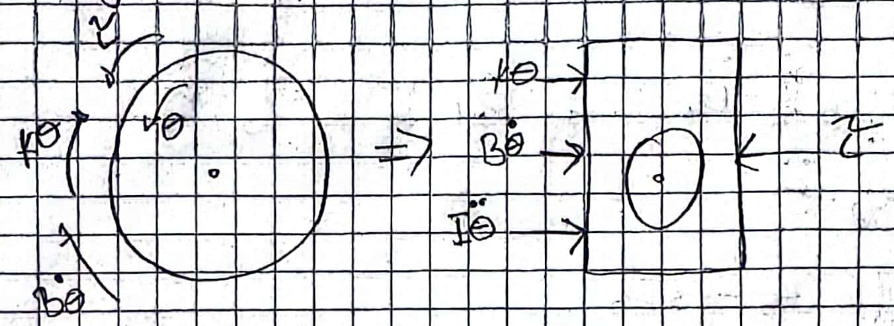
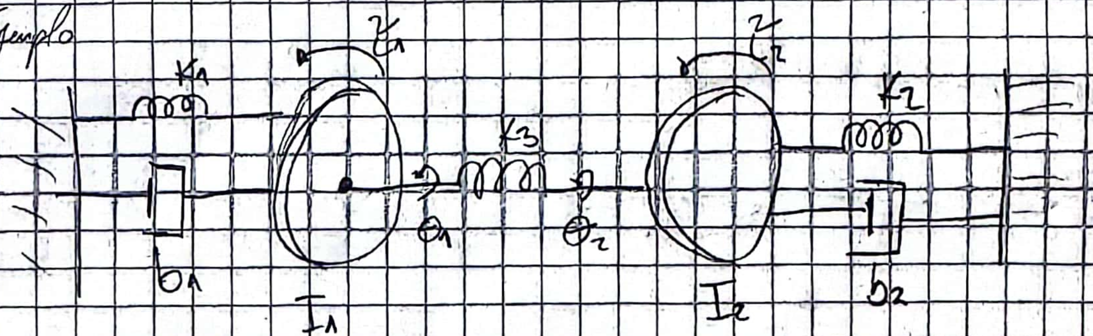
- Diagrama de fuerza sobre el disco

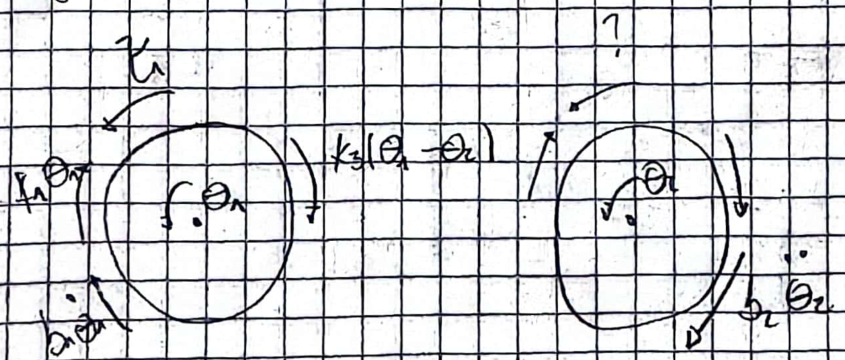



$$\therefore I\ddot{\theta} + B\dot{\theta} + K\theta = \tau$$

- Ejemplo




→ Diagrama de fuerzas




$$\rightarrow I_1\ddot{\theta}_1 + B_1\dot{\theta}_1 + K_1\theta_1 + K_3(\theta_1-\theta_2) = \tau_1 = I_1\ddot{\theta}_1 + b\dot{\theta}_1 + \theta_1(K_1+K_3) - K_3\theta_2$$

$$\rightarrow I_2\ddot{\theta}_2 + b_2\dot{\theta}_2 + K_2\theta_2 + K_3(\theta_1-\theta_2) = 0 = I_2\ddot{\theta}_2 + b\dot{\theta}_2 + \theta_2(K_1+K_3) - K_3\theta_2$$

→ Variables de estado

$q_1 = \theta_1$ ; $q_2 = \dot{q}_1 = \dot{\theta}_1$ ; $\dot{q}_2 = \ddot{\theta}_1$ ; $q_3 = \theta_2$ ; $q_4 = \dot{q}_3 = \dot{\theta}_2$

$\dot{q}_4 = \ddot{\theta}_2$

$\rightarrow \tau_1 = I_1 \dot{q}_2 + b_1 q_2 + q_1(k_1 + k_3) - k_3 q_3$

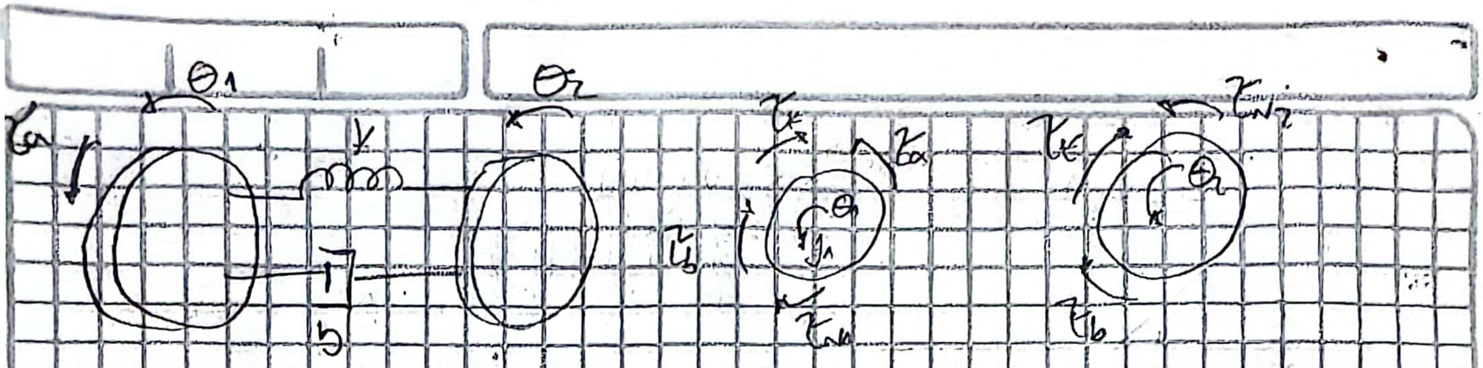
$\rightarrow \dot{q}_2 = -\frac{k_1 + k_3}{I_1} q_1 - \frac{b_1}{I_1} q_2 + \frac{k_3}{I_1} q_3 + \frac{1}{I_1} \tau_1$

$\therefore 0 = I_2 \dot{q}_4 + b_2 q_4 + q_3(k_1 + k_3) - k_3 q_1$

$\rightarrow \dot{q}_4 = \frac{k_3}{I_2} q_1 - \frac{k_1 + k_3}{I_2} q_3 - \frac{b_2}{I_2} q_4$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1+k_3}{I_1} & -\frac{b_1}{I_1} & \frac{k_3}{I_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_3}{I_2} & 0 & -\frac{k_1+k_3}{I_2} & -\frac{b_2}{I_2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/I_1 \\ 0 \\ 0 \end{bmatrix} \tau_1$$

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$



$\theta_1 > \theta_2$

$x_1 = \theta_1 \quad x_2 = \dot{x}_1 \quad u = \tau_a \qquad \tau_b = b(x_2 - x_4) \quad \tau_{N2} = J_2 \dot{x}_4$

$x_3 = \theta_2 \quad x_4 = \dot{x}_3 \quad \tau_k = k(x_1 - x_3) \qquad \tau_{N1} = J_1 \dot{x}_2$

$u = k(x_1 - x_3) + b(x_2 - x_4) + J_1 \dot{x}_2 \Rightarrow \dot{x}_2 = -\frac{k}{J_1}x_1 - \frac{b}{J_1}x_2 + \frac{k}{J_1}x_3 + \frac{b}{J_1}x_4 + \frac{u}{J_1}$

$J_2 \dot{x}_4 = b(x_2 - x_4) + k(x_1 - x_3)$

$\dot{x}_4 = \frac{k}{J_2}x_1 + \frac{b}{J_2}x_2 - \frac{k}{J_2}x_3 - \frac{b}{J_2}x_4$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{J_1} & -\frac{b}{J_1} & \frac{k}{J_1} & \frac{b}{J_1} \\ 0 & 0 & 1 & 0 \\ \frac{k}{J_2} & \frac{b}{J_2} & -\frac{k}{J_2} & -\frac{b}{J_2} \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ \frac{1}{J_1} \\ 0 \\ 0 \end{bmatrix} u$$

$$\vec{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \vec{x} + \vec{0}\, u$$