CS375 HW13

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1. $\begin{cases} y'(t) = (y(t))^2 - y(t) & t \in [0,2] \\ y(0) = 0.5 \end{cases}$

$h = 1.0$

t_i	y_i	$f(t_i, y_i)$	$y_{i+1} = y_i + h \cdot f(t_i, y_i)$
0	0.5	-0.25	0.25
1	0.25	-0.1875	0.0625
2	0.0625	-0.0586	0.0039

2. $\begin{cases} y'(t) = y(t)^{1/3} \\ y(0) = 0 \end{cases}$

$$\frac{dy}{dt} = y^{1/3}$$

$$\int \frac{dy}{y^{1/3}} = \int dt \Rightarrow \frac{3}{2} y^{2/3} = t + c$$

$$y(t) = \left(\frac{2}{3}(t+c)\right)^{3/2}$$

$$y_{i+1} = y_i + h \; f(t_i, y_i)$$

base case:

$$y_1 = 0 + h(0)^{1/3} = 0$$

then:

$y_2 = 0 + h(0)^{1/3} = 0$

$y_3 = 0$

$y_4 = ''$

$y_{i+1} = 0 \ldots$

Solution converges to 0

3. Matlab

4. a) $E[f(x)]$ on $[0,2]$

$$E[f(x)] = E\left[\sqrt{4-x^2}\right] = \int_0^2 \sqrt{4-x^2}\, p(x)\, dx$$

$$p(x) = \frac{1}{b-a} = \frac{1}{2} \longrightarrow = \int_0^2 \frac{1}{2}\sqrt{4-x^2}\, dx$$

$$= \frac{1}{2}\int_0^2 \sqrt{4-x^2} = \frac{\pi}{2}$$

the same as the average

b.

$$\sigma^2[f(x)] = \int_a^b \left(f(x) - E[f(x)]\right)^2 p(x)\, dx$$

$$\sigma^2[f(x)] = \int_0^2 \left(\sqrt{4-x^2} - \frac{\pi}{2}\right)^2 \frac{1}{2}\, dx$$

$$= \frac{1}{2}\int_0^2 4 - x^2 - \pi\sqrt{4-x^2} + \frac{\pi^2}{4}\, dx$$

$$= \frac{1}{2}\left[\cancel{8} - \cancel{8} - \pi^2 + \frac{\pi^2}{2}\right] = -\frac{1}{4}\pi^2$$

c) } Matlab
d) }