Evolutionary Game Theory

Motivation/Background Material

Game theory is the mathematical study of strategic decision-making, in which the outcome of a decision depends not only on one's choices but also on the choices of others. The game theory contains an important concept which is the concept of Nash Equilibrium. We have looked at various coordination games, and we have analyzed the static dynamics of these games. By this, I mean figuring out the pure strategy and the mixed strategy Nash equilibrium. Now we want to understand how different strategies evolve and persist over time, in a population of individuals. Essentially, we want to study the evolutionary dynamics of a game, to understand not only the mathematical analysis of strategic decision-making but also the evolutionary factor. This leads us to the study of evolutionary game theory.

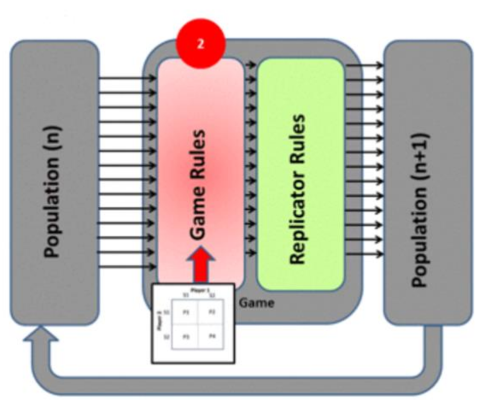
An Important concept of evolutionary game theory is finding the evolutionarily stable strategy(ESS), if present. An evolutionarily stable strategy is a strategy within a game that is impermeable when adopted by a population and will remain dominant even in the presence of other strategies. This concept is the center of evolutionary game theory because ESS can provide insight into how strategies within a game will evolve and interact in a population over time. It will also allow us to make predictions about the long-term outcomes of different games. Furthermore, this will give us insight into how strategies within a game will be favored by natural selection over time, and how cooperation and altruism emerge and be sustained in a population.

Lastly, we want to discover whether game models are more likely to converge to a payoff or risk-dominant strategy. Because even to this day, there exists the debate of believing that a group of people playing a game will more likely choose the payoff-dominant strategy than a risk-dominant strategy. As it is assumed that people are braver to betting higher than playing with a risky mentality. We will discover more about this in the scientific paper "Risk Dominance and Coordination Failures in Static Games" by Paul G. Straub. This will illustrate if people play a game with a mentality of not wanting to risk it all or if people are more willing to aim higher and put everything on the line.

Problem Statement

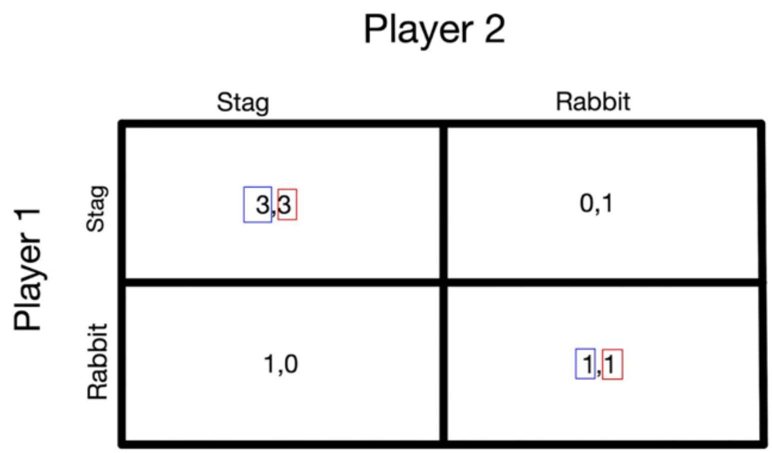
We will analyze and compare the evolutionary dynamics of a least 4 games which are the following: the rock scissors paper game, the chicken game, the stag hunt game, and the battle of the sexes games. A quick description of each game, the rock scissors paper game, it's a game played by two players using their hands to form one of three shapes: a rock, paper, or scissors. The objective of the game is for each player to select one of the three options, and the winner is determined by the game rules. Rock beats scissors, scissors beat paper, and paper beats rock. The chicken game, this game consists of two people Player 1 and Player 2. Each Player will position their cars on the opposite sides of a one-way street, where the front of their cars will point directly at each other. Then, both players will step on the accelerator, and drive right at each other. Right before colliding, both players will simultaneously have to decide whether they want to keep driving or swerves off the street. Whoever swerve is considered the chicken. The battle of sexes game is a game that consists of two players, who must choose between two options/strategies: attending a ballet event or attending a football game. They both want to attend the same event but don't have any sort of communication. The stag hunt game is a two-player game that must choose between cooperating to hunt a stag or acting independently to hunt a rabbit.

Hence, to analyze the evolutionary dynamics we need a model that conducts the evolutionary game theory algorithm, shown in Figure 1. In short, the evolutionary game theory algorithm is when you have a population of individuals who play a game in pairs, and the game has unique rules and payoffs structure. After they have finished playing the game, they will receive a reward. Then they compare their reward with a randomly selected individual from the population. If the random individual had a better payoff/reward, then they copy that behavior. Then the population is updated and we repeat the same procedure for numerous iterations. Essentially, it is the same concept in the evolutionary algorithm, but these game rules take the place of the fitness function. Having a model that conducts the evolutionary game theory algorithm, we need to analyze whether all four games have an evolutionarily stable strategy despite each game having a unique payoff structure and game dynamics. Also, keep in mind, "strategies are not assumed to be selected by players but rather hardwired into the individual's genetic makeup"(Dr.Thanos). This is an important factor to take into consideration when planning out the model, as this determines the evolutionary aspect when running each game in the model. Do you think that each game will have an evolutionarily stable strategy when each game is run for a period of time?



(Figure 1)

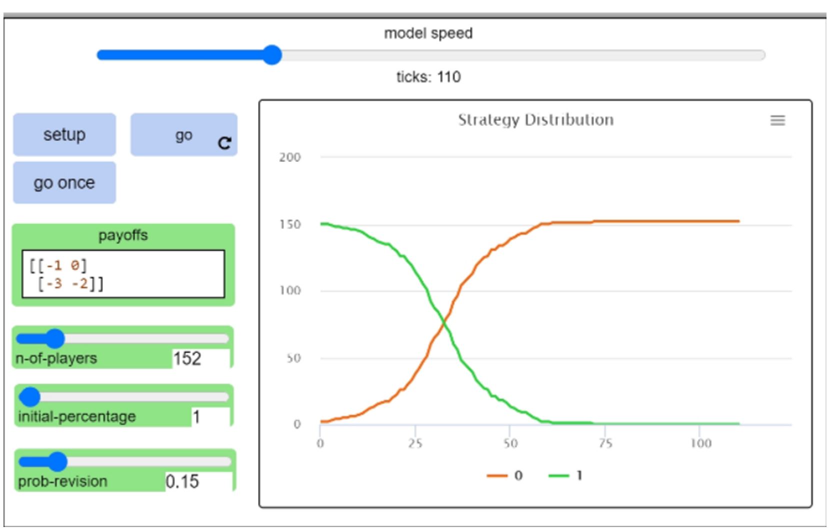
Lastly, visualize that you are playing the stag hunt game with another random player. Now take into consideration the payoff structure for the stag hunt game, which is illustrated in Figure 2. If you were to play this game for n number of periods, are you more likely to choose the stag strategy or the rabbit strategy? Hence, player 2 will choose randomly and you don't know his incentives, he might play it safe and choose the rabbit strategy. Keep in mind that if you and Player 2, choose different strategies, then you or Player 2 will receive a payoff of zero and the other gets a payoff of 1. Do you think that both you and Player 2 will go for the greater payoff which is the stag strategy and is also known as the payoff dominant strategy? Or do you think that both you and player 2 will go for the rabbit strategy which has the highest minimal payoff and is known as the risk-dominant strategy? Also, put yourself as the spectator do you think this game model will converge to a payoff-dominant strategy or risk-dominant strategy?



(Figure 2)

Approach

To analyze and compare the evolutionary dynamics within the four games, we need to construct a model that conducts the evolutionary game theory algorithm. Therefore, I have reverse-engineered Dr.Thanos Agent Based evolutionary model. This model shown in Figure 3, conducts the evolutionary game theory algorithm, for 2x2 games.



(Figure 3)

Evolutionary Game Theory

Looking at Figure 3, this model was constructed using NetLogo code and consists of 5 inputs. The four inputs are payoffs, which is the payoff structure for the game. Also since we are only dealing with symmetrical games, we only need the payoffs of one of the players. Then the second input is “n-of-players”, which is the population size you want to work with for your game. Then you have the initial percentage, which is the probability of the population that plays strategy 1. The fourth input is the prob-revision, which corresponds to the probability that an individual will copy a randomly selected player in every iteration if the randomly selected player did better. Lastly, you have the model speed, which will run the number of ticks per iteration.

The NetLogo code is going to take all the values of these 5 inputs and perform the evolutionary game theory algorithm, and illustrate the strategy distribution through a line graph. Hence, the orange line is the first strategy of the game, and the green line is the second strategy of the game. Also, note this game will only run the evolution game theory for 2x2 symmetrical games. Thus, to analyze the evolutionary dynamics for the paper rock scissors game, I had to also construct an Agent-Based evolutionary model for 3x3 symmetrical games. The logic behind the code is the same as the 2x2 Agent-based evolutionary model. However, I did create two different models, where one was programmed in Python and the other was programmed in NetLogo code. The NetLogo code is using the same logic behind Dr.Thanos Agent Based Evolutionary Model (provided by NetLogo code page). The Python program is using a deferential equation and an infinite population, which will perform the evolutionary game theory algorithm. Finally, we will run each game in these Agent Based evolutionary models, and then analyze the evolutionary dynamics.

Lastly, we will critically analyze the scientific paper “Risk Dominance and Coordination Failures in static games” by Paul G. Straub (1995) and other semi-recent scientific papers. This will overall give us a better understanding of whether game models are more likely to converge to a risk-dominant strategy or payoff-dominant strategy.

Experiment Set-Up

First, you will need to launch the different evolutionary models, which are attached to this folder. Then run each game through the evolutionary models with different population sizes, initial percentages, prob-revision, and the number of ticks to thoroughly analyze the evolutionary dynamics. Then for the scientific paper, we will critically analyze the paper and outline the different arguments that are addressed in the paper.

Results and Discussions

After running all games through the Agent-Based evolutionary model, we have discovered unique evolutionary dynamics among all four games. I will go through each one and illustrate the findings. One thing that I want to note is that for every game I also showed the static dynamics to give us some insight into possible strategies that can be the ESS and also the possible behavior of the players over time.

Evolutionary Game Theory

The Chicken Game

Player 2

	Keep Going	Swerve
Keep Going	-50,-50	4,-4
Swerve	-4,4	1,1

(Figure 4) : Chicken Game payoff structure

- *Player 1 Mixed Strategy :*
- $-50p + 4(1-p) = -4p + 1(1-p)$

$$p = \left(\frac{3}{49}\right)$$

- *Player 2 Mixed Strategy :*
- $-50p + 4(1-p) = -4p + 1(1-p)$

$$p = \left(\frac{3}{49}\right)$$

Thus, the mixed strategies $\left(\frac{3}{49}, \frac{46}{49}\right), \left(\frac{3}{49}, \frac{46}{49}\right)$ are Nash Equilibrium (Figure 4.1)

Starting with the static dynamics of the chicken game in Figure 4 and Figure 4.1, you can notice that there are three Nash equilibriums. Where we have two pure strategies Nash equilibrium and we have a mixed strategy Nash equilibrium. Looking at the two pure strategies of Nash equilibrium we can notice a bargaining conflict, where both players have the incentive to convince the other player to play the swerve strategy to gain maximum payoff. Knowing this behavior from the static dynamics we see a unique behavior in the evolutionary dynamics of the chicken game.

(Figure 4.2)

Evolutionary Game Theory

In Figure 4.2, here we see the chicken game being run through the agent-based evolutionary model. After running the game in the model with different population sizes, initial percentages, and prob-revision values. It is concluded that by setting the initial percentage greater than fifty percent you will see that the population finds coordination and the swerve strategy becomes the evolutionarily stable strategy. Based on Figure 4.2, we set the population to 100, the initial percentage set to 72, and the prob-revision set to 0.15, and we see that the swerve becomes the evolutionarily stable strategy.

Thinking about the results rationally, it is clear why the swerve strategy becomes the evolutionarily stable strategy. That is because the individuals in the population will realize that playing the keep-going strategy will lead to a much greater deficit loss compared to the swerve strategy throughout time. The individuals in the population would much rather take the title of being a chicken rather than have to face a fatal crash and causing them to face a greater loss in payoff over time.

Battles of sexes

		Player 2	
		Ballet	Football
Player 1	Ballet	$\boxed{6},\boxed{4}$	0,0
	Football	0,0	$\boxed{4},\boxed{6}$

(Figure 5): Payoff structure for the battle of sexes game

- *Player 1 Mixed Strategy :*
- $4p + 0(1-p) = 0p + 6(1-p)$

$$p = \left(\frac{3}{5}\right)$$

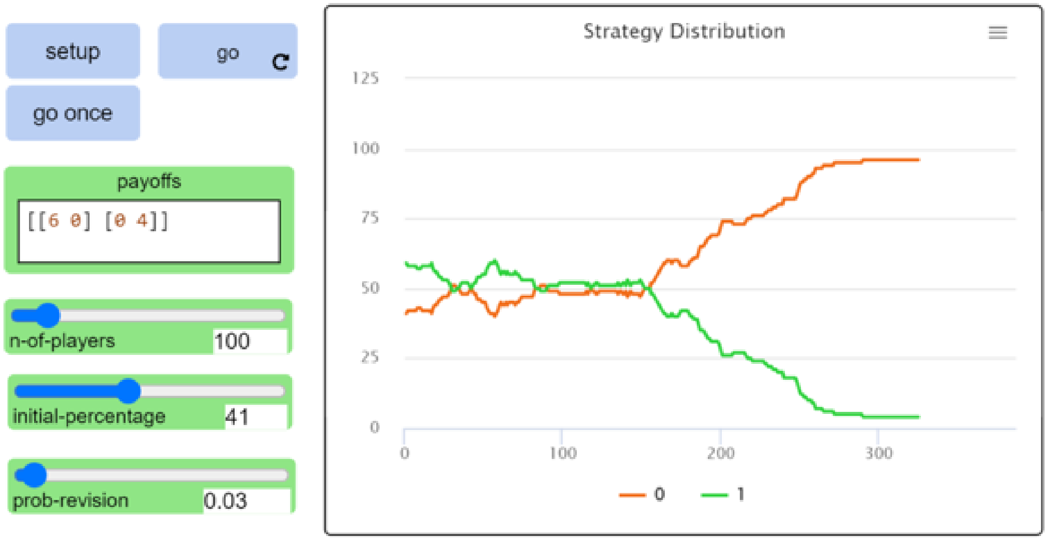
- *Player 2 Mixed Strategy:*
- $6q + 0(1-q) = 0q + 4(1-q)$

$$q = \left(\frac{2}{5}\right)$$

Thus, the mixed strategies $\left(\frac{3}{5}, \frac{2}{5}\right), \left(\frac{2}{5}, \frac{3}{5}\right)$ *are Nash Equilibrium*

(Figure 5.1)

Analyzing the static dynamics of the Battle of sexes game which can be seen in Figure 5 and Figure 5.1, you can notice the same number of Nash equilibriums as we saw in the chicken game. However, the payoff structure for the battle of sexes game(Figure 5) is what makes this game unique and different compared to the other games. It is unique because compared to the chicken game, you don't have a large deficit loss as we saw in the chicken game, where if both players choose the keep-going strategy they each face a payoff of negative fifty. That's not the case in the battle of sexes game payoff structure, because neither player can face a negative payoff and also there isn't a large difference between the payoffs for both players. Hence, the payoff structure in Figure 5, creates conflict between coordination and competition as both players have a preference for coordination, but they also have a preference for different outcomes. This conflict/behavior can be noticeable in the evolutionary dynamics when running this game in the 2x2 agent-based evolutionary model, which can be seen in Figure 5.2.



(Figure 5.2)

After running the game in the model with different population sizes, initial percentages, and prob-revision values. It's concluded that by setting the initial percentage between 30 percent and 50 percent, you will notice that the population finds coordination and that the ballet strategy becomes the evolutionarily stable strategy. However, if you run this game multiple times, you will notice that it is not always the case that the population finds coordination, it varies. By varies I mean that by setting the initial percentage between 30 and 50 percent, you will get cases when the population does find coordination and sometimes they don't. This has to do with the conflict of preferences and competition as discussed in the static dynamics of this game. We can see this competition behavior in Figure 5.2, starting a tick zero through 150, you see a looping pattern that illustrates the competition behavior in the population. However, after tick 150, you see that the population finds coordination and the ballet strategy becomes the evolutionarily stable strategy. As mentioned, it is not always the case they find coordination.

Stag Hunt Game

Player 2

Player 1		Stag	Rabbit
	Stag	3,3	0,1
	Rabbit	1,0	1,1

(Figure 6): Payoff structure for stag hunt game

- *Player 1 Mixed Strategy :*
- $0p - 4(1-p) = -2p + 0(1-p)$

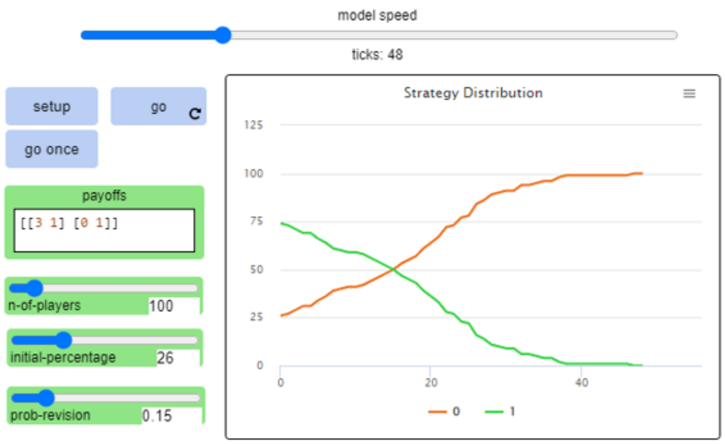
$$p = \left(\frac{2}{3}\right)$$

- *Player 2 Mixed Strategy :*
- $0q + 2(1-q) = 4q + 0(1-q)$

$$q = \left(\frac{1}{3}\right)$$

Thus, the mixed strategies $\left(\frac{2}{3},\frac{1}{3}\right),\left(\frac{1}{3},\frac{2}{3}\right)$ *are Nash Equilibrium* *(Figure 6.1)*

Analyzing the static dynamics of the Stag Hunt game which can be seen in Figure 6 and Figure 6.1, you notice the same number of Nash equilibriums as in the previous games we analyzed. Specifically looking at the pure strategies Nash equilibrium in Figure 6, here we see the case of choosing between a payoff dominant strategy and risk dominant strategy. If both players choose the stag strategy then they receive the highest payoff within the payoff structure. If both choose the Rabbit strategy then they receive the highest minimal payoff. However, if both players choose different strategies then either player 1 or player 2 may not gain any payoff. Essentially, it comes down to guessing the preferences of the other player, whether they want to aim for the highest payoff or lowest payoff. Thus, this is what makes this game different compared to the previous games that we have analyzed.



(Figure 6.2)

In Figure 6.2, here we see the Stag Hunt game being run through the agent-based evolutionary model. After running the game in the model with different population sizes, initial percentages, and prob-revision values. It's concluded that by setting the initial percentage between 0 and 50 percent you will see that the population finds coordination and the stag strategy becomes the evolutionarily stable strategy. The population seems to converge to the payoff dominant strategy, and that is because of how the payoff structure is set up. Individuals in a population, are willing to go for the higher payoff just because they don't run the risk of facing a deficit loss, and fits their preferences.

Rock Scissors Paper Game

		Player 2		
		L Rock	C Paper	1-L-C Scissors
Player 1	U Rock	0,0	-1,$\boxed{1}$	$\boxed{1}$,-1
	C Paper	$\boxed{1}$,-1	0,0	-1,$\boxed{1}$
	$1-U-C$ Scissors	-1,$\boxed{1}$	$\boxed{1}$,-1	0,0

(Figure 7): Payoff structure for Rock scissors paper game

$$EU_U = P_L(0) + P_C(-1) + (1)(1 - P_L - P_c)$$

$$EU_C = P_L(1) + P_C(0) + (-1)(1 - P_L - P_c)$$

$$EU_D = P_L(-1) + P_C(1) + (0)(1 - P_L - P_c)$$

$$EU_U = EU_C = EU_D$$

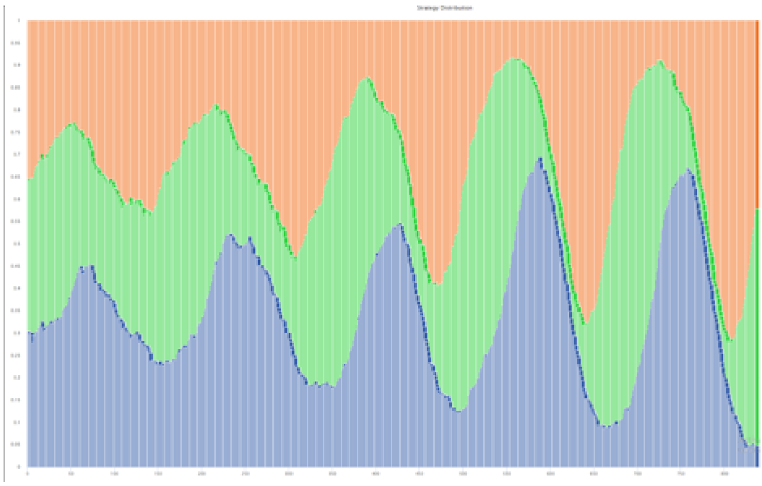
$$EU_U = \frac{1}{3}$$

$$EU_U = \frac{1}{3}$$

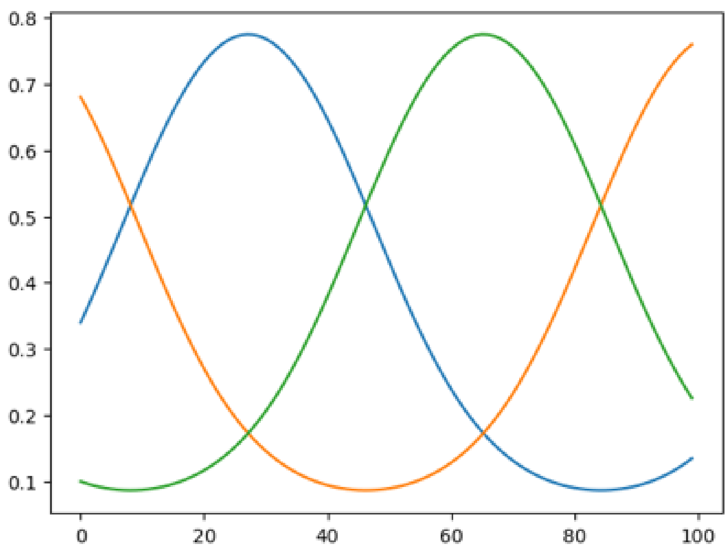
$$EU_D = \frac{1}{3}$$

(Figure 7.1): Mixed strategy Nash Equilibrium

Analyzing the static dynamics of the Rock Scissors Paper game which can be seen in Figure 7 and Figure 7.1. You can quickly notice that based on this payoff structure we don't have a pure strategy Nash equilibrium, and only have a mixed strategy Nash equilibrium. In the previous games that we analyzed, all games had a pure strategy and a mixed strategy Nash equilibrium. However, not the case in this game, as both players will play on a probability of 1/3 for each strategy. This probabilistic and randomizing behavior gives us an insight into the evolutionary dynamics of this game which can be seen in Figure 7.2 and Figure 7.3.



(Figure 7.2)



(Figure 7.3)

In Figures 7.2 and 7.3, here we see the Rock Scissors Paper game being run through the two agent-based evolutionary models. After running the game in both models with different population sizes, initial percentages, and prob-revision values. We see a unique pattern illustrated in the strategy distribution, which we haven't seen in the previous games. We see this looping pattern which is called unstable cycling, this occurs when one dominant strategy is overtaken/displaced by another strategy. Thinking of the results in a biological perspective, it makes sense. For instance, take a population of Paper, and say that a scissors mutant enters the paper population, the scissors mutant will over dominate the paper population, based on the game rules. Thus we now have a population of scissors. Hence, say a rock mutant enters the scissors population, that rock mutant will over dominate the scissors population and convert it into a rock population. Then the cycle repeats as a paper mutant will over dominate the rock population and later get over dominate it. Thus the evolutionary dynamic of the rock scissors paper game is dynamically unstable.

The scientific paper "Risk Dominance and Coordination Failures in Static Games" by Paul G. Straub (1995)

Straub begins by addressing how game theorists and economists are not coming to a consensus in predicting the outcome of simple games that have multiple Nash equilibria. From this point, Straub then presents several examples of coordination games, such as the battle of the sexes game, where the issue of coordination failure seems to occur often. Also, this is one of the reasons that has caused this conflict among game theorists. Thus, Strabs then explores mechanisms to help explain why coordination failure occurs and also helps predict equilibrium selection. This led him to analyze Harsanyi and Selten's theory which is a hypothetical model that claims to help predict equilibrium coordination and explain why coordination failure occurs.

However, Straub doesn't fully agree with Harsanyi and Selten's theory, mainly because they exclude the influences of equilibria's being Pareto ranked and out-of-equilibrium payoffs. Straub then suggests that "risk dominance," which considers the influence of out-of-equilibrium payoffs is the appropriate mechanism. Using this mechanism he believes that it's more likely to solve equilibrium selection problems in single-period, simultaneous move, noncooperative games with multiple Nash equilibria, even when the equilibria can be Pareto ranked.

This then led Straub to look into past experiments but claims that past experiments did not explicitly test the hypothesis of risk dominance being a mechanism to explain coordination failures. Because of this unexplicit examination, Straub introduces his new experiments. They run this new experiment to examine if risk dominance is an effective mechanism to help explain coordination failure and help predict equilibrium selection. A total of eleven games were constructed and each game was designed to test a specific hypothesis one of them was to check if game models were likely to converge to risk-dominant or payoff-dominant.

Table 5

Session	Game.	Type	Pareto Dominant Equilibrium	Risk Dominant Equilibrium	Number of Players	Scheduled Number of Periods	Actual Number of Periods
1	1	CG	DR	UL	10	9	9
2	2	CG	DR	DR	10	9	9
3	3	CG	DR	Mixed	10	9	9
4	4	CG	DR	DR	10	9	9
5	5	BOS	—	UL	10	9	9
6a	6	BOS	—	DR	10	9	9
6b	6	BOS	—	DR	10	18	10
6c	6	BOS	—	DR	10	18	18
7a	7	BOS	—	Mixed	10	18	18
7b	7	BOS	—	Mixed	10	18	18
8	8	CG	DR	UL	10	9	9

Notes: [1]Coordination Game is abbreviated CG.
[2]Battle of the Sexes Game is abbreviated BOS.

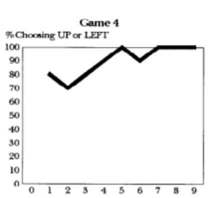
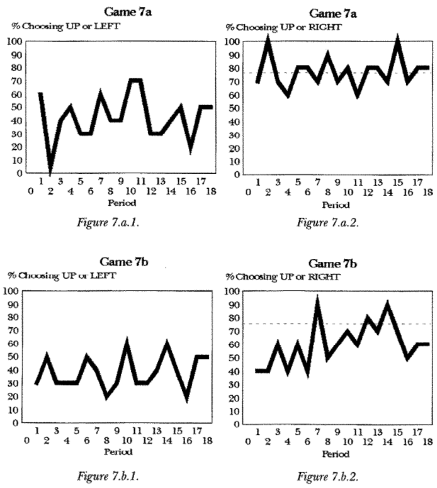
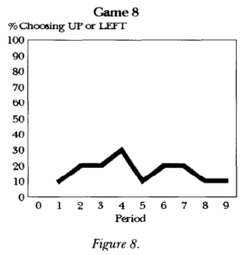


Figure 4.

(Figure 8)



(Figure 8.1)



(Figure 8.2)

After conducting the experiment they concluded that risk dominance predicts behavior more successfully than payoff dominance in the class of coordination games based on Figure 8,8.1, and 8.2. Straub also suggests that risk dominance is moderately successful in predicting behavior in this class of battle of the sexes games. However, there are circumstances in which payoff dominance influences the selection of equilibrium more than risk dominance, which is believed to be influenced by yet another undiscovered factor. Thus, he concluded that risk dominance is a necessary mechanism but there more factors that need to be examined to fully understand the coordination failure among games.

Related work - Do other game theorists Agree with Straub's theory and Results?

Scientific papers that Agree

- "Risk Dominance and Coordination Failure in Complete Information Games" by Uwe Dulleck and Rudolf Kerschbamer (2003).
- "On the Evolution of Strategic Behavior: Some Insights from Evolutionary Game Theory" by Vincent Buskens and Werner Raub (2002).
- "Convergence to risk-dominant equilibria in games with stochastic perturbations" by John Stachurski and Tomasz Strzalecki (2005)
- "Risk-Dominance and Nash Equilibrium: Evidence from Public-Good Experiments" by John Duffy and Huan Xie (2013).

Scientific papers that Disagree

- "Payoff Dominance and Risk Dominance in Experimental Games" by John H. Kagel and Alvin E. Roth (1995)

Straub claimed that game models are more likely to converge to a risk-dominant strategy. However, the paper was published in 1995, therefore it was necessary to look into more recent publications and see what other theorists' experiments claim. After doing my research, I found several semi-new publications where their analysis and experiments agreed with Straub's claim. That is that the game models are more likely to converge to a risk-dominant strategy than a payoff-dominant strategy. Also, I was able to find a scientific paper that didn't agree with Straub's claims and results. However, this paper was published in 1995, and I could not find any semi-recent scientific papers that didn't align with Straub's claim.

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Word Cited

Straub, Paul G., Risk Dominance and Coordination Failures in Static Games. QUARTERLY REVIEW OF ECONOMICS AND FINANCE, Vol 35 No 4, Winter 1995, Available at SSRN: https://ssrn.com/abstract=6808