Graz University of Technology

Institute of Geodesy

Torsten **Mayer-Gürr**
Steyrergasse 30/A308
A–8010 Graz

Tel. +43(0)316 873-6359

Fax +43(0)316 873-6845
mayer-guerr@tugraz.at

ifg.tugraz.at

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Practical: Simulated Earth rotation

A numerical geophysical model enables us to understand and distinguish effects in the complex dynamic system Earth. The task of this lab is to simulate the Earth's rotation by means of numerical integration of the Euler-Liouville equation.

Tasks

1. Solve the modifed Euler-Liouville equation (first-order ordinary differential equation) by an appropriate numerical integration method.
2. Compute and visualize the solution in terms of LOD (length of day) and polar motion.
3. Compare the results with the real observed Earth rotation.
4. Investigate the influence of the different contributors to the solution (e.g. lunar torque, solar torque, motion term, real and imaginary part of the Love numbers, size of the core, tides, ...)
5. Interpret and discuss your results.

The Earth rotation model

The modified Euler-Liouville equation is an ordinary differential equation (ODE):

$$\dot{\omega} = \mathbf{F}^{-1}\left[\mathbf{M} - \frac{D\mathbf{T}_G}{Dt}\omega - \omega \times (\mathbf{T}\omega) - \omega \times \mathbf{h} - \frac{D\mathbf{h}}{Dt}\right], \tag{1}$$

where the time variable inertia tensor can be computed by

$$\mathbf{T}(t) = \mathbf{T}_G(t) + \mathbf{T}_R(t) \tag{2}$$

with

$$\mathbf{T}_G(t) = \sqrt{\frac{5}{3}}MR^2 \begin{pmatrix} \frac{1}{\sqrt{3}}c_{20} - c_{22} & -s_{22} & -c_{21} \\ -s_{22} & \frac{1}{\sqrt{3}}c_{20} + c_{22} & -s_{21} \\ -c_{21} & -s_{21} & -\frac{2}{\sqrt{3}}c_{20} \end{pmatrix} + \frac{tr}{3}\mathbf{I} \tag{3}$$

and

$$\mathbf{T}_R(t) = \frac{\Omega_N R^5}{3G}\begin{pmatrix} 0 & 0 & k^{Re}\omega_x + k^{Im}\omega_y \\ 0 & 0 & k^{Re}\omega_y - k^{Im}\omega_x \\ k^{Re}\omega_x + k^{Im}\omega_y & k^{Re}\omega_y - k^{Im}\omega_x & 0 \end{pmatrix}. \tag{4}$$

The component of the tensor can are determined by the degree 2 spherical harmonic coefficients of the gravitational potential except the trace tr. In this simulation the trace is assumed to be constant

$$tr = \bar{A} + \bar{B} + \bar{C}. \tag{5}$$

The k^{Re} and k^{Im} are the Love numbers used to describe the reaction of an anelastic Earth.

The matrix $\mathbf{F}$ is defined as

$$\mathbf{F}(t) = \mathbf{T}(t) + \frac{\Omega_N^2 R^5}{3G}\begin{pmatrix} k^{Re} & k^{Im} & 0 \\ -k^{Im} & k^{Re} & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{6}$$

For the torques of third bodies (sun and moon, $j = 1, 2$), the following relation can be used:

$$\mathbf{M}_j(t) = \frac{3GM_j}{r_j^5}\begin{pmatrix} y_j z_j(\bar{C} - \bar{B}) \\ x_j z_j(\bar{A} - \bar{C}) \\ x_j y_j(\bar{B} - \bar{A}) \end{pmatrix}. \tag{7}$$

Constants

- Gravitational constant: $G = 6.674 \cdot 10^{-11}$ m^3/kg/s^2
- Love number: $k^{Re} = 0.3077$ and $k^{Im} = 0.0036$

- Gravitational parameter of the Sun: $GM_{\text{sun}} = 1.32712442076 \cdot 10^{20}$ m^3/s^2
- Gravitational parameter of the Moon: $GM_{\text{moon}} = 4.9027779 \cdot 10^{12}$ m^3/s^2
- Mass of the Earth: $M = 5.9737 \cdot 10^{24}$ kg
- Radius of the Earth: $R = 6378136.6$ m
- Moments of inertia: $\bar{A} = \bar{B} = 0.3296108\, MR^2, \bar{C} = 0.3307007\, MR^2$
- Nominal Earth rotation rate: $\Omega_N = 7.2921151467064 \cdot 10^{-5}$ rad/s.

Data

The following data are provided at the TeachCenter. Each file contains a time series which cover the period 01/01/2004 to 01/01/2015 with a sampling rate of 1 hour. In the first column is the time in Modified Julian Date (MJD, number of days since 11/17/1858).

- sun.txt, moon.txt:
  The position coordinates of Sun and Moon (x, y, z) with respect to the Earth-fixed reference system in units of meter.
- potentialCoefficientsAOHIS.txt, potentialCoefficientsTides.txt:
  time variable spherical harmonics coefficients of the gravitational potential of the Earth $(c_{20}, c_{21}, s_{21}, c_{22}, s_{22})$. The first file contains the complete degree 2 potential as observed by GRACE (ITSG-Grace2016_dailyKalman) complemented by high frequency variations of the AOD1B RL05 model. In the second file the potential variations are caused by solid Earth tides (IERS2010) and ocean tides (EOT11a).
- earthRotationVector.txt:
  For comparison the real observed Earth rotation vector in radian per second $(\omega_x, \omega_y, \omega_z)$. The first vector can also serve as initial value for the simulation.

Models for the relative angular momentum $\mathbf{h}(t)$ (motion term) can be downloaded at the German Centre for Geodscience (GFZ) gfz-potsdam.de/en/esmdata/eam Please use the operational data for atmosphere, ocean and atmosphere (operational_AAM, operational_OAM, operational_HAM).

The data are given as effective angular momentum functions (EAMF). The relation between EAMF and the relative angular momentum is

$$\begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix} = \begin{pmatrix} \frac{1.610}{\Omega_N(\bar{C}-\bar{A})} h_x \\ \frac{1.610}{\Omega_N(\bar{C}-\bar{A})} h_y \\ \frac{1.125}{\Omega_N \bar{C}} h_z \end{pmatrix}, \tag{8}$$

see ftp://ig2-dmz.gfz-potsdam.de/EAM/README for more details.

Analysis

The Earth rotation vector

$$\omega(t) = \begin{pmatrix} \omega_x(t) \\ \omega_y(t) \\ \omega_z(t) \end{pmatrix} \tag{9}$$

can be described at one hand by the polar motion at the Earth surface

$$x_p(t) = \frac{R}{\Omega_N}\omega_x(t) \tag{10}$$

$$y_p(t) = \frac{R}{\Omega_N}\omega_y(t), \tag{11}$$

and on the other hand by the Length of Day (LOD)

$$\Delta LOD = 86.400\,\mathrm{s}\frac{\Omega_N - \omega_z}{\Omega_N}. \tag{12}$$

Submission

The presentation of the results is Monday, 20th November 2017, 14:15.