NEWTON'S THERMOMETER: A MODEL FOR TESTING NEWTON'S LAW OF COOLING

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I. Introduction. In order to forestall misinterpretation it is best to begin by explaining our use of the word "cooling," because this word is often confused with the process of heat transfer, as when we say the body cools by radiation, convection, etc. When heat input exactly equals output, there is no change in body temperature; the body is in the steady state. When heat input exceeds output, the body temperature rises; the body warms. When heat input is less than output, the body temperature falls; the body cools. In all three cases there is heat transfer by radiation, convection, etc. In only the third case, however, does the body undergo cooling. By cooling we mean only the fall in temperature, not the transfer of heat. We are concerned with degrees of temperature, not with calories of heat. In particular we are concerned with the time course of the falling body temperature and the factors which affect this course. Newton's law of cooling is of interest because it expresses the simplest and most basic time course possible. Other time courses can be considered to be deviations from this basic trend caused by one or more extraneous factors.

Newton's law of cooling is an assumption and incredible though it may seem to the reader, whether physicist or physiologist, there is no definitive body of experimental data which defines the limits within which reality conforms to the "law". For the past 200 years it has been suspected that the law holds for only a small range of temperature, but this range has never been established, nor the magnitude of error to be encountered beyond the range. It is possible that this range is sufficiently broad to make the application of the law to biothermal problems both practical and useful. An examination of Newton's law of cooling, its limitations, and its applicability in biothermal investigations is therefore in order.

The history of Newton's law is very interesting but a complete account is a report in itself. Briefly Newton published anonymously in 1701 a paper entitled "Scala graduum Caloris. Calorum Descriptiones & Signa" (6). As the title indicates, he was concerned with the establishment of a scale of temperature. In describing his methods he stated his assumptions about heat transfer and cooling in a somewhat cursory fashion, but he presented no evidence in support of his assumptions. It so happens that Newton's thermometer is an adequate device for testing certain aspects of his law of cooling. It is therefore of both scientific and historical interest to examine the results obtained with his thermometer.

II. Newton's Thermometer. To establish the melting point temperatures of metals and alloys up to that of tin, Newton used a thermometer containing linseed oil. He gave no further description. On a recent trip abroad, this writer learned that there is to be found neither relic nor additional information at either the Royal Society of London or at Cambridge University (Trinity College and the Whipple Science Museum).

On returning to New York and browsing in the New York Public Library, he found the following description by Desaguliers (erstwhile chaplain to the Prince of Wales). (1).

"But as I mention Sir Isaac Newton's Thermometer, I think it will not be improper to give an account of the manner of making of it, as I made three of them once by Sir Isaac's direction. I took a tube of half an inch bore 3 feet long, with a ball of two inches diameter at one end of it, and to the tube pasted a list of paper in order to mark a scale upon it. Then with a measure containing 1/4 of a cylindrick inch, I first fill'd the ball with quicksilver, which contained 21 of those measures (67.6 ml); then at every measure of mercury pour'd into the tube I made a mark upon the paper to form the scales, finding those marks commonly about an inch from one another, but a little farther asunder where the bore of the tube was narrowest, and a less distance than an inch where the bore was bigger, and for greater exactness subdivided all those divisions of the scale into decimals. Then the mercury being well taken out of this thermometer, linseed-oil was pour'd into it up to the 10th or 12th division on the scale of the tube ..."

A replica prepared by a glass blower is shown in Figure 1. By toying around with it we have acquired some understanding of its oddities and appreciation for its virtues. The amber color of the linseed oil makes it clearly visible. (Roemer, living at the same time as Newton, colored the spirits of wine in his thermometer with saffron (5). The wide bore makes it easy for one to pour the oil into the stem. For the filling of a thermometer with a quarter inch stem, we found it necessary to direct the oil down a still smaller tube inserted into the stem all the way down to the bulb. Linseed oil has a coefficient of expansion about four times that of mercury and gives a satisfactory expansion in the large bore thermometer, about three centigrade degrees per millimeter. Because of its high boiling point, linseed oil can be used for measuring temperatures up to the melting point of tin (232^{o} C).

The value of the long stem becomes obvious from a consideration of the technique Desaguliers described that Newton used for determining the hardening points of metals. The bulb of the thermometer was preheated in a sand bath resting on a furnace. Then:

"...the Crucible containing the mixture of lead, tin, and tin-glass "bismuth", was taken off the fire and set upon the ground. We took the thermometer out of its sand cricuble, and thrust its ball into the mixture and took it out again immediately, and this for several times till the mixture in cooling made a skin about the ball of the thermometer; and this we call'd the Degree of Heat capable of melting the Mixture."

For the performance of this maneuver from the standing position, the long stem serves as a convenient handle much as with a golf club.

Finally in 1738 Martine (4) reported some observations on an oil thermometer. He gave no dimensions although he had Newton's thermometer in mind.

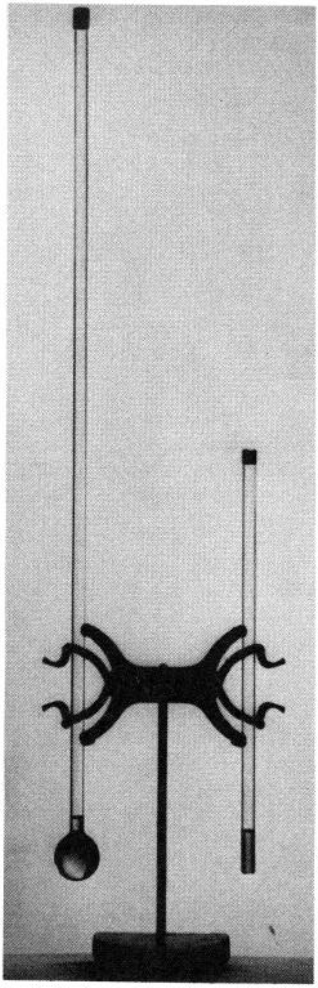

Fig.1. Left, replica of Newton's thermometer made according to the specifications reported by Desaguliers in 1744. Right, the same without the bulb (and shorter).

"But there is another difficulty which will hold in all oil thermometers, or made with a viscid liquor, that it adheres too much to the sides of the tube. In a sudden cold or fall of the oil a good deal sticks by the way, and only sinks gradually after, so that at first the surface appears really lower than the present temperature requires. And beside, as at all times some must continue to stick and moisten the inside of the tube, in different degrees of heat and cold, the oil becoming alternately more or less viscid, will adhere sometimes more and sometimes less; and therefore will inevitably disturb the regularity and uniformity of the thermometer."

We did not find this stickiness to be a problem so long as the glass was first burned clean in the annealing oven.

III. Testing of Newton's Law of Cooling. If Newton had taken the trouble to test his ideas about heat transfer and cooling with his thermometer, he could very easily have become too discouraged to publish his paper even anonymously. Fortunately he refrained from performing the crucial experiment. We did this by heating our thermometer in a sand bath and then timing its cooling in a small wind tunnel with laminar air flow. The tunnel air temperature was maintained constant above general room temperature at about $27^{o} \pm 0.1^{o}$ C. The digital readout timer registered time to 0.01 minute. Cooling was followed both by the descent of the oil with a hand lens and by a thermocouple inserted down the stem into the bulb. The results obtained by the two methods were the same. The data were treated as follows: Newton's law of cooling is expressed by the equation,

$$(T - T_a)_t = (T - T_a)_o \, e^{-kt}$$

where T = temperature of the thermometer

T_a = temperature of the air

t = time

k = cooling constant

$$\frac{(T - T_a)_t}{(T - T_a)_o} = e^{-kt}$$

$$\ln \frac{(T - T_a)_t}{(T - T_a)_o} = -kt$$

The successive temperature differences $(T - T_a)_t$ were divided by the initial difference $(T - T_a)_o$, and these ratios were plotted on semi-log paper. Therefore all plots, regardless of the values for T and T_a, had a common Y-intercept and they differed only in the direction of the trend. According to Newton's assumption the trend should be linear with a slope, -k. The indication that cooling has followed Newton's law therefore is the linearity of the semi-log plot of the successive temperature difference ratios. Deviation from linearity is evidence that "extraneous" factors exerted an influence during cooling.

An example of the results obtained with Newton's thermometer is shown by the lower trend (solid circles) in Figure 2. The points do not follow a single straight line; instead they follow a curve onto which one can impose three lines with successively smaller slopes, -k. Thus Newton's thermometer appears not to "obey" his "law" of cooling.

Three reasons can be adduced to account for the deviation of the trend from linearity in Figure 2.

1. The oil cooled faster in the stem than in the bulb because the surface-area/volume ratio was greater in the stem than in the bulb (roughly 3 times greater when heated to about 200° C). As cooling proceeded and the oil contracted into the bulb, this difference diminished and cooling therefore slowed. The result was that the consecutive temperature excesses were in progressively larger, instead of constant, ratio.

2. Convection currents caused mixing of the cooler oil in the stem with the warmer oil in the bulb. This process accelerated cooling but as cooling proceeded the currents diminished and cooling slowed *pari passu*. As a result again the consecutive temperature excesses were not in constant ratio.

3. Heat transfer by radiation is proportional to the difference between the fourth powers of the absolute temperatures of the thermometer and of the air, and not the first powers as Newton assumed. The difference between the 4th powers diminishes faster than the difference between the 1st powers as cooling proceeds, and so the rate of cooling progressively diminishes more than by a constant ratio of consecutive

differences.

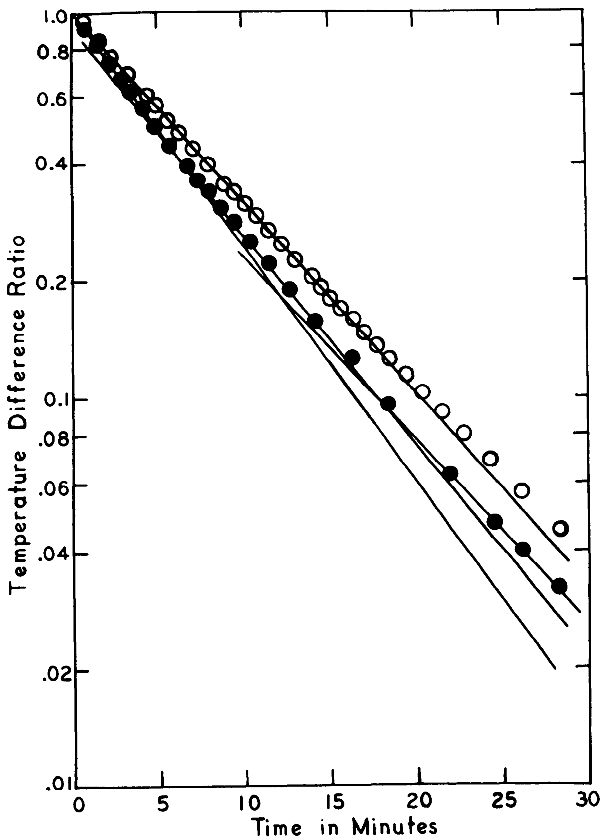

Fig.2. Semi-log plot of the course of cooling of Newton's thermometer in air moving 5 miles/hr. O = with a porous plug at the bottom of the stem. ● = without a porous plug, from 183° toward 25° C.

The first possibility was tested by following the cooling of linseed oil in a half-inch tube without a bulb, shown on the right in Figure . The results are shown in Figure 3 by the lower curve (solid circles). After eight minutes the points deviate slightly from the straight line fitted to the points up to that time. The deviation, however, is much less than it was with a bulb attached as in Figure 2. Hence it appears likely that faster cooling in the stem than in the bulb accounts for most of the deviation from linearity in Figure 2.

To test the possibility that the residual deviation in Figure 3 was due to convection currents, a polyurethane foam plug was placed down in the oil in the tube. The porous plug impeded currents but not the expansion and contraction of the oil. The results are shown by the upper curve (open circles) in Figure 3. The points now fall all on the straight line. Hence with a uniform and constant surface-area to volume ratio, and with minimization of thermal currents, the linseed oil cooled geometrically from 130° to 25° C in air moving 5 miles/hr.

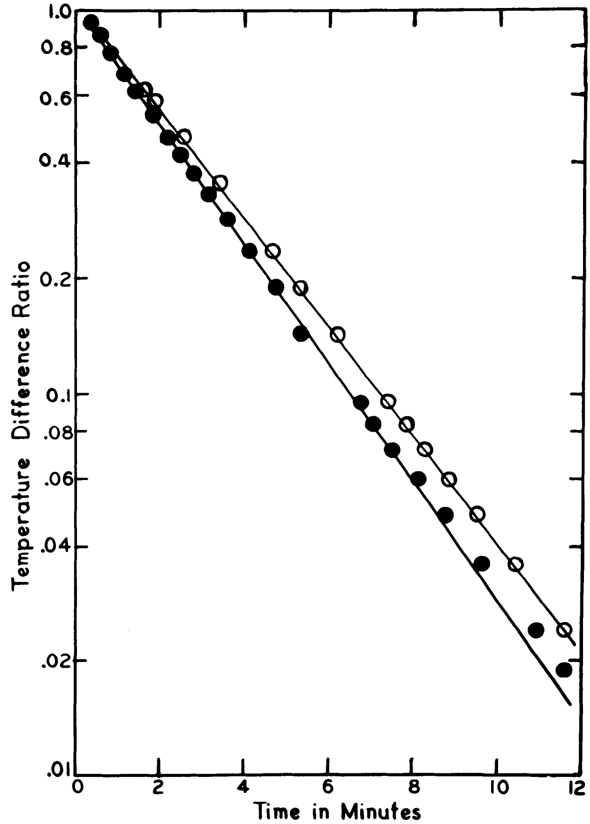

Fig.3. Semi-log plot of the course of cooling of linseed oil in a tube of the same dimensions as the stem of Newton's thermometer. Air flow = 5 miles/hr. O = with a porous plug. ● = without a plug. Cooling from 150° toward 25° C.

In addition to being linear the upper curve in Figure 3 is also of lesser slope (k = -0.326) than is the initial trend of the lower points (k =-0.355). The later trend of the lower points is of an intermediate slope (k = -0.339). This is further evidence that there were mixing currents which by slowing with cooling decelerated cooling and that they were blocked by the foam plug. (The plug was about 2% by weight of oil + plug.)

The experiment was repeated by putting the plug down the stem of the thermometer. As shown by the open circles in Figure 2, the restriction of currents between stem and bulb straightened the trend very considerably by comparison with the trend of the solid circles in Figure 2. Nevertheless there is still a residual terminal deviation with the open circles. Since the plug did not prevent the contraction of the oil from the stem into the bulb, most of which took place initially and little terminally, there was still an interchange of heat between bulb and stem which progressively diminished and thereby slowed cooling.

The slopes give quantitative evidence. For the lower trend they are -0.141, -0.124, and -0.108 for the three segments. With the plug the slope for the initial trend is -0.113 and for the later trend it is -0.103. Thus both the faster cooling in the stem than in the bulb and the interchange between stem and bulb by convection currents and by contraction of the oil account for the deviation of the cooling of Newton's thermometer from Newton's law. The experiments were repeated with water and the results were the same as with oil. The convection currents could be easily made apparent with a drop of India ink.

Confirming evidence for these conclusions is provided by a thermometer with negligible heat exchange via the stem and uniform temperature in the bulb, namely, a mercury thermometer with a small cylindrical bulb (ca. 4x15 mm) and a fine capillary stem. The results of one of three experiments of cooling in the wind tunnel are shown by the lower curve in Figure 4. This mercury thermometer cooled strictly according to Newton's law from 200° to 29° C.

That radiation is seemingly not an important factor in these experiments is probably due to the fact that the fraction of heat transfer by radiation was intentionally minimized by performing the experiments in a wind of 5 miles/hr. The opposite condition, the maximization of the fraction of heat transfer by radiation, was attempted by Ericsson in 1876 (3). He followed the cooling of water in a thin, blackened, copper sphere 2.75 inches in diameter. He placed the sphere within a large double-walled sphere through which he circulated ice water. The intervening air was exhausted but Ericsson did not report the resulting pressure. He presumed that heat transfer was by radiation alone. He stirred the water with a paddle to insure uniformity of temperature from center to surface. Revolving the paddle 30 times per minute did not raise the temperature of the water measurably. He measured the water temperature with a mercury thermometer having a cylindrical bulb.

Figure 5 shows the results from his tabulation for one experiment for cooling from 56.7° toward 0.5° C. The trend is initially linear but then progressively non-linear upward. The deviation at 74 minutes is 0.8°; i.e., when the linearly extrapolated temperature difference would have been 5.7° (0.1 of the initial difference), the temperature difference on the curve of measurements is 6.5° (0.115 of the initial difference). Ericsson ascribed this deviation to a diminution in "emissive power" with fall in temperature. Since he published in the decade preceding the appearance of the Stefan-Boltzmann law, he was unaware of the role of the 4th power of absolute temperatures in heat transfer by radiation. The deviation from linearity in Figure 5 was most probably due to the fact that the difference between the 4th powers diminishes more rapidly than the difference between the 1st powers. Even so under conditions maximizing the fraction of heat transfer by radiation, the deviation was small even after 9/10th of the cooling had taken place. At this time the error of Newton's law was (6.5° - 5.7°)/6.5° x 100 = 12.3%.

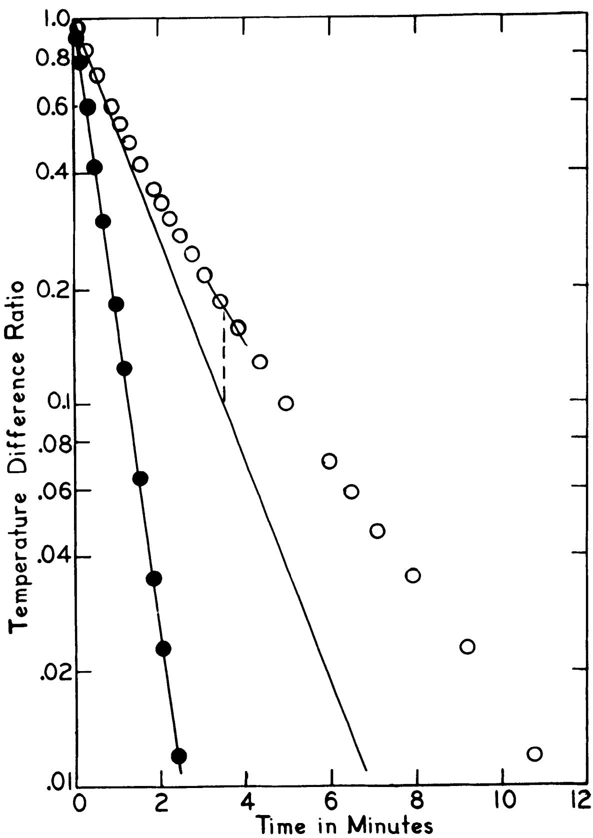

Fig.4. Semi-log plot of the course of cooling of a sphere of water under conditions maximizing the fraction of heat transfer by radiation. Data of Ericsson in 1876 (3).

Finally to ascertain the course of cooling in still air, the mercury thermometer was permitted to cool in the wind tunnel with no air flow through it but with a hole (diameter = 17.5cm) in the top. This arrangement excluded most of the stray currents in the room but permitted the

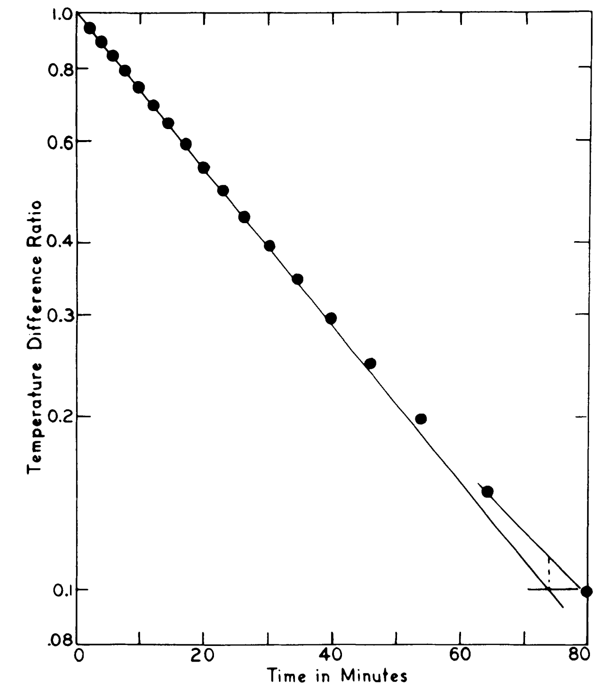

Fig.5. Semi-log plot of the course of cooling of a mercury thermometer with a bulb 4 x 15 mm and a stem with a fine capillary bore. 0 = still air. ● = air moving 5 miles/hr. From 200° toward 29° C.

vertical currents of natural convection. The results are shown in the upper curve of Figure 4. The trend is obviously non-linear; i.e. , cooling did not proceed geometrically as it did for the same thermometer in a wind (lower curve Fig. 4). The linear extrapolation crosses the 0.1 ordinate at 3.5 minutes. At this time the measured ratio is 0.177; i.e., the measured temperature difference at 3.5 minutes was 0.177 x 172° = 30.4° while the linear extrapolation calls for 0.1 x 172° = 17.2°. Thus the deviation of expected from observed was (30.4° - 17.2°)/30.4° x 100 = 43.4%. This is about 3.5 times the deviation from linearity observed by Ericsson with his radiator; therefore the effect of radiation is only a partial explanation. Much more important is the fact that as cooling proceeds in still air and the surface to air temperature difference diminishes, the natural convection generated by this temperature difference diminishes. The heat transfer coefficient therefore progressively diminishes and cooling is retarded more and more.

IV. <u>Discussion.</u> These results with simple devices prove that cooling in air can proceed geometrically over a range of more than 170 C, providing the heat transfer coefficient remains constant as in a steady forced convection. This range of temperature is sufficient for most cooling and warming experiments on organisms. Although mention has been made of Newton's law in the physiological literature (7), it has as yet not been systematically developed and exploited. The limitation however is not one of range of temperature.

The view that Newton's law holds only for a small range of temperature antedates the Stefan-Boltzmann law by at least a century. According to Dulong and Petit (2), Erxleben reported in 1777 that cooling deviates more from Newton's law the greater the temperature range for cooling. Without going into details it is sufficient to say that most if not all of the experiments reported after Newton were performed under still air conditions. Newton had however specified the use of a constant current of air. He placed his cooling object "non in aere tranquillo sed in vento uniformiter spirante" so that equal amounts of air warmed in equal times by the heat from the cooling object (a block of iron for Newton) would be uniformly replaced by cold air and would carry away amounts of heat proportional to the temperature of the cooling object.

In the analysis of cooling data therefore it is legitimate to start with the expectation that the temperature changes have followed Newton's law, i.e., that on a semi-log plot the trend will be linear. Non-linearity will be an indication that "extraneous" factors interfered during cooling.

Two such factors with Newton's thermometer were the difference in the surface-area/volume ratio for the stem and bulb and the presence of convection currents between stem and bulb. These two factors could be eliminated by remaking the thermometer so that it was geometrically uniform and by impeding the convection currents, i.e., by using a tube without a bulb and by inserting a sponge.

A third factor observed with the mercury thermometer was the progressive diminution in the heat transfer coefficient in still air. Under this condition the cooling constant cannot be constant. It is therefore advantageous to perform experiments in moving air, if only a slight draft, to maintain the heat transfer coefficient constant.

The reason that Newton's thermometer can serve as a model is that there are biological structures which are geometrically similar to the stem and bulb of Newton's thermometer, e.g., the finger and hand of man, tail and body of rat, proboscis and head of the elephant, etc. Further the transport of heat by the blood stream is similar to the transport by the convection currents of the oil. Therefore cooling of the hand may possibly be non-Newtonian even as cooling of Newton's thermometer is non-Newtonian. If hand cooling should prove to be Newtonian, then it would be worthwhile to look for the factor which minimized the effect of the fingers on the cooling of the hand.

V. Conclusions

1. Cooling proceeds geometrically from at least 200° C down to

room temperature, i.e., according to Newton's law, if the cooling object is geometrically uniform, the internal conductivity remains constant, and the heat transfer coefficient remains constant. These conditions obtain with a mercury thermometer cooling in a constant stream of air.

2. In the analysis of body temperature changes, it is in order to start with the assumption that Newton's law holds for the conditions of the experiment. To see if the data comply, they should be plotted on a semi-log grid. If the trend deviates from linearity, then the "interfering" factors should be sought for. In the case of Newton's thermometer two factors were found: the greater surface-area/volume ratio of the stem versus the bulb, and the convection currents between stem and bulb. Comparable factors may obtain in an organism as between finger and hand.

3. To insure that the heat transfer coefficient remains constant during an experiment, it is best to proceed even as Newton did, i.e., to permit cooling to occur in a constant stream of air instead of in still air.

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