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# Lattice- Boltzmann para fluidos immiscibles.

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## FLUIDOS

Miscibilidad

## MÉTODO

Colisionador

## SIMULACIÓN

Resultados



- ▶ La capacidad de mezclarse y formar compuestos homogéneos.
- ▶ Una propiedad tanto de fluidos como de gases.
- ▶ Si bien pueden ser solubles en cierta proporción, la miscibilidad implica que lo sea en todas sus proporciones.



Distribución de densidad:

$$f_i(\mathbf{x}, t) = f_i^R(\mathbf{x}, t) + f_i^B(\mathbf{x}, t)$$

Colisionador:

$$f_i^\sigma(\mathbf{x} + \mathbf{c}_i, t+1) = f_i^\sigma(\mathbf{x}, t) + \Omega_i^\sigma(\mathbf{x}, t)$$

Operadores Colisión

$$\Omega_i^\sigma(\mathbf{x}, t) = \Omega_3^\sigma \{ \Omega_1^\sigma + \Omega_2^\sigma \}$$

$$\Omega_1^\sigma = -\frac{1}{\tau^\sigma} (f_i^\sigma - f_i^{\sigma(e)})$$



$$\Omega_i^\sigma = -\omega^\sigma (f_i^\sigma - f_i^{\sigma(e)})$$

Funciones de equilibrio:

$$f_0^{\sigma(e)} = \rho^\sigma \left( \alpha^\sigma - \frac{2}{3} \mathbf{u}^2 \right) \quad (i=0)$$

$$f_i^{\sigma(e)} = \rho^\sigma \left( \frac{1-\alpha^\sigma}{5} + W_i \left[ 3\mathbf{c}_i \cdot \mathbf{u} + \frac{9}{2} (\mathbf{c}_i \cdot \mathbf{u})^2 - \frac{3}{2} \mathbf{u}^2 \right] \right), \quad (i=1,2,3,4)$$

$$f_i^{\sigma(e)} = \rho^\sigma \left( \frac{1-\alpha^\sigma}{20} + W_i \left[ 3\mathbf{c}_i \cdot \mathbf{u} + \frac{9}{2} (\mathbf{c}_i \cdot \mathbf{u})^2 - \frac{3}{2} \mathbf{u}^2 \right] \right), \quad (i=5,6,7,8)$$



Funciones de peso:

$$W_i = \begin{cases} \frac{4}{9}, & (i = 0) \\ \frac{1}{9}, & (i = 1, 2, 3, 4) \\ \frac{1}{36}, & (i = 5, 6, 7, 8) \end{cases}$$



Funciones de densidad:

$$\rho^\sigma = \sum_i f_i^\sigma = \sum_i f_i^{\sigma(e)}$$

Momentum:

$$\rho \mathbf{u} = \sum_i \sum_\sigma f_i^\sigma \mathbf{c}_i = \sum_i \sum_\sigma f_i^{\sigma(e)} \mathbf{c}_i$$



Gradiente de densidad:

$$\mathbf{F}(\mathbf{x}) = \sum_{i=1}^8 \mathbf{c}_i \left[ \rho^R(\mathbf{x} + \mathbf{c}_i) - \rho^B(\mathbf{x} + \mathbf{c}_i) \right]$$

Operador Colision 2:

$$\Omega 2_i^\sigma = \frac{A^\sigma}{2} |\mathbf{F}| \left[ W_i \frac{(\mathbf{c}_i \cdot \mathbf{F})^2}{|\mathbf{F}|^2} - B_i \right]$$





Parámetros del colisionador:

$$B_i = \begin{cases} -\frac{4}{27} & (i = 0) \\ \frac{2}{27} & (i = 1, 2, 3, 4) \\ \frac{5}{108} & (i = 5, 6, 7, 8) \end{cases}$$

Re-Coloreo:

$$f_i^{R'} + f_i^{B'} = f_i^*$$

$$\sum_i f_i^{R'} = \rho^R$$



Función de relajación:

$$\omega = \begin{cases} \omega^R, & \psi > \delta, \\ f^R(\psi), & \delta \geq \psi > 0, \\ f^B(\psi), & 0 \geq \psi \geq -\delta, \\ \omega^B & \psi < -\delta \end{cases}$$

$$f^R(\psi) = \beta + \gamma\psi + \varepsilon\psi^2$$

$$f^B(\psi) = \beta + \eta\psi + \xi\psi^2$$

Parámetros de relajación:

$$\beta = \langle \omega \rangle$$

$$\gamma = \frac{2(\omega^R - \beta)}{\delta}$$

$$\varepsilon = -\frac{\gamma}{2\delta}$$

$$\eta = \frac{2(\beta - \omega^B)}{\delta}$$

$$\xi = \frac{\eta}{2\delta}$$

$$\langle \omega \rangle = \frac{2\omega^R \omega^B}{\omega^R + \omega^B}$$

Deformación de una gota cuadrada de un fluido dentro de otro.

- ▶ Se utiliza una rejilla cuadrada (128x128).
- ▶ Se ajustan las condiciones de frontera para dos fluidos, dispuestos como un cuadrado inmerso dentro de otro ( $\rho = 1$ ).

# Resultados

Gota Cuadrada en dos fases

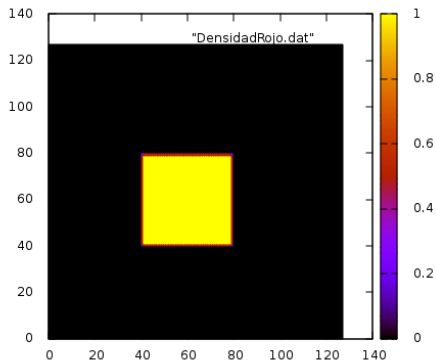


Figure:  $t=0$ ,  $L_x=128$ ,  $L_y=128$ , Lado=40

# Resultados

Gota Cuadrada en dos fases

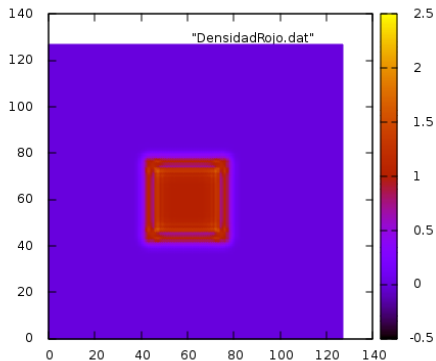


Figure:  $t=10$ ,  $L_x=128$ ,  $L_y=128$ , Lado=40

# Resultados

Gota Cuadrada en dos fases

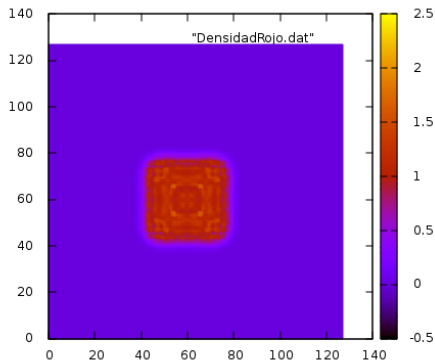


Figure:  $t=50$ ,  $L_x=128$ ,  $L_y=128$ , Lado=40

# Resultados

Gota Cuadrada en dos fases



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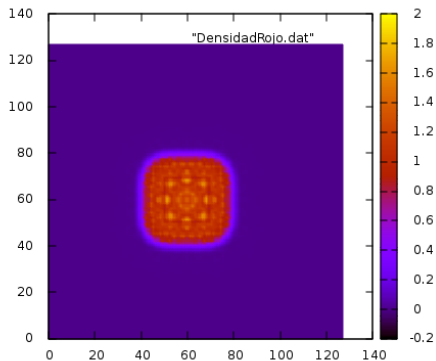


Figure:  $t=100$ ,  $L_x=128$ ,  $L_y=128$ , Lado=40



# Resultados

Gota Cuadrada en dos fases

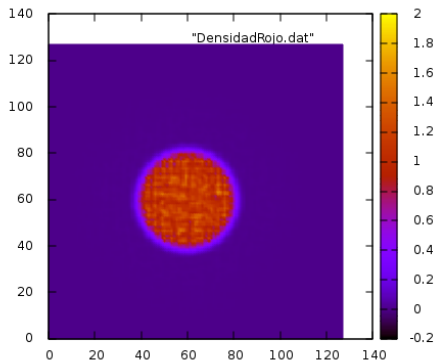


Figure:  $t=400$ ,  $L_x=128$ ,  $L_y=128$ , Lado=40

# Resultados

Gota Cuadrada en dos fases, razón de densidad  $\rho_b/\rho_r = 5$

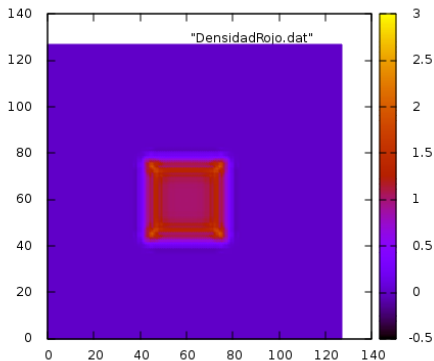


Figure:  $t=10$ ,  $L_x=128$ ,  $L_y=128$ , Lado=40

# Resultados

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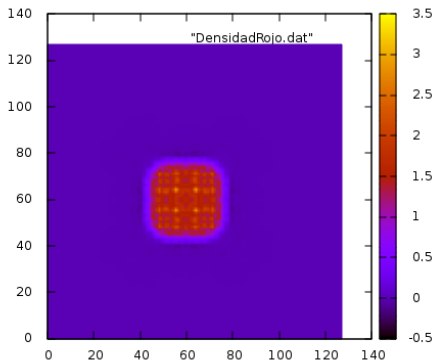


Figure:  $t=100$ ,  $L_x=128$ ,  $L_y=128$ , Lado=40

# Resultados

Gota Cuadrada en dos fases, razón de densidad  $\rho_b/\rho_r = 5$

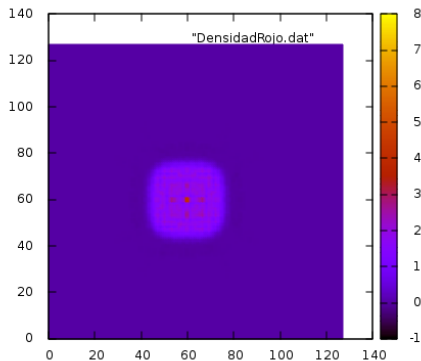


Figure:  $t=150$ ,  $L_x=128$ ,  $L_y=128$ , Lado=40

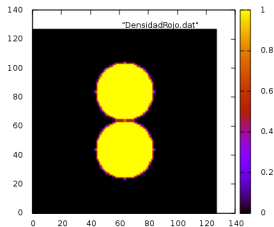


Figure:  $t=0$ ,  $L_x=128$ ,  $L_y=128$ ,  
Radio=20

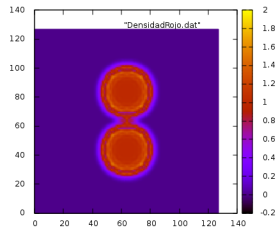


Figure:  $t=10$ ,  $L_x=128$ ,  $L_y=128$ ,  
Radio=20

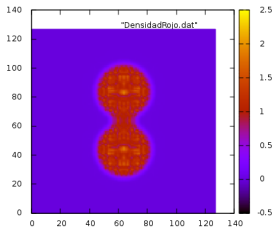


Figure:  $t=50$ ,  $L_x=128$ ,  $L_y=128$ ,  
Radio=20

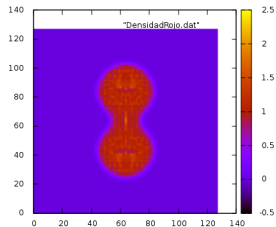


Figure:  $t=100$ ,  $L_x=128$ ,  $L_y=128$ ,  
Radio=20

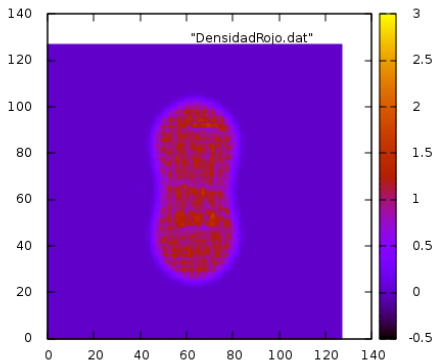


Figure:  $t=400$ ,  $L_x=128$ ,  $L_y=128$ ,  $\text{Radio}=20$

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