→ Ejemplo 12.1 Norman Nise: Diseño de Sistema Control

→ Diseñar un sistema de control con planta $G(s) = \frac{20(s+5)}{s(s+1)(s+4)}$
con $OS = 9{,}5\%$ y $t_s = 0{,}74\ seg$

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$$U(s) \rightarrow \boxed{\frac{1}{s^3+5s^2+4s}} \xrightarrow{X_1} \boxed{0s^2+20s+100} \rightarrow Y(s)$$

$$\frac{X_1(s)}{U(s)} = \frac{1}{s^3+5s^2+4s}$$

$$[s^3+5s^2+4s]\,X_1(s) = U(s)$$

$$\dddot{x}_1 + 5\ddot{x}_1 + 4\dot{x}_1 = u$$

$$q_1 = x_1$$
$$q_2 = \dot{q}_1 = \dot{x}_1$$
$$q_3 = \dot{q}_2 = \ddot{x}_1$$
$$\dot{q}_3 = \dddot{x}_1$$

$$\frac{Y(s)}{X_1(s)} = 20s + 100$$

$$Y(s) = X_1(s)\,[20s+100]$$

$$y = 20\dot{x}_1 + 100x_1$$

$$\rightarrow \dot{q}_3 = u - 5q_3 - 4q_2$$

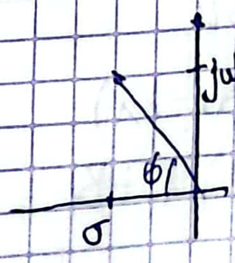
$$y = 20q_2 + 100q_1$$

→ Espacio de estados:

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -4 & -5\end{bmatrix}\cdot\begin{bmatrix}q_1\\ q_2\\ q_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u$$

$$y = \begin{bmatrix}100 & 20 & 0\end{bmatrix}\cdot\begin{bmatrix}q_1\\ q_2\\ q_3\end{bmatrix} + [0]\,u$$

→ $s = \sigma + j\omega_d$, donde $\sigma = \eta\omega_n$



· $\eta = \cos\phi$

→ $\phi = \cos^{-1}(0{,}5996)$
$\approx 53{,}16$

$$\rightarrow OS\% = e^{\left(-\eta\pi/\sqrt{1-\eta^2}\right)} \cdot 100 = 9{,}5$$

$$\eta \approx 0{,}5996$$

$$t_s = \frac{4}{\sigma} = 0{,}74 \quad \rightarrow \sigma\ \frac{4}{0{,}74} \approx 5{,}41$$

- $\sigma = \eta \omega_n$

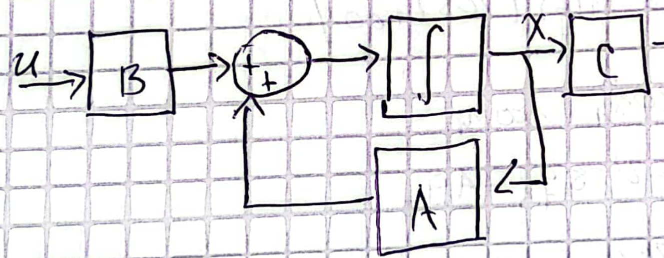
$\hookrightarrow \omega_n = \frac{\sigma}{\eta} = \frac{5,41}{0,5996} = 9,0227 \text{ rad/s}$

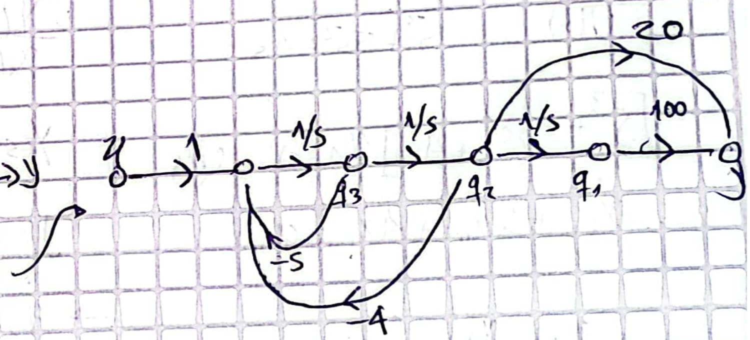
- $\omega_d = \omega_n \sqrt{1-\eta^2} = 9,0227 \sqrt{1-(0,5996)^2} \simeq 7,2207 \text{ rad/s}$

- $\omega_d = \sigma \tan(\phi) = 5,41 \tan(53,16) \simeq 7,2212$

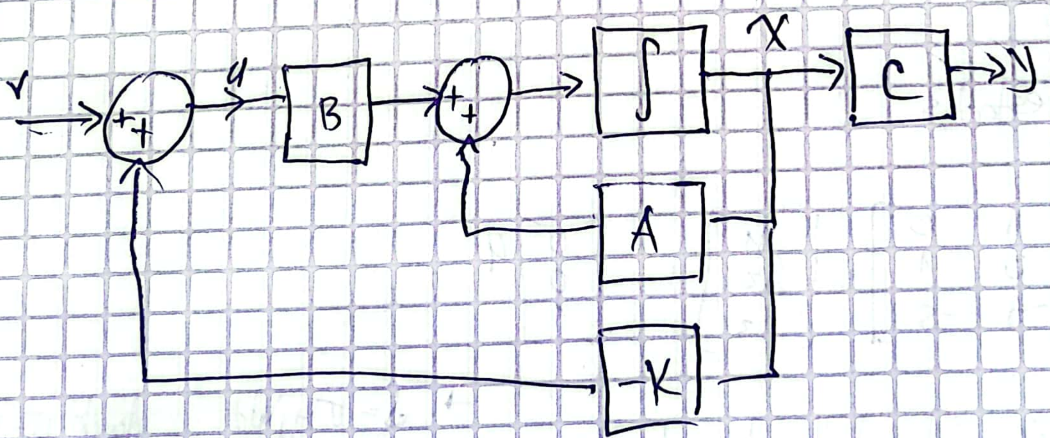
$\dot{x} = Ax + Bu$

$y = Cx$





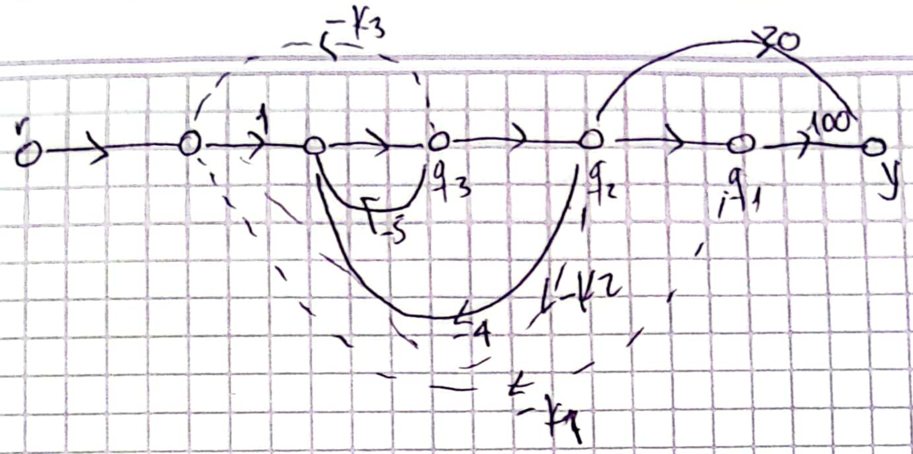
→ Redibujando el diagrama de bloques, agregando realimentación



$\dot{x} = Ax + Bu$

$\dot{x} = Ax + B(r - kx)$

$\dot{x} = (A - Bk)x + Br$



→ A partir del nuevo diagrama de flujo de señal, se debe reescribir el espacio de estados

· $\dot{q}_3 = -5q_3 - 4q_2 + u$

· $\dot{q}_3 = -5q_3 - 4q_2 + (-k_3 q_3 - k_2 q_2 - k_1 q_1) + r$

$$\rightarrow \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ -k_1 & -4-k_2 & -5-k_3 \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

→ Ec. característica del sistema

$$\det[sI - (A - BK)] = S^3 + (5+k_3)S^2 + (4+k_2)S + k_1 S = 0$$

✓ Polo dominante en lazo ~~abierto~~ cerrado en $S = -5,4 + j7,21$

Caro en $S = S$; Con $S = 5,1$

$$(s + 5,4 - j7,2)(s + 5,4 + j7,2)(s + 5,1) \rightarrow S^3 + 15,9S^2 + 136,22S + 413,83 = 0$$

$k_3 = 15,9 - 5 = 10,9$
$k_2 = 136,22 - 4 = 132,22$
$k_1 = 413,83$