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Newton's law of cooling revisited

M Vollmer

Microsystem and Optical Technologies, University of Applied Sciences Brandenburg
Magdeburgerstr. 50, 14770 Brandenburg, Germany

E-mail: vollmer@fh-brandenburg.de

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Abstract

The cooling of objects is often described by a law, attributed to Newton, which states that the temperature difference of a cooling body with respect to the surroundings decreases exponentially with time. Such behaviour has been observed for many laboratory experiments, which led to a wide acceptance of this approach. However, the heat transfer from any object to its surrounding is not only due to conduction and convection but also due to radiation. The latter does not vary linearly with temperature difference, which leads to deviations from Newton's law. This paper presents a theoretical analysis of the cooling of objects with a small Biot number. It is shown that Newton's law of cooling, i.e. simple exponential behaviour, is mostly valid if temperature differences are below a certain threshold which depends on the experimental conditions. For any larger temperature differences appreciable deviations occur which need the complete nonlinear treatment. This is demonstrated by results of some laboratory experiments which use IR imaging to measure surface temperatures of solid cooling objects with temperature differences of up to 300 K.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Temperature differences in any situation result from energy flow into a system (heating by electrical power, contact to thermal bath, absorption of radiation, e.g. microwaves, sun radiation etc) and/or energy flow from a system to the surrounding. The former leads to heating, whereas the latter results in cooling of an object. The cooling of objects is usually considered to be due to three fundamental mechanisms: conduction of heat, convection and radiative transfer of energy [1, 2]. Although these three mechanisms of energy flow are quite different from each other, one often finds a very simple law for their combined action to describe the cooling curves of hot objects if temperature differences are small (see below). This law, mostly referred to as Newton's law of cooling, was originally expressed in a way that states that the temperature difference between an object and its surrounding decreases exponentially, if there is no additional heating involved [3]. Many modern textbooks present this law in another way to describe the physics behind the exponential decrease by stating Newton's law to mean that the heat transfer from an object to the surrounding is proportional to the respective temperature difference (e.g. [1], the very helpful comment by Bohren [3]; see also [4]). The respective time constant τ in the exponential function is characteristic for the object under study, i.e. it depends on its properties (heat capacity, size, geometry [5, 6] etc).

In the last few decades, quite a number of publications dealt with Newton's law of cooling. Some historical notes can be found in [7, 8]. The nearly endless discussions about Newton's law of cooling may be summarized by the statement of O'Connell [9]: 'Newton's law of cooling is one of those empirical statements about natural phenomena that should not work, but does.' Similarly, in 1969 it was stated that 'there is no definitive body of experimental data which defines the limits within which reality conforms to the law' and experiments using steady air flow around hot objects led to the conclusion that cooling in air proceeded geometrically (i.e. following exponential decay) up to 200 °C [7].

In detail, earlier work on Newton's law of cooling dealt with

- undergraduate lab experiments to demonstrate exponential cooling curves [10, 11],
- comparison of the cooling of solids with Newton's law [12],
- the influence of a finite reservoir of lower temperature than the object (e.g. well-defined amounts of hot water surrounded by cold water) [13],
- the mechanical equivalent of heat from Joule's experiment [14],
- the cooling of tea or coffee [15],
- modelling the transient temperature distributions of metal rods heated at one side only [16],
- measuring specific heats of solids and thermal conductivities [17–19],
- the cooling of spherical objects (like fuel droplets) in a gas [20]
- boundary conditions in studies modelling thermos [21],
- the world record for creating the fastest ice cream using liquid nitrogen [22],
- the cooling of incandescent lamp filaments [23] and
- explanations concerning the relevant corrections for the heating curves of water [9].

In most papers, linearization for the radiative contribution of heat transfer was used in order to end up with Newton's law. However, this is only justified for small temperature differences. In any case, one may expect that the strong nonlinearity of the radiative cooling processes should lead to deviations from the simple exponential behaviour.

Although this nonlinearity due to radiative processes was often recognized (e.g. [3, 4, 8, 24]), linearization was often considered to be an appropriate approximation. This was motivated by the fact that experimental investigations usually dealt with the low temperature-difference regime of say $\Delta T < 50$ K, where notable deviations from the exponential decrease due to the linear behaviour were not observed. (Notable deviations in the context of this paper mean that actual values for the temperature differences deviate from expectation according to Newton's law, i.e. a simple exponential, by say 5% or more.) In particular, an extensive study of the easiest student lab experiments like the cooling of flasks filled with hot water never showed any deviations from exponential cooling for $\Delta T < 55$ K [25, 26].

Therefrom several interrelated questions arise: namely (i) what is the magnitude of deviations? (ii) is it possible to define a range of validity for Newton's law of cooling? and (iii) can these deviations be easily observed experimentally?

In the following, all three questions will be addressed. First, a brief theoretical analysis of heat-transfer modes for cooling objects will be given. Second, in order to simplify the theoretical analysis and to properly describe simple experiments, we will discuss the so-called

Biot number. Emphasis will be on objects with a small Biot number. Third, the assumptions underlying Newton's law of cooling will be discussed. Fourth, the correct theoretical treatment of the radiative heat transfer leads to deviations from Newton's law which will be discussed upon variation of a number of parameters. It will be demonstrated that the temperature range where Newton's law is valid does sensitively depend on the relative contributions of convective versus radiative heat transfer. Finally, experimental results with temperature differences ΔT of up to 300 K will be presented which clearly verify the theoretically predicted deviations from Newton's law of cooling.

The problem as well as some experiments seem to be appropriate for undergraduate physics courses.

2. The basic heat-transfer modes: conduction, convection and radiation

Temperature differences in any situation result from energy flows into a system and energy flows from a system to the surrounding. The former leads to heating, whereas the latter results in cooling of an object. In thermodynamics, any kind of energy flow which is due to a temperature difference between a system and its surroundings is usually called heat flow or heat transfer. In physics, one usually distinguishes three kinds of heat flow: conduction, convection and radiation.

2.1. Conduction

Conduction refers to the heat flow in a solid or fluid (liquid or gas) which is at rest. Conduction of heat within an object, e.g. a one-dimensional wall, is usually assumed to be proportional to the temperature difference $T_1 - T_2$ on the two sides of the object as well as the surface area A of the object. This follows from the left-hand side of equation (1) by approximating $\mathrm{d}T/\mathrm{d}s \approx \Delta T/s$:

$$\dot{Q}_{\mathrm{Cond}} = \lambda \cdot A \cdot \frac{\mathrm{d}T}{\mathrm{d}s} \approx \frac{\lambda}{s} \cdot A \cdot (T_1 - T_2) = \alpha_{\mathrm{Cond}} \cdot A \cdot (T_1 - T_2). \tag{1}$$

The heat-transfer coefficient within the object is defined as $\alpha_{\mathrm{Cond}} = \frac{\lambda}{s}$, where λ is the thermal conductivity and s is a measure for the object size. For the one-dimensional wall it would be the wall thickness. The heat-transfer coefficient α_{Cond} describes heat transfer in W $(\mathrm{m}^2\ \mathrm{K})^{-1}$. Hence the heat flux through the wall $\dot{Q}_{\mathrm{Cond}}$ in W gives the energy flow per second through the wall of the surface area A if the temperature difference between the inner and outer surfaces is given. Typical values of heat-transfer coefficients for $s = 10$ cm 'wall thickness' for pure metals are of the order of 1000 W $(\mathrm{m}^2 \cdot \mathrm{K})^{-1}$, building materials such as concrete, stones or glass range between 5 and 20 W $(\mathrm{m}^2 \cdot \mathrm{K})^{-1}$, water has 0.6 W $(\mathrm{m}^2 \cdot \mathrm{K})^{-1}$, and insulating foams or gases range between 0.2 and 0.5 W $(\mathrm{m}^2 \cdot \mathrm{K})^{-1}$.

2.2. Convection

In general, convection refers to the heat flow between a solid and a fluid in motion. The energy flow $\dot{Q}_{\mathrm{Cond}}$ per second from the surface of an object with temperature T_1 into a fluid of temperature T_2 due to convection is usually assumed to follow a law similar to the one of conduction

$$\dot{Q}_{\mathrm{Conv}} = \alpha_{\mathrm{Conv}} \cdot A \cdot (T_1 - T_2). \tag{2}$$

The heat-transfer coefficient for convection depends on the nature of the motion of the fluid, on the fluid velocity and on temperature differences. One distinguishes free convection where

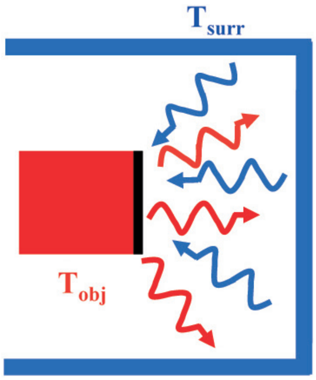

Figure 1. Whenever an object is placed in an environment of different temperature, there will be a net energy transfer due to thermal radiation due to emission as well as absorption of radiation by the object.

the current of the fluid is due to temperature and, hence, density differences in the fluid. In forced convection, the current of the fluid is due to external forces/pressure. Theoretically, convective heat transfer is modelled using dimensionless numbers such as the Nusselt number, the Prandtl number, the Grashof number and the Rayleigh number [1]. These depend on the Reynolds number which defines the kind of flow (laminar versus turbulent). Therefrom it follows that α_{Conv} depends on viscosity, thermal conductivity etc and also nonlinearly on the temperature difference, i.e. $\alpha_{\mathrm{Conv}} \sim \Delta T^x$. The exact power x depends on the flow conditions, e.g. for free convection around a horizontal plate, it changes from $x = 0.25$ for laminar flow to 0.33 for turbulent flow [1].

Typical values for free-convective heat-transfer coefficients of gases above solids are cited to range between 2 and 25 W $(\mathrm{m}^2\ \mathrm{K})^{-1}$, the exact value depending on flow conditions, wind speed and moisture of the surface; for liquids they can be in the range 50–1000 W $(\mathrm{m}^2\ \mathrm{K})^{-1}$. Here, we neglect any effects of latent heat associated with convective heat transfer.

2.3. Radiation

The emission of thermal radiation by an object is usually expressed as the product of a material property, the emissivity ε and the blackbody radiation due to the object temperature T (e.g. [1, 2, 27–29]. For many objects (in particular all studied in this work), the emissivity can be assumed to be independent of wavelength, i.e. having a constant value $\geqslant$0.85.

In any realistic situation, an object of temperature T_{obj} is surrounded by other objects of background temperatures T_{surr}. For simplicity, we assume an object (figure 1) which is completely surrounded by an enclosure of constant temperature (if the surrounding consists of objects with different temperatures, one needs to compute the respective view factors to find the net radiation transfer [1, 2]).

In addition to the emission of radiation from the object there is radiation from the surroundings incident onto the object. This finally leads to a net energy transfer from the object with the surface area A to the surroundings

$$\dot{Q}_{\mathrm{Rad}} = \varepsilon \cdot \sigma \cdot A \cdot \left(T_{\mathrm{obj}}^4 - T_{\mathrm{surr}}^4\right). \tag{3}$$

Since any quantitative analysis concerning the heat transfer is much easier for linear temperature differences, it is customary to approximate the radiative contribution also with a linear equation. This makes sense, if temperature differences are small ($T_{obj} \approx T_{surr}$, i.e. $\Delta T \ll T_{surr}, T_{obj}$), since in this case

$$\left(T_{obj}^4 - T_{surr}^4\right) = k_{appr}(T) \cdot (T_{obj} - T_{surr}), \tag{4}$$

where $k_{appr}(T) \approx 4T_{Surr}^3$. Using $\alpha_{Rad} = \varepsilon \cdot \sigma \cdot k_{appr}$, equation (3) can be rewritten as

$$\dot{Q}_{Rad} = \alpha_{Rad} \cdot A \cdot (T_{obj} - T_{surr}), \tag{5}$$

which is of the same type as the heat-transfer equations for conduction and convection.

3. Conduction within solids: the Biot number

In the experiments described below, information about the temperatures of solid objects will be gained from the surface temperatures of these objects. It is important to know how these surface temperatures are related to average temperatures in order to correctly model the cooling process.

Consider a solid, which is between two fluids of different but constant temperatures T_1 and T_2. Assuming steady state conditions the heat flows due to conduction, convection and radiation will lead to a spatial temperature distribution within the object. It is possible to get some idea on this temperature distribution within the solid by using the so-called Biot number Bi.

$$\mathrm{Bi} = \frac{\alpha_{Conv}}{\alpha_{Cond}} = \frac{\alpha_{Conv}}{\lambda/s}. \tag{6}$$

The Biot number is a dimensionless quantity, usually describing the ratio of two adjacent heat-transfer rates. In the present case, it describes the ratio of the outer heat flow from the surface to the surrounding, characterized by the convective heat-transfer coefficient α_{Conv} at the surface, and the inner heat flow within the object characterized by the conductive heat-transfer coefficient $\alpha_{cond} = (\lambda/s)$. For $\mathrm{Bi} \gg 1$, the outer heat flow is much larger than the inner heat flow. Obviously, this will result in a strong spatial variation of internal temperature within the object. This is typical for walls of buildings. If however, $\mathrm{Bi} \ll 1$, the internal heat flow is much larger than the heat loss from the surface. Therefore, there will be temperature equilibrium within the object, i.e. a homogeneous temperature distribution within the solid and the drop at the boundary of the object to the surrounding fluid is much larger [1].

As an example of time-dependent effects, we discuss the situation of the cooling of objects. Consider e.g. an initially hot one-dimensional object of temperature T_{obj} which is in contact with surroundings of lower temperature. Figure 2 gives a schematic representation of temperature within the object as a function of time for different Biot numbers. The temperature drops outside the boundaries were omitted here for clarity. For the Biot numbers <0.1, the temperature differences between the exact solution and the one assuming equilibrium within the object only leads to 2% deviation [2]. Hence, whenever $\mathrm{Bi} < 0.1$ [1], one may assume constant temperature throughout the solid.

For larger Biot numbers, the conduction heat transfer within the solid proceeds more slowly than the convective heat transfer from the surface boundary. Therefore, the outside parts of the solid cool faster than the inside and a spatial temperature profile results. In this case, the surface temperature does not resemble a useful measure for the inside temperature or even an average temperature of the solid. Figure 2(b) shows that for small Biot numbers, $T_{surface}$ still resembles a more or less reasonable approximation for the average temperature $T_{average}$. For very large Biot numbers, the convective heat transfer dominates, the surface temperature drops very rapidly and then stays low, while the internal temperature drops only very slowly.

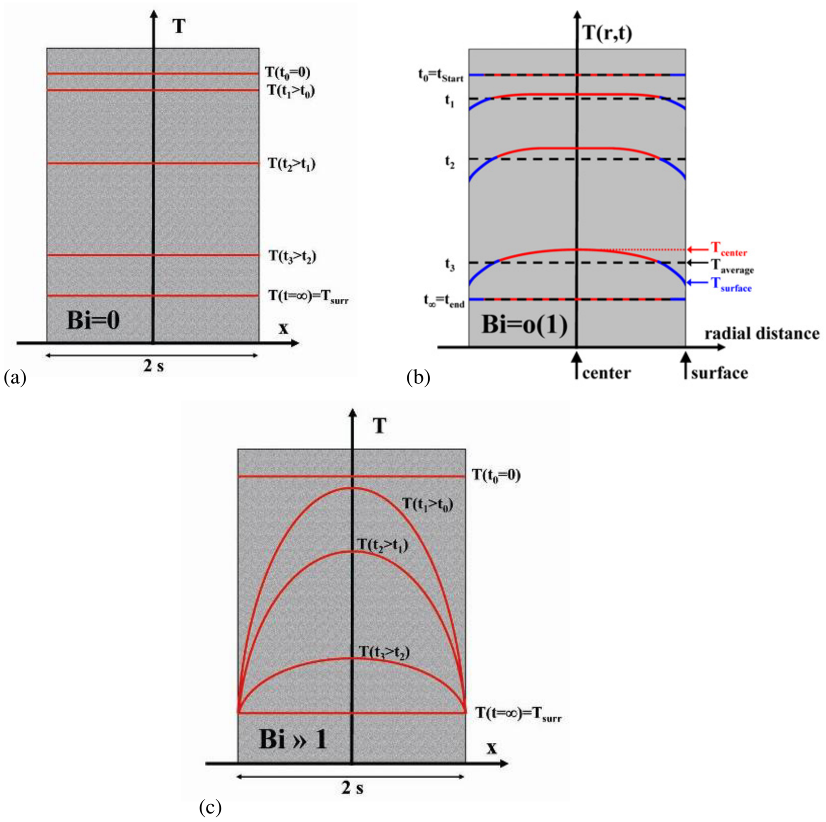

Figure 2. Schematic temperature distributions within solid objects upon cooling as a function of the increasing Biot number (Bi = 0, Bi = 0(1), i.e. of the order of unity, Bi $\gg$ 1). For finite object temperature, there is an additional temperature drop at the boundary to the surrounding fluid, which was omitted here. The intermediate situation (b) depicts also the differences between surface, centre and average temperature of a sphere.

Table 1 gives a summary of Biot numbers for some objects used in our experiments (see below). For small objects of metal the condition Bi < 0.1 is usually fulfilled. The same holds for cans/bottles filled with water for both the liquid inside and the walls of the container. This means that we can simplify the models by assuming thermal equilibrium within the objects, i.e. by describing the cooling process with average temperatures of the objects. In the case of bottles/cans, any internal convection of the water would increase λ, hence decrease Bi further.

4. Simplified model for cooling of objects for Bi $\ll$ 1

Consider a homogeneously heated object, which can be described by a small Biot number, i.e. an average temperature is sufficient to characterize the cooling of the object. If the object is just placed in air, we assume typical values for heat-transfer coefficients for free convection (solids to gases) in the range 2–25 W (m^2 K)$^{-1}$. Supposing an initial temperature T_{init} of the objects, energy conservation requires that any heat loss will lead to a decrease of the thermal

Table 1. Some material properties of objects and respective Biot numbers.

Object	Material(s)	α_{Conv} (W/(m^2 K)$^{-1}$)	s in m	λ (W (m · K)$^{-1}$)	$\alpha_{\mathrm{Cond}} = \lambda/s$ (W (m^2 K)$^{-1}$)	Biot number
Metal cubes	Aluminium, paint	2–25	$20–60 \times 10^{-3}$	220	11 000–3670	<0.01
Soft drink	Aluminium with	2	$\leqslant 1 \times 10^{-3}$	220	>220 000	$\ll 1$
can (0.5 l)	water inside		3.3×10^{-2} radius	0.6	18.2	≈ 0.1
Light bulbs	Glass	2–25	1×10^{-3}	≈ 1	1000	0.002–0.025
Bottle (0.5 l)	Glass with	2	$\approx 3 \times 10^{-3}$	≈ 1	333	≈ 0.006
in fridge	water inside		3.3×10^{-2} radius	0.6	18.2	≈ 0.1

energy of the object, i.e.

$$mc\frac{\mathrm{d}T_{\mathrm{obj}}}{\mathrm{d}t} = -\dot{Q}_{\mathrm{Conv}} - \dot{Q}_{\mathrm{Rad}}, \tag{7}$$

where m is the mass of the object, c is the specific heat (here assumed to be independent of T) and $\mathrm{d}T_{\mathrm{obj}}/\mathrm{d}t$ denotes the decrease in the (uniform) temperature of the objects due to the losses. Conduction within the solid is only important for establishing a homogeneous temperature profile within the objects; the heat transfer from the object to the surrounding air is due to convection and radiation.

Using equations (2) and (3) for the heat losses would lead to a nonlinear differential equation which cannot be easily solved analytically. However, if the radiative cooling contribution can be used in the linearized form (equation (5)), equation (7) turns into a conventional linear differential equation

$$mc\frac{\mathrm{d}T_{\mathrm{obj}}}{\mathrm{d}t} = -\alpha_{\mathrm{total}} \cdot A \cdot (T_{\mathrm{obj}} - T_{\mathrm{surr}}), \qquad \text{where} \quad \alpha_{\mathrm{total}} = \alpha_{\mathrm{C}} + \alpha_{\mathrm{R}} = \alpha_{\mathrm{C}} + \varepsilon \cdot \sigma \cdot k_{\mathrm{appr}}. \tag{8}$$

Here α_{C} accounts for the sum of conduction and convection. α_{total} in addition includes the linearized radiative heat transfer. The solution for $t_0 = 0$ is usually written as

$$T_{\mathrm{obj}}(t) = T_{\mathrm{surr}} + (T_{\mathrm{Init}} - T_{\mathrm{surr}}) \cdot \mathrm{e}^{-t/\tau} \quad \text{with time constant} \quad \tau = \frac{\rho \cdot c}{\alpha_{\mathrm{total}}} \cdot \frac{V}{A}. \tag{9}$$

Here, ρ is the density of the object material, c is the specific heat and the volume-to-surface ratio V/A is proportional to the size of the object. Equation (9) predicts that the difference between the initial temperature T_{Init} and surrounding air temperature T_{surr} drops exponentially (this dependence is denoted as Newton's law). Many experiments seem to support the applicability of this simplified theory for temperature difference.

We briefly summarize the assumptions which led to equation (7), i.e. to Newton's law of cooling.

(1) The object is characterized by a single temperature (Bi $\ll 1$).

(2) For small temperature differences ΔT ($\Delta T \ll T_{\mathrm{obj}}, T_{\mathrm{surr}}$ with absolute temperatures in K) the radiative heat transfer may be approximated by its linearized form (equation (5)) where the heat-transfer coefficient is constant (does not depend on temperature). Below we will discuss in detail how small ΔT may be.

(3) The convective heat-transfer coefficient is assumed to stay constant during the cooling process.

(4) The temperature of the surrounding stays constant during the cooling proceeds, this means that the surroundings must be a very large thermal reservoir.
(5) The only internal energy source of the object is the stored thermal energy.

It is quite easy to experimentally fulfil requirements (1), (4) and (5). The convective heat transfer (3) assumption is more critical. In experiments it can be kept constant using steady airflow around objects, i.e. for forced convection. If experiments use free convection, α_C may depend on the temperature difference. In the following theoretical analysis, we will however focus on the influence of the linearization of the radiative heat transfer. In particular, we will discuss the question, whether the linearization of equation (3) (to give equation (5)) does also work over extended temperature ranges.

5. Modelling with the correct radiative heat transfer and arbitrary temperature differences for the cooling of objects for Bi ≪ 1

The cooling of objects can be studied by theoretical modelling using the complete nonlinear heat transfer.

$$mc\frac{\mathrm{d}T_{\mathrm{obj}}}{\mathrm{d}t} = -\alpha_{\mathrm{Con}} \cdot A \cdot (T_{\mathrm{obj}} - T_{\mathrm{surr}}) - \varepsilon \cdot \sigma \cdot A \cdot \left(T_{\mathrm{obj}}^4 - T_{\mathrm{surr}}^4\right). \tag{10}$$

Equation (10) was numerically solved for a specific example of painted aluminium cubes of 40 mm size since such cubes were used in one of the experiments. They may serve as a theoretical model system with simple geometry.

The cooling, as described by equation (10), depends on three parameters, first the cube size (in general this relates to the energy storage capability of the object), second the convective heat-transfer coefficient and third the emissivity of the object, i.e. the contribution of the radiative heat transfer. Figure 3 depicts the results while varying these parameters independently in semi logarithmic plots. Newton's law would be represented with a straight line.

The variation of cube size (a) in particular the variation of the slope of the plots directly relates to the fact that the time constant for cooling is linearly proportional to size. The other parameters for convection and emissivity lead to curved cooling plots, i.e. deviations from the simple exponential behaviour (straight line) for all investigated sizes. The variation of the convective heat-transfer coefficient (b) does have a strong impact on the linearity of the plot. The larger α_{conv}, the more the plot follows a straight line. Similarly, the variation of emissivity (c) shows that very small emissivities (e.g. $\varepsilon < 0.2$), i.e. small contributions of the radiative heat transfer, clearly favour Newton's law, i.e. exponential cooling. In practice, ε cannot be varied in such wide ranges. Polished metal cubes will have ε-values of the order of 0.1, whereas those painted with high emissivity paint will have emissivities around 0.9. For experiments, intermediate values for the total radiative heat transfer may be realized by only painting a few sides of the cubes and leaving the others polished.

The numerical results of figure 3 were of course expected as they are a direct consequence of equation (10) which already explains the conditions for having either linear or nonlinear behaviour. If the convection term is large and the radiation contribution small, linear plots are expected and vice versa, whereas changing the size (ratio of mass and area) only has an effect on the timescale of the cooling process.

The degree of deviations from a straight line is depicted for three different convective heat-transfer coefficients (3 W m^{-2} K, 10 W m^{-2} K and 30 W m^{-2} K) for 40 mm size cubes and $\varepsilon = 0.9$ in figure 4. The initial temperature differences were assumed to be 700 K with regard to ambient temperature.

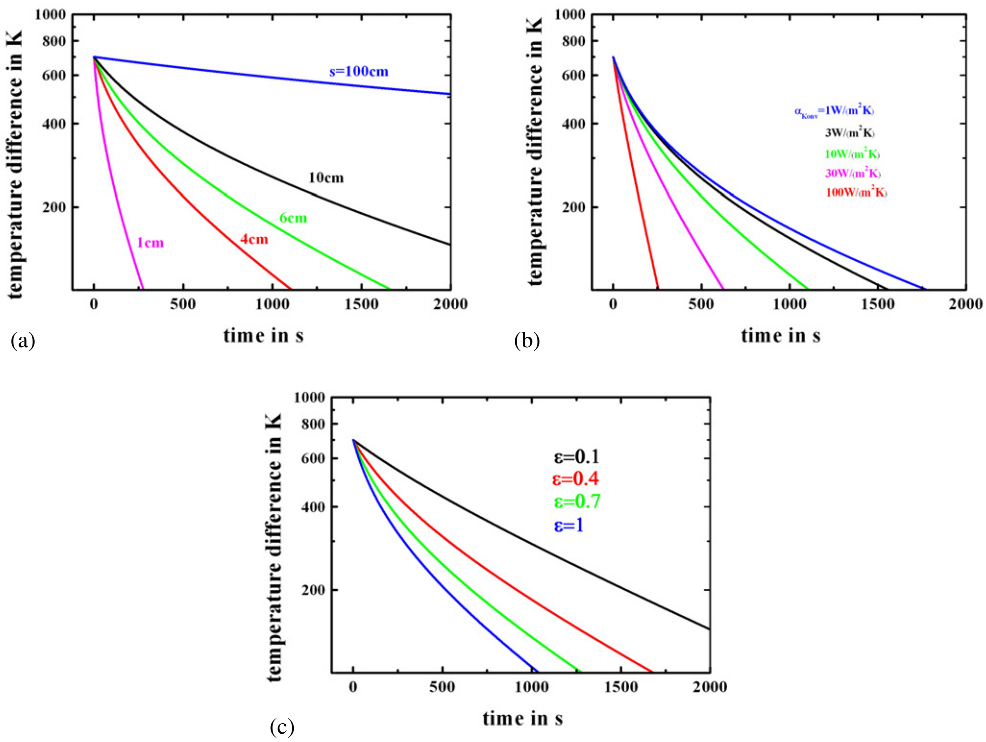

Figure 3. Numerical results of equation (10) for Al metal cubes. (a) Variation of cube size from 1 to 100 cm for fixed $\alpha_{\mathrm{Conv}} = 10$ W (m^2 K)$^{-1}$ and $\varepsilon = 0.9$. (b) Variation of the convective heat transfer coefficient α_{Conv} from 1 to 100 W (m^2 K)$^{-1}$ for fixed size ($s = 4$ cm) and $\varepsilon = 0.9$. (c) Variation of the emissivity for fixed $\alpha_{\mathrm{Conv}} = 10$ W (m^2 K)$^{-1}$ and cube size $s = 4$ cm. In (b) and (c), numbers from top to bottom refer to curves from top to bottom.

It is quite obvious that all plots show deviations from straight lines (broken lines), which nicely fit the low temperature data. The larger the convective heat transfer, the smaller the deviations. For low convective losses of only 3 W m^{-2} K, deviations can already be expected for temperature differences as small as 40 K. In contrast for very high convective losses of 30 W m^{-2} K, simple exponential cooling seems to work quite well for $\Delta T < 100$ K.

The results demonstrate that there is no general number for ΔT describing the range of validity of Newton's law. Rather, the respective temperature range depends on the experimental conditions and how close one looks for deviations. Plotting data only for small temperature differences can lead to the impression that the straight line works quite well since deviations are not as pronounced as for the high temperature range.

The results from figure 3 can be used to consider the two extreme cases for the cooling of objects, one where radiation dominates and another where convection dominates the cooling process. Results are depicted in figure 5. The smallest imaginable realistic value for the convective heat transfer is in the range of 1 W m^{-2} K; the largest respective radiative heat transfer occurs for black bodies, i.e. setting $\varepsilon = 1.0$ as may be realized by metal cubes covered with high emissivity paint. In this case, the cooling curve already starts to deviate from Newton's law for $\Delta T \approx 30$ K. In contrast, polished metal cubes with low emissivity reduce radiation losses. The convective losses may simultaneously be enhanced by directing fans

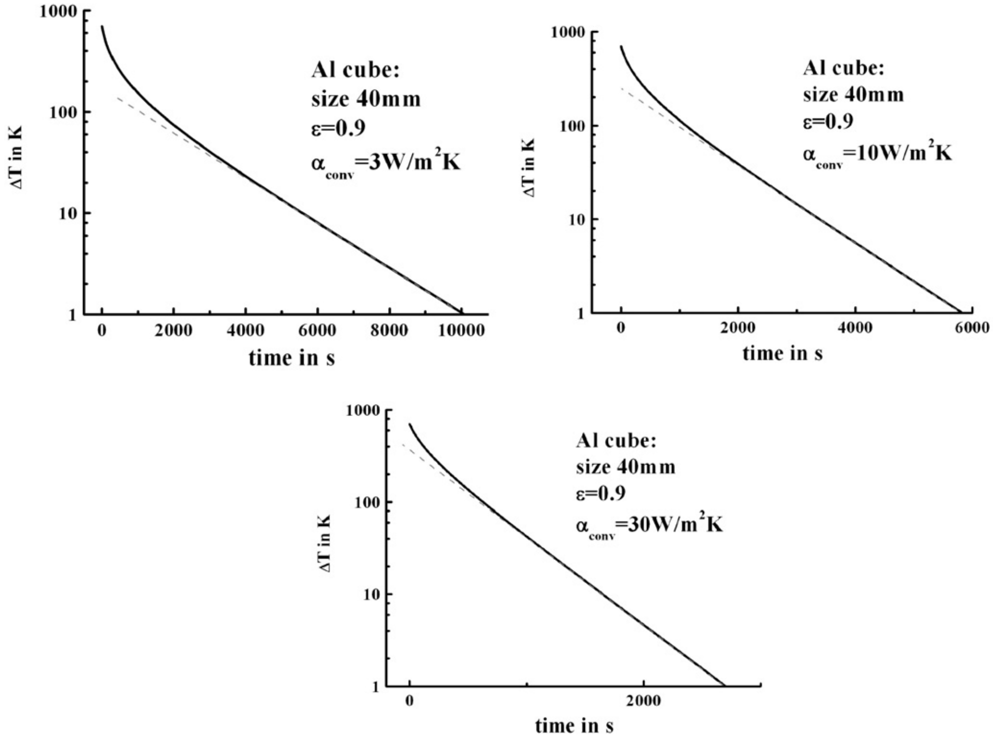

Figure 4. Theoretical cooling of Al metal cubes with size 40 mm and $\varepsilon = 0.9$ for different convective heat transfer coefficients. Newton's law would be a straight line such as the broken lines, which closely describe the low temperature data, but show deviations for larger temperatures.

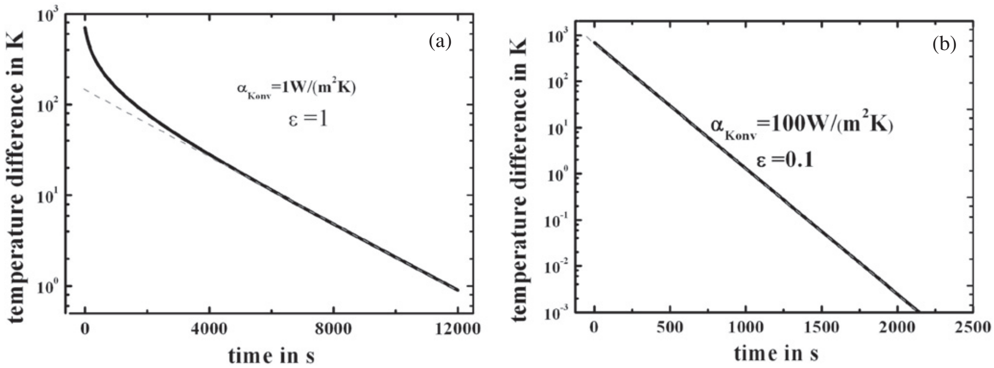

Figure 5. Extreme cases of cooling of Al metal cubes ($s = 4$ cm): small convection with large radiation heat transfer (a) and large convection with small radiative heat transfer (b).

with high air speed onto the objects. In this case, very high values of up to 100 W m^{-2} K seem possible. As a result, no deviation from the straightline plot is observable, i.e. Newton's law would hold for the whole temperature range of $\Delta T = 500$ K.

In order to further understand the relevance of the nonlinearities during the cooling of objects, we now consider the relative contributions of radiation and convection heat transfer.

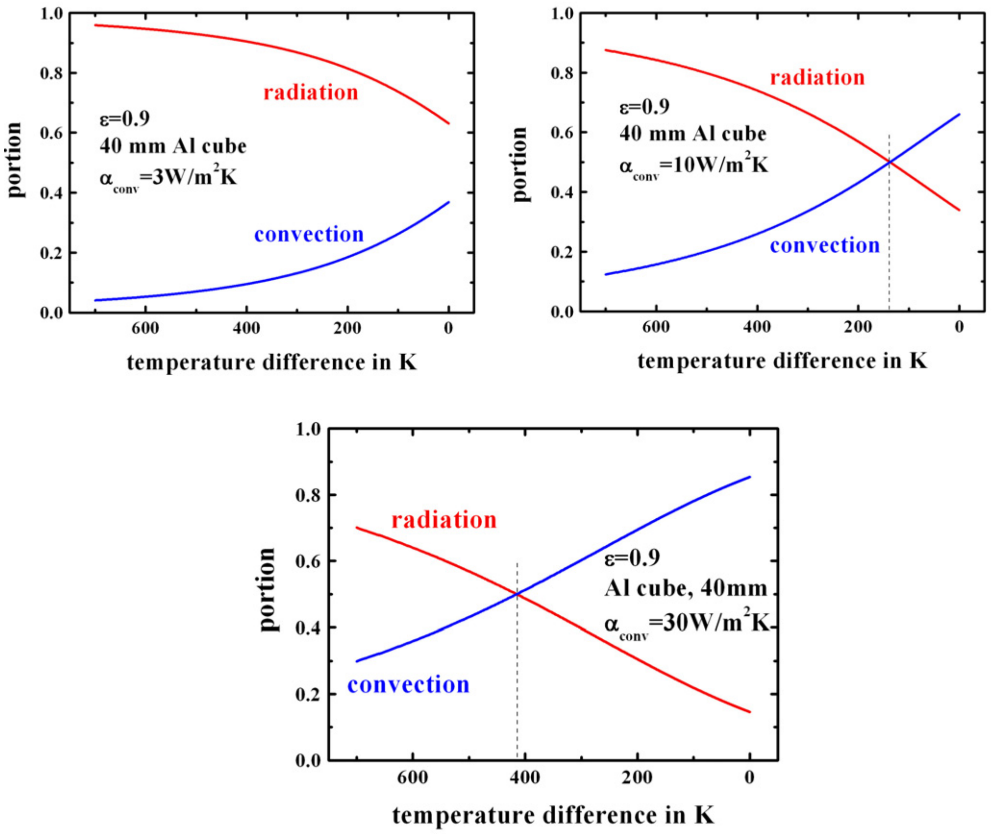

Figure 6. Relative contributions of convection and radiative cooling for Al cubes of 40 mm size with $\varepsilon = 0.9$ as a function of temperature. The initial temperature was 993 K, i.e. $\Delta T = 700$ K. For small convective heat transfer coefficients, radiative cooling is dominant throughout the cooling process, whereas for larger convection, there will be a change of the dominant cooling contribution at a certain temperature.

Quick estimates for special cases are possible using equations (2) and (3). Even close to room temperature, radiative losses are surprisingly large. This must be noted since many people argue that radiation losses can be neglected close to room temperature which is just wrong! Let us assume a background temperature of ≈22 °C, i.e. $T_{\text{surr}} = 293$ K. At 300 K, a blackbody ($\varepsilon = 1$) will then emit about 41 W m^{-2} which is of the same order of magnitude as typical convection losses of 63 W m^{-2} (for $\alpha_{\text{con}} = 9$ W $(\text{m}^2 \cdot \text{K})^{-1}$ or 14 W m^{-2} (for $\alpha_{\text{con}} =$ 2 W $(\text{m}^2 \cdot \text{K})^{-1}$).

A small radiative or a large convective contribution reduces the nonlinear effects in equation (10). This can also be seen in figure 6, which depicts the relative contributions of the convective and radiative heat transfer for the 40 mm Al cubes by assuming $\varepsilon = 0.9$ and three different values $\alpha_{\text{conv1}} = 3$ W m^{-2} K, $\alpha_{\text{conv2}} = 10$ W m^{-2} K and $\alpha_{\text{conv1}} = 30$ W m^{-2} K, for convective heat transfer.

For very small convective heat transfer of 3 W m^{-2} K, radiation dominates the total energy loss of the object from the beginning to the end. This easily explains why one necessarily expects strong deviations from Newton's law already for small temperature differences in this case. For larger convection coefficients like 10 W m^{-2} K or 30 W m^{-2} K, there is a cooling time, i.e. transition temperature difference, where the dominant cooling changes from radiation to convection. For 10 W m^{-2} K this happens at $\Delta T = 137$ K (after ≈840 s in a $T(t)$ plot)) and for 30 W m^{-2} K it happens at $\Delta T = 415$ K (after 112 s). Qualitatively it makes sense that the higher this transition temperature difference, the larger the range of validity of Newton's law of cooling. For the convective heat transfer of 100 W m^{-2} K and $\varepsilon = 0.1$ (see figure 5), convection would dominate radiation right from the beginning, which easily explains the linear plot.

From the theoretical analysis, it is clear that radiative cooling should lead to deviations from Newton's law of cooling above critical temperature differences, which in some of the discussed cases were below 100 K. This raises the question of why many experiments reported the applicability of Newton's law for a temperature difference range of up to 100 K. The answer which is proposed here is simple: if one waits long enough, any cooling process can probably be described by a simple exponential function.

To my knowledge, this statement has not been proven theoretically for the cooling of objects involving nonradiative heat transfer. The idea behind it may be motivated for convective cooling of simple shaped objects irrespective of the size or Biot number by the following argument. The cooling of objects such as spheres, cylinders, or plates of any size (not just small objects), which start at a given initial temperature and which are in contact with a fluid (radiative heat transfer is neglected) can be described by a series expansion of exponential functions [2]. It is found that for sufficiently long times, a single term in the series, i.e. a single exponential function, describes the temperature distribution within the objects. If a suitable average temperature is defined, the single exponential function will therefore describe the cooling process.

With this result from pure convective cooling in mind, theoretical cooling curves involving nonlinear radiative heat transfer were analysed by using series expansions of exponential functions as fit functions.

Figure 7 (left) depicts the theoretical temperature plot for $\alpha_{\mathrm{Conv}} = 10$ W m^{-2} K, $\varepsilon = 0.9$ and 40 mm Al cubes of figure 5. These theoretical data were fitted (figure 7 (right)) using a third-order exponential fit of the form

$$\Delta T(t) = T_0 + A_1 \cdot \mathrm{e}^{-t/\tau_1} + A_2 \cdot \mathrm{e}^{-t/t\tau_2} + A_3 \cdot \mathrm{e}^{-t/t\tau_3}. \tag{11}$$

The agreement of such a simple fit is extremely good as can be seen from figure 8, which depicts the difference between the theoretical plot and the third-order exponential fit. Disregarding the first 10 s, deviations are below 1 K.

The fit includes three functions with different amplitudes and time constants (the constant term of 0.07 is practically zero and unimportant for the discussion). The first one has the smallest amplitude (≈162) and a time constant of only 78.7 s. The second contribution has an appreciable amplitude (≈253) but also a relatively small time constant of 297 s. The third contribution has an amplitude slightly larger than the second one (281) but decays with a much longer time constant of about 1010 s. Due to exponential decay an amplitude is suppressed to less than 1% after five time constants. Hence, the first contribution already contributes less than about 1 K after 400 s of cooling. Similarly, the second contribution becomes less important with time. For times longer than 1500 s it only contributes less than 1.7 K. This happens at a temperature difference of about 65 K, which means that for longer times, i.e. smaller temperature differences, a single exponential provides a very good approximation to the cooling curve. Hence, if we assume that it is a general property of the combined convective and radiative cooling that the resulting $T(t)$ curve can be approximated by a superposition of exponential functions with different time constants, one does indeed expect that after the exponentials with small time constants have died away, a simple exponential

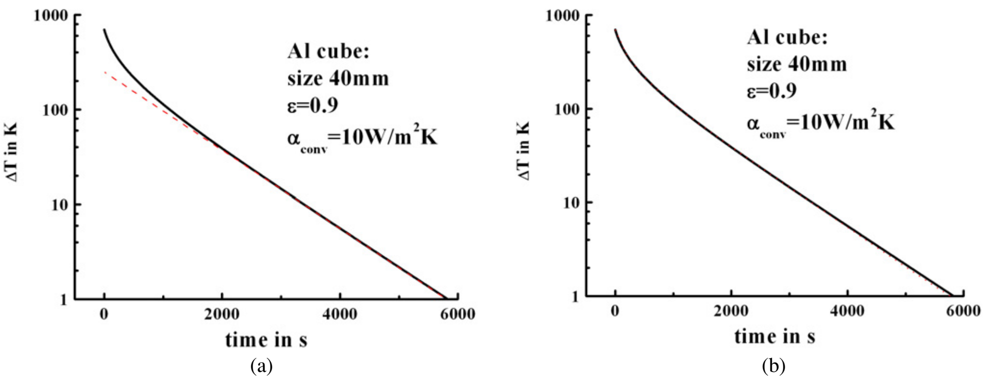

Figure 7. Computed cooling curve for combined convection and radiative cooling. For temperature differences below 100 K, a simple exponential fit, i.e. Newton's law, is a reasonable approximation (a). Such cooling curves can usually be approximated by a second- or third-order exponential fit (b), the first contribution of which will be dying off before ΔT decreases below 100 K.

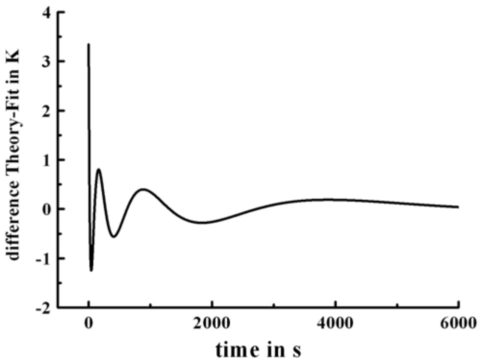

Figure 8. Difference between theoretical cooling curve computed from equation (10) (for $\varepsilon = 0.9$, 40 mm size Al cubes and $\alpha_{\mathrm{Conv}} = 10$ W m^{-2} K) and the third-order exponential fit (parameters $T_0 = 0.072$ K, $A_1 = 281.06$ K, $\tau_1 = 1009.72$ s, $A_2 = 253.27$ K, $\tau_2 = 297.04$ s, $A_3 = 162.25$ K and $\tau_3 = 78.68$ s).

will dominate and describe the cooling curve. This means that Newton's law of cooling is always an appropriate description for adequately long times. Unfortunately, the adequate time interval and the respective relevant critical temperature difference depend on the experimental conditions and must be evaluated each time. One example: cooling water with $\Delta T < 50$ K in beakers usually follows Newton's law.

We finally note that when using a simple exponential and explaining this with equation (9), one usually assumes a single coefficient for the heat transfer which is very often misleadingly called the convective heat-transfer coefficient, although it represents a combination of convection and radiation.

6. Experiments

A number of different objects were studied experimentally for variable conditions. First, liquids in bottles or cans were cooled in fridges, freezers or air convection coolers. With these

Figure 9. Set up for cooling cans and bottles of liquids in a freezer.

everyday objects and situations, relatively small temperature differences could be realized. These were expected to be accurately described by Newton's law of cooling. Second, we studied metal cubes as were treated in the theoretical analysis as model systems due to their simple geometry. Experimentally, we could realize temperature differences up to 140 K. Third, in order to achieve higher initial temperature differences, we used halogen light bulbs for temperature differences of up to 300 K. All temperatures were measured using IR imaging (see [29]). The fridge and the metal cube experiments could also easily be measured using conventional thermocouples; however, the much faster drop in temperature in the light bulb experiments requires fast contactless temperature measurements such as with an IR camera or a pyrometer. Nowadays rather cheap and small IR camera systems are available (some cost less than 3000€ for 80 × 80 pixels), therefore all experiments may be suitable for undergraduate student laboratories.

6.1. Liquids in refrigerator: $\Delta T \leqslant 50$ K

The daily life experiences of cooling are often related to fridges or freezers. As two examples we measured the cooling of cans and bottles filled with water (or other liquids) as a function of time for different cooling methods, using first a conventional fridge with a low temperature of 6 °C, second a (∗∗∗) freezer with a low temperature of −21 °C to –22 °C and third an air convection cooler set to a low temperature of −5.5 °C. As an example, figure 9 depicts the experimental set-up for the freezer. The Biot numbers for cans and bottles (see table 1) were below 0.1; therefore, we assumed the surface temperatures to be a useful measure for the average temperatures of the objects (it was shown that the actual average temperatures and the ones assuming Bi $\ll$ 1 only lead to deviations of 2% between the real average temperature and the one based on the approximation for geometric forms of sphere, cylinder, prisms and cube [2]).

The cooling power of the systems is expected to be quite different. The conventional fridge and the freezer both have objects surrounded by still air, since the temperature differences are usually too low to generate natural convection. Hence, the heat-transfer coefficients and the cooling time constants of both should be the same. However, the refrigerator has a smaller temperature difference than the freezer; therefore, the cooling power of the freezer is larger and the effective cooling times (times to reach a certain low temperature upon cooling) are smaller

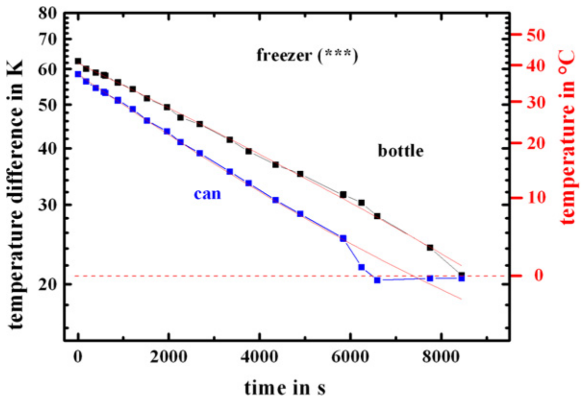

Figure 10. Cooling curve of bottle (0.5 l) and can (0.35 l) in the freezer as a function of cooling time. The temperature difference between can/bottle and freezer temperature decreases exponentially close to the point where freezing starts.

than with the refrigerator. The air convection cooler should have the fastest cooling since the convective heat-transfer coefficient increases strongly with airflow velocity. Therefore, also the time constant should decrease. As samples we used glass bottles and aluminium cans. Temperatures were measured with IR cameras [5, 28, 29]. Therefore a (blue) tape was attached in order to ensure equal emissivity values for all samples. The containers were filled with water slightly above room temperature and placed into the refrigerator, freezer and air convection cooler. During temperature recordings with the IR camera, taken every few minutes, the cooling unit doors were opened for at most 20 s each (thereby we introduce a small error since the ratio of open door to closed door is around 6%–7%, this will lead to a slightly longer cooling time constant). As an example, figure 10 depicts the resulting cooling curve for the freezer.

Due to the small temperature differences, it is expected that Newton's law should be a good description. Indeed, a straight line works well down to around 0 °C, where the phase transition from water to ice imposes a natural limit. Theoretically, it is also possible to understand the differences (factor in time constants: about 1.2) between cans and bottles (more information, see [6]).

The user of cold drinks is usually not interested in time constants, rather the time after which a certain temperature of a drink has been reached. Figure 11 depicts the experimental cooling curves for the 0.5 l bottles (linear scale). The initial temperature was about 28 °C. The fridge has the longest cooling time, whereas the air convection system cools fastest, e.g. in about 30 min from 28 °C to below 13 °C. Obviously, from daily experience, a still faster way of cooling would use forced convective cooling with liquids rather than gases, due to the much large heat transfer between solid and liquid compared to solid with gas. Putting bottles in a cold water stream has the additional advantage that a major problem of freezers is avoided: in freezers one is not allowed to forget the bottles, they may turn into ice and burst.

6.2. Metal cubes: $\Delta T \leqslant 140$ K

One of the simplest geometries for experiments—besides spheres—is cubes. Aluminium metal cubes of 20 mm, 30 mm, 40 mm and 60 mm side lengths were heated up in a conventional oven on a metal grid (details, see [5]). The cubes were partially covered with a high temperature

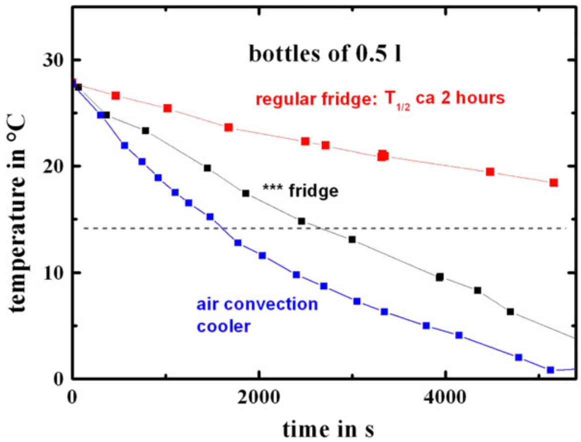

Figure 11. Cooling curves of (0.5 l) bottles for a regular fridge, a freezer and an air convection cooler. The typical timescale for cooling from 28 °C to below 14 °C (which may be a suitable drinking temperature) is about 2 h for the fridge, about 3/4 of an hour for the freezer but less than 1/2 h for the air convection cooler.

stable paint to achieve the required high emissivity. Specifically, three sides were covered with the paint, while the remaining three were left as polished metal (due to the extremely small values for the Biot number, we still assume a very fast thermal equilibrium within the cube and describe the total radiative cooling to first order using an average value for the emissivity of around 0.45; later experiments performed with all six sides covered with the paint justified this approach). Sufficient time for the heating was given such that all cubes were in thermal equilibrium within the oven at a temperature of 180 °C. The IR-imaging experiment started after opening the oven and placing the metal grid with the cubes onto some thermal insulation on a table. The time needed to do this usually led to a strong cooling effect already such that the maximum temperature differences in this experiment were around 140 K.

Figure 12 depicts two snapshots of the cooling process.

The smallest cubes cool best as can also be seen in the temperature profiles as a function of time (figure 13). A thorough discussion on time constants and size effects in this experiment can be found in [5], here we want to focus on the question whether Newton's law can be used to describe the results.

For the metal cubes, the Biot numbers follow from the values of heat conductivity $\lambda \approx 220\ \mathrm{W\ (m\cdot K)^{-1}}$, size $s =$ between 20 mm and 60 mm and typical values for heat-transfer coefficients for free convection (solids to gases) in the range 2–25 W $(\mathrm{m^2\ K})^{-1}$. We find λ/s between 11000 W $\mathrm{m^{-2}}$ K and 3667 W $\mathrm{m^{-2}}$ K giving Bi $\ll 1$, i.e. we can expect a temperature equilibrium within any of the metal cubes. This means that the measured surface temperatures should quite well resemble the average temperatures.

Using the theory outlined above for the metal cubes, the best fit to the experimental data was used to find a value for the convective heat-transfer coefficient (which was assumed to be constant during the cooling process). The radiation losses do not involve any additional parameter since ε is known. Figure 14 depicts the result for the 40 mm cube. The value $\alpha_{\mathrm{Conv}} = 9\ \mathrm{W\ m^{-2}\ K}$ for the best agreement with the data lies in the typical range for convective heat-transfer coefficients.

Previously it was stated [5] that the cube temperatures as a function of cooling time can be fitted extremely well with a simple exponential function. After considering the validity

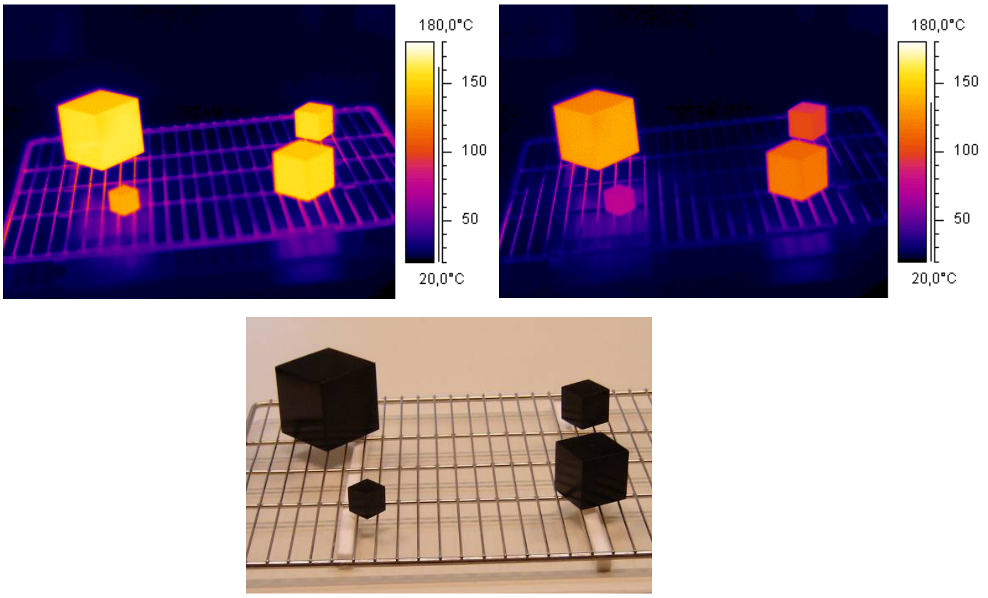

Figure 12. Two thermal imaging snapshots during the cooling of paint covered aluminium cubes of different sizes and visible image, showing the cubes on a grid and thermal insulation on the lab table.

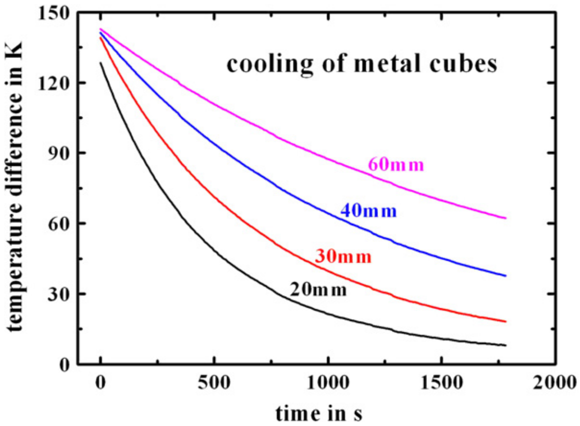

Figure 13. Temperature as a function of cooling time for the aluminium cubes of various sizes.

of Newton's law, the cube data [5] were reanalysed to find out whether any deviations from Newton's law could be observed.

The possibility of fitting cooling curves of cubes with a simple exponential to high accuracy is indeed true. However, Newton's law will only be fulfilled if the constant term in the exponential fit is zero. This is, however, not the case. As an example, figure 15 depicts the cooling curve of the 30 mm cube as a function of time. Similar to figure 14 for the 40 mm cubes, it can be nicely fitted with a theoretical fit using the full radiative heat transfer (which is however not shown in the plot).

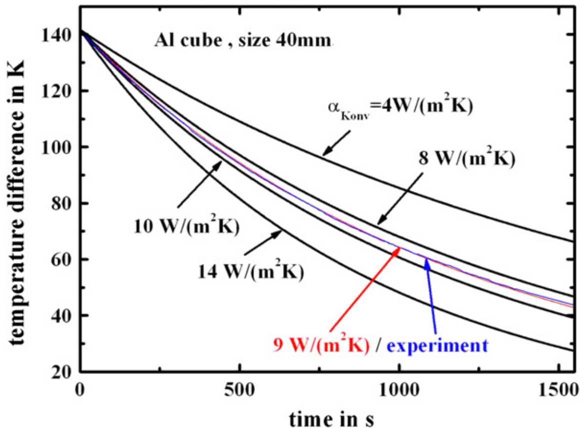

Figure 14. Comparison of measured temperatures (experiment) with numerical calculations of $T(t)$. Geometry (size 40 mm), ΔT_{init}, and material properties ($\varepsilon_{\mathrm{av}} = 0.45$) were given, the only free parameter was the heat transfer coefficient α_{Con}. The best-fit results from a value of 9 W m^{-2} K^{-1}, which lies in the interval for typical values. Experimental values and theory with 9 W m^{-2} K^{-1} lie so close to each other that they more or less form a single line in the plot.

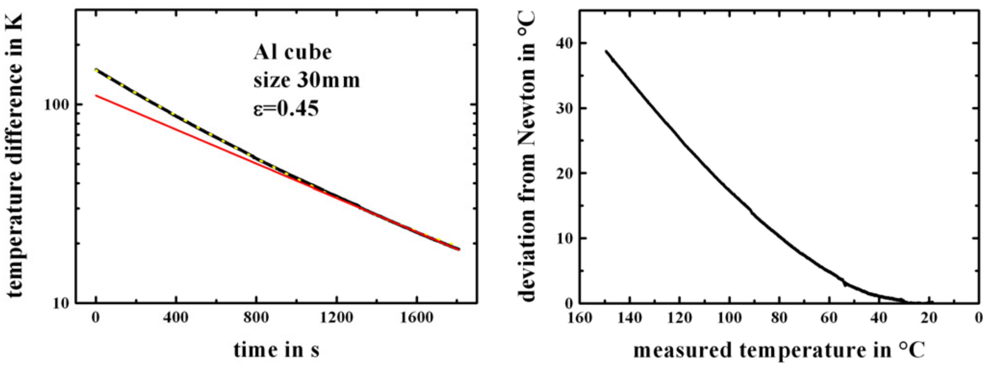

Figure 15. Left: experimental cooling curve of 30 mm Al metal cubes (thick black line), linear fit to temperature data $\Delta T < 30$ K (straight line) and simple exponential fit (dots within black). Right: deviations between measured values (thick black) and linear fit (straight line) as a function of measured temperature.

In order to point out the deviations from Newton's law, a straight line along the low temperature data (red broken line) starts to deviate from the measurement already at around $\Delta T = 40$ K. The (yellow) dots, which coincide nicely with the measurement, resemble a simple exponential decay fit (like equation (11) with $A_2 = 0$ and $A_3 = 0$); however, the constant term T_0 which should be zero for Newton's law (since we already plot the temperature difference) is around 8.8 K. This leads to the curved line in the semi-logarithmic plot, i.e. Newton's law is not fulfilled. However, quite surprisingly, a simple exponential fit with an additional constant term provides a good description of the cooling curve!

The deviations between measured temperatures and a simple fit to the low temperature data ($\Delta T < 30$ K) is also shown in figure 15. The absolute values of the deviations depend on the chosen temperature range of the fit. The fit would still look fine for $\Delta T = 40$ K, maybe

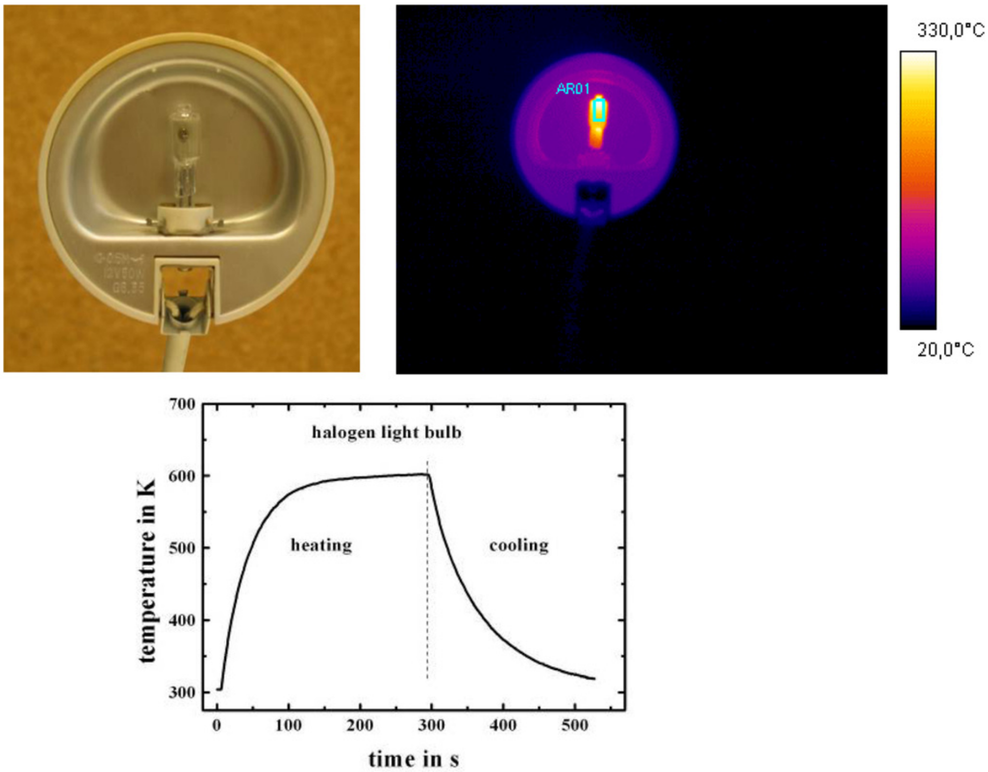

Figure 16. Examples of investigated light bulbs. Samples were placed in front of a room temperature cork board (top, left). Analysis of IR images (top right)reveal that halogen light bulbs reach maximum temperatures >330 °C (bottom) with small relative temperature variations. The heating and cooling of the halogen light bulb covers a temperature difference range of more than 300 K.

even at 50 K. In the case of figure 15, deviations already amount to more than 15 K at $\Delta T = 100$ °C. For temperature differences below 50 °C, deviations are below 2.5 °C. These numbers would decrease if the upper temperature difference for the fit increased.

6.3. Light bulbs: $\Delta T \leqslant 300$ K

The Al metal cubes could not be heated to the high temperatures needed to observe really large deviations of $T(t)$ from the simple exponential law. In order to study higher temperatures experimentally, we used light bulbs. Several light bulbs of different power consumption and size were tested. Experiments were performed with small halogen light bulbs (near cylinder diameter 11 mm and height 17 mm). Their Biot number is much smaller than unity (table 1). Figure 16 shows the lamp and an IR image of the halogen bulb while hot as well as measured surface temperatures during a heating and cooling cycle.

In the following analysis, the maximum temperatures in a small area around the top of the light bulb were used (a test using average rather than maximum temperatures within the indicated area showed that the general form of normalized $T(t)$ curves as in figure 16 changed very little).

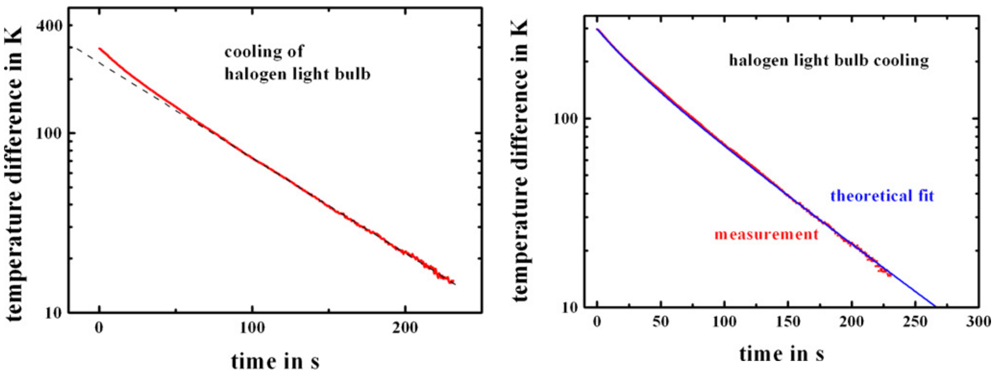

Figure 17. Cooling of the halogen light bulb. Deviations between measured temperatures and expectations from Newton's law occur for $\Delta T > 100$ K as indicated by the straight line (left). The data could be fitted by a double exponential function (solid line, right). For the smallest temperatures, the signal (dots) is very noisy. This is due to the fact that the temperature range of the camera was fixed to 80–500 K during the measurement, hence data below 80 K must be considered with care.

For the measurement in figure 16, the halogen light bulb was powered until an equilibrium temperature was reached, then, the power was turned off and the cooling curve was recorded. Compared to the metal cubes, the cooling of the light bulb occurs much faster due to the small amount of stored thermal energy in the mass of the light bulb. The smaller the system, the smaller the respective time constants (see [5] and figure 13).

Experimental results for the cooling of the halogen light bulb (figure 17) are nicely represented by a simple exponential function for small ΔT values, but deviations occur for larger ΔT values. Theoretically—in comparison to the cubes—we expect larger natural convection heat-transfer coefficients due to the larger temperature difference. Therefore, convection will start to dominate much earlier during the cooling process. Therefore, Newton's law seems to be fulfilled for higher temperature differences of up to about 100 K. However, deviations for larger temperatures are clearly observed.

Similar to figure 7, the cooling curve can be approximated by a higher order exponential function. Since deviations are less pronounced than in the model for figure 7, a second-order fit with time constants of 16.7 s (initial part, amplitude 37.7) and 77.3 s (slower decrease with amplitude 259.3) provides already reasonable results.

7. Summary and conclusions

Nonlinear radiative heat transfer in cooling processes of objects with small Biot numbers does lead to systematic deviations from simple exponential cooling curves. Three related questions were investigated theoretically as well as experimentally: (i) what is the magnitude of deviations? (ii) is it possible to define a range of validity for Newton's law of cooling? and (iii) can these deviations be easily observed experimentally?

The first and second questions are connected to each other. Theoretical studies which assumed a constant convective heat transfer revealed that the magnitude of the deviations does sensitively depend on the ratio between convective and radiative heat-transfer rates. If radiation dominates, deviations from Newton's law become obvious already at low temperature differences, of say 30 K. If, however, convection dominates over the radiative heat transfer, Newton's law may be valid for a much larger range of temperature differences of 500 K (and may be more). In conclusion, there is no single limit of validity, rather it is necessary to discuss the relative contributions of convective and radiative heat-transfer rates.

Therefore, it is easily possible that Newton's law may be observed for temperature differences as high as, e.g., 200 K, in particular, since in some experimental studies fans were used to achieve high convective heat transfer. It is also logical that in many low temperature experiments like with water in flasks, cans, or bottles and $\Delta T < 50$ K, it is found to be a very good approximation.

However, it is also easily possible to experimentally detect deviations from Newton's law. For the experiments performed for this study with Al cubes, it worked quite well for up to 40 or 50 K. For higher temperature differences, deviations were clearly present. Similarly, the high temperature region was investigated with halogen light bulbs, with which ΔT up to 300 K could be realized. Here, Newton's law provided a reasonable approximation up to about $\Delta T = 100$ K, whereas deviations were obvious for larger temperature differences.

In conclusion, Newton's law of cooling does successfully describe cooling curves in many low temperature applications. At a first glance this is indeed surprising, in particular when considering the fact that even around room temperature, radiative heat loss is of the same order of magnitude as convective heat loss. At a closer look, however, this result was to be expected: the range of validity of Newton's law does more or less just depend on the ratio of convective to radiative heat transfer.

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