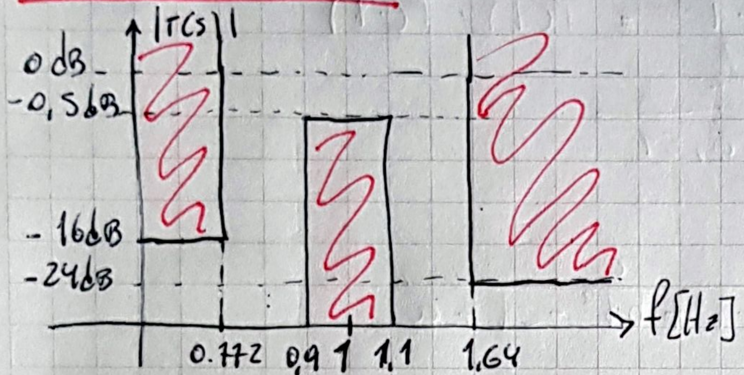
Tarea Semanal 5



$f_0 = 22\,kHz$

$Q = 5$

$|T(f_{s1})| = -16\,dB \quad ; \quad f_{s1} = 17\,kHz$

$|T(f_{s2})| = -24\,dB \quad ; \quad f_{s2} = 36\,kHz$

$$BW = \frac{\omega_0}{Q} = \frac{1}{5} = 0{,}2 = \omega_2 - \omega_1 \qquad \omega_2 = 1{,}1$$

$$\omega_0^2 = 1 = \omega_2 \cdot \omega_1 \qquad \omega_1 = 0{,}9$$

Buscamos $\Omega_S = \frac{1}{BW} \frac{(\omega_s^2 - \omega_0^2)}{\omega_s}$

$$\Omega_{S1} = 5 \cdot \frac{(0{,}77^2 - 1)}{0{,}77} = -2{,}6 \ \frac{rad}{s}$$

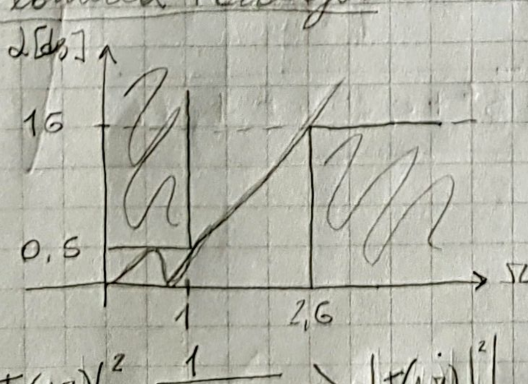
$$\Omega_{S2} = 5 \ \frac{(1{,}64^2 - 1)}{1{,}64} = 5{,}15 \ \frac{rad}{s}$$

Utilizamos la ecuación 7.20 del Schaumann para hallar el n

$$n = \frac{\ln \sqrt{4\left(10^{0,1\alpha_{min}} - 1\right) / \left(10^{0,1\alpha_{max}} - 1\right)}}{\ln\left(\omega_s + \sqrt{\omega_s^2 - 1}\right)}$$

$$\left.\begin{array}{l} n_1 = 3 \\ n_2 = 2 \end{array}\right] \longrightarrow n_1 > n_2 \longrightarrow n = 3$$

Plantilla Pasabajos



$\alpha_{max} = 10 \log(1 + \varepsilon^2) \rightarrow \varepsilon^2 = 10^{\frac{0,5}{10}} - 1$

$\varepsilon^2 = 0{,}122 \rightarrow \varepsilon = 0{,}35$

$C_3(\Omega) = 4\Omega^3 - 3\Omega$

$C_3(\Omega)^2 = 16\Omega^6 - 24\Omega^4 + 9\Omega^2$

$$|T(j\Omega)|^2 = \frac{1}{1 + \varepsilon^2 C_n^2} \Rightarrow \left.|T(j\Omega)|^2\right|_{\Omega = \frac{s}{j}} = |T(s)|^2 = \frac{1}{1 + 0{,}122\left(16\frac{s^6}{j^6} - 24\frac{s^4}{j^4} + 9\frac{s^2}{j^2}\right)}$$

$$|T(\$)|^2 = \frac{1}{1 + 0,122\left(-16\$^6 - 24\$^4 - 9\$^2\right)}$$

$$|T(\$)|^2 = \frac{1}{-1,95\$^6 - 2,93\$^4 - 1,1\$^2 + 1} = T(\$) \cdot T(-\$)$$

$$|T(\$)|^2 = \frac{0,51}{-\$^6 - 1,5\$^4 - 0,564\$^2 + 0,51}$$

$$|T(\$)|^2 = \frac{a}{\$^3 + b\$^2 + c\$ + a} \cdot \frac{a}{-\$^3 + b\$^2 - c\$ + a}$$

$$-\$^6 + \cancel{b\$^5} - c\$^4 + \cancel{a\$^3} - \cancel{b\$^5} + b^2\$^4 - \cancel{bc\$^3} + ba\$^2 - c\$^4 + \cancel{cb\$^3} - c^2\$^2 + \cancel{ca\$}$$
$$- \cancel{\$^3 a} + ab\$^2 - \cancel{ca\$} + a^2$$

$$a^2 = 0,51 \quad \Bigg| \quad \begin{aligned} -c\$^4 + b^2\$^4 - c\$^4 &= -1,5\$^4 \\ b^2 &= -1,5 + 2c \end{aligned} \quad \Bigg| \quad \begin{aligned} 2ab - c^2 &= -0,564 \\ 1,43b &= -0,564 + c^2 \end{aligned}$$

$$a = 0,714 \qquad c = 1,53 \qquad b = 1,25$$

$$\boxed{T(\$) = \frac{0,714}{\$^3 + 1,25\$^2 + 1,53\$ + 0,714}}$$

Prototipo Pasabajo

$$T(s) = T(\$)\Big|_{\$ = Q\left(s + \frac{1}{s}\right)}^{s} = \frac{0,714}{Q^3\left(s+\frac{1}{s}\right)^3 + 1,25\,Q^2\left(s+\frac{1}{s}\right)^2 + 1,53\,Q\left(s+\frac{1}{s}\right) + 0,714}$$

$$T(s) = \frac{0,714}{125\left(\frac{s^6 + 3s^4 + 3s^2 + 1}{s^3}\right) + 31,3\left(\frac{(s^2+1)^2}{s^2}\right) + 7,65\left(\frac{s^2+1}{s}\right) + 0,714}$$

$$T(s) = \frac{s^3 \cdot 0,714}{125\left(s^6 + 3s^4 + 3s^2 + 1\right) + 31,3 \cdot s\left(s^4 + 2s^2 + 1\right) + 7,65 \cdot s^2\left(s^2 + 1\right) + 0,714}$$

$$\boxed{T(s) = \frac{5,7 \cdot 10^{-3}\, s^3}{s^6 + 0,25s^5 + 3,06s^4 + 0,5s^3 + 3,06s^2 + 0,25s + 1}}$$

Con ayuda computacional finalmente se obtiene:

$$\boxed{T(s) = \frac{s \cdot 1{,}23 \cdot \frac{1}{7{,}79}}{s^2 + s\frac{1}{7{,}79} + 1} \cdot \frac{s \cdot 2{,}05 \cdot \frac{1{,}1}{16{,}5}}{s^2 + s\frac{1{,}1}{16{,}5} + 1{,}1^2} \cdot \frac{s \cdot 4{,}8 \cdot \frac{0{,}9}{16{,}5}}{s^2 + \frac{0{,}9}{16{,}5} s + 0{,}9^2}}$$

Lo cual corresponde correctamente a la parametrización:

$$T(s) = \frac{k_1 \cdot s \cdot \frac{1}{Q}}{s^2 + s\frac{1}{Q} + 1} \cdot \frac{k_2 \cdot s \cdot \frac{\omega_{01}}{q}}{s^2 + s\frac{\omega_{01}}{q} + \omega_{01}^2} \cdot \frac{k_3 \cdot s \cdot \frac{\omega_{02}}{q}}{s^2 + s\frac{\omega_{02}}{q} + \omega_{02}^2}$$