2 \sum_{i=1}^1 f(a + 2ih) \right]$$

1 eval 2 eval 4 eval 5 eval

Since there is 5 evaluations of $f(x)$ per interval and 2 intervals total, there will be 10 evaluations of $f(x)$ to approximate the integral