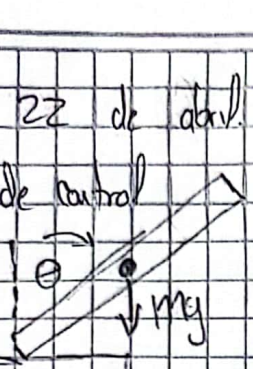
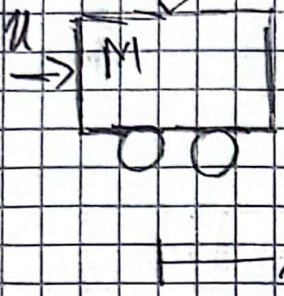
Clase 22 de abril

Línea de control





→ Movimiento horizontal de carro

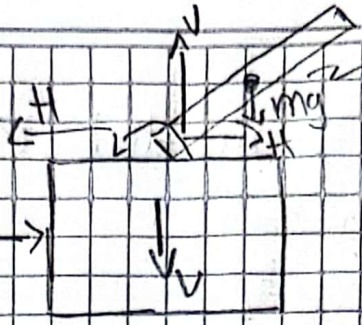
$$M\ddot{x} = u - H$$

→ Se debe linealizar

Ángulos de oscilación pequeños

$\theta \to 0$

Poder controlar



→ Movimiento rotacional

$$I\ddot{\theta} - V l \operatorname{Sen}\theta - H l \operatorname{Sen}\theta$$

→ Movimiento Vertical

$$V - mg = \frac{m d^2 (l \operatorname{Cos}(\theta))}{dt^2}$$

→ Teniendo en cuenta las relaciones

$$\theta \to 0 \begin{cases} \operatorname{Sen}\theta \approx \theta \\ \operatorname{Cos}\theta \approx 1 \\ \theta\dot{\theta}^2 \approx 0 \end{cases}$$

$$\to I\ddot{\theta} = V l \operatorname{Sen}\theta - H l \operatorname{Cos}\theta$$

$$I\ddot{\theta} = V l\theta - H l_1$$

$$\to H = m\ddot{x} - m l \operatorname{Sen}(\theta)\dot{\theta}^2 + m l \operatorname{Cos}(\theta)\ddot{\theta}$$

$$H = m\ddot{x} + m l\ddot{\theta}$$

$$H = m(\ddot{x} + l\ddot{\theta})$$

$$\frac{m d^2 (l \operatorname{Cos}\theta)}{dt^2} = V - mg$$

$$\theta = V - mg$$

(x_g, y_g)

$x_g = x + l \operatorname{Sen}\theta$

$y_g = l \operatorname{Cos}\theta$

→ Movimiento horizontal

$$H = \frac{m d^2 (x + l \operatorname{Sen}\theta)}{dt^2}$$

$\downarrow \frac{\partial}{dt}$

$$H = m\ddot{x} + \frac{m d^2 (l \operatorname{Sen}\theta)}{dt^2}$$

$$H = m\ddot{x} + \frac{m d (l \operatorname{Cos}(\theta)\dot{\theta})}{dt}$$

$$H = m\ddot{x} + m l \frac{\operatorname{Cos}(\theta)\dot{\theta}}{dt}$$

$$H = m\ddot{x} + m l [-\operatorname{Sen}(\theta)\dot{\theta}\dot{\theta} + \operatorname{Cos}(\theta)\ddot{\theta}]$$

$$H = m\ddot{x} - m l \operatorname{Sen}(\theta)\dot{\theta}^2 + m l \operatorname{Cos}(\theta)\ddot{\theta}$$

2. $M\ddot{x} = u - H$

$H = m(\ddot{x} + l\ddot{\theta})$

$M\ddot{x} = u - m(\ddot{x} + l\ddot{\theta}) + H$

$M\ddot{x} = u + m\ddot{x} - ml\ddot{\theta} + H$

$M\ddot{x} + m\ddot{x} + ml\ddot{\theta} = u$

$(M + m)\ddot{x} + ml\ddot{\theta} = u$

$V = mg$

$\rightarrow I\ddot{\theta} = Vl\theta - Hl \quad H = m(\ddot{x} + l\ddot{\theta}) \quad 0 = V - mg$

$\downarrow I\ddot{\theta} = mgl\theta - m(\ddot{x} + l\ddot{\theta})l$

$I\ddot{\theta} = mgl\theta - m(\ddot{x} + l\ddot{\theta})l$

$I\ddot{\theta} = mgl\theta - l(m\ddot{x} + ml\ddot{\theta})$

$\rightarrow$ Obtenendo o espaço de estados

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(I+ml^2)b}{I(M+m)+Mml^2} & \frac{m^2gl^2}{I(M+m)+Mml^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-mlb}{I(M+m)+Mml^2} & \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{I+ml^2}{I(M+m)+Mml^2} \\ 0 \\ \frac{ml}{I(M+m)+Mml^2} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \vec{0}u$$