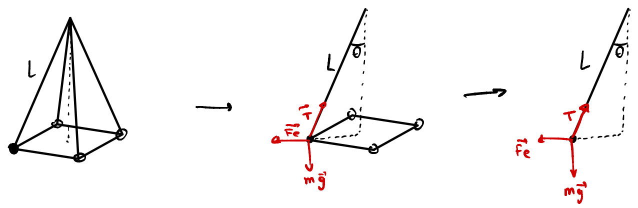
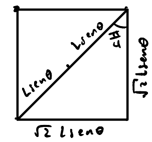
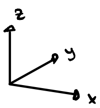
Preparcial





Equilibrio de fuerzas en eje z y plano XY:



$$z: \quad |\vec{T}|\cos\theta - mg = 0 \quad (g = |\vec{g}|)$$
$$XY: \quad |\vec{T}|\,\text{sen}\,\theta - |\vec{Fe}| = 0$$

$$F = \frac{1}{4\pi\varepsilon_0}\,\frac{q^2}{2L^2\text{sen}^2\theta} = \frac{1}{8}\cdot\frac{q^2}{\pi\varepsilon_0 L^2\text{sen}^2\theta}$$

$$|\vec{F}_{diagonal}| = \frac{\sqrt{2}}{2}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{4L^2\text{sen}^2\theta} = \frac{\sqrt{2}}{4}F$$

$$|\vec{F}_{lateral_x}| = |\vec{F}_{lateral_y}| = F$$

Fuerza eléctrica neta:

$$\begin{aligned}\vec{Fe} &= \vec{F}_{lateral_x} + \vec{F}_{lateral_y} + \vec{F}_{diagonal}\\ &= \left(\pm F \pm \tfrac{\sqrt{2}}{4}F,\ \pm F \pm \tfrac{\sqrt{2}}{4}F\right)\\ &= \left(F + \tfrac{\sqrt{2}}{4}F\right)(\pm1, \pm1)\\ &= F\left(1+\tfrac{\sqrt{2}}{4}\right)(\pm1,\pm1)\end{aligned}$$

(± Dependiendo sobre cuál carga se observa. Se ignora por la simetría del sistema

$$\begin{aligned}|\vec{Fe}| &= \sqrt{2}\cdot F\left(1+\tfrac{\sqrt{2}}{4}\right)\\ &= \left(\sqrt{2}+\tfrac{1}{2}\right)F\\ &= \left(\sqrt{2}+\tfrac{1}{2}\right)\cdot\frac{1}{8}\,\frac{q^2}{\pi\varepsilon_0 L^2\text{sen}^2\theta}\end{aligned}$$

Así, reemplazando en las ecuaciones de equilibrio:

$$z: \quad |T|\cos\theta = mg$$
$$XY: \quad |T|\,\text{sen}\,\theta = \left(\sqrt{2}+\tfrac{1}{2}\right)F$$

Se sigue que:

$$\frac{\text{sen}\,\theta}{\cos\theta} = \frac{\left(\sqrt{2}+\frac{1}{2}\right)\cdot F}{mg}$$

$$\frac{\text{sen}\,\theta}{\cos\theta} = \frac{\left(\sqrt{2}+\frac{1}{2}\right)}{mg}\cdot\frac{1}{8}\cdot\frac{q^2}{\pi\varepsilon_0 L^2\text{sen}^2\theta}$$

$$\frac{\text{sen}^3\theta}{\cos\theta} = \left(\frac{\sqrt{2}}{2}+\frac{1}{4}\right)\cdot\frac{1}{mg}\cdot\frac{q^2}{4\pi\varepsilon_0 L^2}$$

$$\frac{\text{sen}^3\theta}{\sqrt{1-\text{sen}^2\theta}} = \left(\frac{\sqrt{2}}{2}+\frac{1}{4}\right)\cdot\frac{1}{mg}\cdot\frac{q^2}{4\pi\varepsilon_0 L^2}$$

$$\frac{\text{sen}^6\theta}{1-\text{sen}^2\theta} = \left[\left(\frac{\sqrt{2}}{2}+\frac{1}{4}\right)\cdot\frac{1}{mg}\cdot\frac{q^2}{4\pi\varepsilon_0 L^2}\right]^2$$

$$\Rightarrow \quad \text{sen}^6\theta + \left[\left(\frac{\sqrt{2}}{2}+\frac{1}{4}\right)\cdot\frac{1}{mg}\cdot\frac{q^2}{4\pi\varepsilon_0 L^2}\right]^2\text{sen}^2\theta - \left[\left(\frac{\sqrt{2}}{2}+\frac{1}{4}\right)\cdot\frac{1}{mg}\cdot\frac{q^2}{4\pi\varepsilon_0 L^2}\right]^2 = 0$$