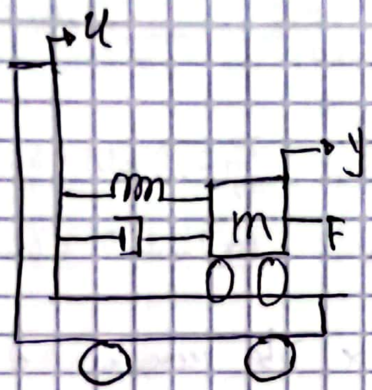
Ejemplo 3.3 Ogata 5ta edición



$$0 = m\ddot{y} + b(\dot{y}-\dot{u}) + k(y-u)$$

$$m\ddot{y} = -b\dot{y} + b\dot{u} - ky + ku$$

$$\ddot{y} = \frac{-b}{m}\dot{y} - \frac{ky}{m} + \frac{b}{m}\dot{u} + \frac{k}{m}u$$

→ Separando variables

$$\ddot{y} + \frac{b}{m}\dot{y} + \frac{k}{m}y = \frac{b}{m}\dot{u} + \frac{k}{m}u$$

→ Aplicando Laplace

$$\left(s^2 - \frac{b}{m}s - \frac{k}{m}\right)y(s) = \left(\frac{-b}{m}s - \frac{k}{m}\right)u(s) \quad ; \quad G(s) = \left(-\frac{1}{m}\right)\left(\frac{bs+k}{s^2 - \frac{b}{m}s - \frac{k}{m}}\right)$$

✓ Es sistema es de la forma

$$\ddot{y} + a_1\dot{y} + a_2 y = b_0\ddot{u} + b_1\dot{u} + b_2 u$$

→

$$a_1 = \frac{b}{m}, \quad a_2 = \frac{k}{m}, \quad b_0 = 0, \quad b_1 = \frac{b}{m} \quad b_2 = \frac{k}{m}$$

→ Definimos estados

$$x_1 = y - \beta_0 u \qquad x_2 = \dot{x}_1 - \beta_1 u$$

$$\beta_0 = b_0 = 0 \qquad \beta_1 = b_1 - a_1\beta_0 = \frac{b}{m} \qquad \beta_2 = b_2 - a_1\beta_1 - a_2\beta_0 = \frac{k}{m} = \left(\frac{b}{m}\right)^2$$

$$x_1 = y \quad ; \quad x_2 = \dot{x}_1 - \frac{b}{m}u \qquad \dot{x}_1 = x_2 + \frac{b}{m}u$$

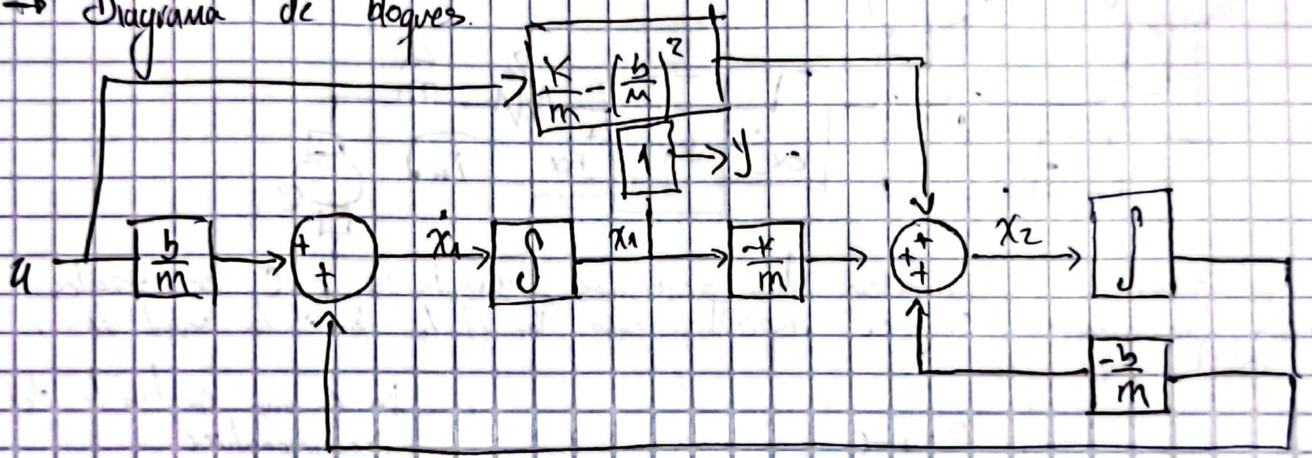
$$\dot{x}_2 = -a_2x_1 - a_1x_2 + \beta_2 u = \frac{-k}{m}x_1 - \frac{b}{m}x_2 + \left[\frac{k}{m} - \left(\frac{b}{m}\right)^2\right]u$$

→ Espacio de estados

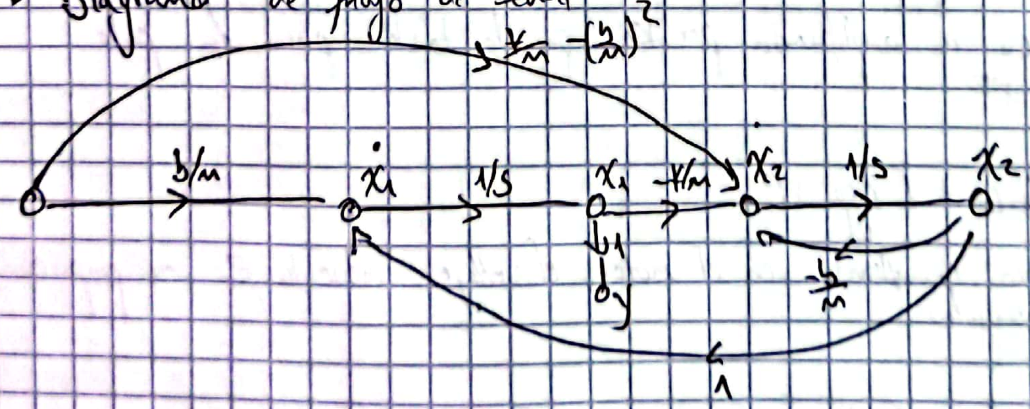
$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \vec{x} + \begin{bmatrix} \frac{b}{m} & 0 \\ \frac{k}{m} & -\left(\frac{b}{m}\right)^2 \end{bmatrix} u$$

$$y = x_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} \vec{x} + \vec{0}\, u$$

→ Diagrama de bloques.



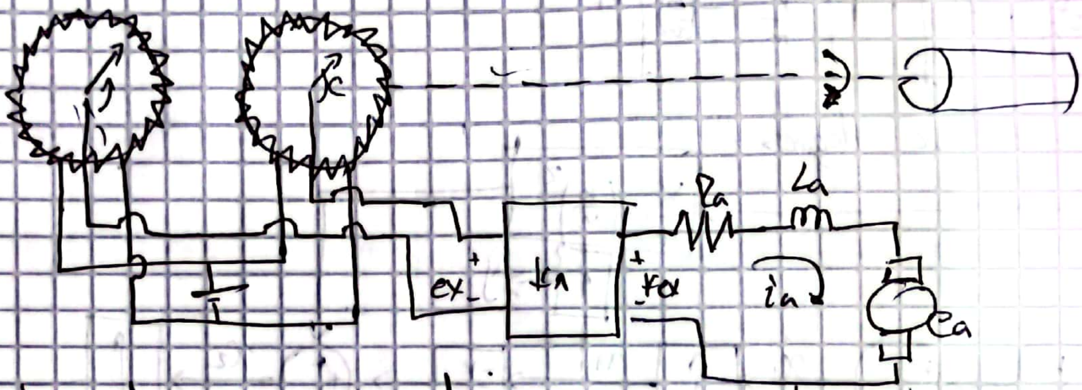
→ Diagrama de flujo de señal



Juan Ramirez
Cod: 20182005115

Parcial 2 Sistemas Dinamicos

Ejercicio A-3-9- Ogata.



→ Los potenciometros convierten las posiciones angulares r y c, en señales electricas proporcionales er y ec respectivamente. Por lo tanto la señal de error está dada por:

$$e = r - c$$

Ka es una constante de proporcionalidad

y el error de voltaje es: $e_v = e_r - e_c = K_a[r - c]$

→ La señal de error e_v es amplificada por K_1, por lo tanto para la parte electrica se tiene que:

$$K_1 e_v = R_a i_a + L\frac{di_a}{dt} + e_a$$

Para un flujo magnetico constante en el motor, el voltaje inducido e_a es proporcional a la velocidad angular

$$K_1 e_v = R_a i_a + L\frac{di_a}{dt} + K_3\frac{d\theta}{dt} \quad [1]$$

→ Para la parte mecánica, se tiene que:

$$T_a = J\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} \longrightarrow K_2 i_a = J\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} \quad [2]$$

→ Laplace en [2]

$$I_a = \left[\frac{J}{K_2}s^2 + \frac{bs}{K_2}\right]\Theta$$

→ Reemplazamos en [1] y aplicamos Laplace

~~Ea~~ -

$$K_1 E_v = Ra\left[\frac{j}{K_2}s^2 + \frac{bs}{K_2}\right]\theta + ia\,s\left[\frac{js^2}{K_2} + \frac{bs}{K_2}\right]\theta + K_3 s\theta$$

$$K_1 E_v = \left[(ia s + Ra)\left(\frac{j}{K_2}s^2 + \frac{b}{K_2}s\right) + K_3 s\right]\theta$$

$$K_1 K_2 E_v = \left[(ia s + Ra)(j s^2 + bs) + K_2 K_3 s\right]\theta$$

$$\frac{\theta(s)}{E_v(s)} = \frac{K_1 K_2}{s[(ia s + Ra)(js + b) + K_2 K_3]}$$

→ Debido al tren de engranes, el eje de salida gira n veces en cada revolución del eje del Motor, entonces

$$C(s) = n\,\theta(s)$$

El error de voltaje está dado por

$$E_v(s) = K_0[R(s) - C(s)] = K_0 E(s)$$

por lo tanto $\frac{C(s)}{E(s)} = \frac{E_v}{E} \cdot \frac{\theta}{E_v} \cdot \frac{C}{\theta} = \frac{n K_0 K_1 K_2}{s[(ia s + Ra)(js + b)] + K_2 K_3}$

A partir de [1] y [2] se pueden definir los estados:

$x_1 = \theta \qquad x_2 = \dot{x}_1 \qquad x_3 = ia \qquad u = e \qquad y = c = n x_1$

$$K_0 K_1 u = Ra\,x_3 + ia\,\dot{x}_3 + K_3 x_2 \qquad K_2 x_3 = j\dot{x}_2 + b x_2$$

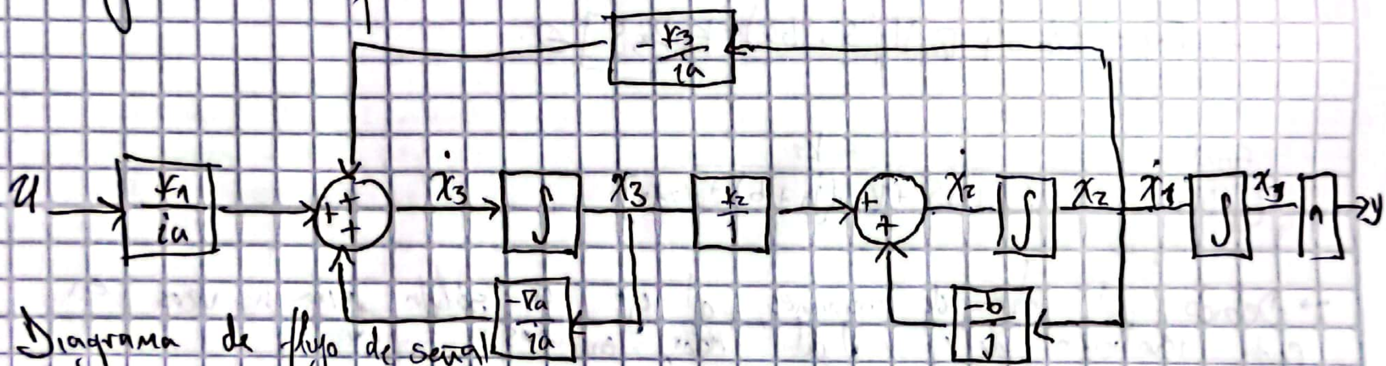
$$\dot{x}_2 = \frac{-b}{j}x_2 + \frac{K_2}{j}x_3$$

$$\dot{x}_3 = \frac{-K}{ia}x_2 - \frac{Ra}{ia}x_3 + \frac{K_0 K_1 u}{ia}$$

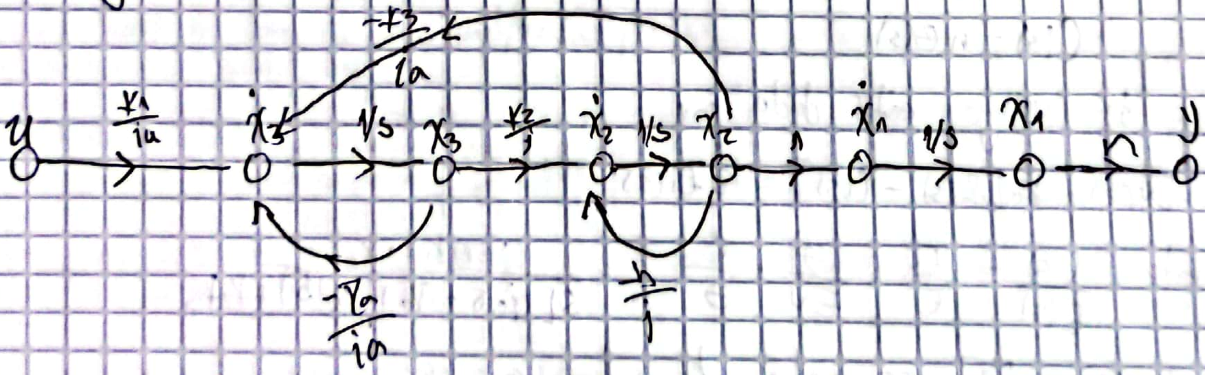
→ Espacio de estados

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{b}{J} & \frac{K_2}{J} \\ 0 & -\frac{K_3}{i_a} & -\frac{R_a}{i_a} \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ 0 \\ \frac{K_0 f_a}{i_a} \end{bmatrix} u \qquad y = [n \;\; 0 \; 0]\,\vec{x} + \vec{0}\, i$$

→ Diagrama de bloques



→ Diagrama de flujo de señal



→ Si i_a se hace despreciable entonces [1]

$$K_1 e v = R_a i_a + K_3 \frac{d\theta}{dt} \quad [3]$$

→ Definimos estados:

$x_1 = \theta$, $x_2 = \dot{x}_1$ $\quad u = e \quad y = n x_1$

$k_0 k_1 u = R_a i_a + k_3 x_2$ $\qquad$ $k_2 I_a = j\dot{x}_2 + b x_2$

$$I_a = \frac{j}{k_2}\dot{x}_2 + \frac{b}{k_2}x_2$$

$\cancel{k_0 k_1 u = R_a I_a}$

$$k_0 k_1 u = \frac{R_a j}{k_2}\dot{x}_2 + \frac{R_a b}{k_2}x_2 + k_3 x_2 \; ; \; k_0 k_1 k_2 u = R_a j \dot{x}_2 + (R_a b + k_2 k_3)x_2$$

$$\dot{x}_2 = -\frac{1}{R_a j}(R_a b + k_2 k_3)x_2 + \frac{k_0 k_1 k_2}{R_a j}u = \frac{-1}{j}\left(b + \frac{k_2 k_3}{R_a}\right) + \frac{k_0 k_1 k_2}{R_a j}u$$

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{j}\left(b + \frac{k_2 k_3}{R_a}\right) \end{bmatrix}\vec{x} + \begin{bmatrix} 0 \\ \frac{k_0 k_1 k_2}{R_a j} \end{bmatrix}u \qquad y = [n \;\; 0]\vec{x} + \vec{0}u$$

Diagrama de bloques

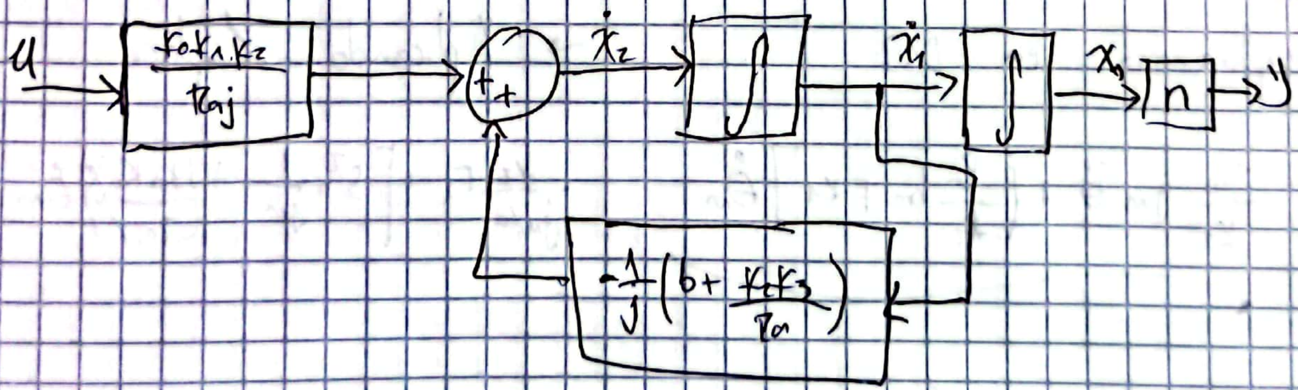
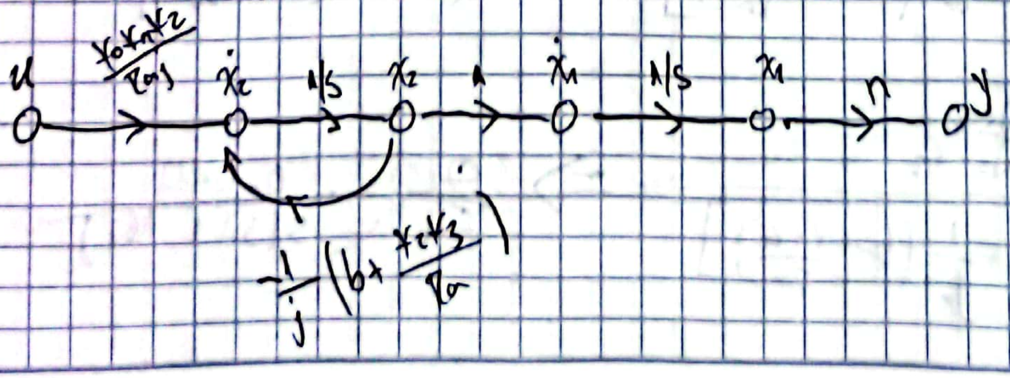
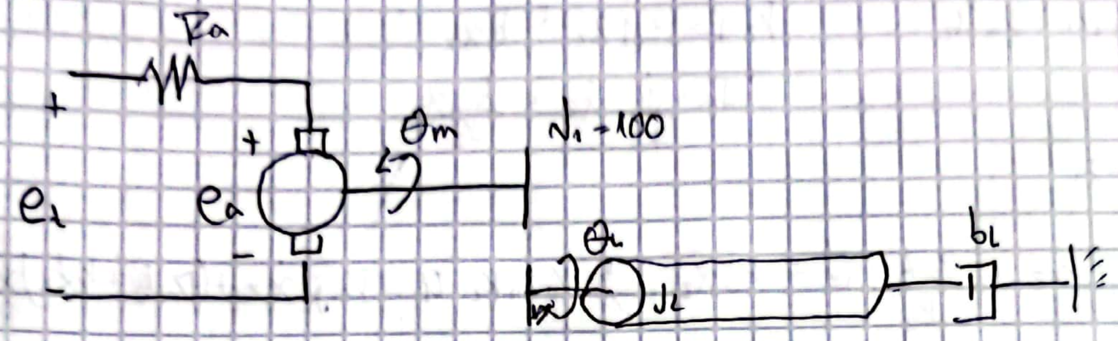


Diagrama de flujo de señal:





$J_L = 400 \, kg\,m^2$

$b_L = 800 \, Nm \, s/rad$

$\frac{N_1}{N_2} = \frac{1}{10}$

$j_m = j_a + \left(\frac{N_1}{N_2}\right)^2 J_L = 12$

$b_m = b_a + \left(\frac{N_1}{N_2}\right)^2 b_L = 10$

$\frac{K_2}{R_a} = \frac{T_{stall}}{e_t} = 5$

$K_1 = \frac{e_t}{\omega_n} = 2$

Para la parte electrica

$e_t = R_a i_a + e_a = R_a i_a + K_1 \dot{\theta}_m \quad [1]$

→ Mecanica

$T_m = j_m \ddot{\theta} + b_m \dot{\theta} \Rightarrow T_m = K_2 i_a$

→ Reemplazamos en [1]

$e_t = \frac{R_a}{K_2} j_m \ddot{\theta} + \left(\frac{R_a}{K_2} b_m + K_1\right] \dot{\theta}_m$

→ Aplicando Laplace

$\frac{K_2}{j_m R_a} E_t = \left[s^2 + \frac{1}{j_m} + \left(\frac{K_1 K_2}{R_a}\right) s\right) \theta_m$

$$\frac{\theta_m}{E_t} = \frac{K_2}{R_a j_m s\left[s + \frac{1}{j_m}\left(b_m + \frac{K_1 K_2}{R_a}\right)\right]} \Rightarrow \frac{\theta_m}{E_t} = \frac{0,417}{s(s+1,667)}$$

$$\frac{\theta_L}{E_t} = \frac{\frac{N_1}{N_2} K_2}{R_a J_a s\left(s + \frac{1}{j_m}\left(b_m + \frac{K_1 K_2}{R_a}\right)\right]} \Rightarrow \frac{\theta_L}{\theta_t} = \frac{0,0417}{s(s+1,667)}$$

de [2] se tiene

$$\ddot{\theta}_m = -\frac{1}{J_m}\left(b_m + \frac{K_1K_2}{R_a}\right)\dot{\theta} + \frac{K_2}{R_a J_m} e_t$$

$$\ddot{\theta}_m = -1{,}669\,\dot{\theta} + 0{,}417\, e_t$$

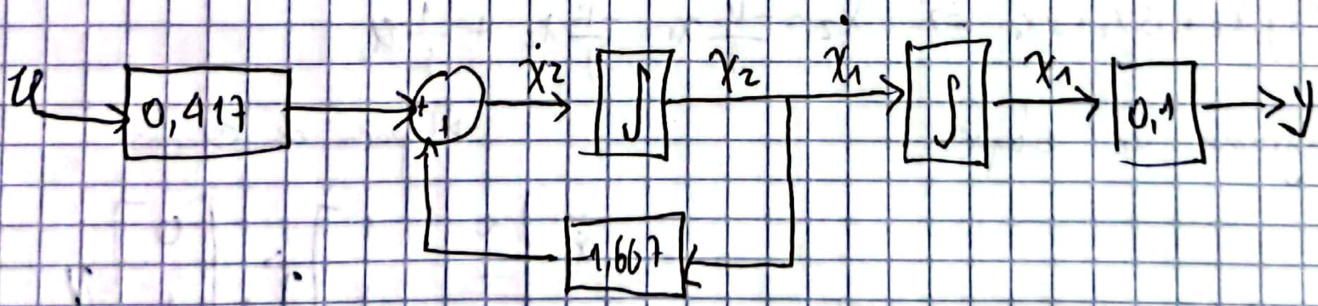
→ Definimos estados

$x_1 = \theta \quad x_2 = \dot{x}_1 \quad u = e_t \quad y = 0{,}1 \quad x_1 = \theta_1$

$$\dot{x}_2 = -1{,}667\, x_2 + 0{,}417\, u$$

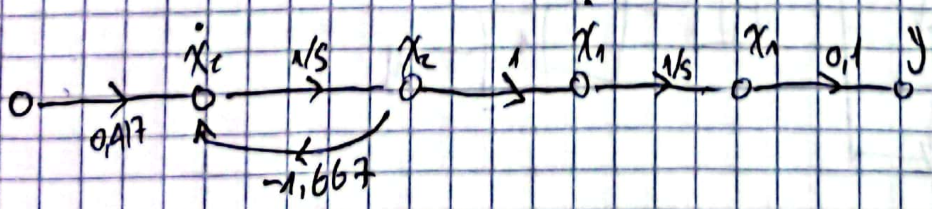
$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 \\ 0 & -1{,}667 \end{bmatrix}\vec{x} + \begin{bmatrix} 0 \\ 0{,}417 \end{bmatrix} u$$

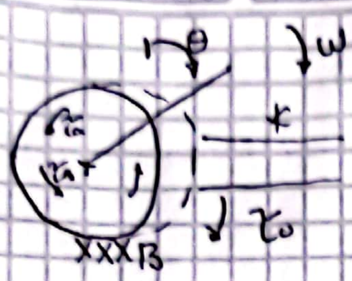
→ Diagrama de bloques



→ Diagrama de flujo de señal correspondiente

* Son sistemas equivale





→ Ley de Newton

$$\tau_a = \tau_n + \tau_B + \tau_K$$

Donde:

$$\tau_n = j\ddot{\theta} \quad \tau_B = B\dot{\theta} \quad \tau_K = K\theta$$

→ Reemplazando

$$\tau_a = j\ddot{\theta} + B\dot{\theta} + K\theta$$

→ Laplace

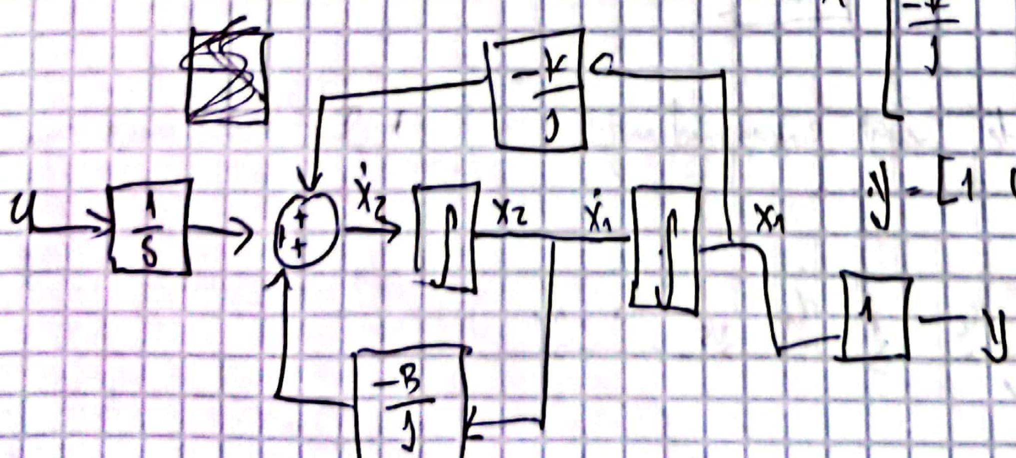
$$\tau_a = (js^2 + Bs + K)\theta \Rightarrow \frac{\theta}{\tau_a} = \frac{1/j}{s^2 + \frac{B}{j}s + \frac{K}{j}}$$

→ Definimos Estados:

$$x_1 = \theta \quad x_2 = \dot{x}_1 \quad u = \tau_a \quad y = x_1$$

$$u = j\dot{x}_2 + Bx_2 + Kx_1 \Rightarrow \dot{x}_2 = -\frac{K}{j}x_1 - \frac{B}{j}x_2 + \frac{1}{j}u$$

→ Diagrama de bloques

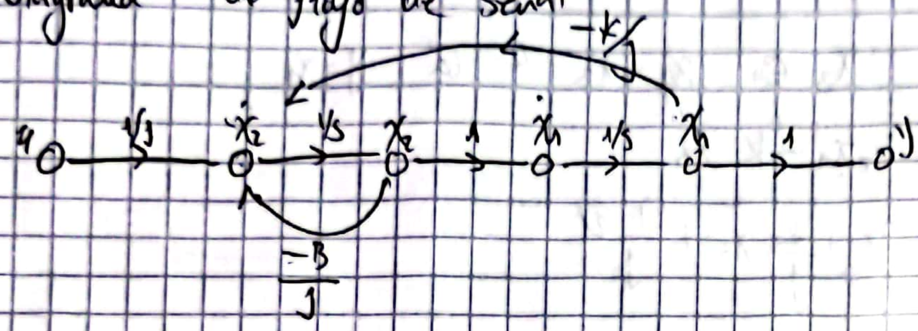


→ Espacio de estados

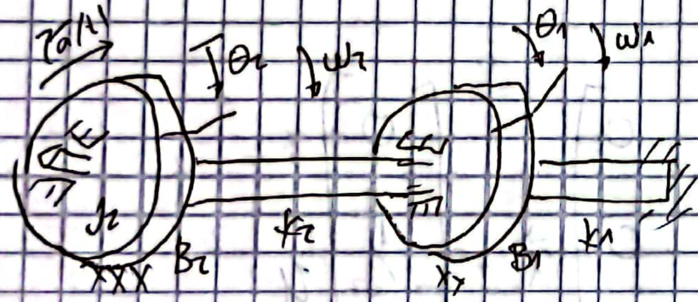
$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{J} & -\frac{B}{J} \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \vec{x} + \vec{0}\, u$$

Diagrama de flujo de señal



2) $\theta_2 > \theta_1$



$$\tau_{J_1} = J_1 \ddot{\theta}_1$$

$$\tau_{B_1} = B_1 \dot{\theta}_1$$

$$\tau_{K_1} = K_1 \theta_1$$

$$\tau_{J_2} = J_2 \ddot{\theta}_1$$

$$\tau_{B_2} = B_2 \dot{\theta}_2$$

$$\tau_{K_2} = K_2(\theta_2 - \theta_1)$$

$$\tau_a = \tau_{J_2} + \tau_{B_2} + \tau_{K_2}$$

$$\tau_a = J_2 \ddot{\theta}_2 + B_2 \dot{\theta}_2 + K_2(\theta_2 - \theta_1)$$

$$K_2(\theta_2 - \theta_1) = J_1 \ddot{\theta}_1 + B_1 \dot{\theta}_1 + K_1 \theta_1$$

→ Laplace

$$K_2(\theta_2 - \theta_1) = (J_1 s^2 + B_1 s + K_1)\theta_1$$

$$K_2\theta_2 = (J_1 s^2 + B_1 s + K_1 + K_2)\theta_1$$

$$\theta_1 = \frac{K_2}{J_1 s^2 + B_1 s + K_1 + K_2}\,\theta_2$$

$$\tau_a = \left(J_2 s^2 + B_2 s + \frac{K_2(J_1 s^2 + B_1 s + K_1)}{J_1 s^2 + B_1 s + K_1 + K_2}\right)\theta_2$$

$$\frac{\theta_2}{\tau_a} = \frac{J_1 s^2 + B_1 s + K_1 + K_2}{(J_2 s^2 + B_2 s)(J_1 s^2 + B_1 s + K_1 + K_2) + K_2(J_1 s^2 + B_1 s + K_1)}$$

→ Definimos estados

$x_1 = \theta_2 \quad x_2 = \dot{x}_1 \quad x_3 = \theta_1 \quad x_4 = \dot{x}_3 \quad u = \tau_a \quad y = x_1$

$u = J_2 \dot{x}_2 + B_2 x_2 + K_2 x_1 - K_2 x_3$

$\dot{x}_2 = \frac{-K_2}{J_2} x_1 - \frac{B_2}{J_2} x_2 + \frac{K_2}{J_2} x_3 + \frac{1}{J_2} u$

$\dot{x}_4 = \frac{K_2}{J_1} x_1 - \frac{(K_1+K_2)}{J_1} x_3 - \frac{B_1}{J_1} x_4$

→ Espacio de estados

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-K_2}{J_2} & \frac{-B_2}{J_2} & \frac{K_2}{J_2} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K_2}{J_1} & 0 & \frac{-(K_1+K_2)}{J_1} & \frac{-B_1}{J_1} \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ 1/J_2 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0 \quad 0]\, \vec{x} + \vec{0}\, u$$

→ Diagrama de bloques

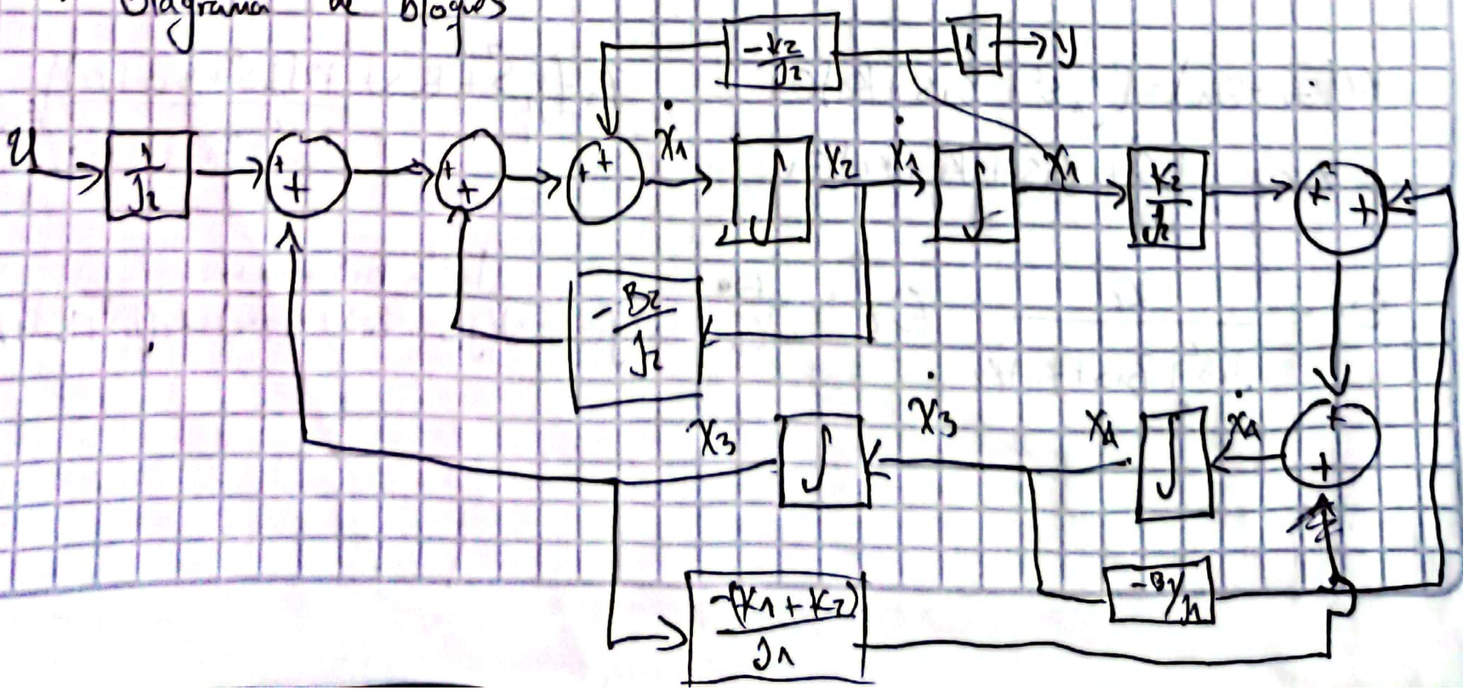
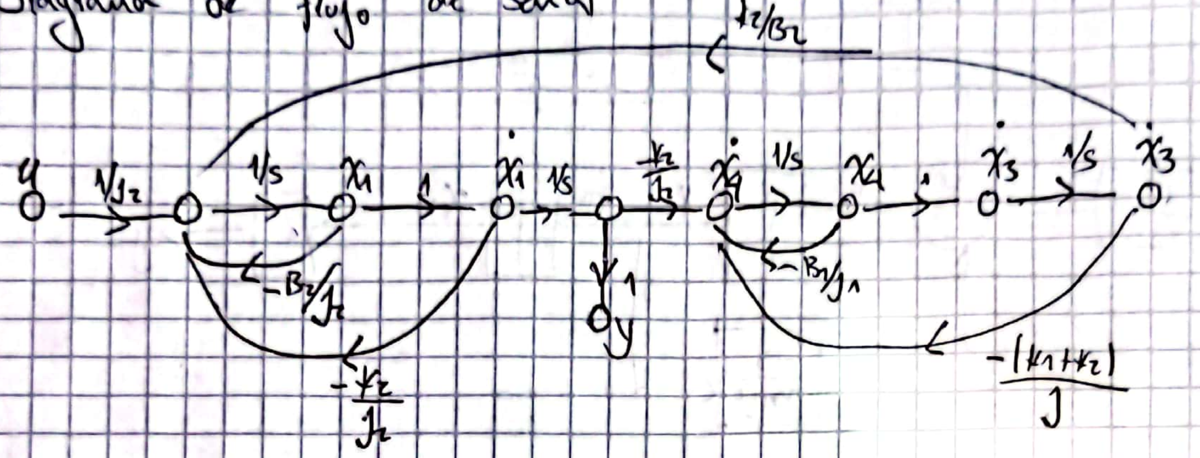
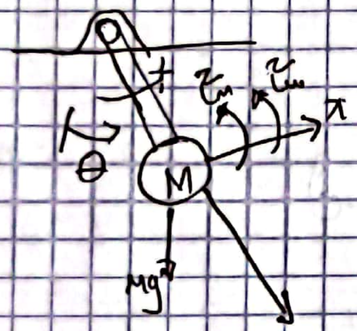


Diagrama de flujo de señal



4.



En y se debe mantener el equilibrio, por lo que la ecuación que interesa es en x^1

$$mL^2\ddot{\theta} + B\dot{\theta} + mgL\operatorname{Sen}\theta = \tau_a$$

$$\ddot{\theta} = \frac{-B\dot{\theta}}{mL^2} - \frac{g}{L}\operatorname{Sen}\theta + \frac{\tau_a}{mL^2}$$

$$\ddot{\theta} = \frac{-B}{mL^2}\dot{\theta} - \frac{g}{L}\theta + \frac{\tau_a}{mL^2} \qquad G(s) = \frac{\theta}{\tau_a} = \frac{1/mL^2}{s^2 + \frac{B}{ML^2}s + \frac{g}{L}}$$

✓Definiendo estados

$x_1 = \theta \quad x_2 = \dot{x}_1 \quad u = \tau_a \quad y = x_1$

$$\dot{x}_2 = \frac{-B}{ML^2}x_2 - \frac{g}{L}x_1 + \frac{1}{ML^2}u$$

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{L} & \frac{-B}{ML^2} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{ML^2} \end{bmatrix} u \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix}\vec{x} + \vec{0}\,u$$

Diagrama de bloques

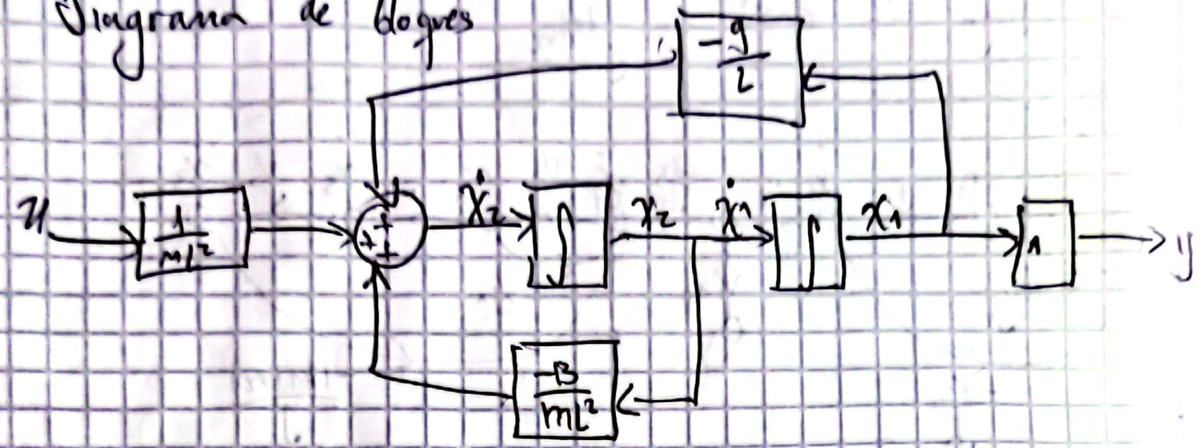


Diagrama de flujo de señal