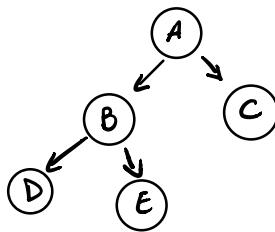


## Tree Traversals



- Preorder: ABDEC
- Inorder: DBEAC
- Postorder: DEBCA
- Level-order: ABCDE

[LC] Left Child:  $\approx n+1$ ;  $N = RC_2^{-1}$   
 [RC] Right Child:  $\approx (n+1)$ ;  $N = (LC \cdot 1)/2$

Recursion simplifies the implementation of tree traversals.

↳ Pre order:

- visit node
- Traverse (left child)
- Traverse (right child)

n | r

↳ Postorder:

- Traverse (left child)
- Traverse (right child)
- visit node

r | n

↳ Inorder:

- Traverse (left child)
- visit node
- Traverse (right child)

l | n

↳ Level-order:

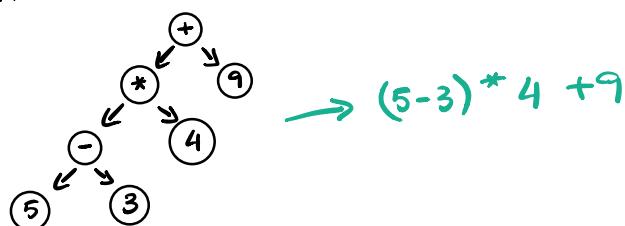
- more complicated.
- requires use of extra data structures (queues and/or lists) to create the necessary order.

↑ depends on type

Binary trees are ADT → has no add/delete

## Expression Trees

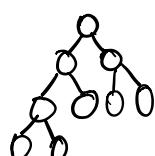
- shows the relationships among operators and operands in an expression
- It's evaluated from the bottom



Full: every node has '0' or '2' children

Complete: all levels are completely filled, except possibly the last level  
 and the last level has all nodes as left as possible. Top-to-bottom, left-to-right

Maximum number of nodes at level  $L$ :  $\approx 2^{(L-1)}$   
 Maximum number of nodes in a tree of height  $H$ :  $\approx 2^{(H+1)} - 1$



# Binary Search Tree (BST)

## • search tree:

Tree whose elements are organized to facilitate finding a particular element when needed.

## • binary search tree:

Binary tree that, for each node  $n$ :

- + the left subtree of  $n$  contains elements less than the element stored in  $n$ .
- + the right subtree of  $n$  contains elements greater than or equal to the element stored in  $n$ .

- The particular shape of a binary search tree depends on the order in which the elements are added.
- The shape may also be dependant on any additional processing performed on the tree to reshape it.
- Binary search trees can hold any type of data, so long as we have a way to determine relative ordering
- Objects implementing the Comparable interface provide such capability.

## ADDING

- Process of adding an element is similar to finding an element  $\Rightarrow$  adding  $\approx$  finding
- New elements are added as leaf nodes
- Start at the root, follow path dictated by existing elements until you find no child in the desired direction.
- Then add the new element.

## REMOVAL

- Removing a target in a BST is not as simple as that for linear data structures.

- After removing the element, the resulting tree must still be valid.

- Three distinct situations must be considered when removing an element:

- The node to remove is a leaf.  $\rightarrow$  It can simply be deleted
- The node to remove has one child.  $\rightarrow$  the deleted node is replaced by the child
- The node to remove has two children.  $\rightarrow$  an appropriate node is found lower in the tree and used to replace the node

↳ # Good choice: inorder successor (node that would follow the removed node in an inorder traversal.  
# The inorder successor is guaranteed not to have a left child.  
# Thus, removing the inorder successor to replace the deleted node will result in one of the first situations (it's a leaf or has a child).

