Taller #1

Sección 1.5.7:

Ejercicio 2

2. a) $\nabla(\phi\psi) = \psi\nabla(\phi) + \phi\nabla(\psi)$

- $[\nabla(\phi\psi)]_1 = \frac{\partial(\phi\psi)}{\partial x_1} + \frac{\partial(\phi\psi)}{\partial x_2} + \frac{\partial(\phi\psi)}{\partial x_3}$

$$= \left(\phi\frac{\partial\psi}{\partial x_1} + \frac{\partial\phi}{\partial x_1}\psi\right) + \left(\phi\frac{\partial\psi}{\partial x_2} + \frac{\partial\phi}{\partial x_2}\psi\right) + \left(\phi\frac{\partial\psi}{\partial x_3} + \frac{\partial\phi}{\partial x_3}\psi\right)$$

$$= \phi\left(\frac{\partial\psi}{\partial x_1} + \frac{\partial\psi}{\partial x_2} + \frac{\partial\psi}{\partial x_3}\right) + \psi\left(\frac{\partial\phi}{\partial x_1} + \frac{\partial\phi}{\partial x_2} + \frac{\partial\phi}{\partial x_3}\right)$$

$$= \phi\nabla(\psi) + \psi\nabla(\phi)$$

2.d) $\nabla \cdot (\nabla \times a)$ ¿Qué puede decir de $\nabla \times (\nabla \cdot a)$?

- $(\nabla \times a) = \epsilon_{ijk}\, \partial_j a_k$

$\nabla \cdot \vec{v} = \partial_i v_i$

- $\nabla \cdot (\nabla \times a) = \partial_i (\epsilon_{ijk}\, \partial_j a_k)$

$= \epsilon_{ijk}\, \partial_i \partial_j a_k$

Por propiedades de simetría, ϵ_{ijk} (Antisimétrico) y $\partial_i \partial_j$ (simétrico) se cancelan.

$= 0$

- El término $\nabla \times (\nabla \cdot a)$ no está definido ya que el rotacional se aplica a vectores, no escalares.

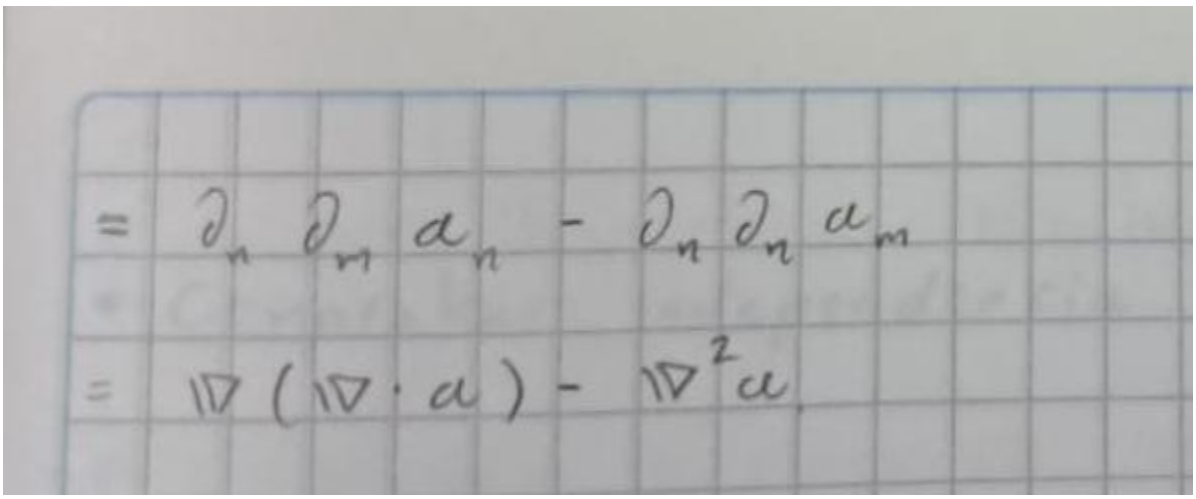
2.f) $\nabla \times (\nabla \times a) = \nabla (\nabla \cdot a) - \nabla^2 a$

$\nabla \times (\nabla \times a) = \epsilon_{ijk}\, \partial_j (\epsilon_{lmn}\, \partial_m a_n)$

$= \epsilon_{ijk}\, \epsilon_{lmn}\, \partial_j \partial_m a_n$

$= \delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}\, (\partial_j \partial_m a_n)$

$= \delta_{im}\delta_{jn}\, \partial_j \partial_m a_n - \delta_{in}\delta_{jm}\, \partial_j \partial_m a_n$


= ∂n ∂m an - ∂n ∂n am
= ∇(∇ · a) - ∇²a