Discrete 2 (Chapter 7)

Noti għal ~~Qabel~~ Waqt l-Eżami

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1 Relations

Totality: $r \in X \leftrightarrow Y$ is said to be total iff $\text{total}(r) \stackrel{\text{def}}{=} \forall\, x \in X, \exists\, y \in Y : (x, y) \in r$ it true.

Functionality: $r \in X \leftrightarrow Y$ is said to be functional iff
$\text{functional}(r) \stackrel{\text{def}}{=} \forall\, y_1, y_2 \in Y, \forall\, x \in X : (x, y_1) \in r \land (x, y_2) \Rightarrow y_1 = y_2$ is true.

Surjectivity: $r \in X \leftrightarrow Y$ is said to be surjective iff $\text{surjective}(r) \stackrel{\text{def}}{=} \forall\, y \in Y, \exists\, x \in X : (x, y) \in r$ is true.

Injectivity: $r \in X \leftrightarrow Y$ is said to be injective iff
$\text{injective}(r) \stackrel{\text{def}}{=} \forall\, x_1, x_2 \in X, \forall\, y \in Y : (x_1, y) \in r \land (x_2, y) \Rightarrow x_1 = x_2$ is true.

2 Multisets

2.1 Definitions

Set of Total and Functional Relations: $X \to Y \stackrel{\text{def}}{=} \{r \in X \leftrightarrow Y : \text{total}(r) \land \text{functional}(r)\}$

Set of Multisets Over X: $\mathbb{M}\, X \stackrel{\text{def}}{=} X \to \mathbb{N}$.

Empty Multiset: $\not\square \stackrel{\text{def}}{=} \lambda\, x \in X \cdot 0$.

Multiset Element Of: $x \sqsubseteq T \stackrel{\text{def}}{=} T(x) > 0$

Multiset Subset: $T \sqsubseteq S \stackrel{\text{def}}{=} \forall\, x \in X \cdot T(x) \leq S(x)$.

Multiset Equality: $T \stackrel{\text{mst}}{=} S \stackrel{\text{def}}{=} \forall\, x \in X \cdot T(x) = S(x)$.

Multiset Union: $T \sqcup S \stackrel{\text{def}}{=} \lambda\, x \in X \cdot T(x) + S(x)$.

Multiset Intersection: $T \sqcap S \stackrel{\text{def}}{=} \lambda\, x \in X \cdot \min\{T(x), S(x)\}$.

2.2 Theorems

Theorem 7.4: $T \sqcup S \stackrel{\text{mst}}{=} S \sqcup T$.

Theorem 7.6: $T \sqcap S \sqsubseteq T \sqcup S$.

2.3 Exercises

Exercise 7.1: $T \sqsubseteq T$ (reflexive) and $T \sqsubseteq R \wedge R \sqsubseteq S \Rightarrow T \sqsubseteq S$ (transitive).

Exercise 7.2: $\emptyset \sqcap T \stackrel{\text{mst}}{=} T \sqcap \emptyset \stackrel{\text{mst}}{=} \emptyset$ and $\emptyset \sqcup T \stackrel{\text{mst}}{=} T \sqcup \emptyset \stackrel{\text{mst}}{=} T$.

Exercise 7.3: $T \sqcap T \stackrel{\text{mst}}{=} T$ (idempotent), $T \sqcap S \stackrel{\text{mst}}{=} S \sqcap T$ (commutative) and $(T \sqcap S) \sqcap R \stackrel{\text{mst}}{=} T \sqcap (S \sqcap R)$ (associative).

Exercise 7.4: $(T \sqcup S) \sqcup R \stackrel{\text{mst}}{=} T \sqcup (S \sqcup R)$ (associative).

Exercise 7.5: $T - S \stackrel{\text{def}}{=} \lambda\, x \in X \cdot \max\{0, T(x) - S(x)\}$ (multiset difference), $T - T \stackrel{\text{mst}}{=} \emptyset$ and $T - \emptyset \stackrel{\text{mst}}{=} T$.

3 Sequences

4 Graph Theory