Negative log likelihood of

$$f(x) \begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Calculate NLL:

1) loss function $= -\log f_{x_1 \cdots\cdots x_n}$

$$\sum_1^n \lambda e^{-\lambda x} = \lambda^n e^{-\lambda \sum_1^n x}$$

$$-\ln\left(\lambda^n e^{-\lambda \sum_1^n x}\right) = \ln(\lambda^n) + \ln e^{-\lambda \sum_1^n x}$$

$$-n \ln(\lambda) + \lambda \sum_1^n x \ln e = -n \ln(\lambda) + \lambda \sum_1^n x$$

$$\lambda \sum_1^n x - n \ln(\lambda)$$

2) Take deritive in terms of λ set result to

$$\frac{d}{d\lambda}\left(\lambda \sum_{1}^{n} x - n \ln(\lambda)\right) =$$

$$\sum_{1}^{n} x - \frac{n}{\lambda} = 0 = \sum_{1}^{n} x = \frac{n}{\lambda}$$

$$\lambda \sum_{1}^{n} x = n = \lambda = \frac{n}{\sum_{1}^{n} x}$$

$$\lambda = \frac{n}{\sum_{1}^{n} x}$$