

$$\exists c, d \text{ in}(c, \text{South America}) \wedge \text{in}(d, \text{Europe}) \wedge \text{Borders}(c, d)$$

$$\equiv ①$$

$$\exists c, d \neg \left( \left( \text{in}(c, \text{South America}) \wedge \text{in}(d, \text{Europe}) \right) \wedge \text{Borders}(c, d) \right)$$

$$\equiv ②$$

$$\exists c, d \left( \neg \left( \text{in}(c, \text{South America}) \wedge \text{in}(d, \text{Europe}) \right) \vee \neg \text{Borders}(c, d) \right)$$

$$\equiv \text{Det} \rightarrow$$

$$\forall c, d \left[ \text{in}(c, \text{South America}) \wedge \text{in}(d, \text{Europe}) \rightarrow \neg \text{Borders}(c, d) \right]$$

$$\neg \left[ \exists c, d \text{ in}(c, \text{South America}) \wedge \text{in}(d, \text{Europe}) \wedge \text{Borders}(c, d) \right]$$

$$\equiv ①$$

$$\forall c, d \neg \left( \left( \text{in}(c, \text{South America}) \wedge \text{in}(d, \text{Europe}) \right) \wedge \text{Borders}(c, d) \right)$$

$$\equiv ②$$

$$\forall c, d \left( \neg \left( \text{in}(c, \text{South America}) \wedge \text{in}(d, \text{Europe}) \right) \vee \neg \text{Borders}(c, d) \right)$$

$$\equiv \text{Det} \rightarrow$$

$$\forall c, d \left[ \text{in}(c, \text{South America}) \wedge \text{in}(d, \text{Europe}) \rightarrow \neg \text{Borders}(c, d) \right]$$

$$\neg [\exists c, d \text{ in}(c, \text{South America}) \wedge \text{in}(d, \text{Europe}) \wedge \text{Borders}(c, d)]$$

$$\equiv ①$$

$$\forall c, d \neg [(\text{in}(c, \text{South America}) \wedge \text{in}(d, \text{Europe})) \wedge \text{Borders}(c, d)]$$

$$\equiv ②$$

$$\forall c, d \neg [\text{in}(c, \text{South America}) \wedge \text{in}(d, \text{Europe})] \vee \neg \text{Borders}(c, d)$$

$$\equiv \text{DeM} \rightarrow$$

$$\forall c, d [\text{in}(c, \text{South America}) \wedge \text{in}(d, \text{Europe}) \rightarrow \neg \text{Borders}(c, d)]$$

$$b) Q(y, G(A, B)), Q(G(x, x), y')$$

$$\{ \overset{y}{G(x, x)}, \overset{y'}{G(A, B)} \}$$

$$\alpha (G(x, x), G(A, B))$$

$$c) Older(Father(y), y), Older(Father(x), John)$$

$$\{ \overset{y}{John}, \overset{y}{John} \}$$

$$Older(Father(John), John)$$

$$d) knows(Father(y), y), knows(x, x)$$

NO!

$$\forall g.c [GP(g.c) \leftrightarrow [\exists p P(g.p) \wedge P(p.c)]]$$

$\equiv (1)$

$$\forall g.c [GP(g.c) \rightarrow \exists p P(g.p) \wedge P(p.c)] \wedge$$

$$[\exists p P(g.p) \wedge P(p.c) \rightarrow GP(g.c)]$$

$\equiv (2)$

$$\forall g.c [\neg GP(g.c) \vee \exists p P(g.p) \wedge P(p.c)] \wedge$$

$$[[\neg \exists p P(g.p) \wedge P(p.c) \vee GP(g.c)]]$$

$\equiv$  Morgan

$$\forall g.c [\neg GP(g.c) \vee \neg \exists p P(g.p) \vee P(p.c)] \wedge$$

$$[\neg \exists p P(g.p) \wedge P(p.c) \vee GP(g.c)]$$

$\equiv (3)$

$$\forall g.c [\neg GP(g.c) \vee [\forall p \neg (P(g.p) \vee P(p.c))]] \wedge$$

$$[\neg \forall p [P(g.p) \wedge P(p.c)] \vee GP(g.c)]$$

$$\begin{aligned} & \equiv \\ & \forall_{y,c} [ \cancel{G(p,y,c)} \wedge \neg p(p,y,p) \vee p(p,c) ] \wedge \forall p [ p(y,p) \wedge \neg p(p,c) ] \wedge \neg \cancel{G(p,y,c)} \\ & \stackrel{(x)}{=} \end{aligned}$$