CS375: HW11

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1.

$$A = \begin{bmatrix} 1 & \epsilon \\ \epsilon & 1 \end{bmatrix}$$

a) Characteristic polynomial of A

$$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & \epsilon \\ \epsilon & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - \epsilon^2$$

$$= \lambda^2 - 2\lambda + 1 - \epsilon^2$$

b) What are the eigenvalues and eigenvectors of A.

$$\lambda_{1,2} = \frac{2 \pm \sqrt{4 - 4(1-\epsilon^2)}}{2}$$

$$\lambda_{1,2} = 1 \pm \sqrt{\cancel{1} - \cancel{1} + \epsilon^2}$$

$$\lambda_{1,2} = 1 \pm \epsilon$$

$$\boxed{\lambda_1 = 1+\epsilon \quad \lambda_2 = 1-\epsilon}$$

$$A - \lambda_1 I = \begin{bmatrix} \cancel{1}-\cancel{1}-\epsilon & \epsilon \\ \epsilon & -\epsilon \end{bmatrix} = \begin{bmatrix} -\epsilon & \epsilon \\ \epsilon & -\epsilon \end{bmatrix}$$

Singular matrix

∞ Solutions

$$(A - \lambda_1 I)V = 0$$

$$\boxed{V_{\lambda_1 = 1+\epsilon} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad V_{\lambda = 1-\epsilon} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

$$A - \lambda_2 I = \begin{bmatrix} \epsilon & \epsilon \\ \epsilon & \epsilon \end{bmatrix}$$

c. See Matlab code...

d. Since ϵ is $\sqrt{\epsilon_m}/4$

let's plug this in our equation for the characteristic polynomial

$\lambda^2 - 2\lambda + 1 - \epsilon^2$

$\lambda^2 - 2\lambda + 1 - \frac{\epsilon_m}{16}$

Since $\frac{\epsilon_m}{16} < \epsilon_m$

It essentially can be thought of as 0 in the computer, yielding:

$\lambda^2 - 2\lambda + 1$

$(\lambda - 1)(\lambda - 1)$

$\lambda_1 = 1 = \lambda_2$

$(A - \lambda_{1,2} I)v = \begin{pmatrix} 0 & \epsilon \\ \epsilon & 0 \end{pmatrix} v$

real eigenvalues and eigenvectors are not being calculated because of.

e.
f. } See MatLab code...

2.
a) $B = A - \frac{\lambda_1}{v^{(1)t} v^{(1)}} v^{(1)}(v^{(1)})^t \quad (1)$

$Bv^{(1)} = 0$

$(v^{(i)})^t v^{(i)} = 0$ for $i \neq j \quad (2)$

let's multiply both sides of eq (1) by $v^{(1)}$

$$Bv^{(1)} = \left(A - \frac{\lambda_1}{v^{(1)t} v^{(1)}} v^{(1)} (v^{(1)})^t \right) v^{(1)}$$

$$Bv^{(1)} = \left(Av^{(1)} - \frac{\lambda_1}{\cancel{v^{(1)t} v^{(1)}}} v^{(1)} \cancel{(v^{(1)t} v^{(1)})} \right)$$

$$Bv^{(1)} = Av^{(1)} - \lambda_1 v^{(1)}$$

by definition:

$$Bv^{(1)} = \cancel{Av^{(1)}} - \cancel{Av^{(1)}} = 0$$

$$Bv^{(1)} = 0$$

let us try another number now $\neq 1$ and let's call it j

let us plug $v^{(j)}$ on both sides

$$Bv^{(j)} = Av^{(j)} - \frac{\lambda_1}{v^{(1)t} v^{(1)}} v^{(1)} \cancelto{0}{\underbrace{v^{(1)t} v^{(j)}}_{\text{using orthogonality.}}} \qquad 1 \neq j$$

$$Bv^{(j)} = Av^{(j)}$$

This means A will have the same eigenValues as B for all $\lambda_{j \neq 1}$

6. See MatLab code...