Contents

CS375 HOMEWORK 9

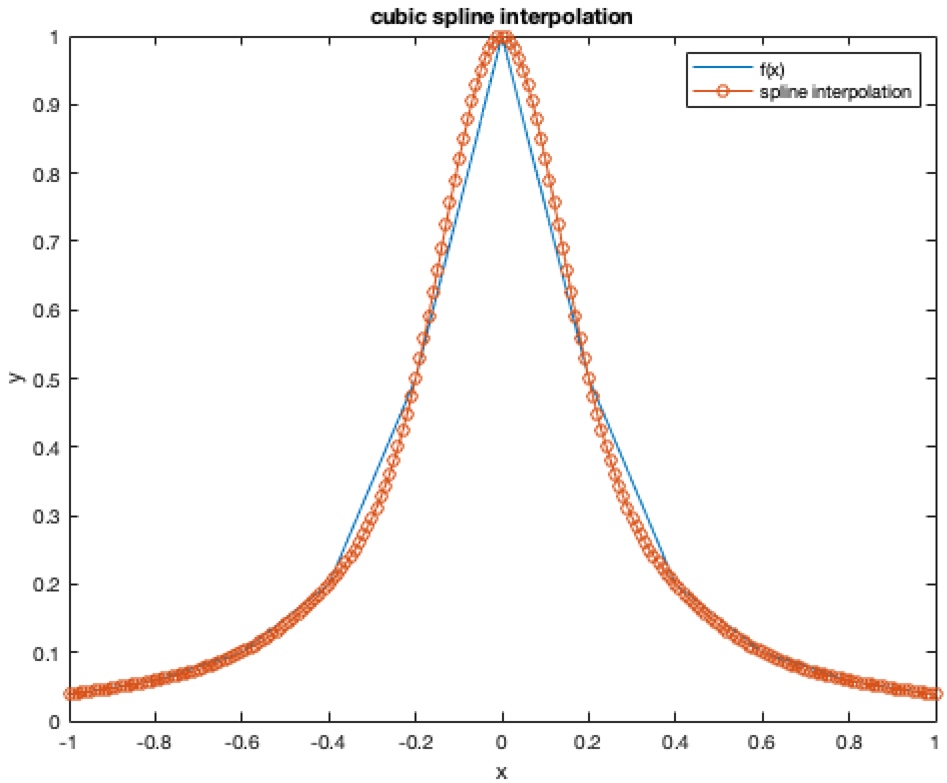
Juan Alejandro Ormaza November 2, 2021


```
clear all;clc;close all;
format long g;
```

Problem 1

```
t=linspace(-1,1,11);
f=@(x) 1./(1+25*x.^2);
y=f(t);

z=spline3_coeff(t,y);

x_eval=-1:0.01:1;
y_eval=zeros(1,length(x_eval));
for i=1:length(x_eval)
    y_eval(i)=eval_spline(x_eval(i),t,y,z);
end

plot(t,y);
hold on
plot(x_eval,y_eval,'-o');
xlabel('x')
ylabel('y')
title('cubic spline interpolation')
legend('f(x)','spline interpolation')

fprintf('the spline interpolation works really well near the endpoints and does not\n')
fprintf('appear to have the same issues as the lagrange or polynomial interpolation\n')
fprintf('with equispaced points.\n ')
```

```
the spline interpolation works really well near the endpoints and does not
appear to have the same issues as the lagrange or polynomial interpolation
with equispaced points.
```



Problem 2

```
% 2.a see attachments

% 2.b see attachments

% 2.c

f_x=@(x) x*sin(x);
exact_int = sin(pi)-pi*cos(pi);

n=[4, 8, 16, 32];
h=(pi-0)./n;
integrals= zeros(1,length(n));
error=zeros(1,length(n));

for i=1:length(n)
    integrals(i)=comp_trap_int(f_x,0,pi,n(i));
    error(i) = abs(exact_int-integrals(i));
end

p=zeros(1,length(n)-1);

for i=1:length(n)-1
    p(i)=log(error(i+1)/error(i))/log(h(i+1)/h(i));
end

p=[0 p];

fprintf(" n  approx integral\t    error\t convergence p\n");
fprintf("%2d  %2.10f\t %2.10f\t %2.10f\n",[n;integrals ;error; p])
```

```
fprintf("since the expected convergence for composite trapezoid is 2, it is possible to see\n")
fprintf("that as n increases, the convergence gets closer and closer to 2. indicating that the\n")
fprintf("expected convergence is being found.\n")

% 2.d see attachments
```

```
 n  approx integral         error         convergence p
 4  1.6698795626        1.4717130910    0.0000000000
 8  2.6880135827        0.4535790709    1.6980704688
16  3.0186727993        0.1229198543    1.8836361114
32  3.1097827181        0.0318099355    1.9501686104
since the expected convergence for composite trapezoid is 2, it is possible to see
that as n increases, the convergence gets closer and closer to 2. indicating that the
expected convergence is being found.
```

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spline degree 3 coefficient

```
function [z] = spline3_coeff(t,y)

ti=t;
ti1=circshift(t,-1);

yi=y;
yi1=circshift(y,-1);


h= ti1(1:length(t)-1)-ti(1:length(t)-1);
b=(1./h).*(yi1(1:length(y)-1)-yi(1:length(y)-1));


A=zeros(length(t),length(t));
A(1,1)=1;
A(end,end)=1;


for i=2:length(t)-1
    for j=1:length(t)
        if(j==i)
            A(i,j)=2*(h(i)+h(i-1));
        end
        if(j==i-1)
            A(i,j)=h(i-1);
        end
        if(j==i+1)
            A(i,j)=h(i);
        end
    end
end

v=zeros(length(t),1);

for i=2:length(t)-1
    v(i)=6*(b(i)-b(i-1));
end


z=A\v;
```

```
Not enough input arguments.

Error in spline3_coeff (line 6)
ti=t;
```

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```
function y_eval = eval_spline(x,t,y,z)

%Inputs:
% x: Scalar where spline is to be eval
% t: Vector of x-values of the data points
% y: Vector of y-values of the data points
% z: Vector of spline coefficients

%Outputs
%y_eval: scalar value giving S(x)

num_pts = length(y);

sp_index = num_pts-1;

for i = num_pts-1:-1:1
    if x >= t(i)
        sp_index = i;
        break;
    end
end
h = t(sp_index+1) - t(sp_index);
tmp = z(sp_index)/2 + (x - t(sp_index))*(z(sp_index+1) - z(sp_index))/(6*h);
tmp = -(h/6)*(z(sp_index+1) + 2*z(sp_index)) + (y(sp_index+1) - y(sp_index))/h + (x-t(sp_index))*tmp;
y_eval = y(sp_index) + (x-t(sp_index))*tmp;

end
```

```
Not enough input arguments.

Error in eval_spline (line 12)
num_pts = length(y);
```

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Composite trapezoid integral

```
function integral = comp_trap_int(f,a,b,n)

h = (b - a)/n;
sum = (f(a) + f(b))/2;
x=a:h:b;
for i = 1:n-1
    sum = sum + f(x(i));
end
integral = sum * h;

end
```

```
Not enough input arguments.

Error in comp_trap_int (line 6)
h = (b - a)/n;
```

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HW9 Nov 2nd 2021

Problem 2.a) How many points are needed to ensure the composite trapezoid rule is accurate to 10^{-6} for

$$I = \int_0^{\pi} x \cdot \sin(x)\, dx$$

let $f(x) = x\sin(x)$

$f'(x) = \sin(x) + x\cos(x)$

$f''(x) = \cos(x) + \cos(x) - x\sin(x)$

$= 2\cos(x) - x \cdot \sin(x)$

$f'''(x) = -2\sin(x) - \sin(x) - x\cos(x)$

$0 = -3\sin(x) - x\cos(x)$

in 0 to π root at $x = 2.4556$ thus max

$|f''(2.4556)| = 3.103$

$$\frac{(b-a)h^2 f''(\eta)}{12} \leq 10^{-6}$$

$$\frac{(b-a)}{n} = h$$

$$\frac{\pi h^2 f''(\eta)}{12} \leq 10^{-6}$$

$$h^2 f''(\eta) \leq \frac{12 \cdot 10^{-6}}{\pi}$$

$$h^2 \leq \frac{12 \cdot 10^{-6}}{\pi(3.103)}$$

$$h \leq \sqrt{\frac{12}{\pi(3.103}}\, 10^{-3}$$

$$\frac{\pi}{n} \leq \sqrt{\frac{12}{\pi(3.103)}} \cdot 10^{-3}$$

$$\frac{\pi \cdot 10^3}{1.109} \leq n$$

$$\boxed{2833 \leq n}$$

problem 2.d Approximate the above integral using the composite Simpson rule with 2 intervals of equal length (take $n=4$) How many evaluations of the function $f(x)$ are required?

$$\int_0^{\frac{\pi}{2}} f(x)\,dx + \int_{\pi/2}^{\pi} f(x)\,dx \qquad f(x) = x\sin(x)$$

$n=4$

$$[\underbrace{x_0, x_1}_{1}, \overset{2}{\overbrace{x_1, x_2}}, \underbrace{x_2, x_3}_{3}, \overset{4}{\overbrace{x_3, x_4}}]$$

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + f(b) + 4\sum_{i=1}^{n/2} f(a+(2i-1)h) + 2\sum_{i=1}^{(n/2)-1} f(a+2ih)\right]$$

Since $n=4$

$$\int_0^{\frac{\pi}{2}} f(x)dx = \frac{h}{3}\left[\overset{1\text{ eval}}{f(0)} + \overset{2\text{eval}}{f(\pi/2)} + \overset{4\text{ eval}}{4\sum_{i=1}^{2} f(a+(2i-1)h)} + \overset{5\text{ eval}}{2\sum_{i=1}^{1} f(a+2ih)}\right]$$

Since there is 5 evaluations of $f(x)$ per interval and 2 intervals total, there will be 10 evaluations of $f(x)$ to approximate the integral