Tarea 1.4

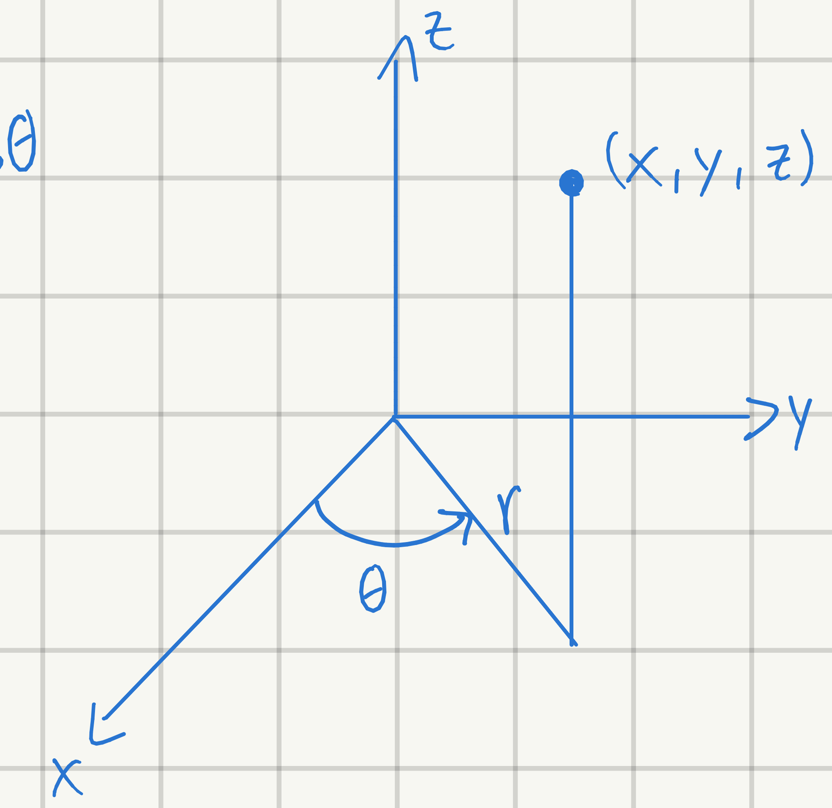
Pozos González Alexis
4CM1

Expresar las conds de CR en coordenadas Cilíndricas (r, θ, z)

$x = r\cos\theta$

$y = r\,\text{sen}\,\theta$

$z = z$



$\to f(z) = u(r,\theta,z) + i v(r,\theta,z)$ $\{$ u y v son funciones cilin.

Así

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial r}$$

$$= \frac{\partial u}{\partial y}\cos\theta + \frac{\partial u}{\partial y}\text{sen}\,\theta + \cancel{\frac{\partial u}{\partial z}}\cdot 0$$

$$= \frac{\partial u}{\partial x}\cos\theta + \frac{\partial u}{\partial y}\text{sen}\,\theta$$

$$\frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial \theta} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial \theta}$$

$$= \frac{\partial u}{\partial x}(-r\,\text{sen}\,\theta) + \frac{\partial u}{\partial y} r\cos\theta + \cancel{\frac{\partial u}{\partial z}}\,0$$

$$= -\frac{\partial u}{\partial x} r\,\text{sen}\,\theta + \frac{\partial u}{\partial y} r\cos\theta$$

$$\frac{\partial v}{\partial r} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial v}{\partial z}\frac{\partial z}{\partial r}$$

$$= \frac{\partial v}{\partial x}\cos\theta + \frac{\partial v}{\partial y}\text{sen}\,\theta + \cancel{\frac{\partial v}{\partial z}}\cdot 0$$

$$= \frac{\partial v}{\partial x}\cos\theta + \frac{\partial v}{\partial y}\text{sen}\,\theta$$

$$\frac{\partial u}{\partial \theta} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial \theta} + \frac{\partial v}{\partial z}\frac{\partial z}{\partial \theta}$$

$$= \frac{\partial v}{\partial x}(-r\,\text{sen}\,\theta) + \frac{\partial v}{\partial y} r\cos\theta + \cancel{\frac{\partial v}{\partial z}}\,0$$

$$= -\frac{\partial v}{\partial x} r\,\text{sen}\,\theta + \frac{\partial v}{\partial y} r\cos\theta$$

$$\frac{\partial u}{\partial x}\cos\theta + \frac{\partial u}{\partial y}\text{sen}\,\theta = -\frac{\partial v}{\partial x} r\,\text{sen}\,\theta + \frac{\partial v}{\partial y} r\cos\theta$$

$$\frac{\partial u}{\partial x}\cos\theta + \frac{\partial u}{\partial y}\text{sen}\,\theta = \frac{1}{r}\frac{\partial y}{\partial \theta}$$

$$-\frac{\partial u}{\partial x} r\,\text{sen}\,\theta + \frac{\partial u}{\partial y} r\cos\theta = \frac{\partial v}{\partial x}\cos\theta + \frac{\partial v}{\partial y}\text{sen}\,\theta$$

$$\frac{1}{r}\frac{\partial u}{\partial \theta} = \frac{\partial v}{\partial r}$$

$$\frac{\partial u}{\partial \theta} = r\frac{\partial v}{\partial r}$$

$$\therefore \frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0$$