



TECNOLÓGICO
NACIONAL DE MÉXICO

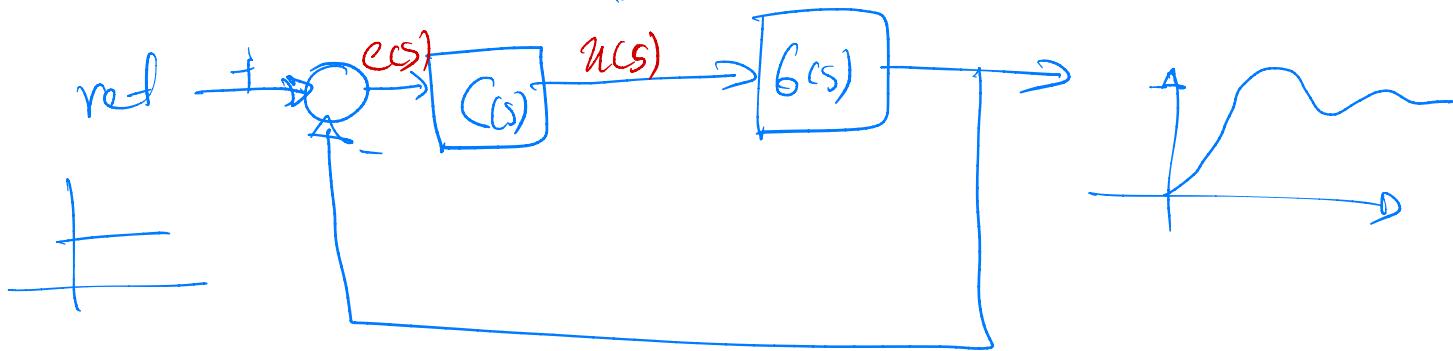
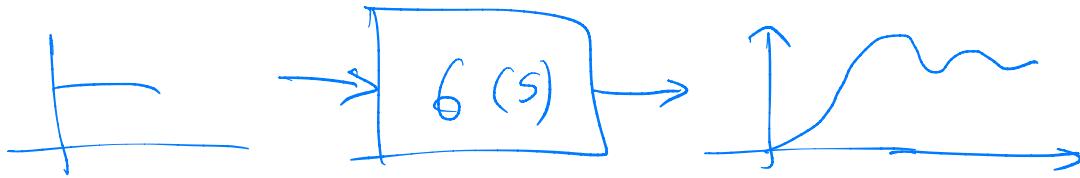
Posgrado en Electrónica

Tema: *Controladores convencionales discretos*

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Instituto Tecnológico de Morelia



Controladores digitales

Control PID continuo

$$u(s) = (K_P + K_I \frac{1}{s} + K_D s) e(s)$$

Algoritmo paralelo

$$u(s) = K_P \left(1 + \frac{1}{\tau_I s} + \tau_D s \right) e(s)$$

Algoritmo no iterativo

Controladores digitales

Control PID continuo

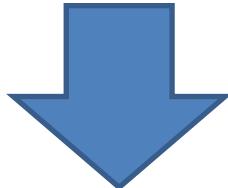
$$u(t) = K_P e(t) + K_I \int e(t) dt + K_D \frac{de(t)}{dt}$$

Algoritmo paralelo

Controladores digitales discretos

Acción proporcional

$$P(t) = K_P e(t)$$



$$P(k) = K_P e(k)$$

Controladores digitales discretos

Acción integral

$$I(t) = K_I \int e(t) dt \rightarrow \frac{dI(t)}{dt} = K_I e(t)$$

$$\frac{I(k) - I(k-1)}{T} = K_I e(k)$$

$$I(k) - I(k-1) = T K_I e(k)$$

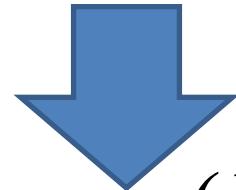
$$I(k) = I(k-1) + T K_I e(k)$$

$$I(k) = I(k-1)$$

Controladores digitales discretos

Acción derivativa

$$D(t) = K_D \frac{de(t)}{dt}$$



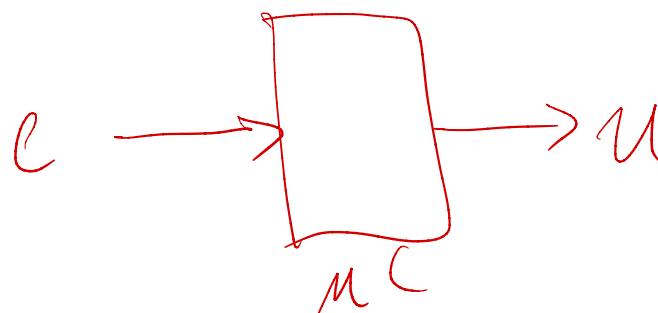
$$D(k) = K_D \frac{e(k) - e(k-1)}{T}$$

Controladores digitales discretos

Algoritmo de posición

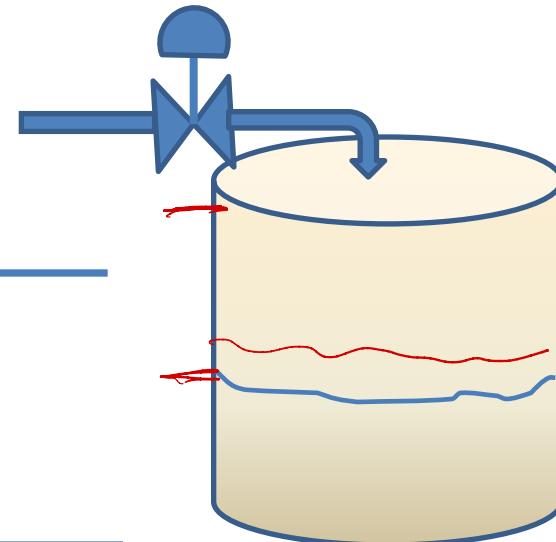
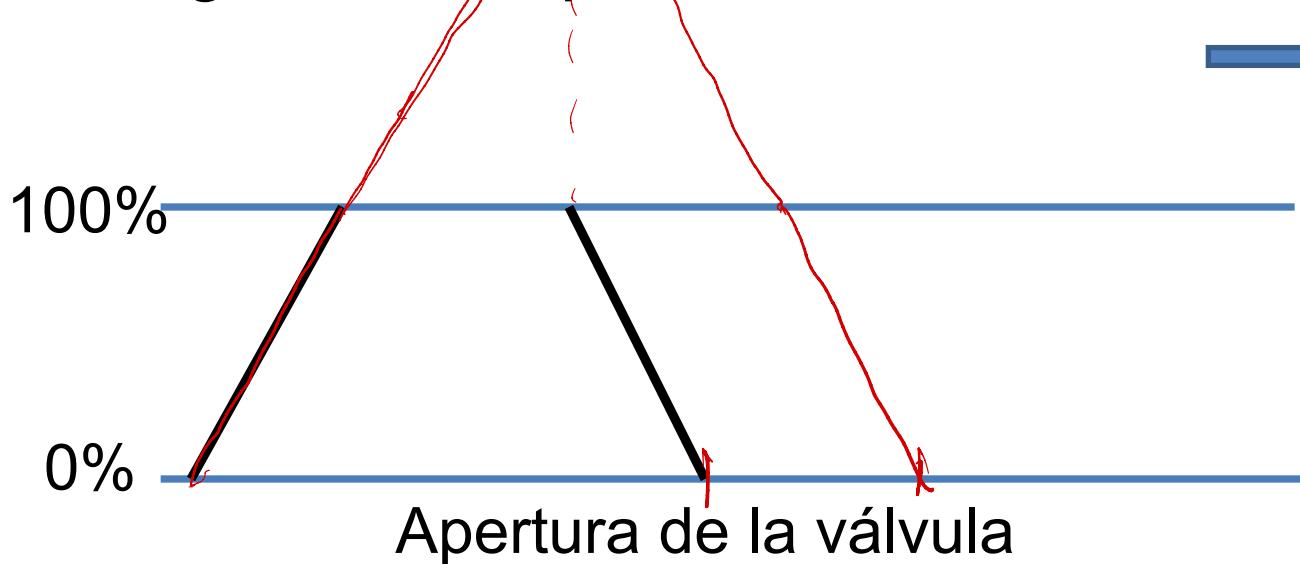
$$u(k) = P(k) + I(k) + D(k)$$

$$u(k) = K_P e(k) + I(k-1) + K_I T e(k) + K_D \frac{e(k) - e(k-1)}{T}$$



Controladores digitales discretos

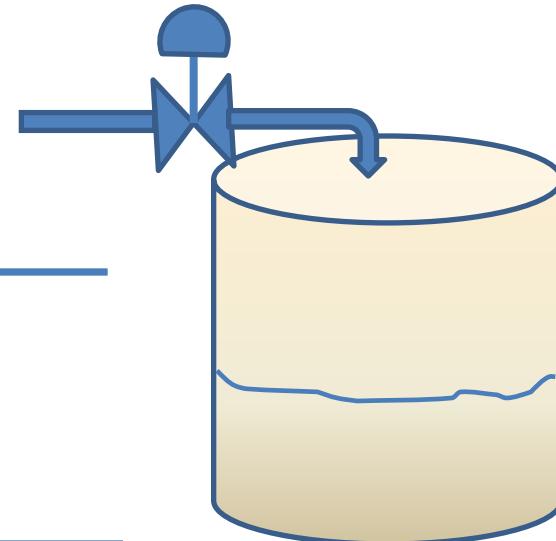
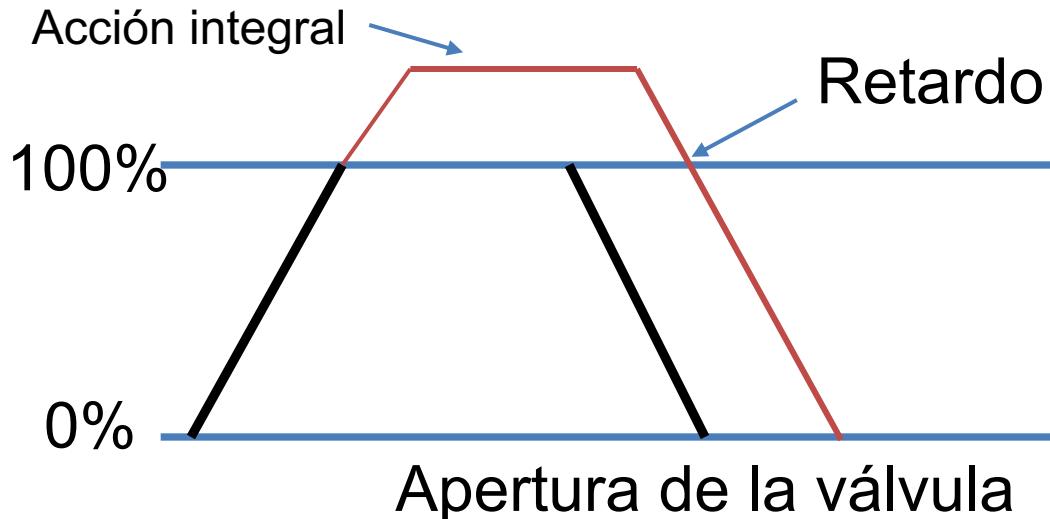
Algoritmo de posición



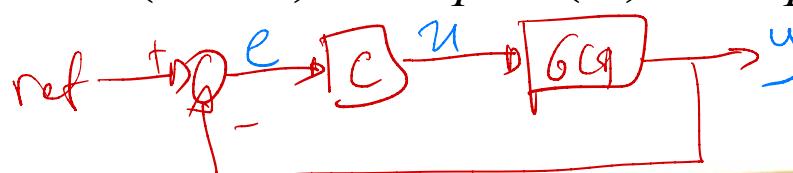
$$u(k) = K_P e(k) + I(k-1) + K_I T e(k) + K_D \frac{e(k) - e(k-1)}{T}$$

Controladores digitales discretos

Algoritmo de posición



$$u(k) = K_P e(k) + I(k-1) + K_I T e(k) + K_D \frac{e(k) - e(k-1)}{T}$$



Controladores digitales discretos

Considerando que

$$\Delta u = u(k) - u(k-1)$$

$$u(k) = K_P e(k) + I(k) + K_D \frac{e(k) - e(k-1)}{T}$$

$$u(k-1) = K_P e(k-1) + I(k-1) + K_D \frac{e(k-1) - e(k-2)}{T}$$

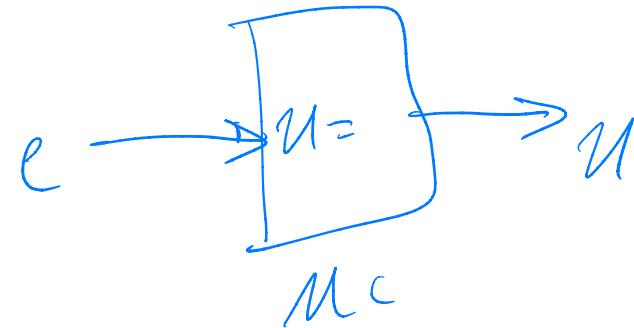
Recordando que $I(k) - I(k-1) = K_I T e(k)$

$$\Delta u = K_P [e(k) - e(k-1)] + K_I T e(k) + \frac{K_D}{T} [e(k) - 2e(k-1) + e(k-2)]$$

Controladores digitales discretos

Algoritmo de velocidad

$$u(k) = u(k-1) + \Delta u$$



$$\Delta u = K_P [e(k) - e(k-1)] + K_I T e(k) + \frac{K_D}{T} [e(k) - 2e(k-1) + e(k-2)]$$

$$\Delta u = [K_P + K_I T + \frac{K_D}{T}] e(k) - [K_P + 2 \frac{K_D}{T}] e(k-1) + \frac{K_D}{T} [e(k-2)]$$

Ajuste de controladores PID

- Ajuste por prueba y error
- Ajuste por curva de reacción (lazo abierto)
- Ajuste de ganancia última (lazo cerrado)

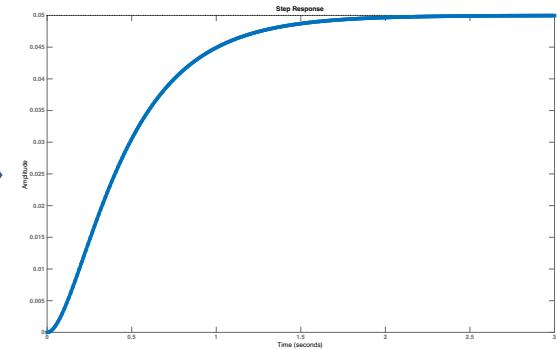
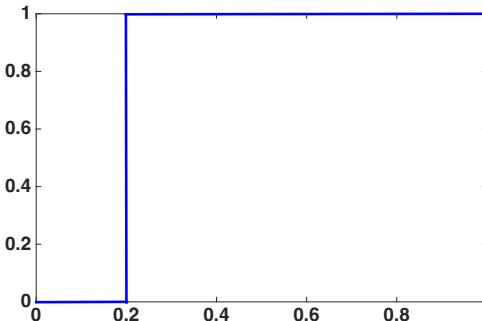
Prueba y error

- Las acciones I y D se ajustan a cero
- Incrementar la ganancia proporcional hasta que el sistema oscile
- Registrar el valor de ganancia y ajustar K_p igual a un medio de la ganancia registrada

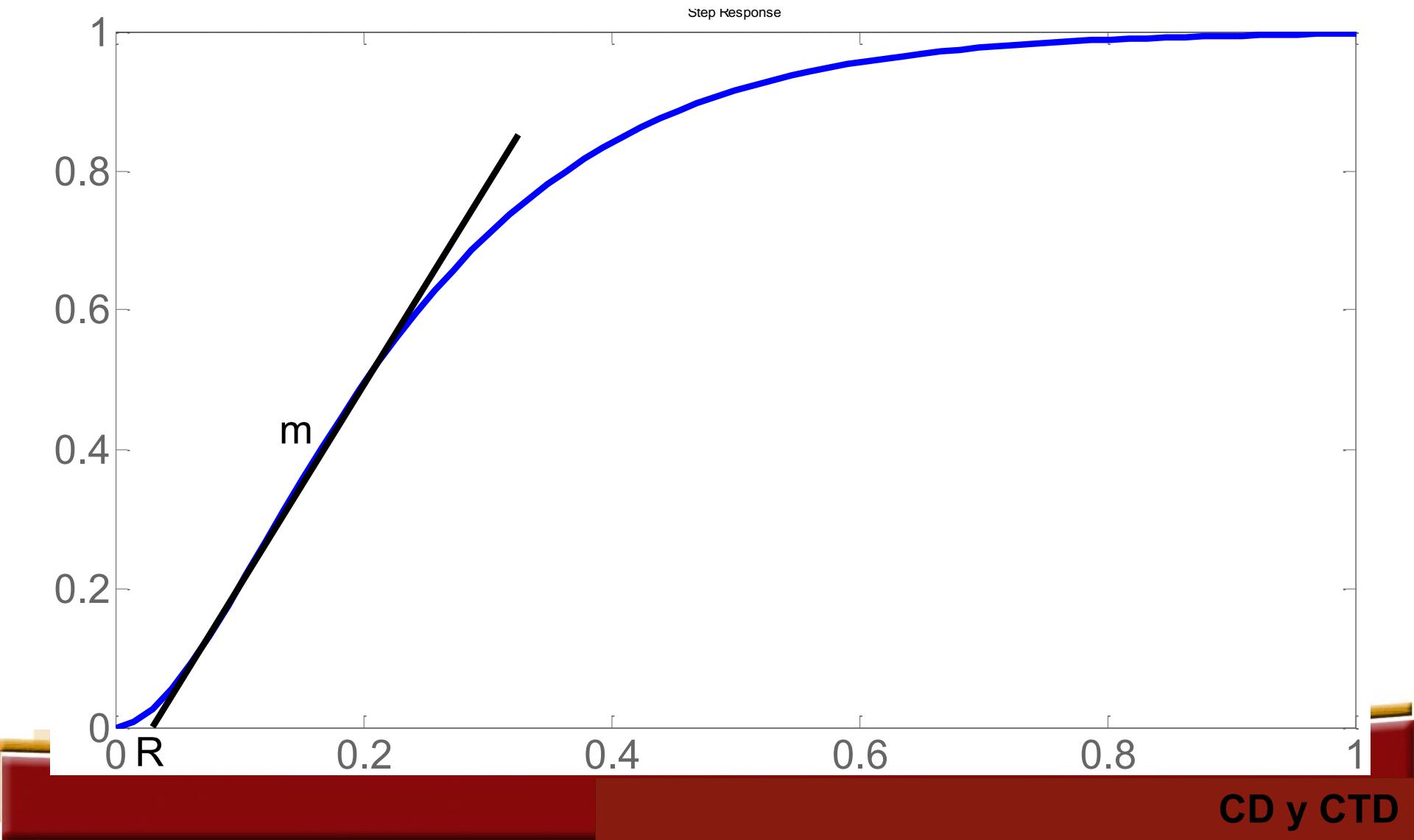
Prueba y error

- Reducir el valor de τ_I hasta lograr que el error desaparezca en un tiempo razonable
- Incrementar τ_D si es necesario para que el controlador responda rápidamente a cambios bruscos en la carga (normalmente $K_d=1/[4K_i]$)

Método de curva de reacción



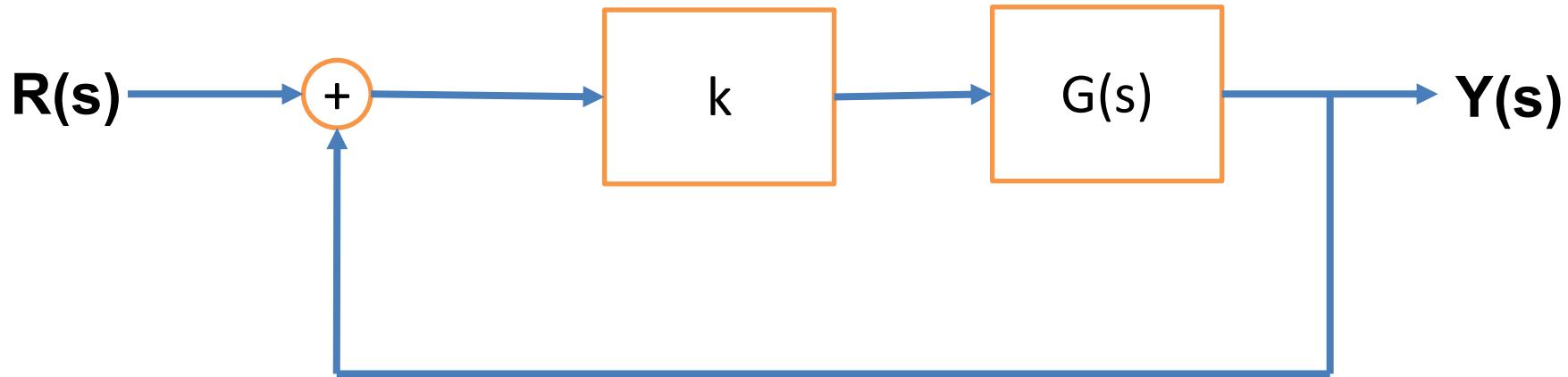
Método de curva de reacción



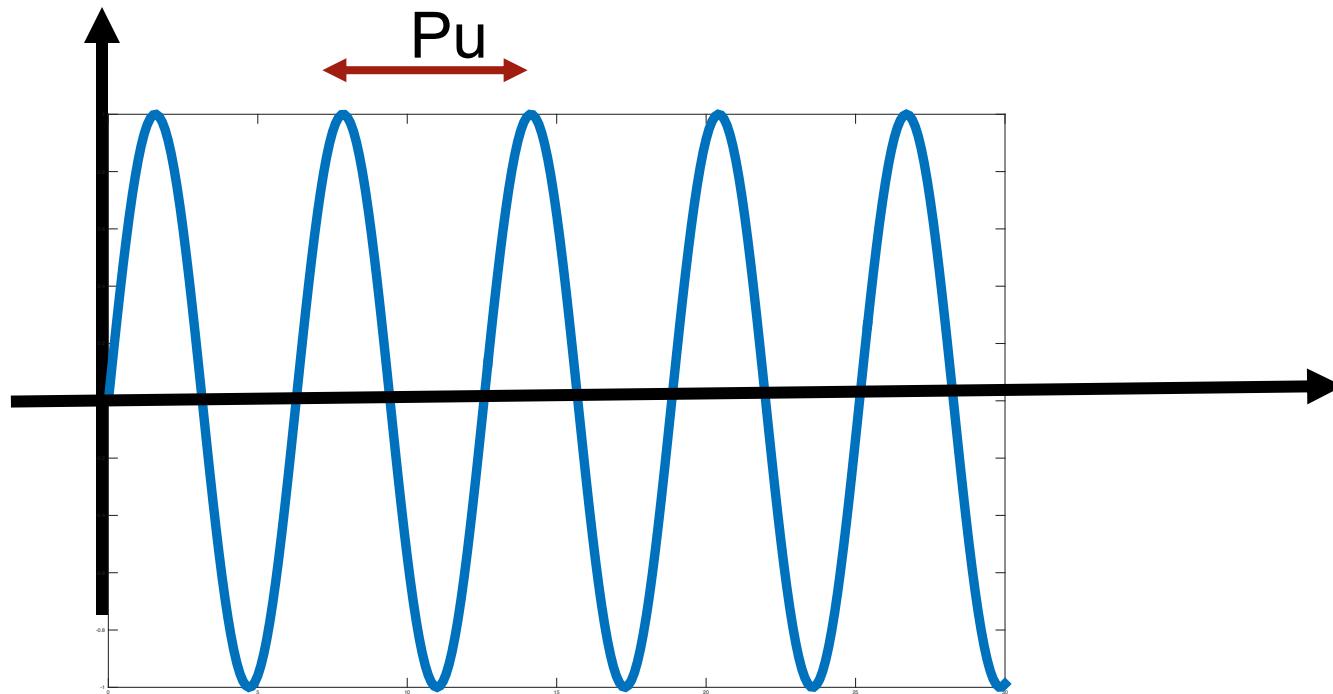
Ajuste por curva de reacción

Controlador	Kp	Ki	Kd
P	$1/(mR)$		
PI	$0.9/(mR)$	$1/(3R)$	
PID	$1.2/(mR)$	$1/(2R)$	$R/2$

Método de ganancia última



Método de ganancia última



K_u = ganancia para obtener oscilación sostenida

Método de ganancia última

Controlador	K _p	K _i	K _d
P	0.5K _u		
PI	0.45K _u	1.2/P _u	
PID	0.6K _u	2/P _u	P _u /8

Ajuste del Controlador Discreto

Método de Takahashi

Curva de reacción

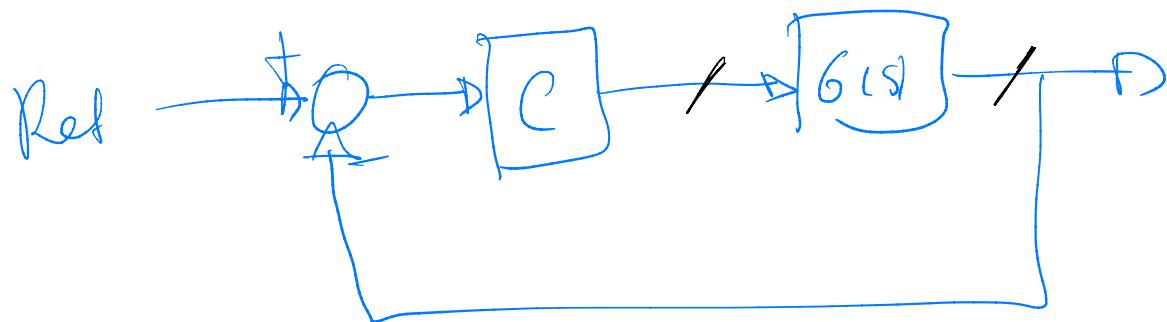
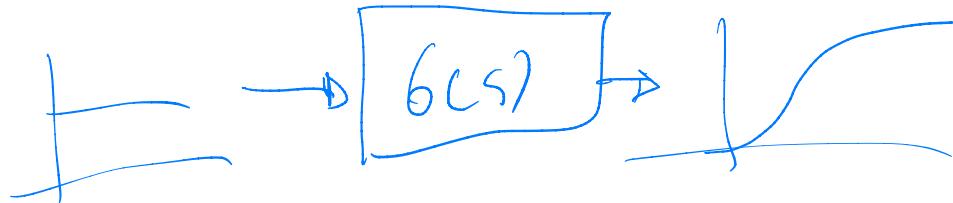
Controlador	Kp	Ki	Kd
P	$\frac{1}{m(R + T)}$		
PI	$\frac{0.9}{m\left(R + \frac{T}{2}\right)} - \frac{Ki}{2}$	$\frac{0.27T}{m(R + T)^2}$	
PID	$\frac{1.2}{m\left(R + \frac{T}{2}\right)} - \frac{Ki}{2}$	$\frac{0.6T}{m(R + T)^2}$	$\frac{0.6}{m(T)}$

Ajuste del Controlador Discreto

Método de Takahashi

Ganancia última

Controlador	K _p	K _i	K _d
P	$\frac{K_u}{2}$		
PI	$0.45K_u - \frac{K_i}{2}$	$0.54 \frac{K_u}{P_u} T$	
PID	$0.6K_u - \frac{K_i}{2}$	$1.2 \frac{K_u}{P_u} T$	$\frac{3}{40} \frac{K_u P_u}{T}$



Ajuste del Controlador Discreto

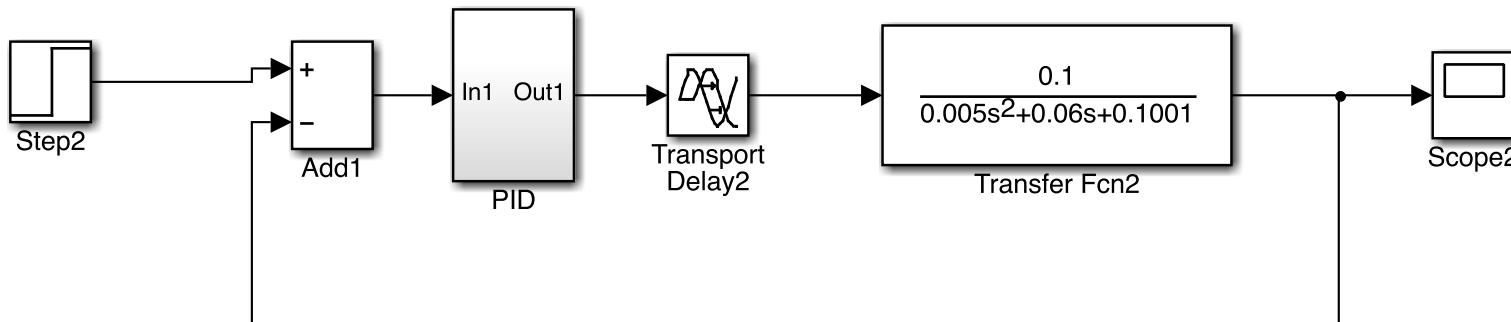
Método de Takahashi

Ganancia última

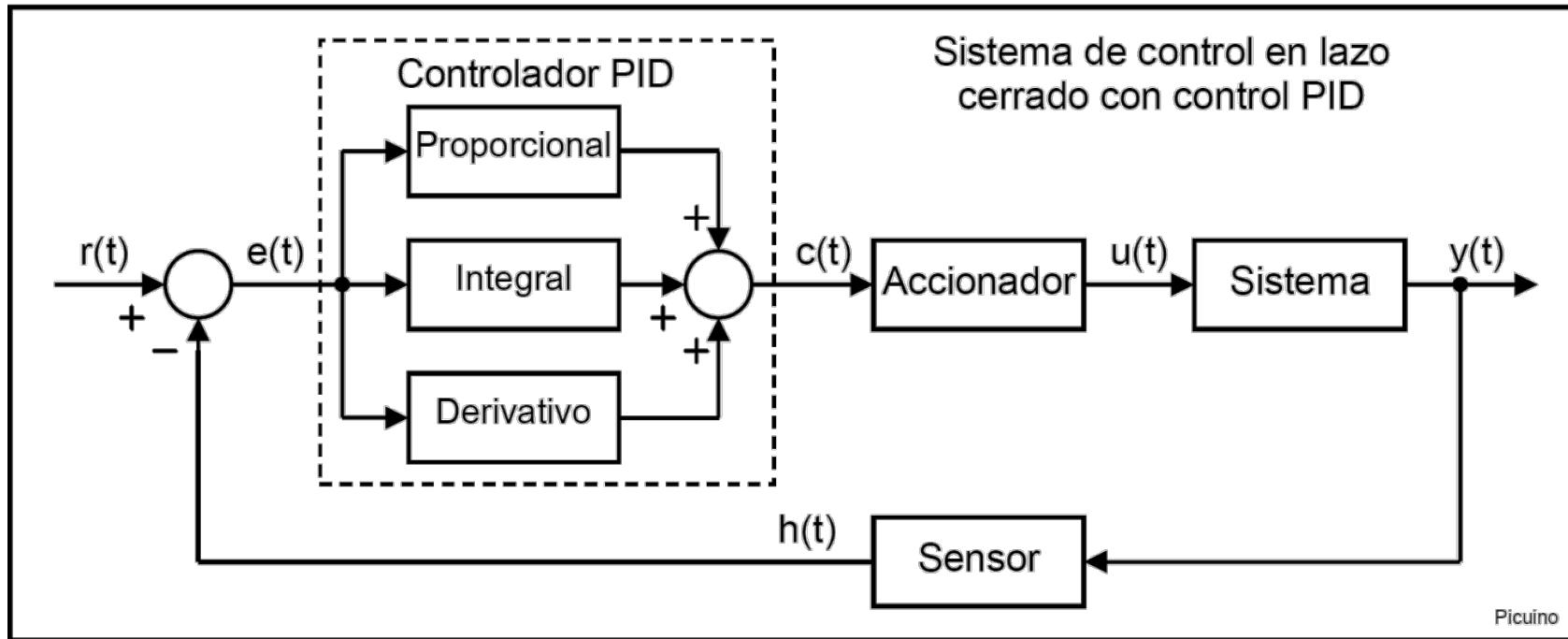
Controlador	Kp	Ki	Kd
P	$\frac{Ku}{2}$		
PI	$0.45Ku - \frac{Ki}{2}$	$0.54 \frac{Ku}{Pu} T$	
PID	$0.6Ku - \frac{Ki}{2}$	$1.2 \frac{Ku}{Pu} T$	$\frac{3}{40} \frac{KuPu}{T}$

Efectos de las acciones del PID

Sistema en lazo cerrado



Efectos de las acciones del PID



Fuente: picuino.com

<https://www.picuino.com/es/arduprog/control-pid.html>

Efectos de las acciones del PID

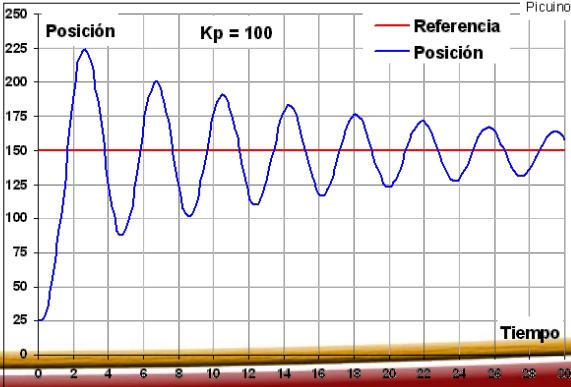
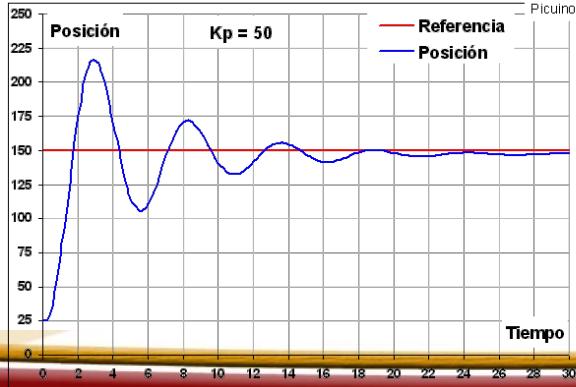
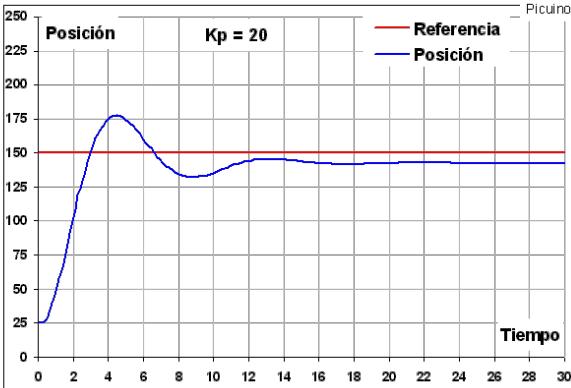
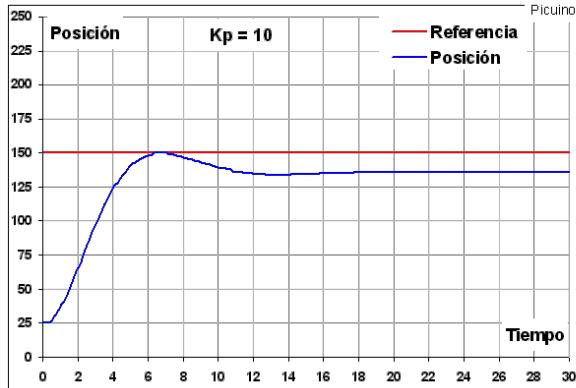
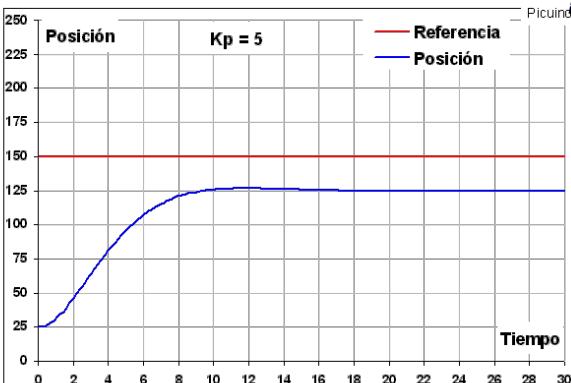
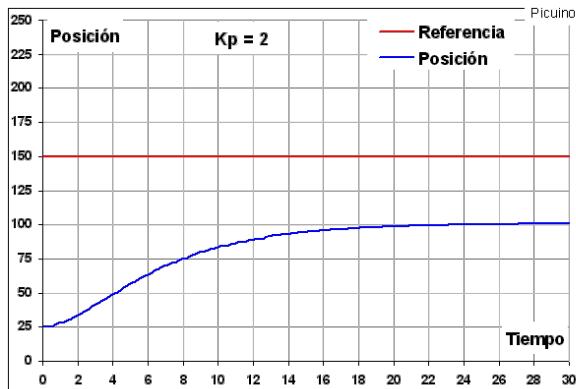
Acción proporcional

Esta acción de control es proporcional a la señal de error $e(t)$. Intenta minimizar el error del sistema.

Al aumentar la acción proporcional K_p :

- Aumenta la velocidad de respuesta del sistema.
- Disminuye el error del sistema en régimen permanente.
- Aumenta la inestabilidad del sistema.

Fuente: picuino.com



Acción proporcional

Fuente: picuino.com

Efectos de las acciones del PID

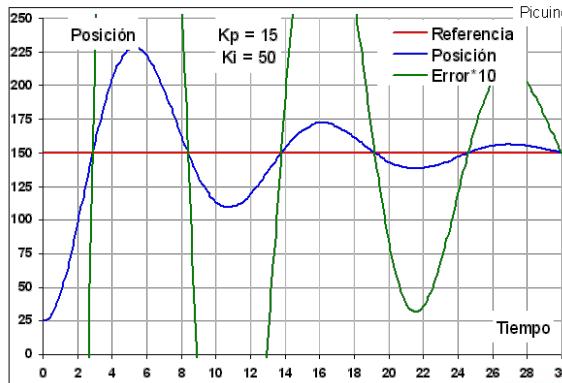
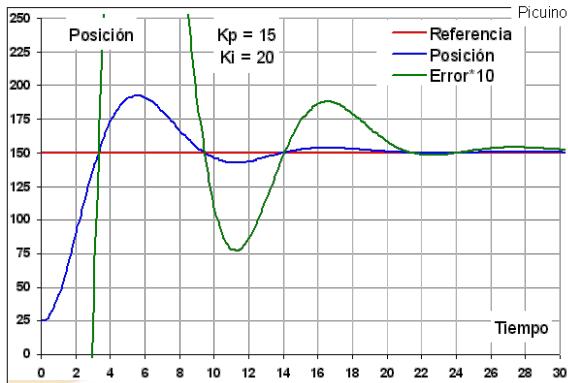
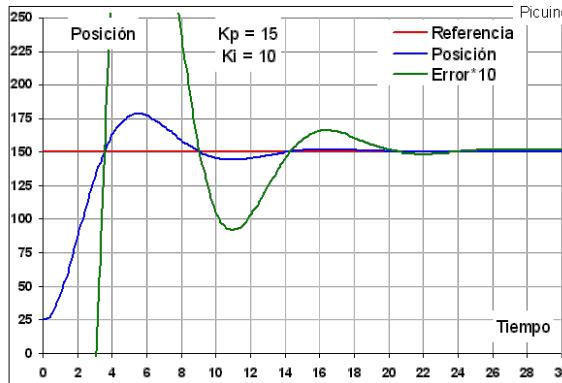
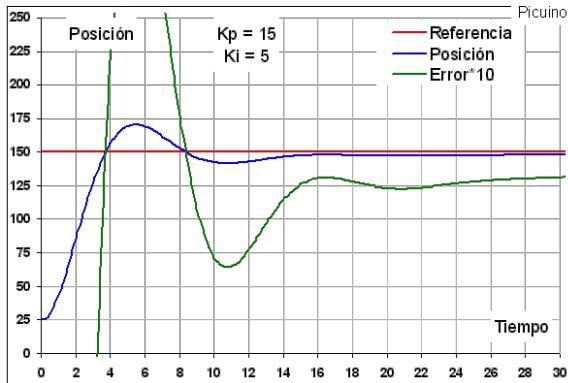
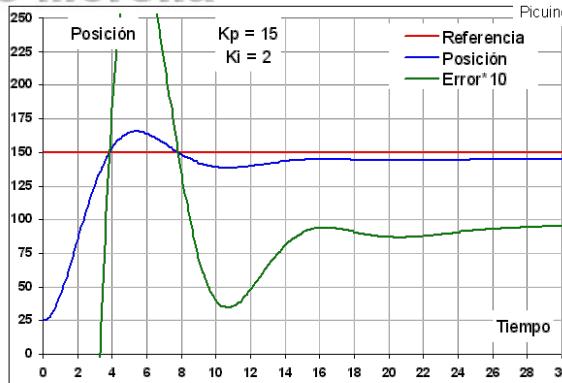
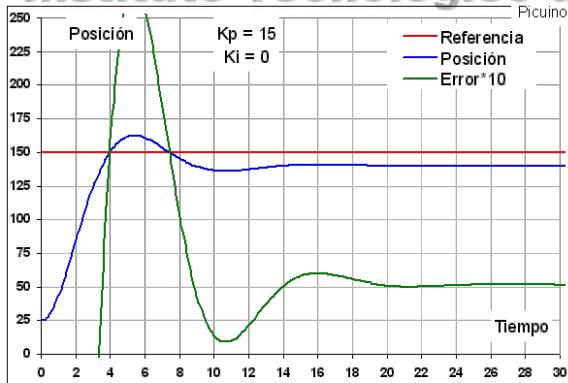
Acción integral

Esta acción calcula la integral de la señal de error $e(t)$.

Aumentar la acción integral K_i tiene los siguientes efectos:

- Disminuye el error del sistema en régimen permanente.
- Aumenta la inestabilidad del sistema.
- Aumenta un poco la velocidad del sistema.

Fuente: picuino.com



Acción Integral

Fuente: picuino.com

Efectos de las acciones del PID

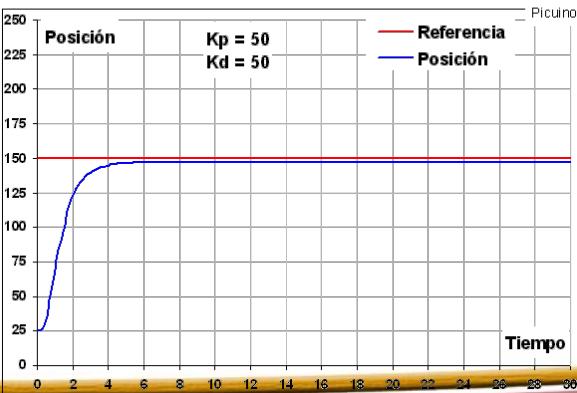
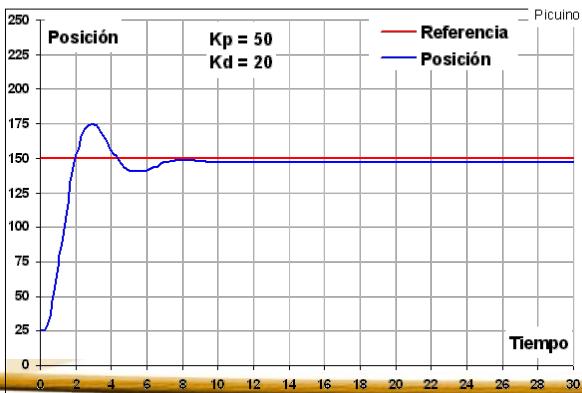
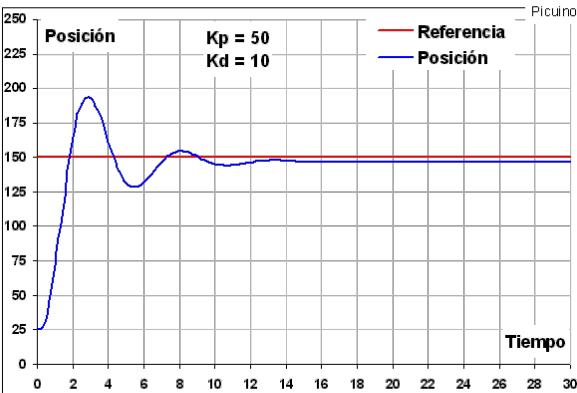
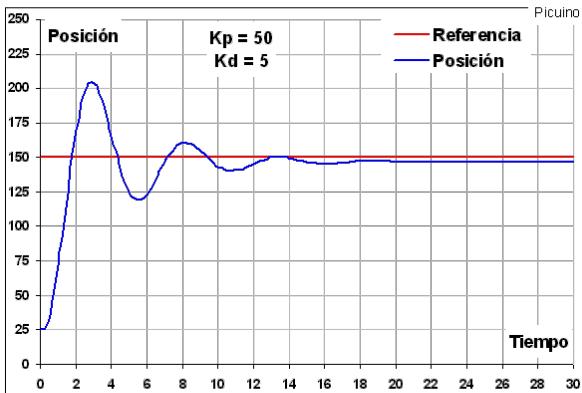
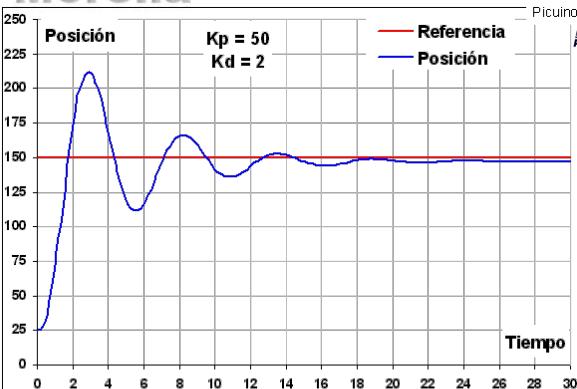
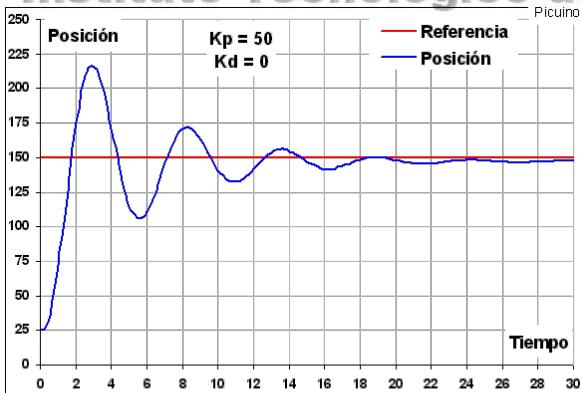
Acción derivativa

Esta acción de control es proporcional a la derivada de la señal de error $e(t)$.

Aumentar la constante de control derivativa K_d tiene los siguientes efectos:

- Aumenta la estabilidad del sistema controlado.
- Disminuye un poco la velocidad del sistema.
- El error en régimen permanente permanecerá igual.
- Amplifica el ruido de alta frecuencia

Fuente: picuino.com

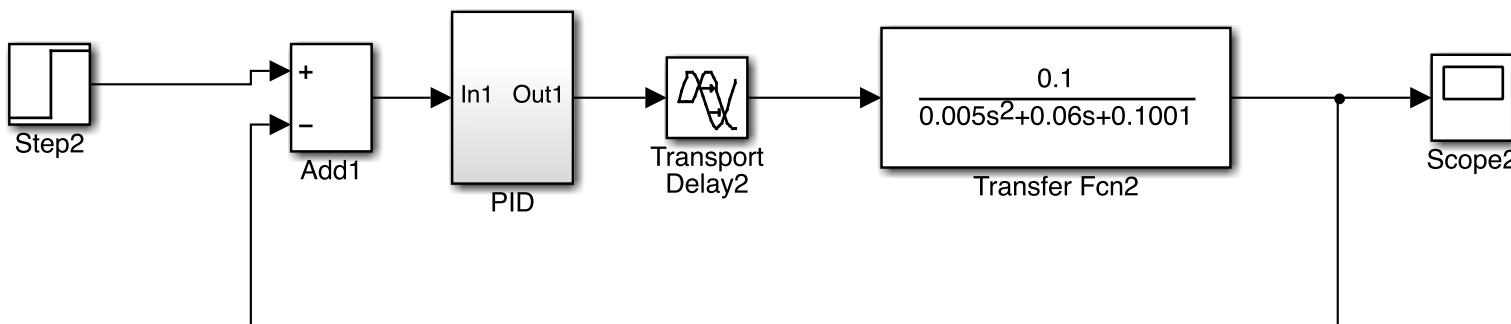


Acción Derivativa

Fuente: picuino.com

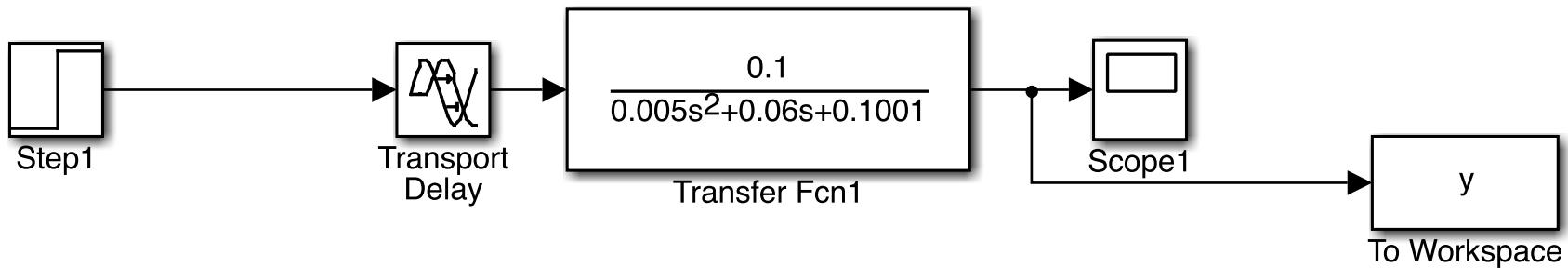
Ejemplos de ajustes del PID

Sintonización de Controles PID

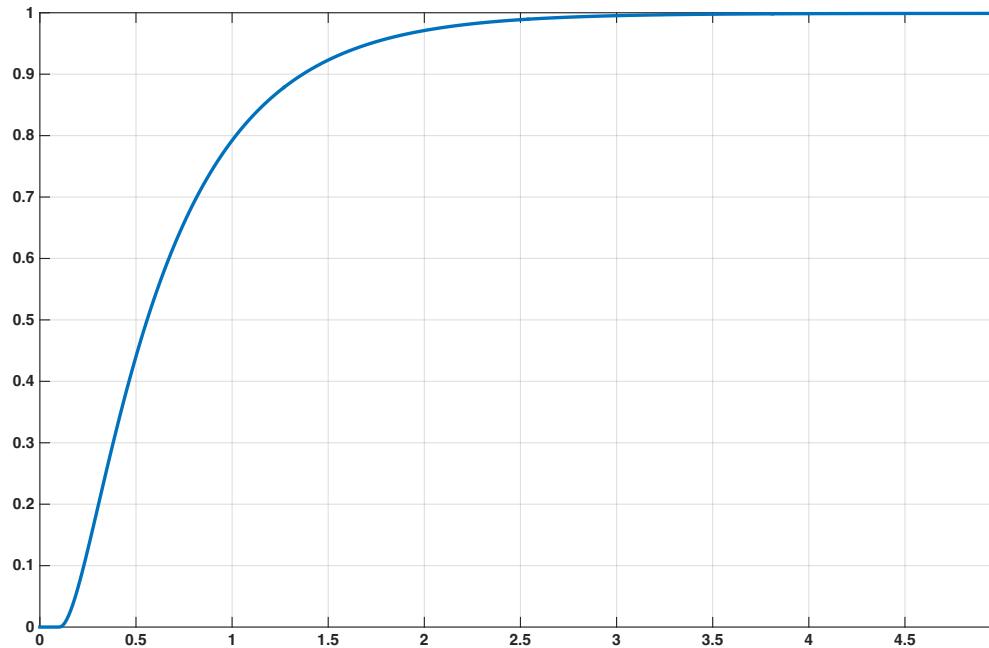
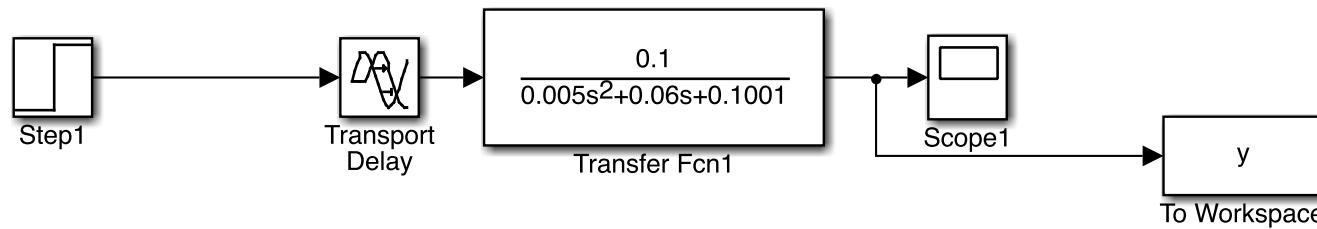


Método de curva de reacción

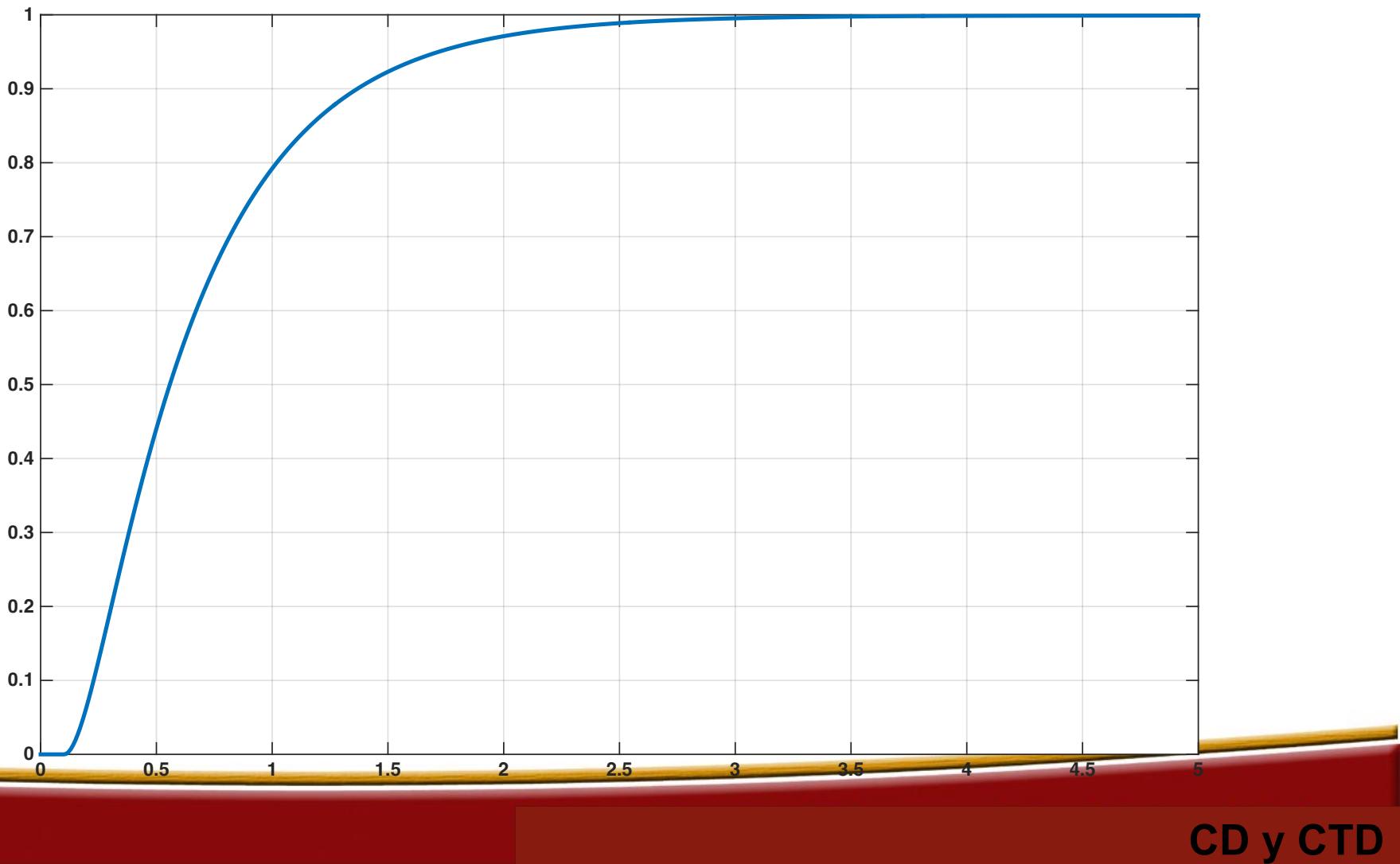
Lazo abierto



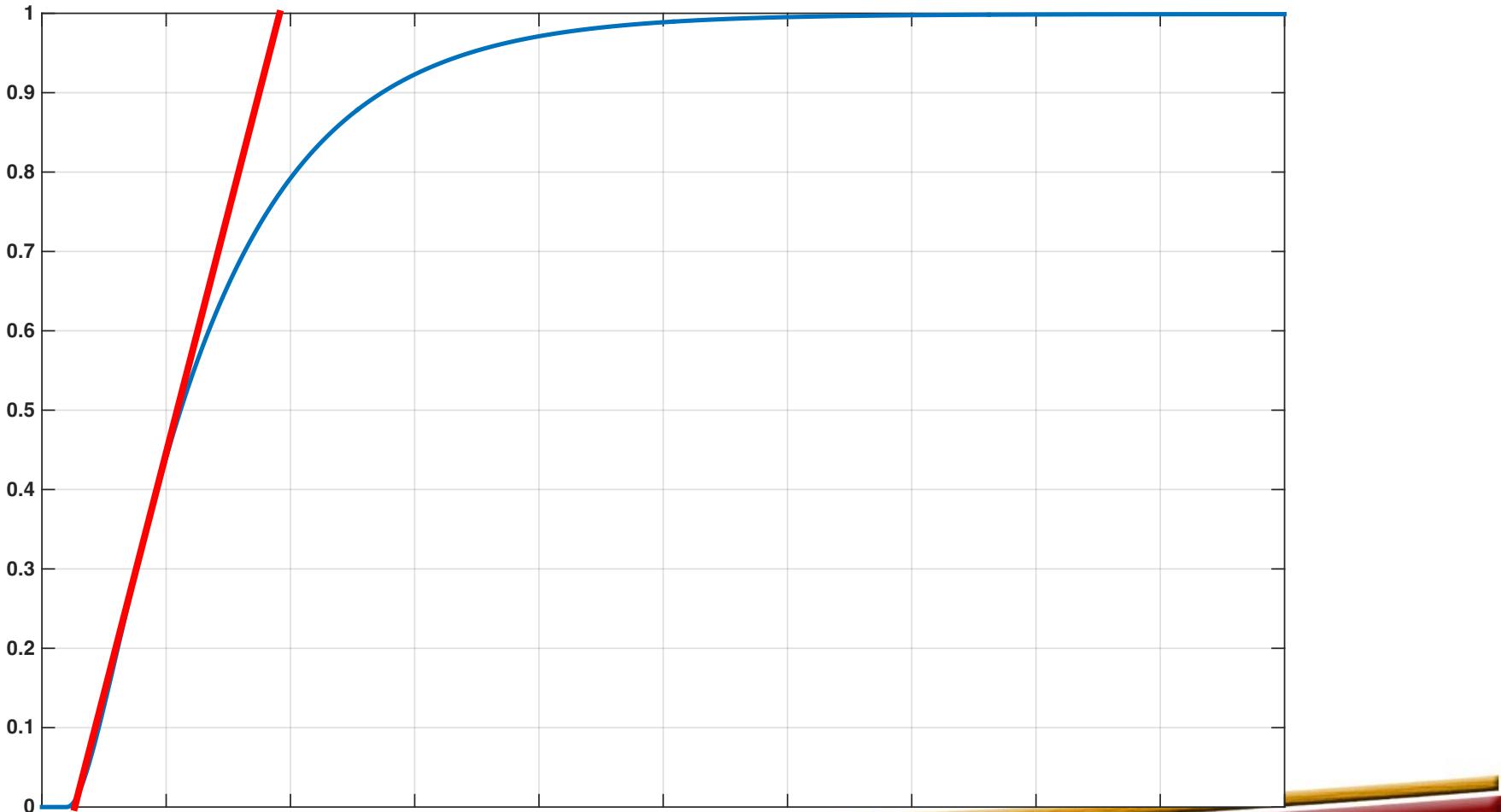
Método de curva de reacción



Método de curva de reacción

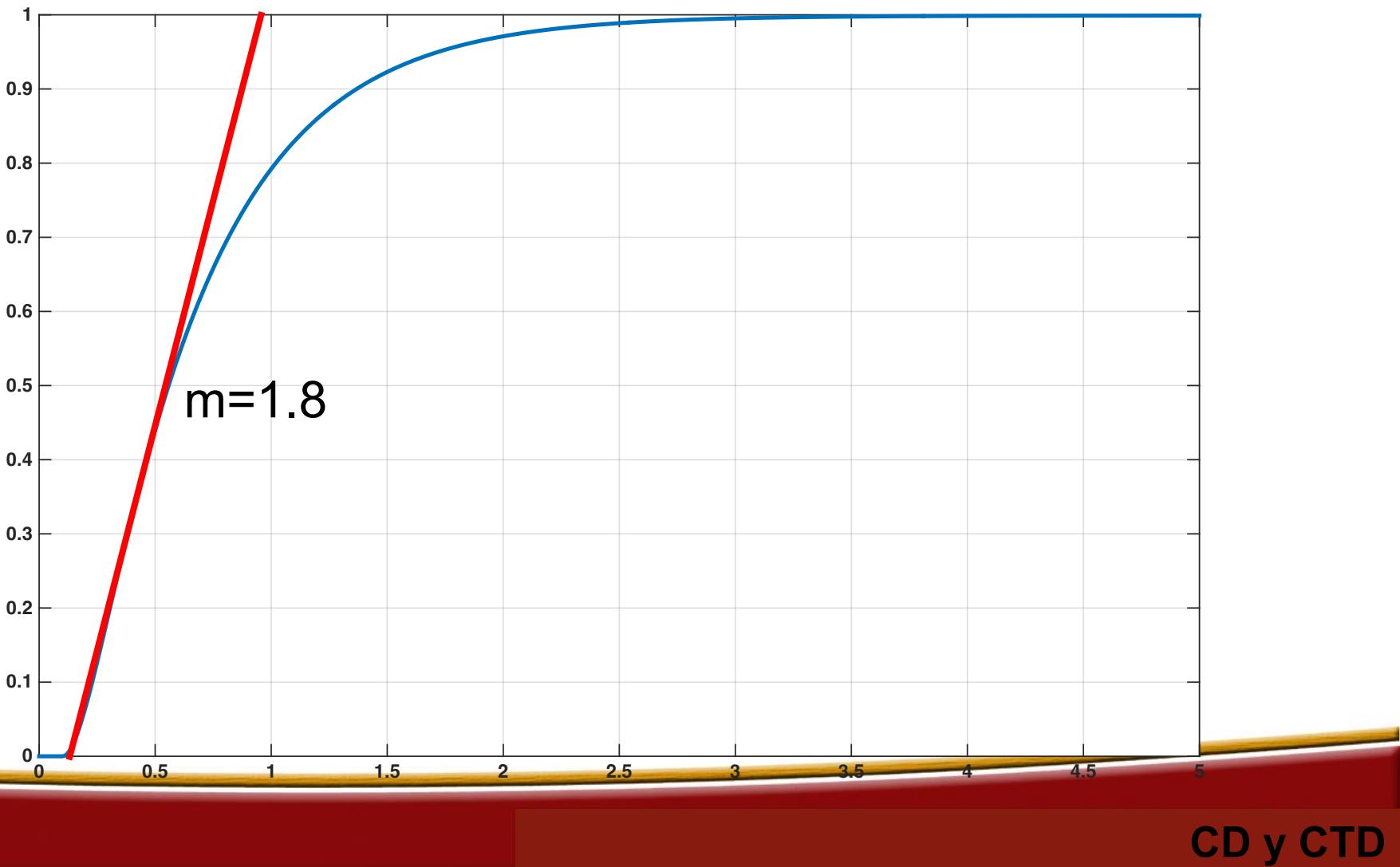


Método de curva de reacción

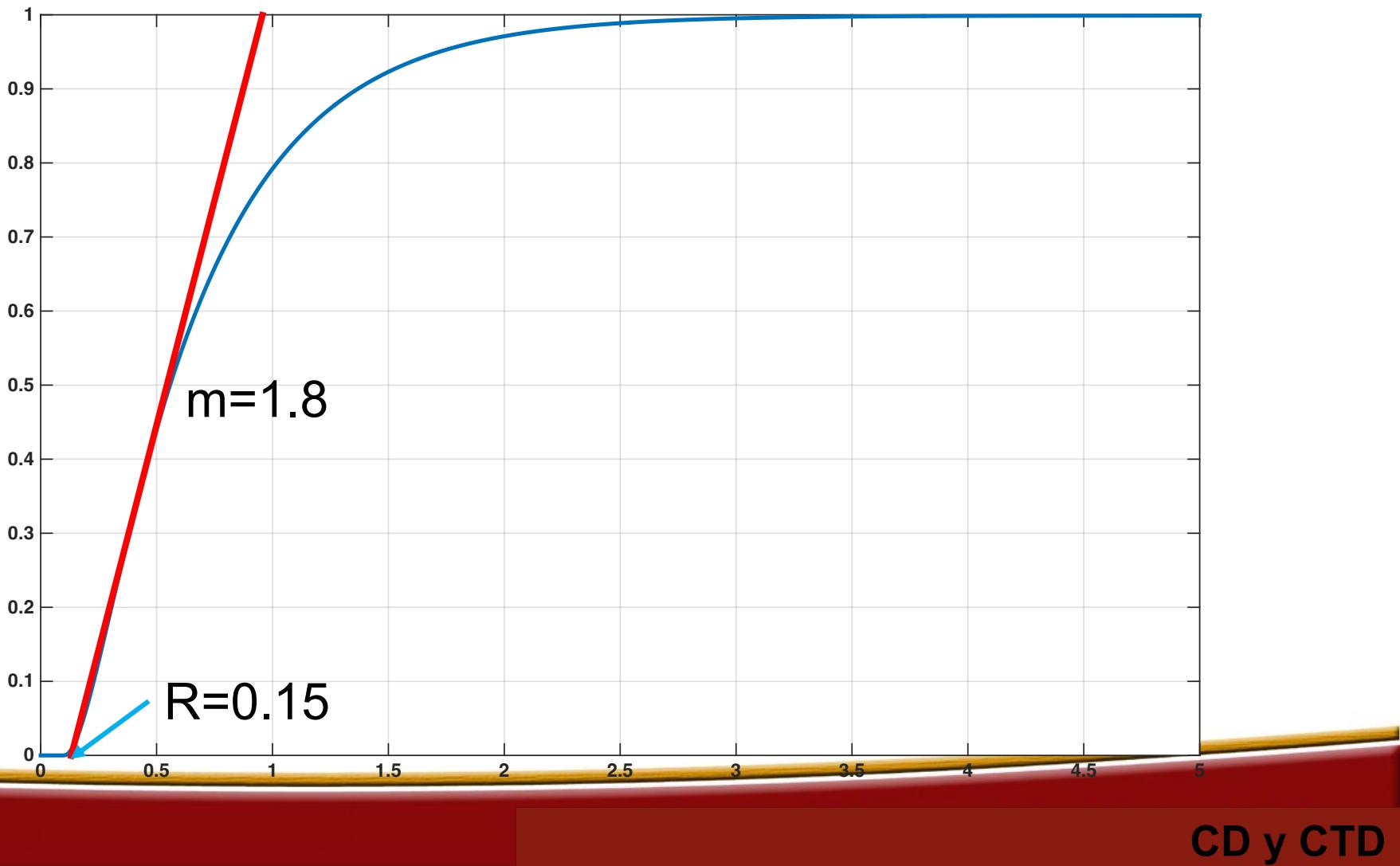


CD y CTD

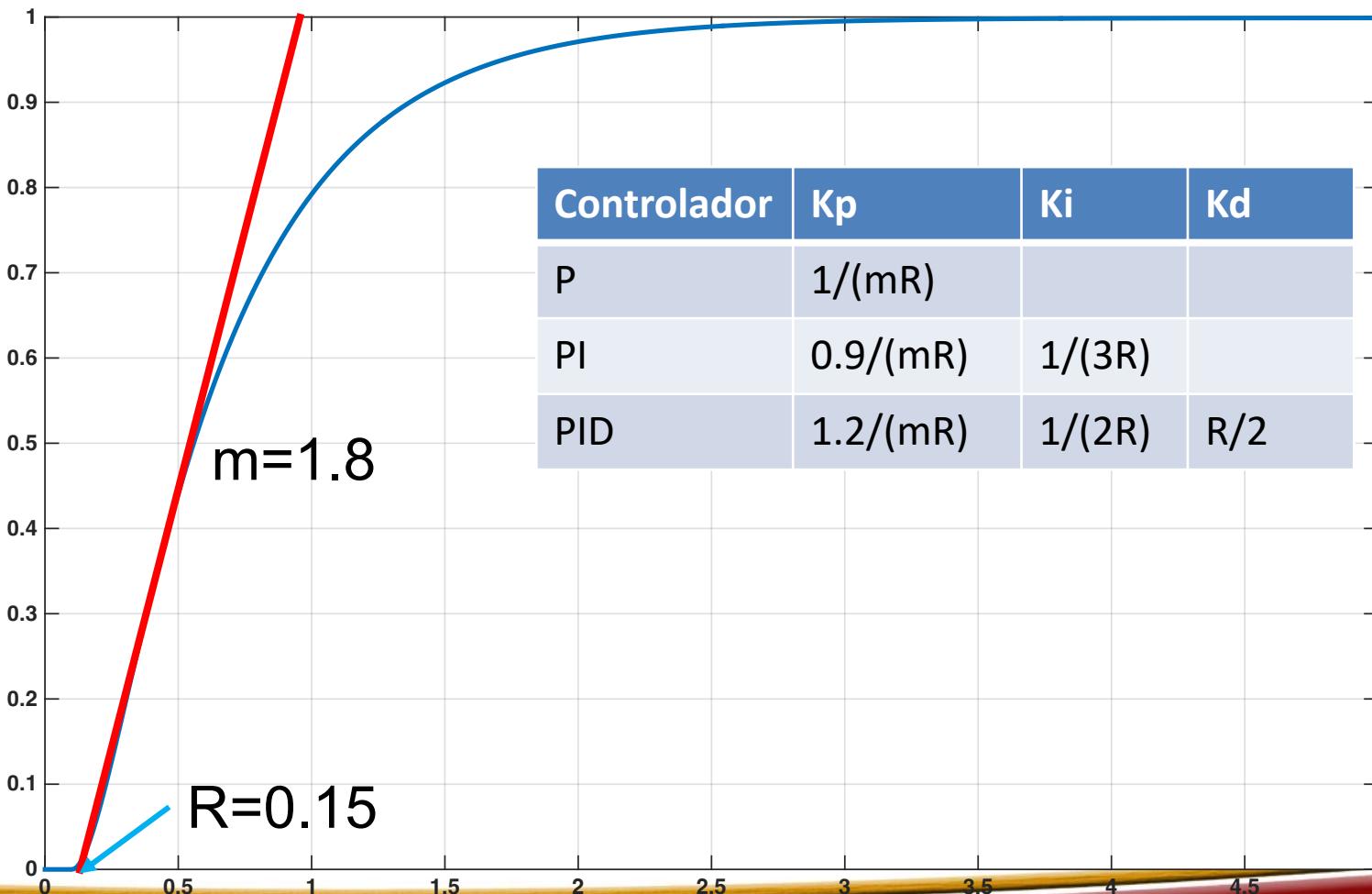
Método de curva de reacción



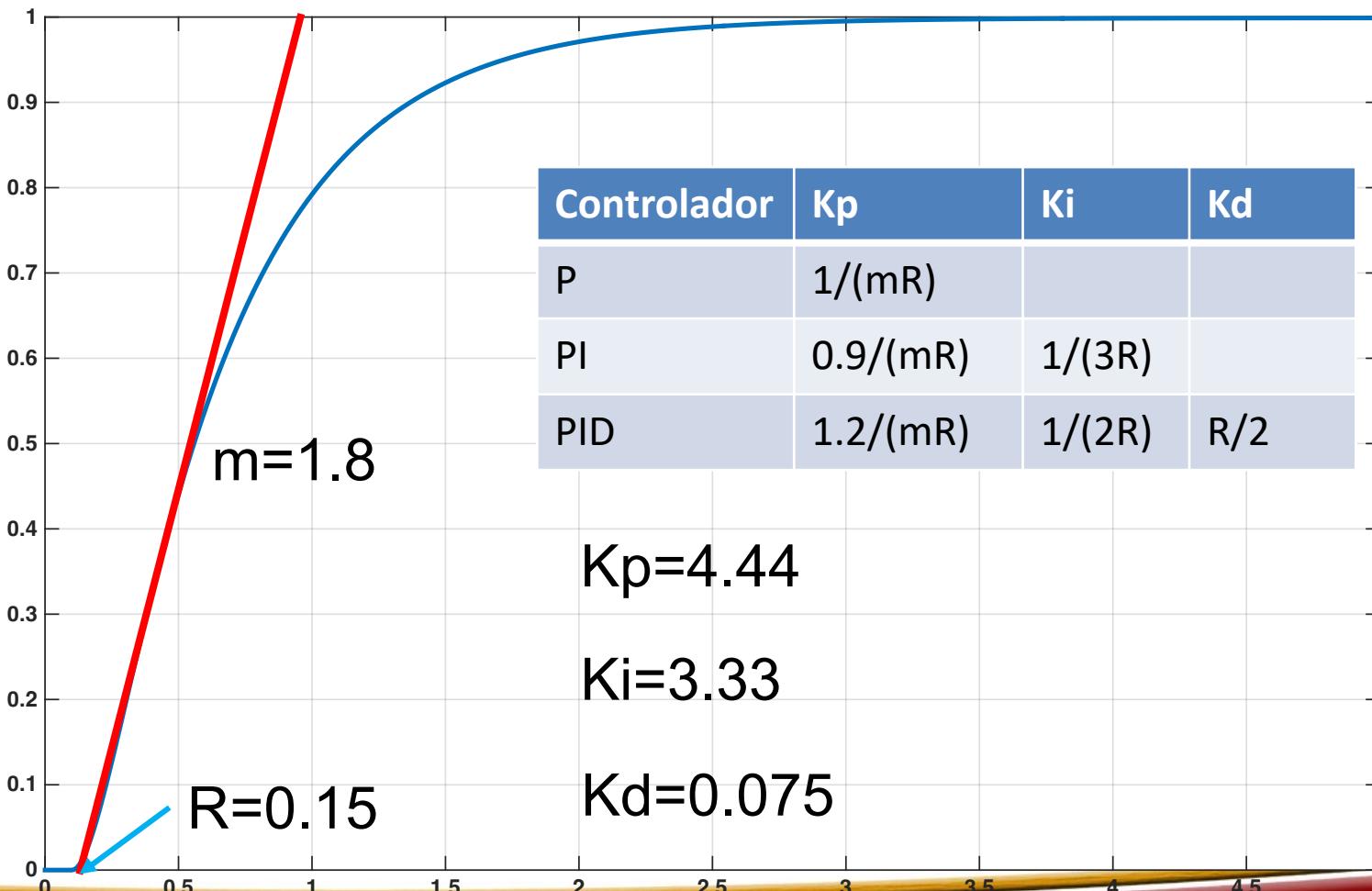
Método de curva de reacción



Método de curva de reacción

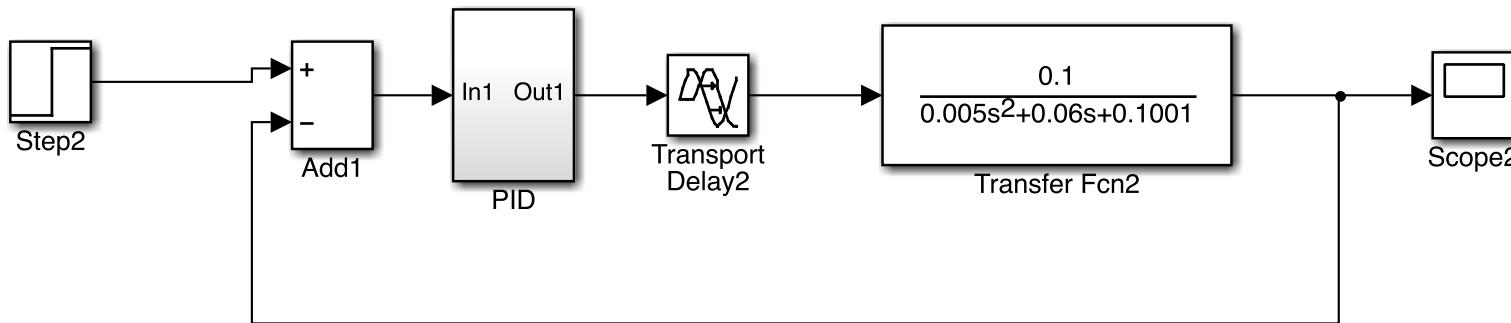


Método de curva de reacción



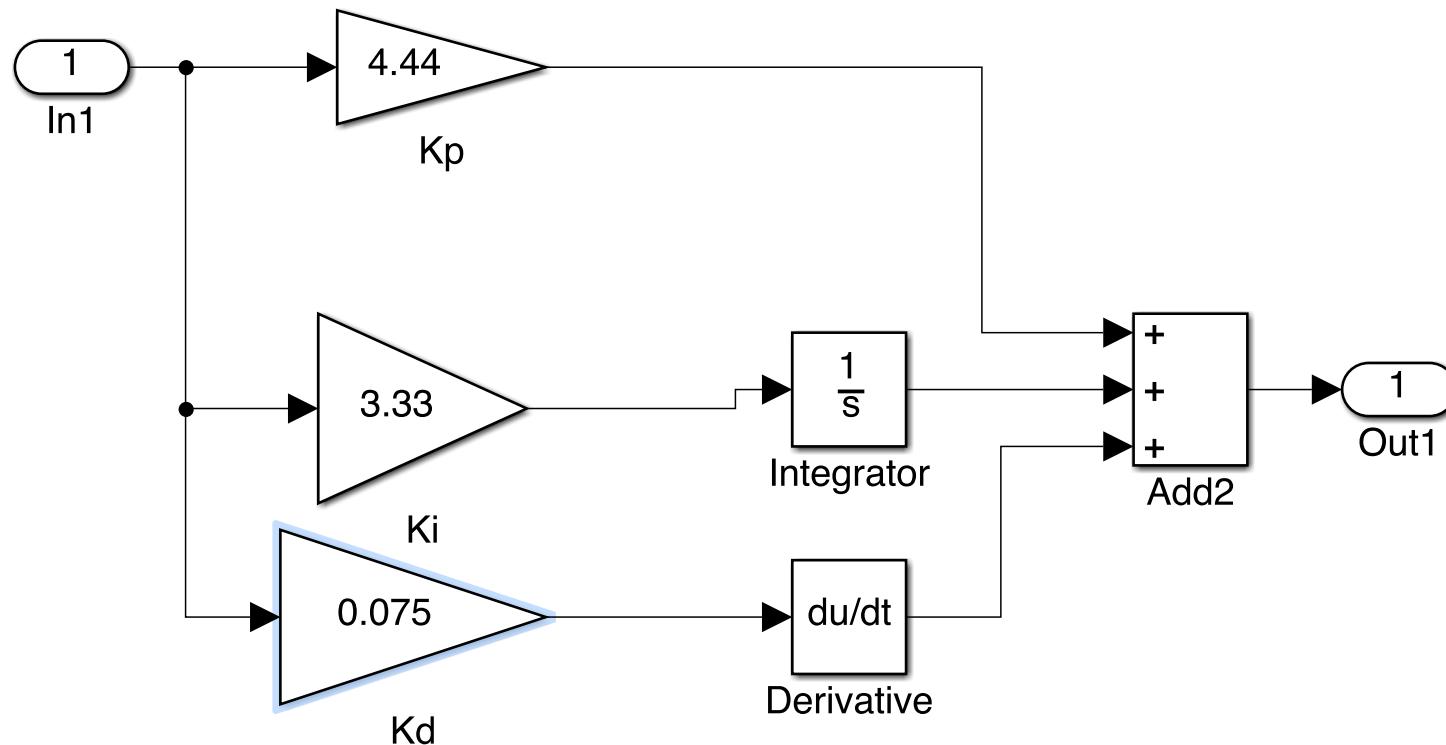
Método de curva de reacción

Controlador PID



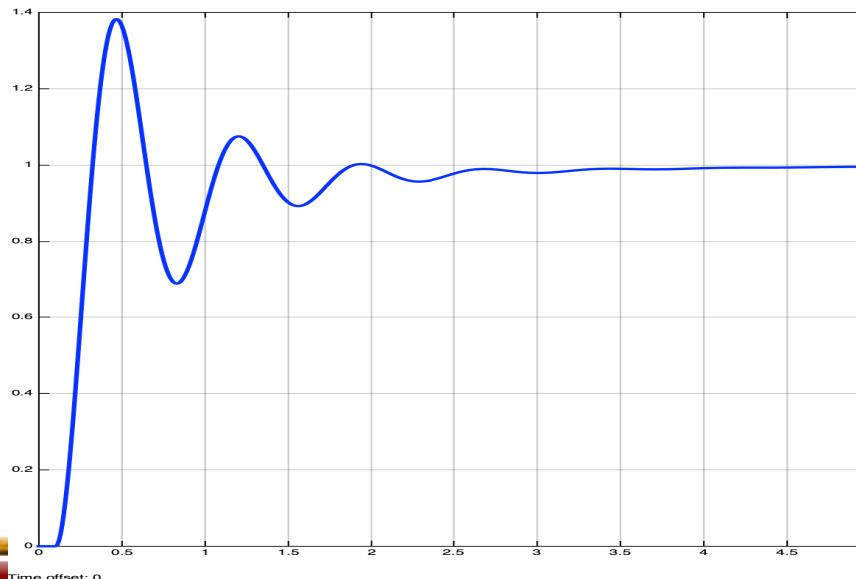
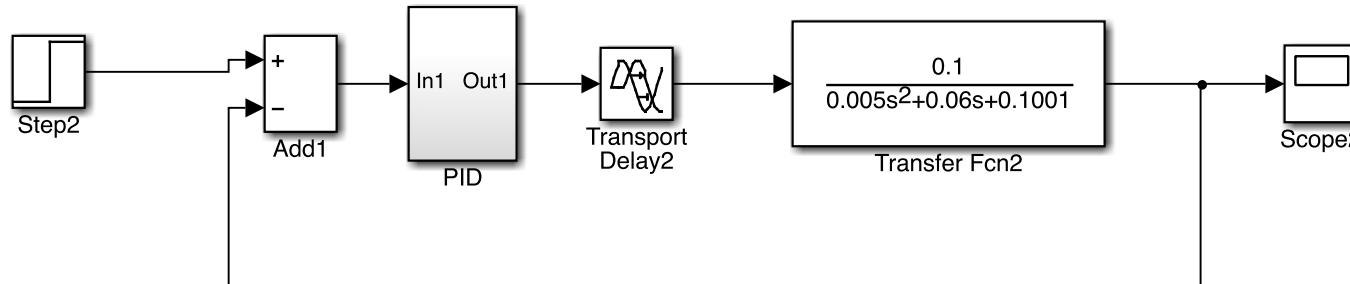
Método de curva de reacción

Controlador PID



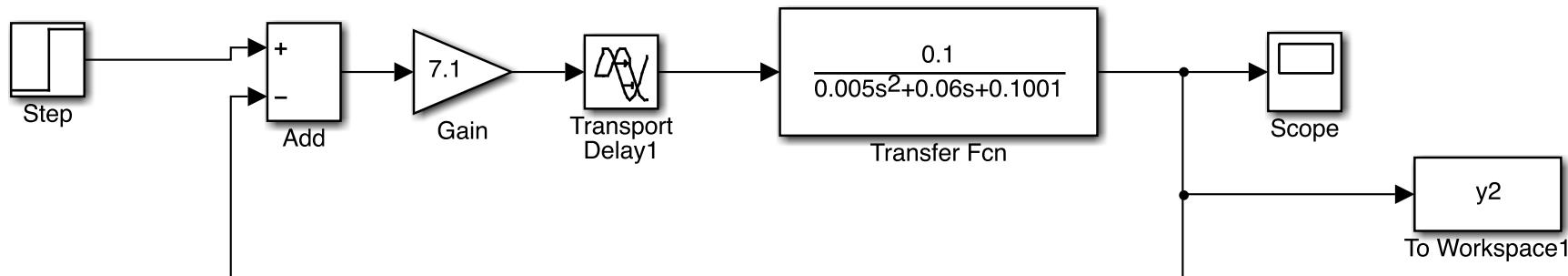
Método de curva de reacción

Controlador PID



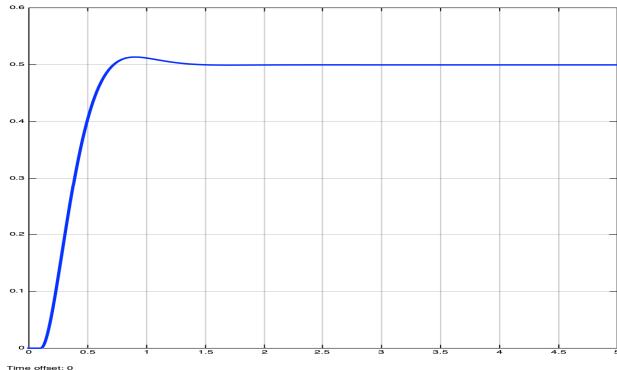
Método de ganancia última

Lazo cerrado



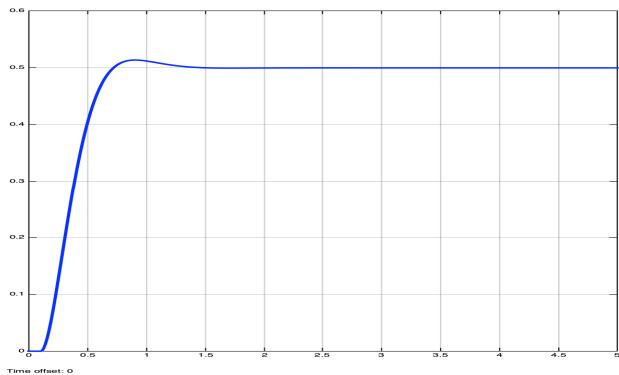
Método de ganancia última

$$k = 1$$

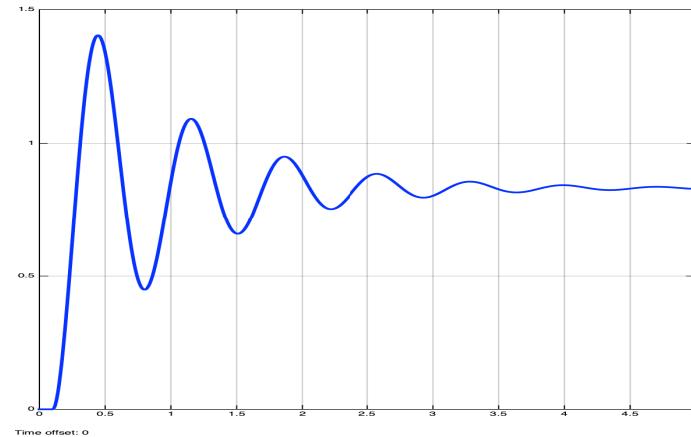


Método de ganancia última

$k = 1$

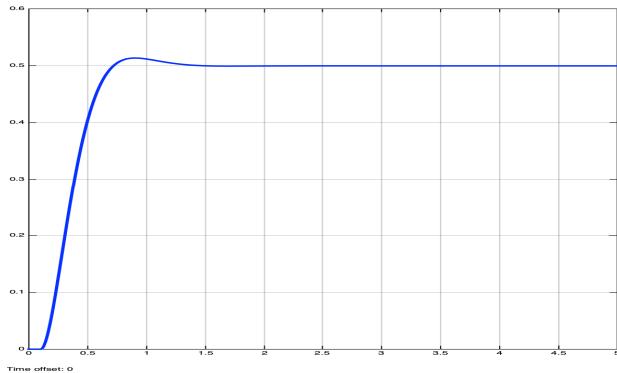


$k = 5$

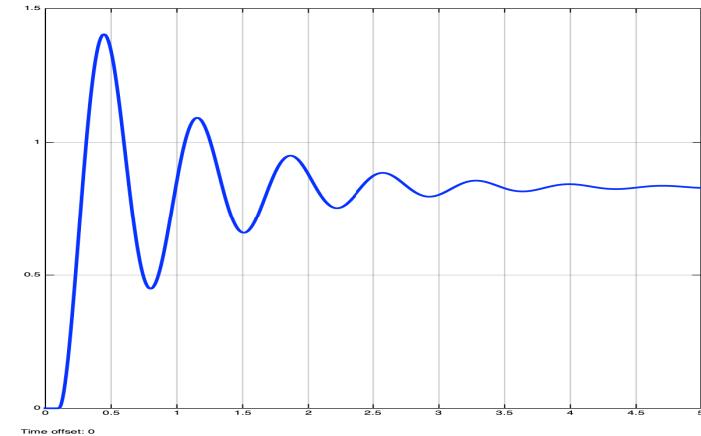


Método de ganancia última

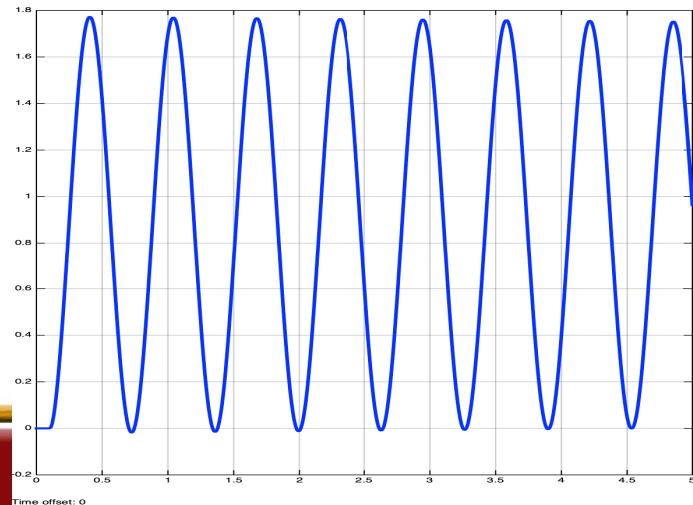
$k = 1$



$k = 5$



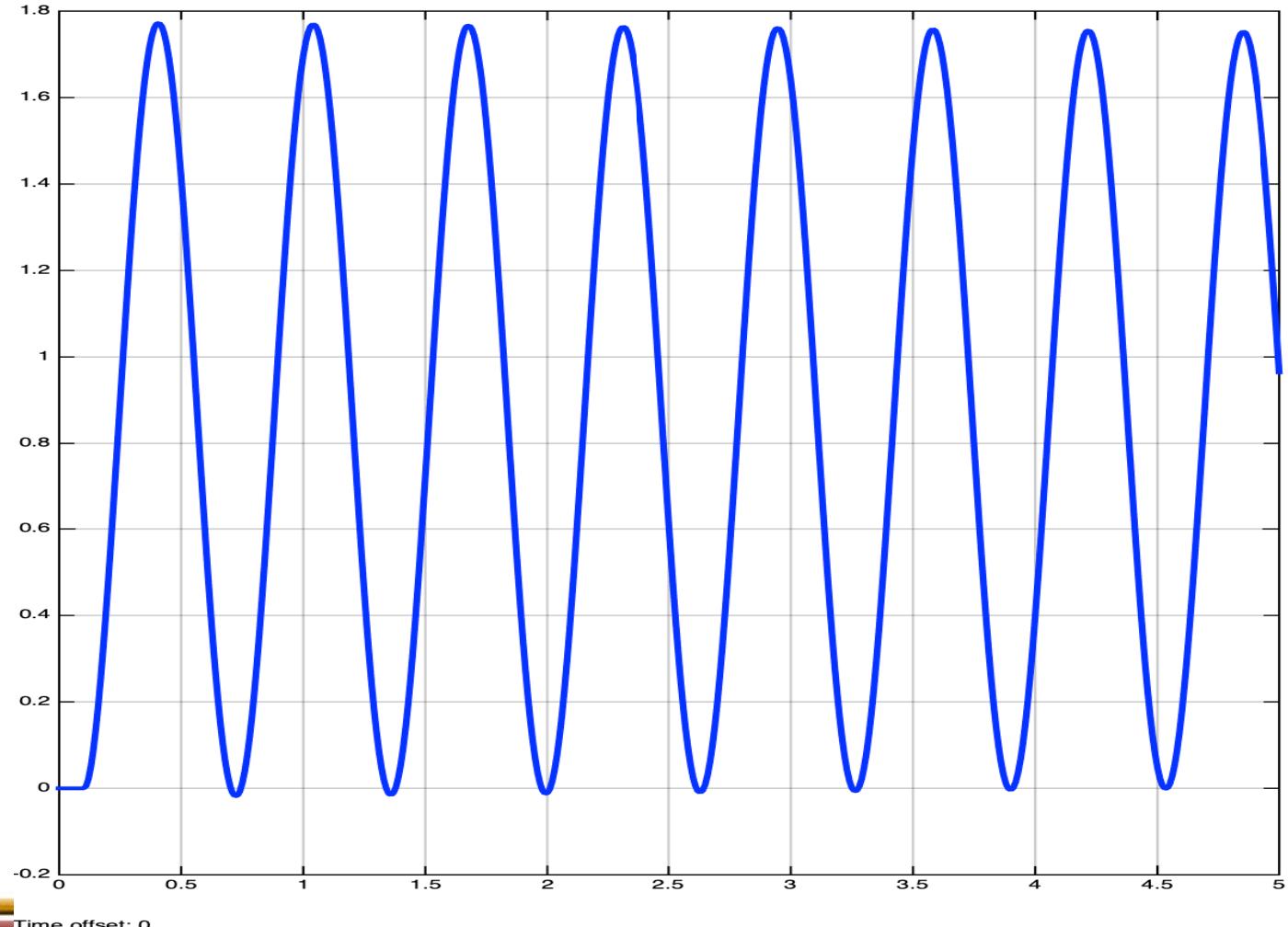
$k = 7.1$



CD y CTD

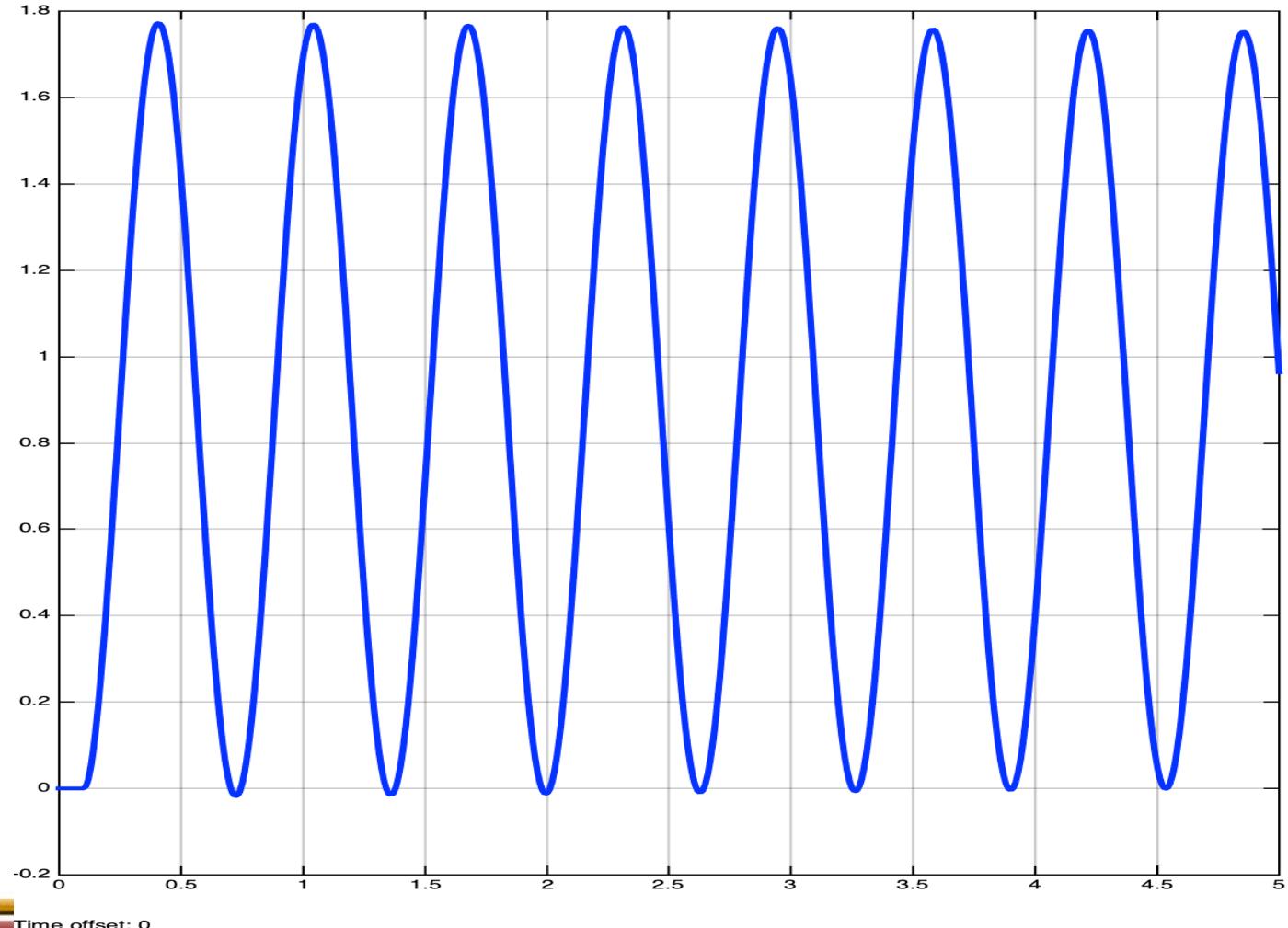
Método de ganancia última

$k = 7.1$



Método de ganancia última

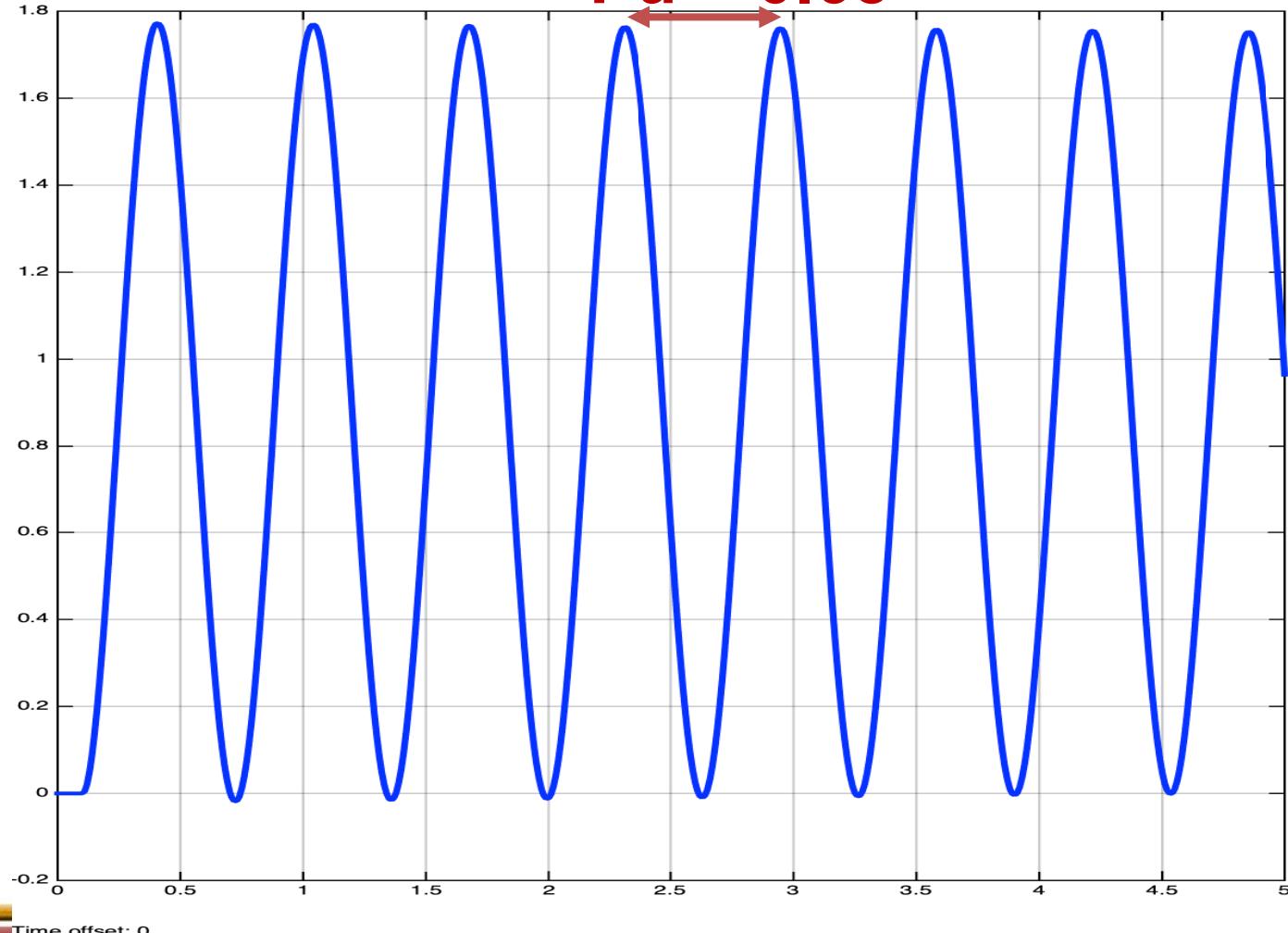
K_U = 7.1



Método de ganancia última

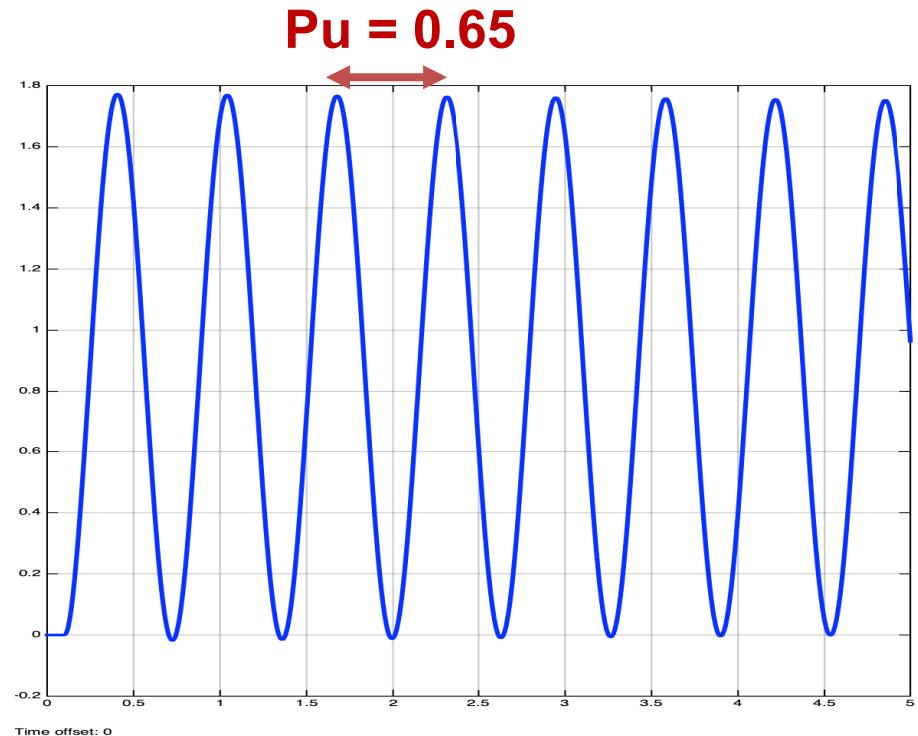
$P_u = 0.65$

$K_u = 7.1$



Método de ganancia última

Controlador	Kp	Ki	Kd
P	$0.5K_u$		
PI	$0.45K_u$	$1.2/P_u$	
PID	$0.6K_u$	$2/P_u$	$P_u/8$



$K_u = 7.1$

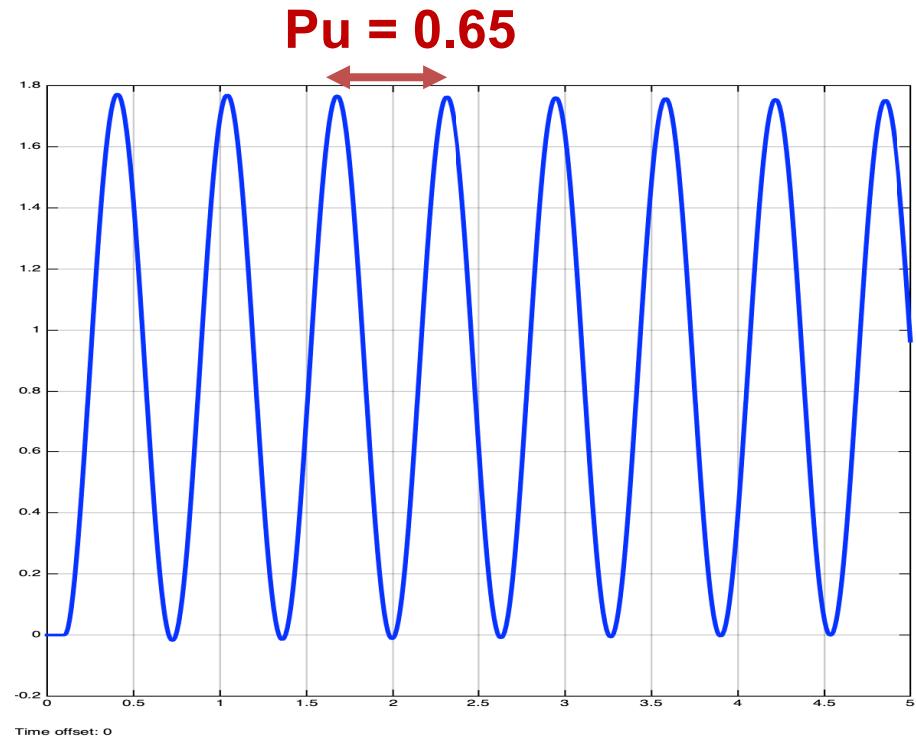
Método de ganancia última

Controlador	Kp	Ki	Kd
P	$0.5K_u$		
PI	$0.45K_u$	$1.2/P_u$	
PID	$0.6K_u$	$2/P_u$	$P_u/8$

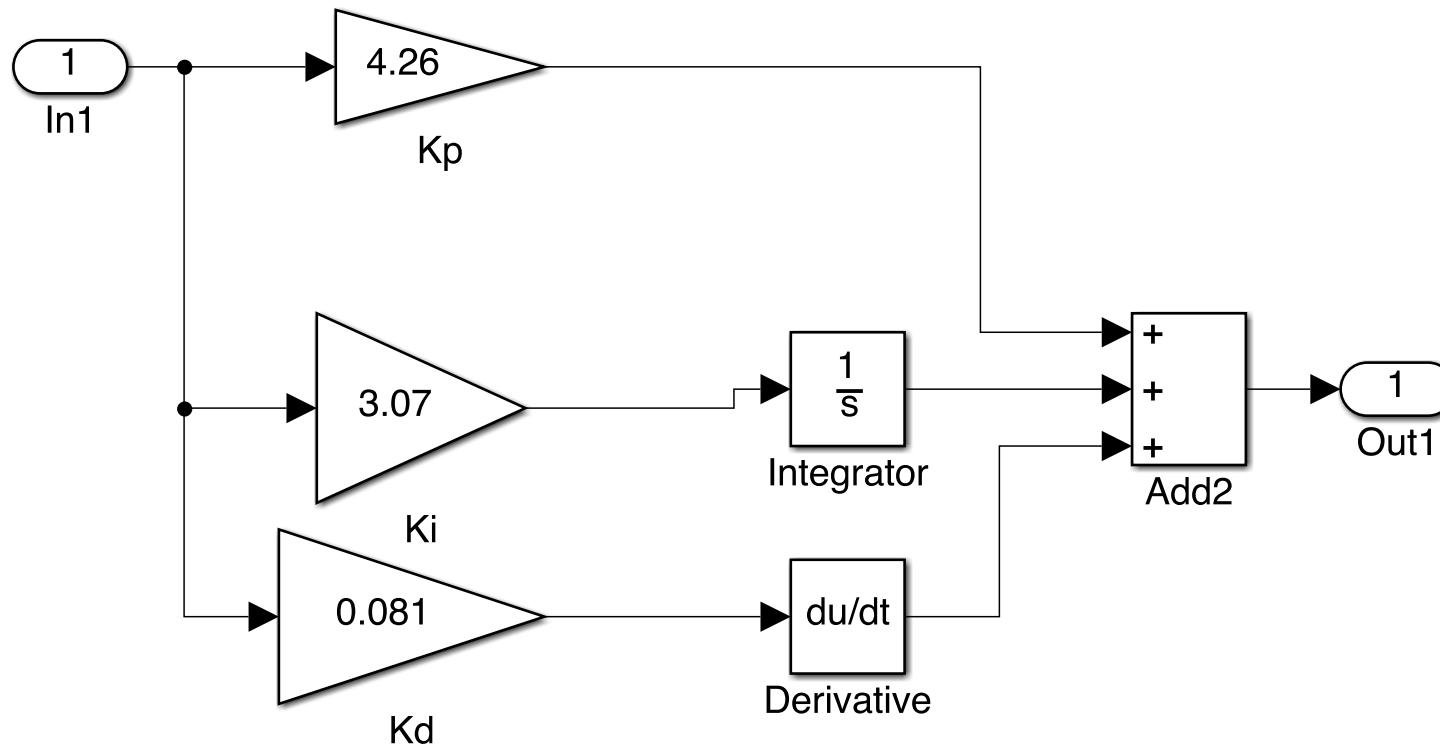
$$K_p = 4.26$$

$$K_i = 3.07$$

$$K_d = 0.081$$

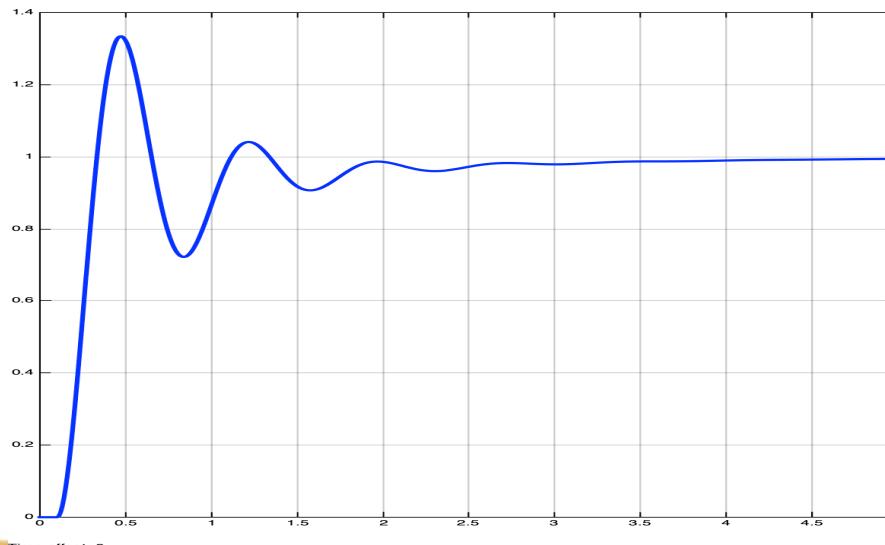
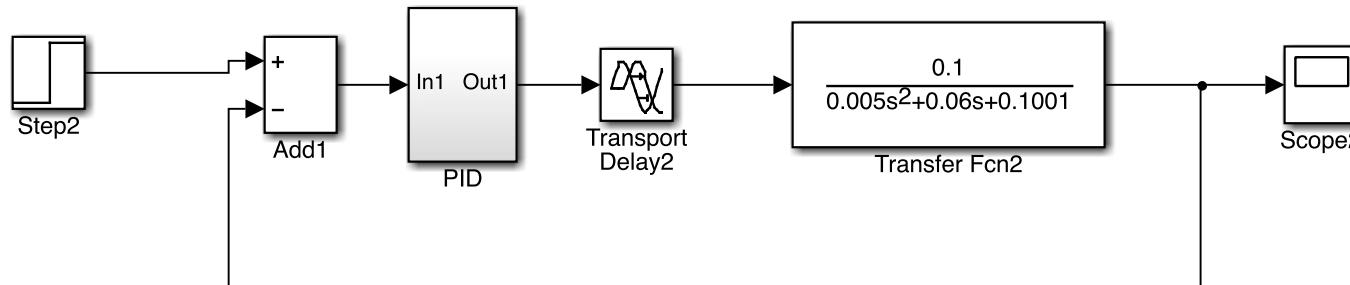


Método de ganancia última

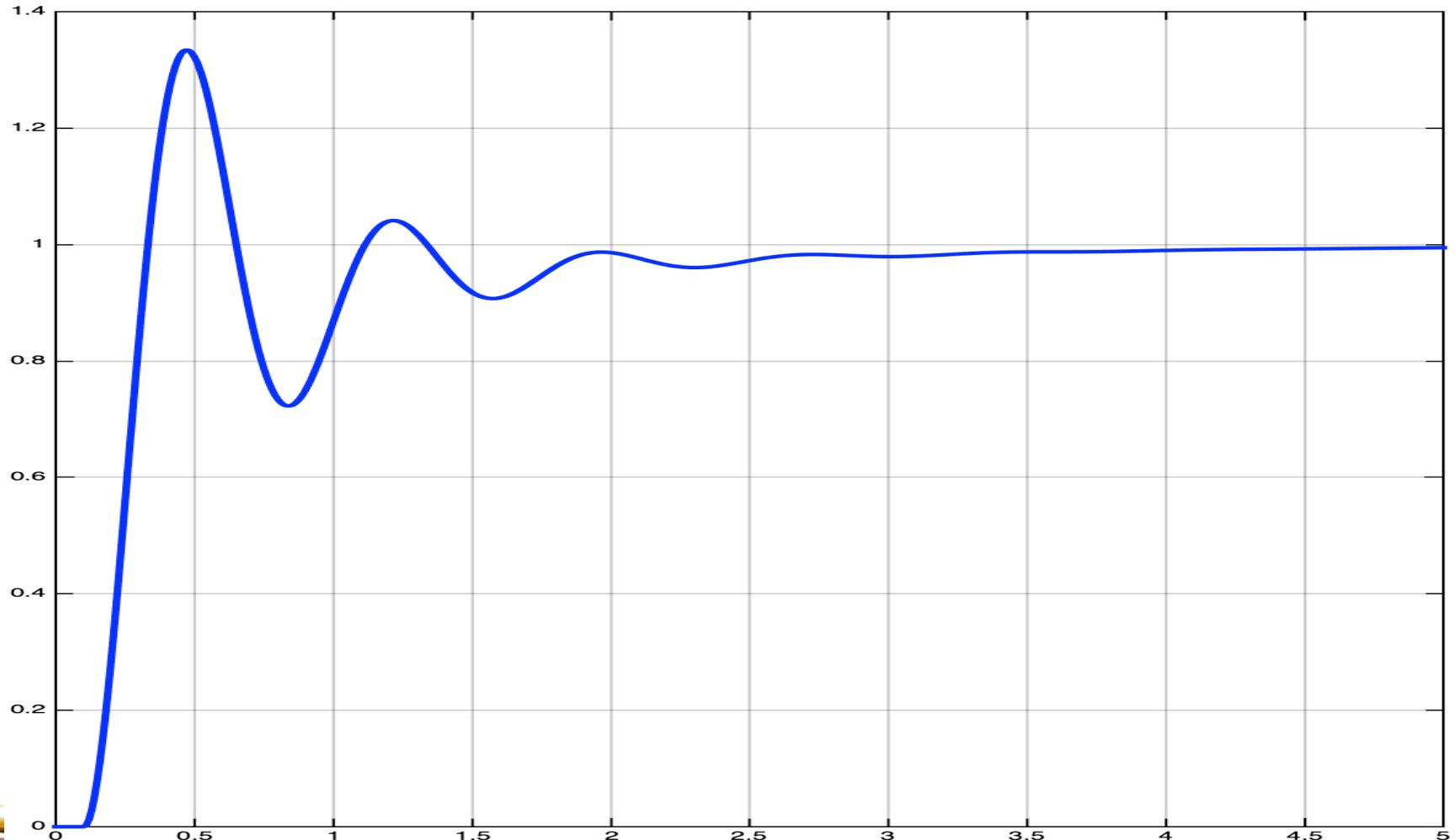


Método de ganancia última

Controlador PID



Método de ganancia última

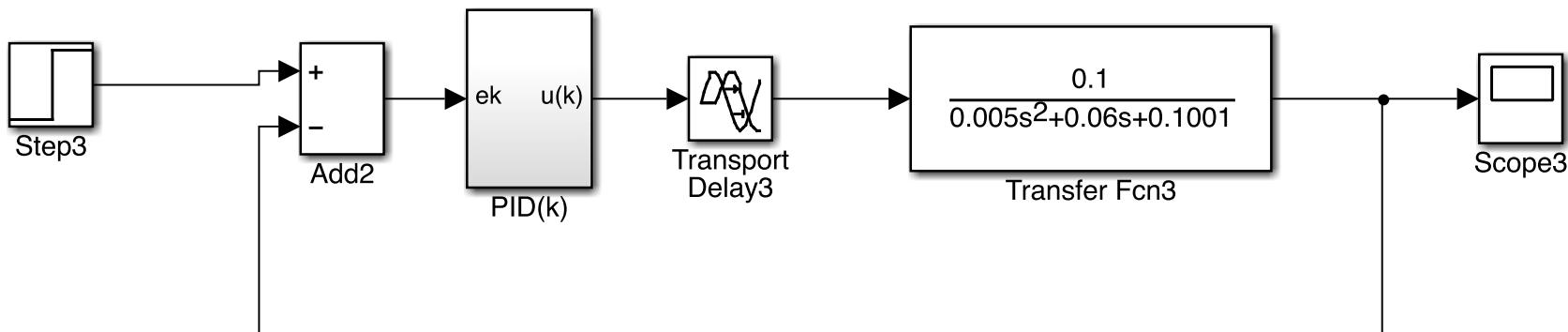


Time offset: 0

CD y CTD

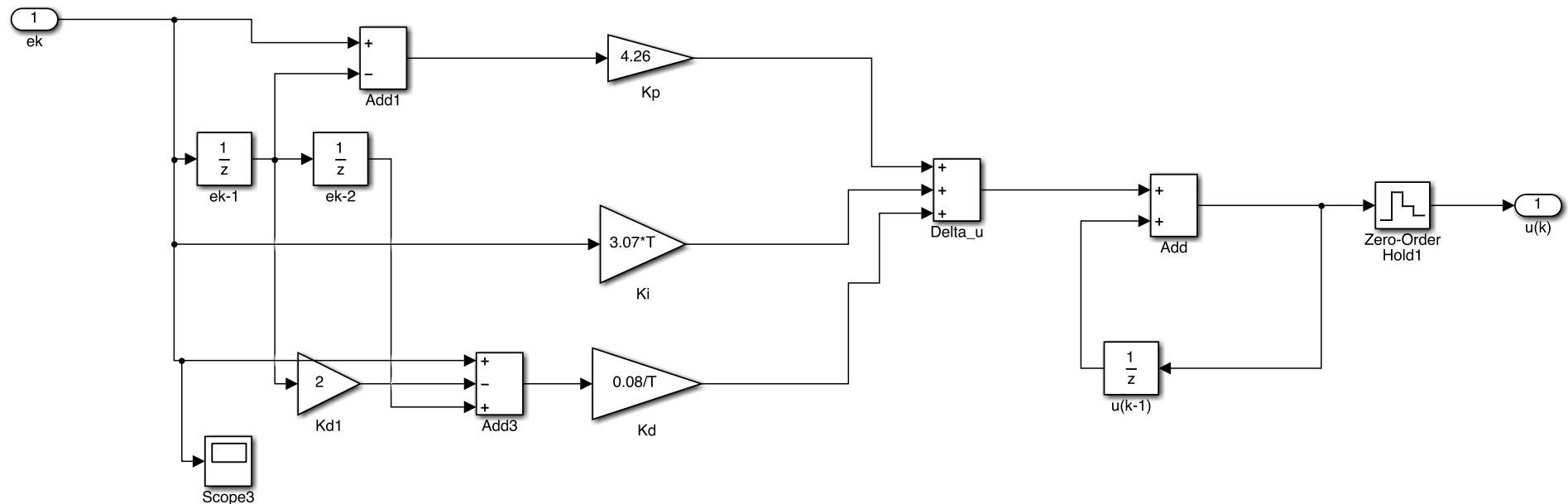
Método de ganancia última

Controlador PID discreto



Método de ganancia última

Controlador PID discreto

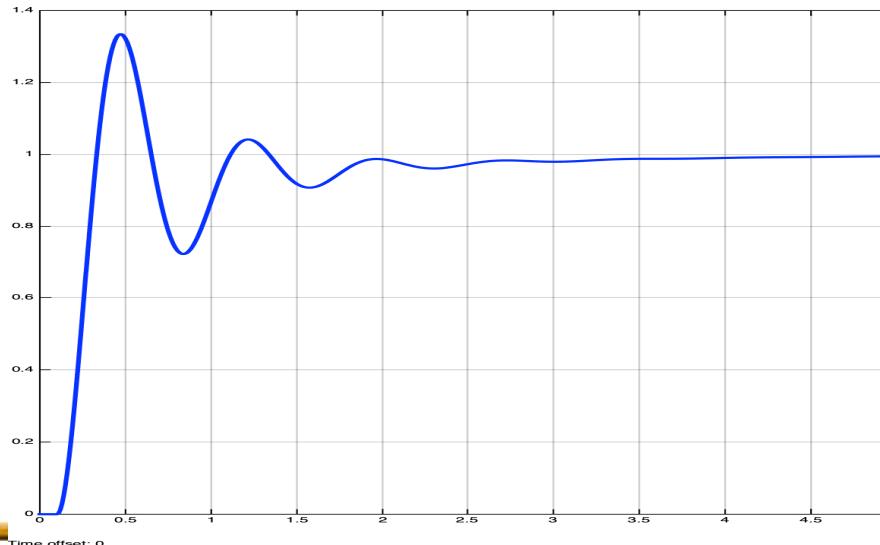
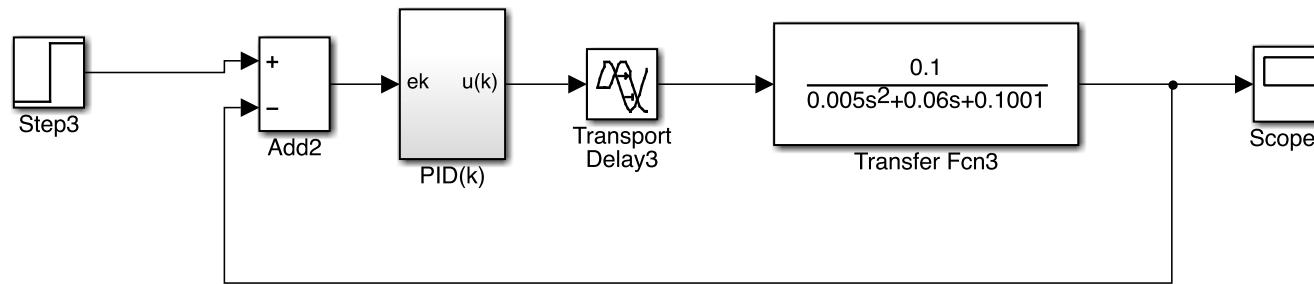


$$u(k) = u(k-1) + \Delta u$$

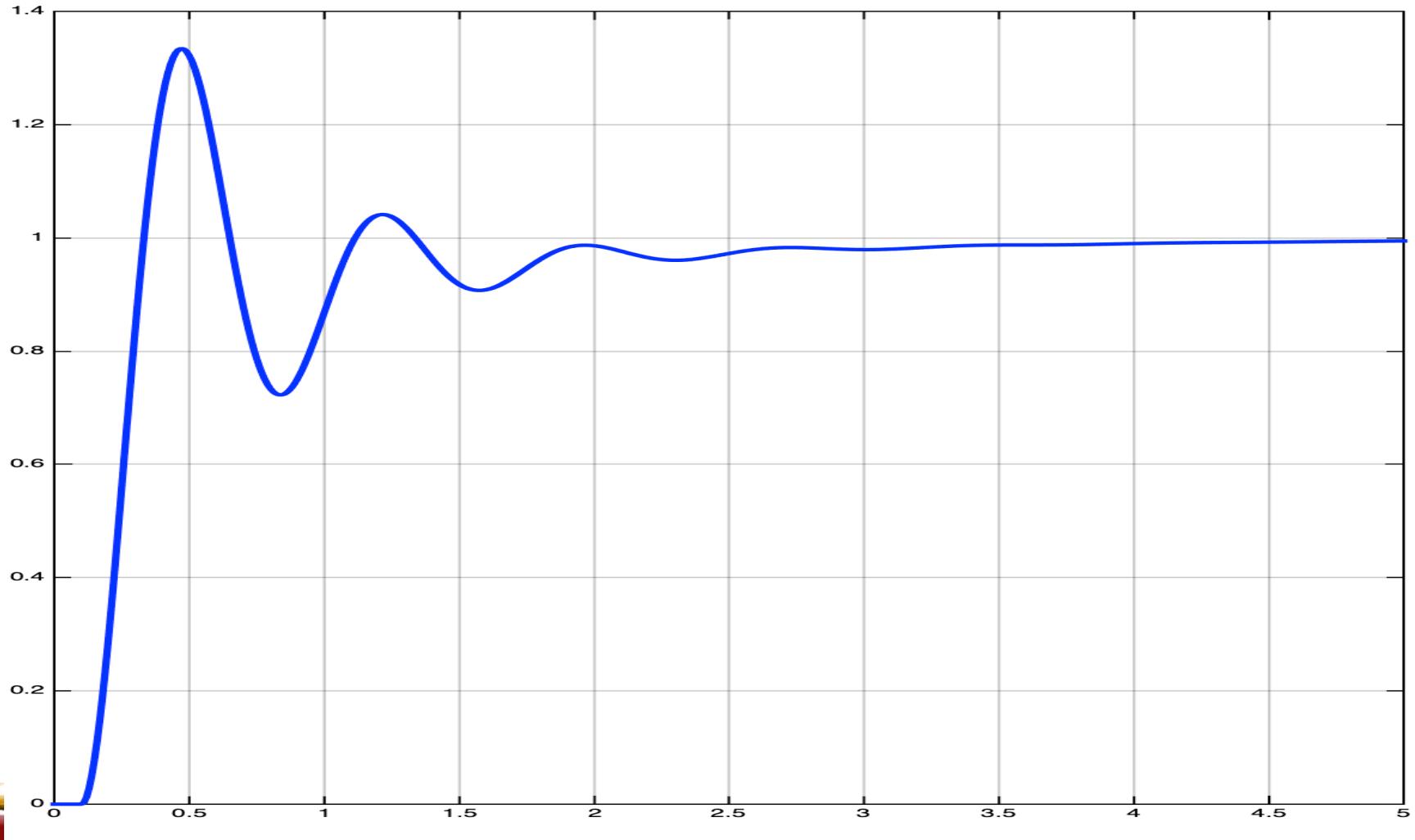
$$\Delta u = K_P [e(k) - e(k-1)] + K_I T e(k) + \frac{K_D}{T} [e(k) - 2e(k-1) + e(k-2)]$$

Método de ganancia última

Controlador PID discreto



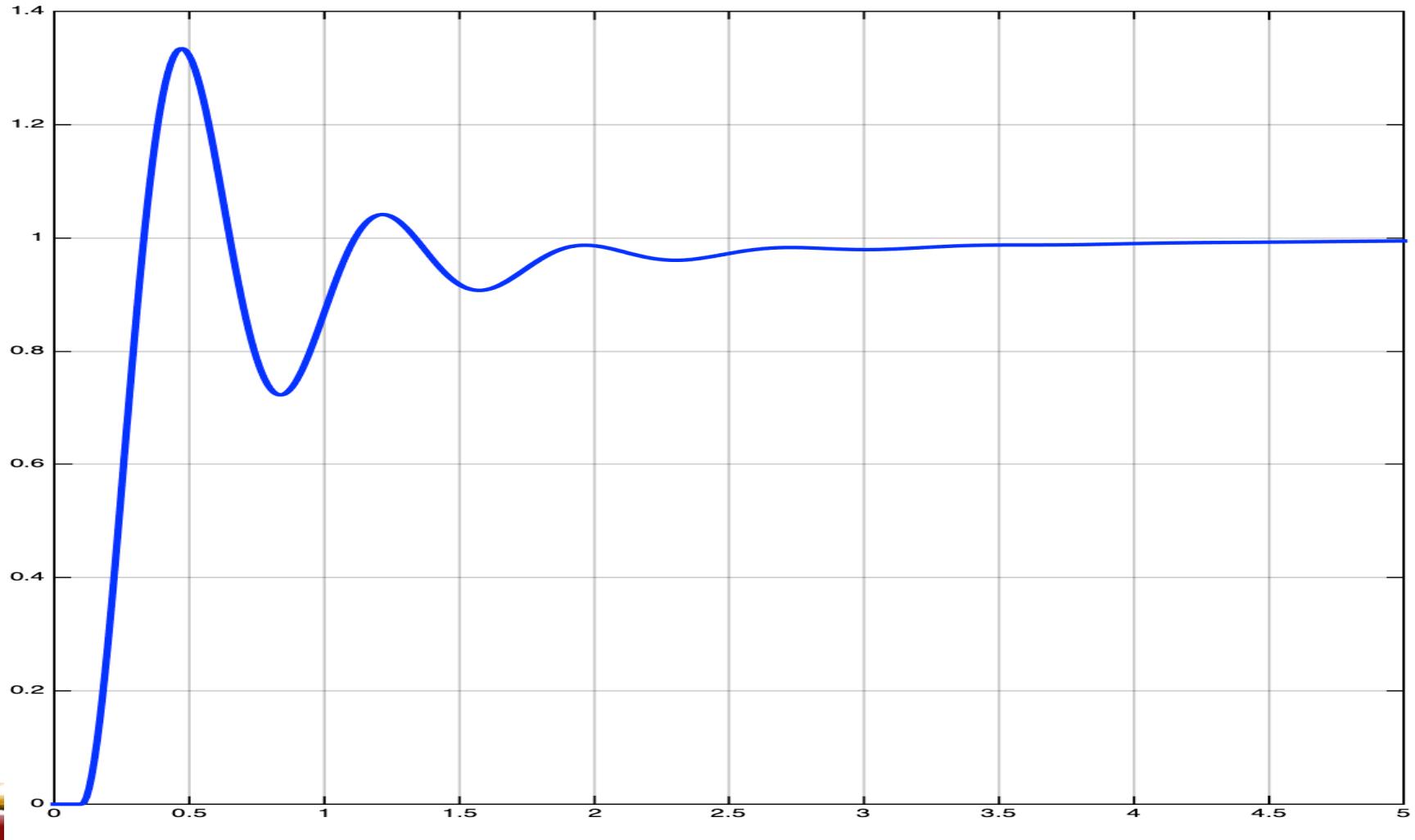
Controlador discreto



Time offset: 0

CD y CTD

Controlador discreto



Time offset: 0

CD y CTD

Sistema propuesto para controlar Sistema de segundo orden

