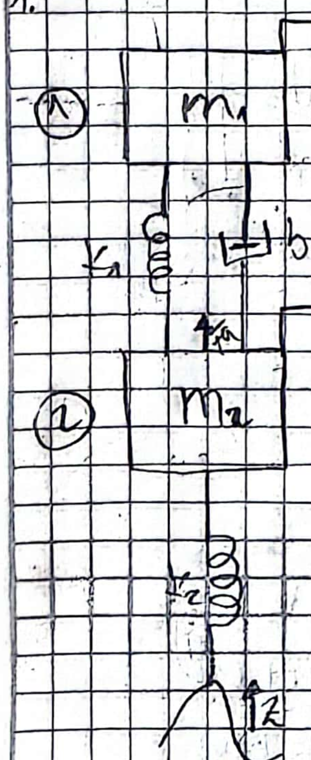
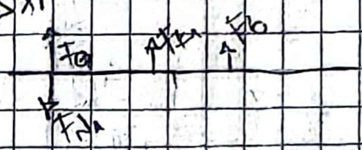
$$\dot{\vec{q}} = A\vec{q} + B\vec{U}$$

Tareas.

1.



$\bullet\ x_1 < x_2 < Z$



$$F_k = kx_1 \qquad F_b = b\frac{\partial(x_2 - x_1)}{\partial t}$$

$$F_{k_1} = k_1(x_2 - x_1)$$

$$F_{k_2} = k_2(Z - x_2)$$

$$F_{N1} = m_1\frac{\partial^2(x_1)}{\partial t^2}$$

$$F_{N2} = m_2\frac{\partial^2(x_2)}{\partial t^2}$$

(1) $Fa + F_{k_1} + F_b - F_{N1} = 0$

(2) $Fa + F_{k_2} - F_{N2} - F_{k_1} - F_b = 0$

$q_1 = x_1$

$q_2 = \dot{q}_1$

$q_3 = x_2$

$q_4 = \dot{q}_3$

(1) $Fa + k_1(x_2 - x_1) + b\dfrac{\partial(x_2 - x_1)}{\partial t} - m_1\dfrac{\partial \dot{x}_1}{\partial t} = 0$

$$Fa + k_1(q_3 - q_1) + b(q_4 - q_2) - m_1\dot{q}_2 = 0$$

(2) $Fa + k_2(Z - q_3) - m_2\dot{q}_4 - k_1(q_3 - q_1) - b(q_4 - q_2) = 0$

$$\dot{q}_2 = \frac{Fa + k_1(q_3 - q_1) + b(q_4 - q_2)}{m_1} = \frac{Fa}{m_1} + \frac{k_1 q_3}{m_1} - \frac{k_1 q_1}{m_1} + \frac{b q_4}{m_1} - \frac{b q_2}{m_1}$$

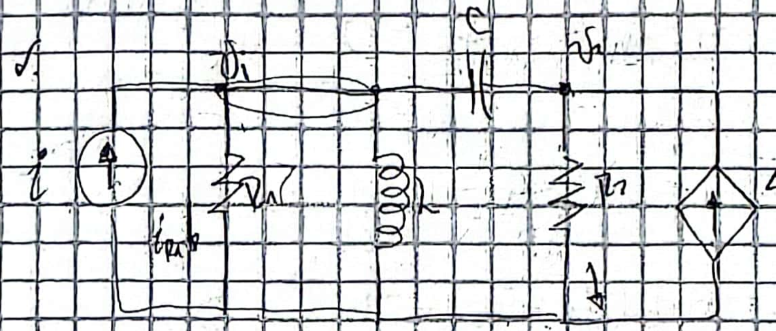
$$\dot{q}_4 = \frac{Fa + k_2(Z - q_3) - k_1(q_3 - q_1) - b(q_4 - q_2)}{m_2}$$

$$\frac{Fa}{m_2} + \frac{k_2 Z}{m_2} - \frac{k_2 q_3}{m_2} - \frac{k_1 q_3}{m_2} + \frac{k_1 q_1}{m_2} - \frac{b q_4}{m_2} + \frac{b q_2}{m_2}$$

→ Espaço de estados

$$\begin{bmatrix}\dot q_1\\ \dot q_2\\ \dot q_3\\ \dot q_4\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0\\ -\frac{k_1}{m_1} & -\frac{b}{m_1} & \frac{k_1}{m_1} & \frac{b}{m_1} \\ 0&0&0&1\\ \frac{k_1}{m_2} & \frac{b}{m_2} & \frac{-k_2-k_1}{m_2} & -\frac{b}{m_2}\end{bmatrix}\begin{bmatrix}q_1\\q_2\\q_3\\q_4\end{bmatrix} + \begin{bmatrix}0&0\\ \frac{1}{m_1}&0\\0&0\\ \frac{1}{m_2} & \frac{k_2}{m_2}\end{bmatrix}\begin{bmatrix}fa\\ z\end{bmatrix}$$

$$\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}1&0&0&0\\0&0&1&0\end{bmatrix}\begin{bmatrix}q_1\\q_2\\q_3\\q_4\end{bmatrix} + \begin{bmatrix}0\end{bmatrix}\begin{bmatrix}fa\\ z\end{bmatrix}$$



LTK

(1) $i = i_{R_1} + i_L + i_C$

(2) $i_C + 4v_i = i_{R_2}$

$i_{R_1} = \frac{v_i}{R_1}$

$i_C = C\frac{dv_C}{dt} = C\frac{d(v_i - v_2)}{dt}$

$i_L = \frac{1}{L}\int v_L\,dt$

$i_{R_2} = \frac{v_2}{R_2}$

$v_i = v_L = L\frac{di_L}{dt}$

$i = \frac{v_i}{R_1} + \frac{v_i}{L} + \frac{v_i - v_2}{}$

$x_1 = i_L = \frac{1}{L}\int v_L\,dt$

$\dot x_1 = \frac{v_L}{L}$

$x_2 = v_C = v_i - v_2 = L\dot x_1 - v_2$

$\boxed{v_2 = x_2 - L\dot x_1}$

$v_2 = L\dot x_1 - x_2$

(1) $i = \frac{L\dot{x}_1}{R_1} + x_1 + C\dot{x}_2$; $\dot{x}_2 = \frac{i}{C} - \frac{L\dot{x}_1}{R_1 C} - \frac{x_1}{C} = \star$

(2) $C\dot{x}_2 + 4L\dot{x}_1 = \frac{x_2 - L\dot{x}_1}{R_2}$

$\star$ en ec (2)

$$\cancel{C}\left(\frac{i}{\cancel{C}} - \frac{L\dot{x}_1}{R_1\cancel{C}} - \frac{x_1}{\cancel{C}}\right) + 4L\dot{x}_1 = \frac{x_2 - L\dot{x}_1}{R_2}$$

$$i - L\dot{x}_1 - x_1 + 4L\dot{x}_1 = \frac{x_2}{R_2} - \frac{L\dot{x}_1}{R_2}$$

$$i - x_1 + 3L\dot{x}_1 = \frac{x_2}{R_2} - \frac{L\dot{x}_1}{R_2}$$

$$i + x_1 - \frac{x_2}{R_2} = -3L\dot{x}_1 - \frac{L\dot{x}_1}{R_2}$$

$$i - x_1 - \frac{x_2}{R_2} = \dot{x}_1\left(-3L - \frac{L}{R_2}\right) = -\dot{x}_1\left(\frac{3R_2 + 1}{R_2}\right) = -\dot{x}_1\left(3 + 1/R_2\right)$$

$$\boxed{\dot{x}_1 = \frac{i R_2}{3LR_2 + L} - \frac{x_1 R_2}{3LR_2 + L} - \frac{x_2 R_2}{R_2(3LR_2 + L)}}$$

$$\dot{x}_2 = \frac{x_2 - L\dot{x}_1}{CR_2} - \frac{4L\dot{x}_1}{C} = \frac{x_2}{CR_2} - \dot{x}_1\left(\frac{L}{CR_2} + \frac{4L}{C}\right) = \frac{x_2}{CR_2} - \dot{x}_1\left(\frac{L + 4LR_2}{CR_2}\right)$$

$$\dot{x}_2 = \frac{x_2}{CR_2} - \left(\frac{iR_2}{3LR_2 + L} - \frac{x_1 R_2}{3LR_2 + L} - \frac{x_2}{3LR_2 + L}\right)\underbrace{\left(\frac{L}{CR_2} + \frac{4L}{C}\right)}_{\left(\frac{L + 4LR_2}{CR_2}\right)}$$

$$\dot{x}_2 = \frac{x_2}{CR_2} - \frac{i\cancel{R_2}(L + 4LR_2)}{(3LR_2 + L)C\cancel{R_2}} + \frac{x_1\cancel{R_2}(L + 4R_2)}{(3LR_2 + L)C\cancel{R_2}} - \frac{x_2(L + 4LR_2)}{(3LR_2 + L)CR_2}$$

$$\dot{x}_2 = x_2\left(\frac{1}{CR_2} - \frac{L + 4LR_2}{(3LR_2 + L)CR_2}\right) - \frac{i(L + 4LR_2)}{C(3LR_2 + L)} + \frac{x_1(L + 4LR_2)}{C(3LR_2 + L)}$$

Matriz de estados

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{R_2}{3LR_2+L} & -\frac{1}{3LR_2+L} \\ \frac{L+4LR_2}{C(3LR_2+L)} & \left(\frac{1}{CR_2} - \frac{L4R_2+L}{(3LR_2+L)CR_2}\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{R_2}{3LR_2+L} \\ \frac{L+4LR_2}{C(3LR_2+L)} \end{bmatrix} \begin{bmatrix} i \end{bmatrix}$$

$$\bar{y} = \begin{bmatrix} \frac{-R_1}{R_1+R_2-4R_1R_2} & \frac{4R_1-1}{R_1+R_2-4R_1R_2} \\ \frac{-R_1R_2}{R_1+R_2-4R_1R_2} & \frac{4R_1R_2-R_2}{R_1+R_2-4R_1R_2} \end{bmatrix} \bar{x} + \begin{bmatrix} \frac{R_1}{R_1+R_2-4R_1R_2} \\ \frac{R_1R_2}{R_1+R_2-4R_1R_2} \end{bmatrix} u$$

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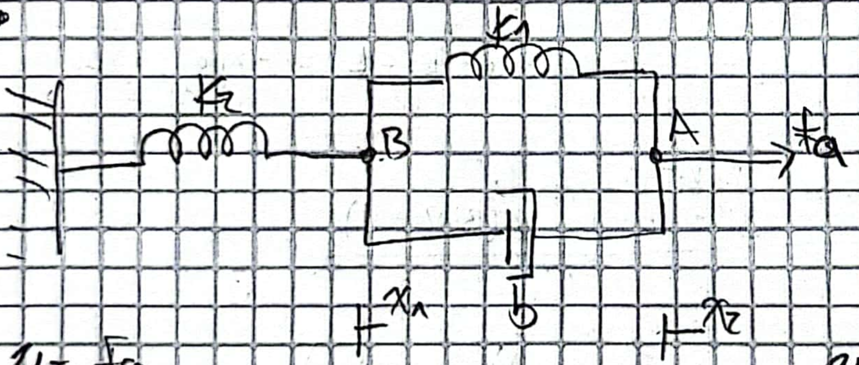

$x_2 > x_1$

$F_a = F_b = F_{k_2}$ (1)

$F_{k_2} = F_{k_1} + b$ (2)

$u = F_a$

$q_1 = F_{k_1}$

$q_1 = k_1(x_2 - x_1)$

$F_b = b(\dot{x}_2 - \dot{x}_1) = \frac{b}{k_1}\dot{q}_1$

$x_2 = \frac{q_1}{k_1} - x_1$

$u = \frac{b}{k_1}\dot{q}_1 + q_1$

$\dot{q}_1 = \frac{k_1}{b}u - \frac{k_1}{b}q_1$

$\rightarrow u = F_a = k_2 x_1$; $x_1 = \frac{u}{k_2}$

$\dot{q}_1 = \frac{-k_1}{b}q_1 + \frac{k_1}{b}u$

$x_2 = \frac{q_1}{k_1} - \frac{u}{k_2}$

$$\bar{y} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1/k_1 \end{bmatrix} q_1 + \begin{bmatrix} 1/k_1 \\ -1/k_2 \end{bmatrix} u$$