

MEASUREMENT OF STATIC AND DYNAMIC PERFORMANCE CHARACTERISTICS OF SMALL ELECTRIC PROPULSION SYSTEMS

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ABSTRACT

Unmanned aerial vehicles are being utilized by numerous groups around the world for various missions. Most of the smaller vehicles that have been developed use commercially-off-the-shelf parts, and little information about the performance characteristics of the propulsion systems is available in the archival literature. In light of this, the aim of the present research was to determine the performance of various small-scale propellers in the 4.0 to 6.0 inch diameter range driven by an electric motor. An experimental test stand was designed and constructed in which the propeller/electric motor was mounted in a wind tunnel for both static and dynamic testing, and the results were compared to those from previous studies. For static testing, the coefficient of thrust, the coefficient of propeller power, and the total propulsive efficiency, defined as the ratio of the propeller output power to the electrical input power, were plotted versus the propeller rotational speed. For dynamic testing, the rotational speed of the propeller was held constant at regular intervals while the airspeed was increased from zero to the windmill state. The coefficient of thrust, the coefficient of propeller power and the propeller efficiency were plotted versus the advance ratio for various rotational speeds. The thrust and torque were found to increase with rotational speed, propeller pitch and diameter, and decrease with airspeed. Using the present results and data from archival and non-archival sources, it was found that the coefficient of thrust could not be correlated with propeller diameter for square propellers where $D = P$. For a family of propellers (same manufacturer and application), correlations for the coefficient of thrust, the coefficient of propeller power and the propeller efficiency could be improved by modifying either the coefficients or the advance ratio with D/P . This dimensionless ratio allows for the propeller pitch to be accounted for in the performance coefficients.

NOMENCLATURE

A_P	propeller disk area, m^2
A_{WT}	wind tunnel test section area, m^2
C_P	coefficient of propeller power
C_Q	coefficient of torque
C_T	coefficient of thrust
D	propeller diameter, m
F_D	fixture drag, N
FOM	figure of merit
H	height of the Pitot tube from the bottom of the wind tunnel test section, m
I	electrical motor current, Amperes
J	advance ratio
n	propeller rotational speed, rev/s
P	propeller pitch, m
P_{atm}	atmospheric pressure, Pa
P_{diff}	Pitot tube differential pressure, Pa
P_e	electrical input power, W
P_P	propeller output power, W
Q	torque, N-m
R	particular gas constant, J/(kg-K)
R^2	goodness of fit parameter
T	measured thrust, N
T'	corrected thrust, N
T_{atm}	atmospheric temperature, K
V	electric motor voltage, Volts
V_∞	free-stream velocity, m/s
V'_∞	corrected free-stream velocity, m/s
W	wind tunnel test section width and height, m
Δ	uncertainty
η_P	propeller efficiency
η_T	total propulsive efficiency
ρ	density, kg/m^3
τ_4	Glauert correction variable

INTRODUCTION

Interest in the performance of small propellers operating at low Reynolds numbers has grown recently. The aerospace industry has developed numerous unmanned aerial vehicles (UAVs) and has kept most of the data about the propulsion systems proprietary. Very little information is available in the archival literature about the performance characteristics of these motor and propeller combinations. The present research and others like it have aimed to gather and compare information about these small propulsion systems so that proper motor and

propeller combinations can be selected for a given mission profile. Several papers were reviewed that relate directly to the present work and provide direction for the research.

Brandt and Selig (2011) experimentally determined efficiency as well as coefficients of thrust and power for low Reynolds number propellers. The parametric ranges were as follows: Propeller diameter $9 \leq D \leq 11$ inches, propeller rotational speed $1500 \leq n \leq 7500$ RPM, and the incoming air velocity V_∞' ranged from zero (static) to the windmill state of each propeller, i.e., that point at which the propeller generates zero thrust. A test stand was built inside the UIUC wind tunnel to measure thrust, torque, and propeller rotational speed. Freestream air velocity was measured using a Pitot tube and one of two differential pressure transducers depending on the airspeed range. Velocity corrections were applied to account for the change in upstream airspeed at the Pitot tube created by the propeller as well as the pressure change created by the fairing and the constriction of the propeller slipstream caused by the walls. In total, 79 propellers from four different manufacturers were tested to find the coefficient of thrust, the coefficient of power and the propeller efficiency, all of which were plotted against advance ratio. The designs of the propellers ranged from those for electric motors to those used for fuel-powered engines. For each test, the rotational speed of each propeller was fixed while the freestream airspeed was varied. Four different values of propeller rotational speed ($n = 3000, 4000, 5000$, and 6000 RPM) were tested for each of the propellers. The results show that the propeller efficiency increases with the propeller speed. This is primarily due to the increase in Reynolds number as the propeller spins faster. Overall, the propeller efficiency ranged from $28 \leq \eta_p \leq 65\%$. The propellers were also tested statically, but the data is only available in the UIUC propeller database (Selig, 2012).

Gamble (2009) designed an intricate LabVIEW program to automatically collect data and generate propeller performance plots. A dynamometer was constructed using beam-type load cells to measure thrust and torque. The development of the LabVIEW program was detailed as well as a procedure for carrying out the experiment. Propellers were tested for repeatability by performing identical experiments over several days with two identical propellers. The results primarily focus on the effect of the Reynolds number on thrust and power coefficients and efficiency versus advance ratio. Thrust versus velocity was compared for propellers with constant diameter and varying pitch. Lastly, advance ratio was modified by replacing diameter with pitch in the equation for advance ratio. The optimal advance ratio is shown using this technique. This allows for the optimal pitch of a model propeller to be selected to achieve

maximum efficiency. The diameter can then be chosen from plots of thrust versus velocity to produce the required thrust for the airframe.

Deters and Selig (2008) performed static tests on smaller propellers ranging from $2.5 \leq D \leq 5$ inches in diameter. Static coefficients of thrust and power as well as the figure of merit ($FOM = C_T^{3/2} / \sqrt{2} C_P$, typically used to measure the efficiency of helicopters) using modified coefficients of thrust and power that use disk area and tip speed were determined experimentally. The test stand utilized a 0.3 kg load cell and a 25 oz-in torque transducer to measure thrust and torque, respectively. Propeller rotational speeds ranging from $2500 \leq n \leq 27,000$ RPM were measured using an infrared detector. A schematic of the test stand indicated the locations of the components and a fairing surrounding the load cell and torque transducer. Calibrations of the components were performed and data was collected using a data acquisition board. The geometry of each propeller was found using PropellerScanner software (Hepperle, 2003) to find the chord and twist distribution. This was used to calculate the Reynolds number at the 75% chord location. Results show that over the rotational speed range tested, the figure of merit remained fairly constant throughout the test. The results also show that a larger diameter propeller is more efficient than a smaller one, and a propeller with a lower pitch is more efficient than one with a higher pitch.

Ol et al. (2008) took a more analytical approach to studying small propellers operating at low Reynolds numbers. Iterative methods were used to calculate the coefficient of thrust, the coefficient of torque, and the propeller efficiency using propeller momentum theory and blade-element methods. Propellers were discretized by cutting and tracing sections as well as digital scans. Leading and trailing edges were fitted to the UIUC propeller library so that the resulting analysis in XFOIL would successfully converge. The iterative process for thrust was dependent on the various Reynolds numbers across the propeller blade at a given rotational speed. Two separate experimental setups were constructed to compare the numerical results. Propellers in the $6 \leq D \leq 12$ inch range were tested in the Langley Research Center Basic Aerodynamics Research Tunnel (BART) and larger propellers in the $14 \leq D \leq 20$ inch range were tested in the AFRL Vertical Wind Tunnel (VWT). Static tests were performed with the wind tunnel sides open to alleviate the induced airflow velocity inside the wind tunnel. Blockage corrections were applied to BART tests but not to VWT tests, since the tunnel diameter of the VWT was greater than five times the diameter of the propellers tested. Drag on the test stand was corrected by sweeping

tunnel velocity and generating curve fits that were used to adjust the actual data. A large sensitivity to twist distribution was observed in the tests and the analysis. Ol et al. postulated that plots of torque coefficient versus advance ratio are sometimes misleading because they do not account for Reynolds number effects. It was also shown that when the ratio of diameter to pitch is scaled (10×10 to 12×12 , for example) the experimental data fits together well within the bounds of error. Modifications to the dimensionless terms to factor in propeller pitch were presented, however more research was deemed necessary to apply this theory.

Corrigan and Altman (2008) examined different methods for wind tunnel blockage corrections. These methods included the Glauert (1926) correction as well as a correction by Hackett et al. (1979). These methods were described in detail and their applications were shown. A wind tunnel experiment was designed and constructed to record the necessary variables to calculate total propulsive efficiency. This is in contrast to other works that primarily explored propeller efficiency. The stand was constructed using a beam-type load cell and a reaction torque sensor. Three propellers ($D = 10, 12$, and 14 inches) were tested using different motors for each propeller. Static pressure taps were used on the wall of the wind tunnel test section to record the changes in pressure forward and aft of the propeller disk plane for the velocity corrections. The Glauert method did not provide sufficient correction for large blockage conditions. The Hackett method yielded more correction at higher airspeeds and larger propeller diameters, but the method could not be validated and therefore further work was found to be necessary.

Merchant and Miller (2006) performed dynamic tests on propellers in the $6 \leq D \leq 22$ inch range. A test stand was constructed to record propeller performance parameters, where the thrust and torque were collected by a combined thrust/torque cell. The load/torque cell was calibrated using dead weights in the axial (thrust) and transverse (torque) directions. Wind tunnel velocity was measured directly using a Pitot probe and a differential pressure transducer. Since the propellers were large compared to the test section, blockage corrections developed by Glauert (1926) were applied to the results. Readings were taken at wind-off-zero conditions before and after each test. These values were then averaged and subtracted from the test data to account for zero drift and temperature effects. Data was collected at constant propeller rotational speeds and the wind tunnel velocity was varied to sweep through values of advance ratio. The results were compared to other works and were shown to be acceptable. The setup was also tested for variations in flow angularity. Pitch and yaw variations between -3 and $+3$ arc degrees were

examined and it was shown that only the coefficient of thrust was affected by a change in pitch. However, it was shown that pitch variations of -3 and $+3$ degrees yielded the same results, which indicated that the system was symmetric in the pitch direction. Lastly, two identical propellers made by the same manufacturer were tested and compared, which showed that for some propellers there may be significant differences in performance due to manufacturing. Very limited results were presented, however, and the results shown only give a small sample of the entire test range.

The objective of the present research was to determine the performance of various commercially-available small-scale propellers driven by an electric motor. An experimental test stand was designed and constructed in which the electric motor was mounted in a wind tunnel at Wright State University for both static and dynamic testing. The freestream airspeed was varied from zero to the windmill state for each propeller. The rotational speed was varied over the operational range recommended by the propeller manufacturers, while ensuring that the electric motor did not overheat. The primary measurement devices were calibrated, and an extensive uncertainty analysis was performed. The results from the present experiment were compared to those from previous studies for both static and dynamic data. For static testing, the coefficient of thrust, the coefficient of propeller power and the total propulsive efficiency were plotted versus the propeller rotational speed. For dynamic testing, the rotational speed of the propeller was held constant at regular intervals while the freestream airspeed was increased from zero to the maximum. The coefficient of thrust, the coefficient of propeller power and the propeller efficiency were plotted versus the advance ratio for various rotational speeds.

BACKGROUND

The performance characteristics to be determined by the experimental setup are as follows. The coefficients of thrust, torque and propeller power, and the propeller efficiency are (Merchant and Miller, 2006):

$$C_T = \frac{T'}{\rho n^2 D^4}, \quad C_Q = \frac{Q}{\rho n^2 D^5}, \quad C_P = \frac{P_P}{\rho n^3 D^5}, \quad \eta_P = \frac{J C_T}{C_P}$$

The three performance coefficients and the propeller efficiency defined above are typically plotted against the advance ratio for dynamic testing:

$$J = \frac{V'_\infty}{nD}$$

where the corrected freestream velocity is (Glauert, 1926):

$$V'_\infty = V_\infty \left[1 - \frac{\tau_4 \left(\frac{A_P}{A_{WT}} \right)}{2\sqrt{1 + 2\tau_4}} \right]$$

The uncorrected freestream velocity is:

$$V_\infty = \sqrt{\frac{2P_{\text{diff}}}{\rho}}$$

The Glauert correction variable is:

$$\tau_4 = \frac{T'}{\rho A_P V_\infty^2}$$

The propeller disk area and wind tunnel area are, respectively:

$$A_P = \frac{\pi D^2}{4}, \quad A_{WT} = W^2$$

The corrected thrust is defined as the measured thrust minus the drag force due to the flow of air over the motor, torque cell and load cell (Selig and Ananda, 2011):

$$T' = T - F_D$$

The total propulsive efficiency is the ratio of the propeller output power to the electrical input power:

$$\eta_T = \frac{P_P}{P_e} = \frac{2\pi n Q}{VI}$$

The density of air is given by the perfect gas law:

$$\rho = \frac{P_{\text{atm}}}{RT_{\text{atm}}}$$

EXPERIMENTAL SETUP

The objective of the present experiment was to determine the performance characteristics of small electric motor/propeller combinations from static conditions to the windmill state. The overall design of the dynamic test rig is shown in Figure 1. The electric motor was directly attached to a 25 oz-in torque cell (Transducer Techniques, Model RTS-25), which was able to withstand 10 kg in thrust and 1.7 kg in shear. The torque cell was in turn mounted onto a 1-kg single point beam-type load cell (Transducer Techniques, Model LSP-1). Each cell was driven

by a signal conditioner (Transducer Techniques, Model TMO-1) that produced a 0 to 5 Volt linear output. The assembly of the motor, torque cell and load cell is shown in Figure 2. The motor was held in place with a custom-designed clam-shell clamp, in which fins were incorporated to increase the convective heat transfer from the electric motor.

The load cell was attached to a section of 1.25-inch square aluminum tubing, which acted as a riser to place the propeller in the middle of the test section. The bottom of the riser was connected to an optical breadboard table (Melles-Griot, Model BBSS-25-610-1219) using flanges of angle aluminum. A hole was milled in the acrylic floor of the wind tunnel for the aluminum riser to pass through. The low-speed wind tunnel at Wright State University is an open circuit design capable of producing speeds from 0.6 to 36 m/s with a contraction ratio of 6.25:1. The square entrance of the wind tunnel has a 3.8 m^2 opening with aluminum hexagonal honeycomb sections that serve as a flow straightener. The height and width of the square test section is $W = 0.6096 \text{ m}$, and its length is 2.438 m. Doors on one side of the test section allow for an entire wall to be opened for easy access.

The data acquisition system used to collect data from the instrumentation consisted of a DAQ board (National Instruments, Model SCC-68) and a DAQ card (National Instruments, Model PCI-6221) installed in a PC. Shielded wires were used to connect the outputs of the transducers to the DAQ board. The electric motor driving the propeller was energized using a precision DC power supply (Hewlett-Packard, Model 6012B). A servo tester (GWS, Model MT-1) was used to control the rotational speed of the propeller (Corrigan and Altman, 2008). The voltage supplied to the electric motor was measured using a digital multi-meter (National Instruments, Model USB-4065). To measure the current, a DC Hall effect current transducer (CR Magnetics, Model CR5210-30) with a range of 0 to 30 A was placed in-line between the power supply and the motor speed controller.

A remote optical sensor (Monarch Instrument, Model ROS-W) connected to a panel meter (Monarch Instrument, Model ACT-3X) was used to measure propeller rotational speed. Reflective tape supplied with the sensor was placed near the hub on the leeward side of the propeller so that the optical sensor did not have to be adjusted between runs.

Atmospheric pressure was measured to determine the density of the air. To record atmospheric pressure, a barometer (Vaisala, Model PTB110) capable of measuring 500 to 1100 mbar with accuracy of $\pm 0.3 \text{ mbar}$ was used. The differential pressure produced by the Pitot tube

was measured using a differential pressure manometer (MKS, Model 226A). The height of the Pitot tube from the floor of the wind tunnel was selected by traversing the boundary layer thickness using the Pitot tube as outlined by Brezina (2012). The height was set to $H = 2.5$ inches, and the Pitot tube was made parallel to the wind tunnel walls by using a bubble level and a custom-made jig.

The temperature of the motor was measured using a Type T thermocouple while the temperature of the air inside the wind tunnel was measured using a Type E thermocouple probe (Omega, Model EMQSS-125G-12). The Type T thermocouple junction was placed on the center of the motor and was held in place by the aluminum clam-shell clamp. The Type E probe was mounted in the floor of the wind tunnel ahead of the motor/propeller so that the sensing junction extended into the airflow. The thermocouples were connected to thermocouple modules (National Instruments, Model SCC-TC01) on the data acquisition board. The signals from the eight sensors were read using custom-designed LabVIEW virtual instruments.

The twenty-four propellers selected for analysis ranged from $4.0 \leq D \leq 6.0$ inches in diameter and $2.0 \leq P \leq 5.5$ inches in pitch, as shown in Table 1. Some of the propellers were selected to overlap with previous research so that the procedures and test setup used for the measurements could be compared and validated. The GWS 4.5×3.0 and GWS 5.0×4.3 inch propellers were tested statically and compared to Deters and Selig (2008). An APC 8.0×3.8 inch Slow Flyer was tested dynamically and compared to the results posted on the UIUC Propeller Database (Selig, 2012), while an APC 6.0×4.0 inch propeller was tested dynamically and compared to the results presented by Ol et al. (2008).

UNCERTAINTY ANALYSIS

The uncertainties of all of the calculated results described in the above equations were determined using the root-sum-square uncertainty method (Kline and McClintock, 1953). During experimentation, eight primary measurements were made using the data acquisition system: Uncorrected thrust (T); torque (Q); propeller rotational speed (n); atmospheric pressure (P_{atm}); atmospheric temperature (T_{atm}); Pitot tube pressure difference (P_{diff}); motor voltage (V); and motor amperage (I). The uncertainty of a given measurement was estimated to be the sum of the calibration uncertainty and the confidence interval of the collected data set at a confidence level of 99%:

$$\Delta U = \Delta U_{\text{CAL}} + \Delta U_{99}$$

In order to calibrate the torque cell, two identical arms were attached to the sides of the motor clamp so that the torque cell could be calibrated in both directions of rotation simultaneously. Varying weights were hung from one of the arms to calibrate in the clockwise direction, and then the process was repeated for the counterclockwise direction. The load cell used to measure thrust was calibrated in situ as follows: A strand of fishing line was attached to the front of the propeller using aircraft wire. This strand was then passed over a smooth cylinder with bearings mounted in the wind tunnel. Varying weights were suspended from the fishing line over the expected range of thrust, and voltage readings were recorded.

The drag of the fixture was measured versus airspeed by removing the propeller and replacing it with a propeller hub with the blades removed. The airspeed was increased systematically while data was collected from the load cell and the Pitot tube. The free-stream velocity was then calculated and the measured drag was plotted against the velocity. A second-order regression was applied to the points and this equation was used in the calculation of the corrected thrust.

Table 2 gives the uncertainties for each device or transducer used to collect the data. The principal equations used for determining the uncertainties of the computed quantities shown in the graphs in the Results and Discussion section are shown below.

Coefficient of Thrust:

$$\Delta C_T = \left[\left(\frac{\Delta T}{\rho n^2 D^4} \right)^2 + \left(\frac{-T \Delta \rho}{\rho^2 n^2 D^4} \right)^2 + \left(\frac{-2T \Delta n}{\rho n^3 D^4} \right)^2 + \left(\frac{-4T \Delta D}{\rho n^2 D^5} \right)^2 \right]^{\frac{1}{2}}$$

Coefficient of Torque:

$$\Delta C_Q = \left[\left(\frac{\Delta Q}{\rho n^2 D^5} \right)^2 + \left(\frac{-Q \Delta \rho}{\rho^2 n^2 D^5} \right)^2 + \left(\frac{-2Q \Delta n}{\rho n^3 D^5} \right)^2 + \left(\frac{-5Q \Delta D}{\rho n^2 D^6} \right)^2 \right]^{\frac{1}{2}}$$

Coefficient of Power:

$$\Delta C_P = \left[\left(\frac{\Delta P_P}{\rho n^3 D^5} \right)^2 + \left(\frac{-P_P \Delta \rho}{\rho^2 n^3 D^5} \right)^2 + \left(\frac{-3P_P \Delta n}{\rho n^4 D^5} \right)^2 + \left(\frac{-5P_P \Delta D}{\rho n^3 D^6} \right)^2 \right]^{\frac{1}{2}}$$

Propeller Efficiency:

$$\Delta\eta_P = \left[\left(\frac{C_T \Delta J}{C_P} \right)^2 + \left(\frac{\Delta C_T J}{C_P} \right)^2 + \left(\frac{-J C_T \Delta C_P}{C_P^2} \right)^2 \right]^{\frac{1}{2}}$$

Advance Ratio:

$$\Delta J = \left[\left(\frac{\Delta V'_\infty}{nD} \right)^2 + \left(\frac{-V'_\infty \Delta n}{n^2 D} \right)^2 + \left(\frac{-V'_\infty \Delta D}{nD^2} \right)^2 \right]^{\frac{1}{2}}$$

Total Propulsive Efficiency:

$$\Delta\eta_T = \left[\left(\frac{2\pi n \Delta Q}{VI} \right)^2 + \left(\frac{2\pi Q \Delta n}{VI} \right)^2 + \left(\frac{-2\pi n Q \Delta V_e}{V^2 I} \right)^2 + \left(\frac{-2\pi n Q \Delta I}{VI^2} \right)^2 \right]^{\frac{1}{2}}$$

EXPERIMENTAL PROCEDURES

Two separate procedures were developed for the static and dynamic tests. For all of the tests, the power supply driving the motor controller for the propeller motor was turned on and set to a nominal output of 11.1 Volts, which matches the voltage output of a standard 3-cell lithium-polymer battery. Then, the data acquisition system and the signal conditioners driving the sensors were powered up for the warm-up periods recommended by the manufacturers.

Static Test Procedure

After the warm-up period, the load cell and torque cell were zeroed by adjusting the balance potentiometers on the signal conditioners so that the voltage outputs were as close as possible to zero. At this point, five hundred data points were collected with the propeller motor off to obtain baseline values for the load cell and torque cell. The propeller motor was then set to the first desired speed setting and one thousand data points were collected. The propeller motor was then turned off and another set of 500 data points was acquired. The average values for thrust and torque from the two propeller-off states were averaged and this value was used to correct the thrust and torque measurements to account for zero drift and temperature effects (Merchant and Miller, 2008). The process was then repeated for increased values of rotational speed until the maximum speed was achieved.

Dynamic Test Procedure

After the warm-up period, the differential pressure transducer reading the Pitot tube and the signal conditioners reading the load cell and the torque cell were zeroed. Five hundred data points were taken with the propeller motor off and the wind tunnel motor off. At the end of the first five hundred points, the propeller motor was set to the desired rotational speed setting and the wind tunnel airspeed was set to the first desired setting. After the system reached steady state, one thousand data points were acquired. Next, the wind tunnel airspeed setting was changed and the propeller rotational speed was adjusted to match the original setting. This process was repeated until the windmill state of the propeller was reached. The propeller motor and the wind tunnel motor were both stopped at this point, and then five hundred data points were collected in order to again account for drift in the sensors. Data sets were collected for approximately ten wind tunnel airspeed settings for each of the four rotational speed settings for each propeller tested.

RESULTS AND DISCUSSION

To ensure that the collected data was repeatable and correct, tests were necessary to validate the static and dynamic results. The first type of test checked for repeatability of the same propeller as well as the repeatability across three identical propellers. The second type of test was to compare the results of the present experiment to published results from researchers using the same propeller. A complete summary of the data collected from the static and dynamic tests is provided by Brezina (2012).

Validation of the Static Test

In order to check the repeatability of the experiment, three identical Graupner 4.7×4.7 inch propellers were tested under static conditions three times each, thus creating a total of nine sets of data. This was done to determine the repeatability of the experiment for multiple runs of the same propeller as well as establishing whether manufacturing variability affected the performance of identical propellers. Figure 3 shows typical results for a static propeller, where both the thrust and torque increase monotonically with rotational speed. The coefficients of thrust and propeller power were relatively constant, whereas the total efficiency reached a peak value at approximately $n = 20,000$ RPM. The uncertainties of the coefficients of thrust and propeller power increased significantly at the lowest propeller rotational speed. This was driven

by the uncertainty of the load cell and the torque cell at relatively small values of thrust and torque. Figure 4 shows that the repeatability of the reduced data (coefficient of thrust, coefficient of propeller power and total propulsive efficiency) was excellent. The data from all nine tests fall within the uncertainty bounds for the first run. The duplicate propellers also fall directly in line, meaning that, at least for this type of propeller, manufacturing differences can be neglected.

Static tests were performed on two propellers ($\text{GWS } 4.5 \times 3.0$ and $\text{GWS } 5.0 \times 4.3$) which matched tests performed by Deters and Selig (2008). The coefficient of thrust and the coefficient of power were compared to data provided by Deters and Selig as shown in Figure 5, where the results for both propellers show good agreement.

Static Test Results

Having established the validity of the experimental results, static test data was collected for all of the propellers shown in Table 1. Figure 6 shows a comparison between propellers with constant diameter and varying pitch, while Figure 7 gives a comparison between propellers with varying diameter and constant pitch. In Figure 6, the coefficients of thrust and propeller power were relatively constant while the propeller efficiency increased with propeller rotational speed. Increasing the pitch (while holding the propeller diameter constant) significantly increased all three measures of performance. This same trend can be found in the data provided by Deters and Selig (2008) for the coefficient of thrust and coefficient of propeller power for the $\text{GWS } 4.0 \times 4.0$ propeller versus that for the $\text{GWS } 4.0 \times 2.5$ propeller. In Figure 7, the variation of the three performance parameters with propeller diameter is also shown to be significant, where increasing the diameter decreased the thrust coefficient and the propeller power coefficient but increased the total propulsive efficiency. This trend is also apparent in the data reported by Deters and Selig for the following propellers: $\text{GWS } 3.0 \times 3.0$, $\text{GWS } 4.5 \times 3.0$, and $\text{GWS } 5.0 \times 3.0$.

Validation of the Dynamic Test

The dynamic test procedure and experiment were validated by comparing the present results to non-archival and archival literature sources. Figure 8 shows typical dynamic results for the thrust and torque generated by one propeller over the full range of airspeed and various levels of rotational speed. Both the thrust and torque increased with rotational speed and decreased with airspeed, as expected. For a given rotational speed, the coefficients of thrust and propeller power

decreased with advance ratio, whereas the propeller efficiency increased with advance ratio. The APC 8.0×3.8 inch Slow Flyer propeller was tested at nominal propeller rotational speeds of $n = 4000$ and 7000 rpm, and the results for the coefficient of thrust, the coefficient of propeller power, and the propeller efficiency versus advance ratio were compared to those reported on the UIUC propeller database (Selig, 2012), as shown in Figure 9. The agreement with the data from the UIUC database is excellent for both rotational speeds, even where the propeller efficiency drops off steeply with advance ratio.

To further validate the dynamic results, an APC 6.0×4.0 inch propeller was tested at nominal propeller rotational speeds of $n = 8000$ to 16000 rpm by intervals of 2000 rpm and compared to the results reported by Ol et al. (2008), as shown in Figure 10. The coefficients of thrust and torque decrease with advance ratio and the propeller efficiency increases to a peak and then decreases. Since the exact propeller rotational speed tested by Ol et al. is unclear, it can only be compared to the trends in the data. The present data agrees well with that shown by Ol et al. At a rotational speed of $n = 8000$ rpm, the propeller was tested by sweeping the advance ratio from low to high values, and then sweeping from high to low values to examine the potential for hysteresis in the experiment. As can be seen, there is not a noticeable difference between these two sets of data.

Dynamic Test Results

With the dynamic results validated, data was collected for all of the propellers. Comparisons were drawn between propellers with constant diameter and varying pitch in Figure 11 and between propellers with constant pitch and varying diameter in Figure 12, both at a nominal rotational speed of $n = 16000$ rpm. In Figure 11, propellers with larger pitch generally had larger coefficients of thrust and power, and the windmill state occurred at higher values of the advance ratio, which indicates that larger pitch values tend to allow for higher airspeed. Figure 11(c) shows that the propeller efficiency decreases with increasing pitch for lower values of advance ratio, and the peak efficiency occurs at higher values of advance ratio. An increase in pitch essentially means that the angle of attack of the airfoil is higher, which should increase both thrust and torque prior to reaching stall. In Figure 12, increasing the propeller diameter for a given pitch tends to decrease the coefficient of thrust and the coefficient of power, and the propeller efficiency increases with diameter for lower values of advance ratio. Increasing the

diameter for a given rotational speed and airspeed actually increases the thrust and torque due to the increased wingspan of the propeller, but this effect is negated due to the factor of D^4 in the denominator of C_T and the factor of D^5 in the denominator of the C_P .

Figure 13 presents results for square propellers, where $D = P$, from Ol et al. (2008), Selig (2012), and the present experiment for a fairly wide range in propeller diameter ($4.0 \leq D \leq 18$ inches). Ol et al. had conjectured that the coefficient of thrust should collapse for square propellers. The results are grouped from small to large propeller diameter, where the three researchers essentially covered different diameter ranges. The data does not appear to collapse in that the propeller diameter still affects the coefficient of thrust. It should be noted, however, that this set of data covers multiple manufacturers and propeller types (i.e., Thin Electric, Sport). Geometric differences in the blades, such as the twist and chord distributions, as well as blade stiffness (and hence deflection under loading) may significantly affect performance.

Figure 14 presents the coefficient of thrust for the same family of propellers (APC Speed 400 Electric). It was assumed that using a group of similar propellers may reduce the effects of geometric blade variations. In this case, the diameter-to-pitch ratio for this family of propellers has a relatively small range ($0.86 \leq D/P \leq 1.5$). Figure 14(a) shows the coefficient of thrust versus advance ratio for all of the collected data for the APC Speed 400 Electric propellers with an uncertainty level of $\Delta C_T \leq 20\%$. As can be seen, the results are not correlated well, as witnessed by the low goodness of fit parameter, $R^2 = 0.529$. Figure 14(b) shows the coefficient of thrust modified by D/P plotted against advance ratio, where the goodness of fit parameter is improved to $R^2 = 0.680$. This correlation can be further improved by modifying the advance ratio by D/P as seen in Figure 14(c), where the goodness of fit parameter increases to $R^2 = 0.720$, which is a significant improvement over the original data correlation. This analysis shows that the inclusion of the propeller pitch into the correlations for the performance coefficients can be beneficial for this family of propellers.

Figure 15(a) shows the coefficient of power versus advance ratio for all of the collected data with an uncertainty level of $\Delta C_P \leq 20\%$. As can be seen, the results are not correlated well, as witnessed by the very low goodness of fit parameter, $R^2 = 0.059$. Figure 15(b) shows a coefficient of power that is modified by D/P , which improves the goodness of fit parameter to $R^2 = 0.538$. This is still relatively low, and probably should not be used in most engineering analyses. Figure 16 shows a similar comparison for the propeller efficiency versus the advance

ratio. Here, the diameter to pitch ratio was used to modify the original advance ratio to increase the goodness of fit parameter from $R^2 = 0.938$ to 0.983, which is deemed to be very accurate for most applications, especially for advance ratios less than $J \leq 0.6$.

CONCLUSIONS

Twenty-four propellers in the range of $4.0 \leq D \leq 6.0$ inches in diameter and $2.0 \leq P \leq 5.5$ inches in pitch were tested statically and dynamically in the Wright State University wind tunnel over a wide range of propeller rotational speeds and air speeds. A detailed experimental procedure for both cases was employed and an extensive uncertainty analysis was performed on the resulting data. The experiments were validated by comparing the results to previous works. The repeatability of the experimental results and the repeatability of the manufacture of the propellers were proven by testing three duplicate propellers three times each. Static tests were performed by varying propeller speed from $n = 4000$ rpm to the maximum speed limited by the manufacturer's specifications or the maximum motor temperature. Dynamic tests were performed by holding the propeller speed constant and varying the wind tunnel airspeed and thus varying the advance ratio.

For a given airspeed and rotational speed, the thrust and torque both increased with propeller pitch and diameter, as expected. For static testing, the coefficient of thrust, coefficient of propeller power and the total propulsive efficiency increased with propeller pitch for a given propeller diameter at all rotational speeds. Alternately, when the diameter was increased and the pitch was held constant, the coefficient of thrust and coefficient of propeller power increased and the total propulsive efficiency decreased. Similar results were found for dynamic testing. The square (D/P) propellers that were tested and compared to the results in the open literature were not correlated well with propeller diameter alone, which was possibly due to variations in the twist and chord distributions, as well as the blade stiffness. Correlations for the coefficient of thrust, propeller power and propeller efficiency for a family of propellers (APC Speed 400 Electric) under dynamic testing were found to be improved when the original coefficient (and/or advance ratio) was modified by D/P . This permitted the pitch to be accounted for in the performance characteristics of propellers in the range of $0.86 \leq D/P \leq 1.5$.

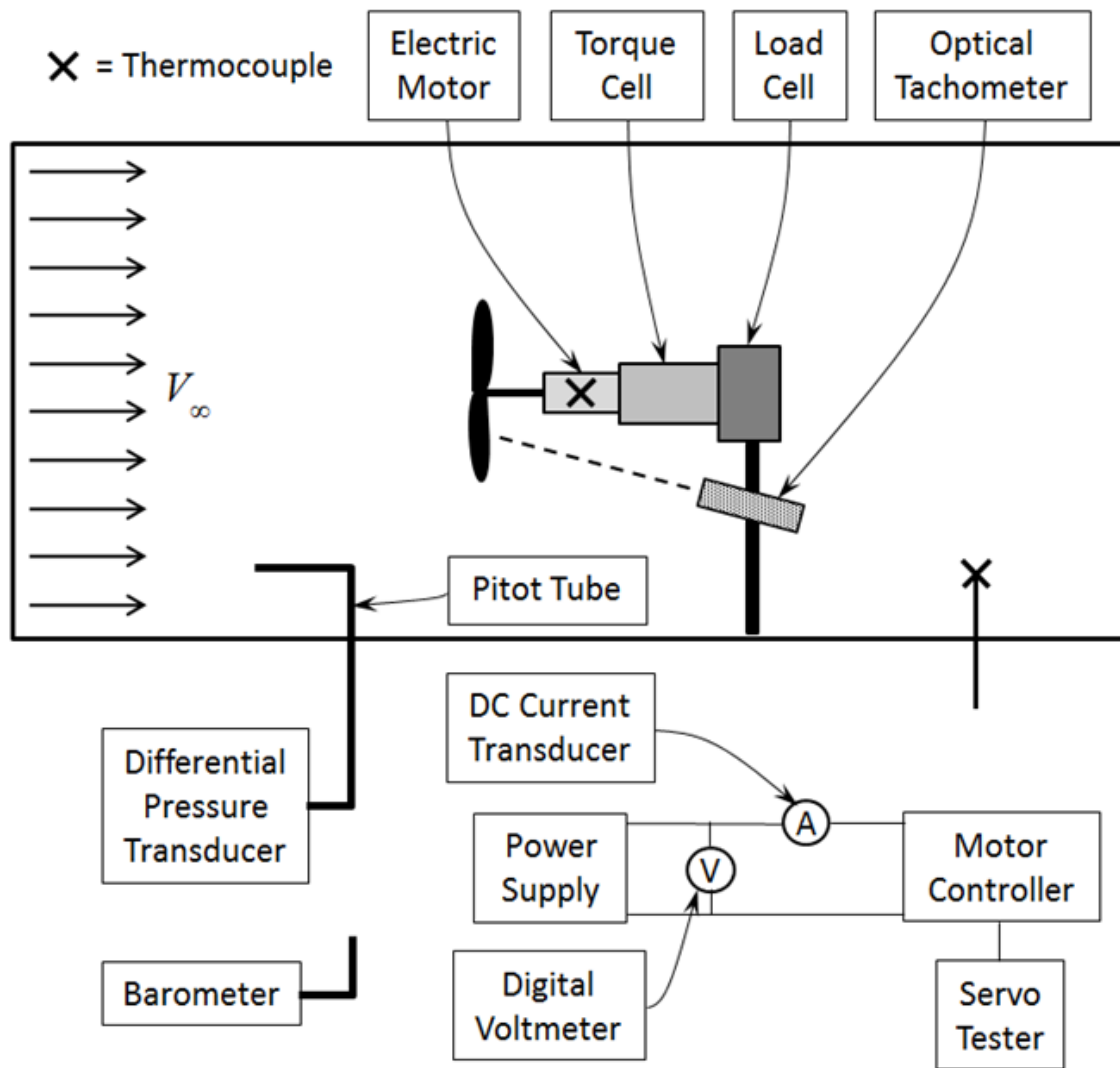


Figure 1: Schematic Diagram of the Experimental Setup.

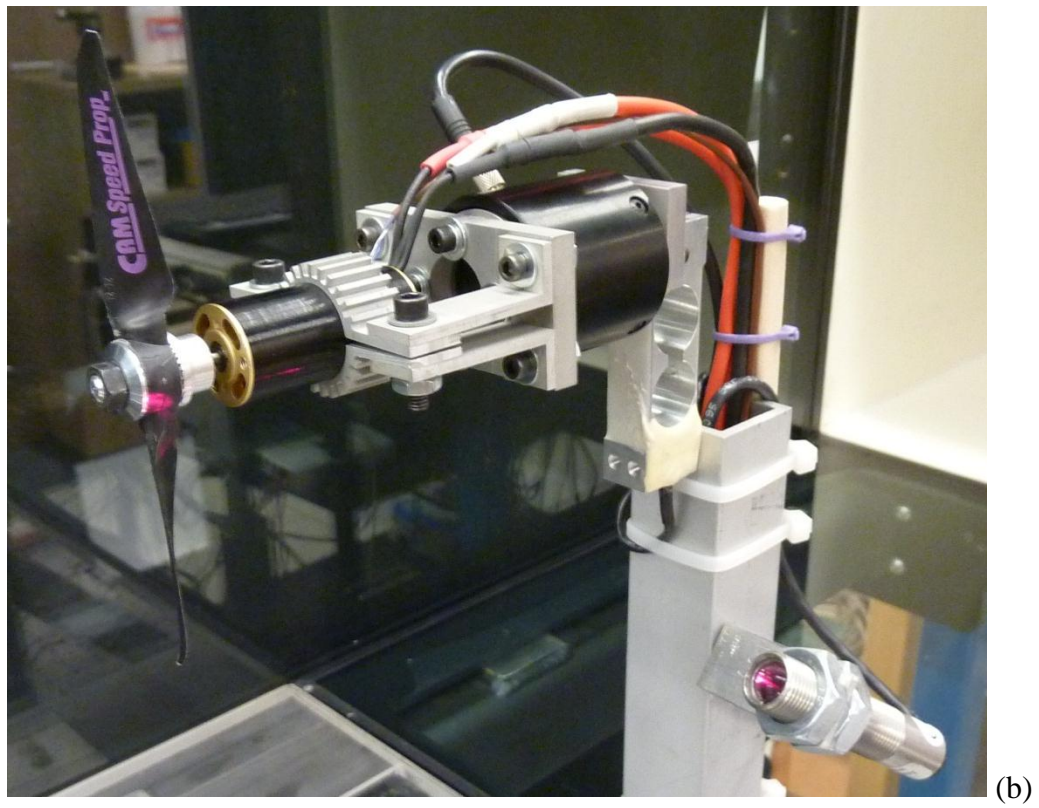
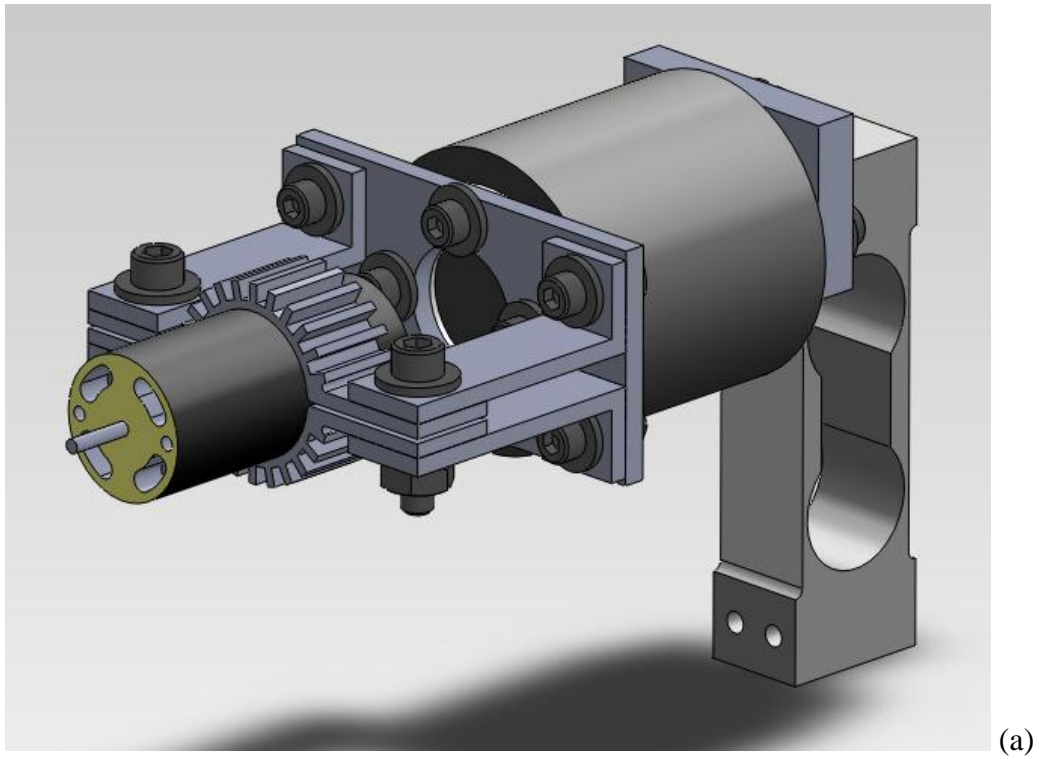


Figure 2: Assembly of Motor, Torque Cell and Load Cell: (a) Solid Model Representation; (b) Photograph.

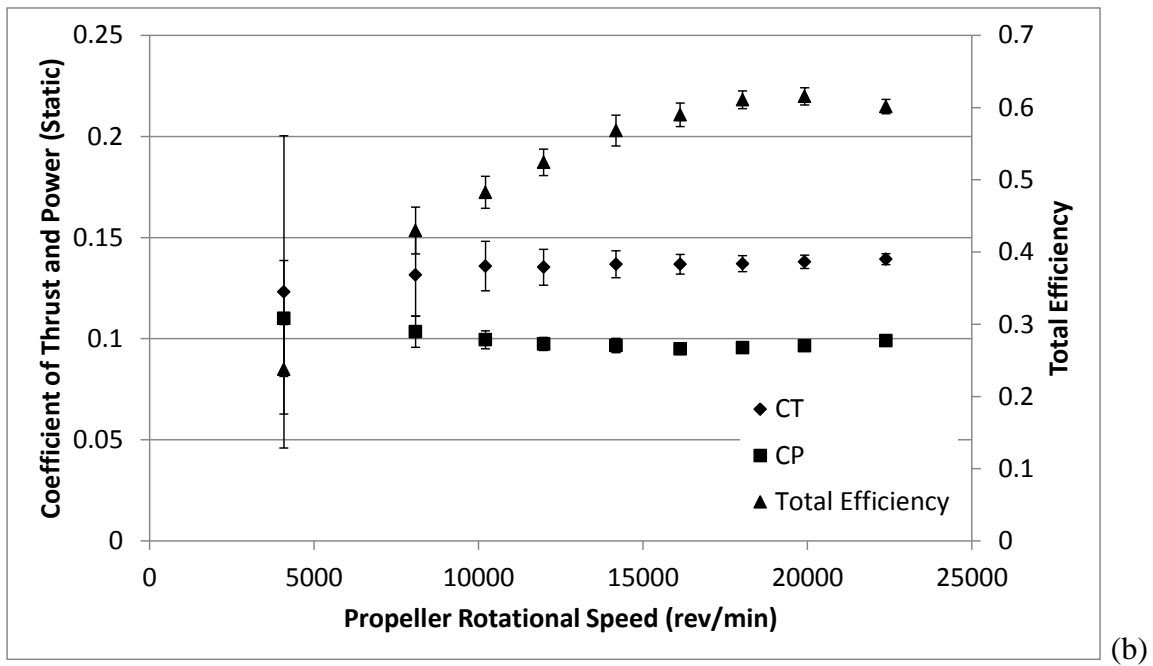
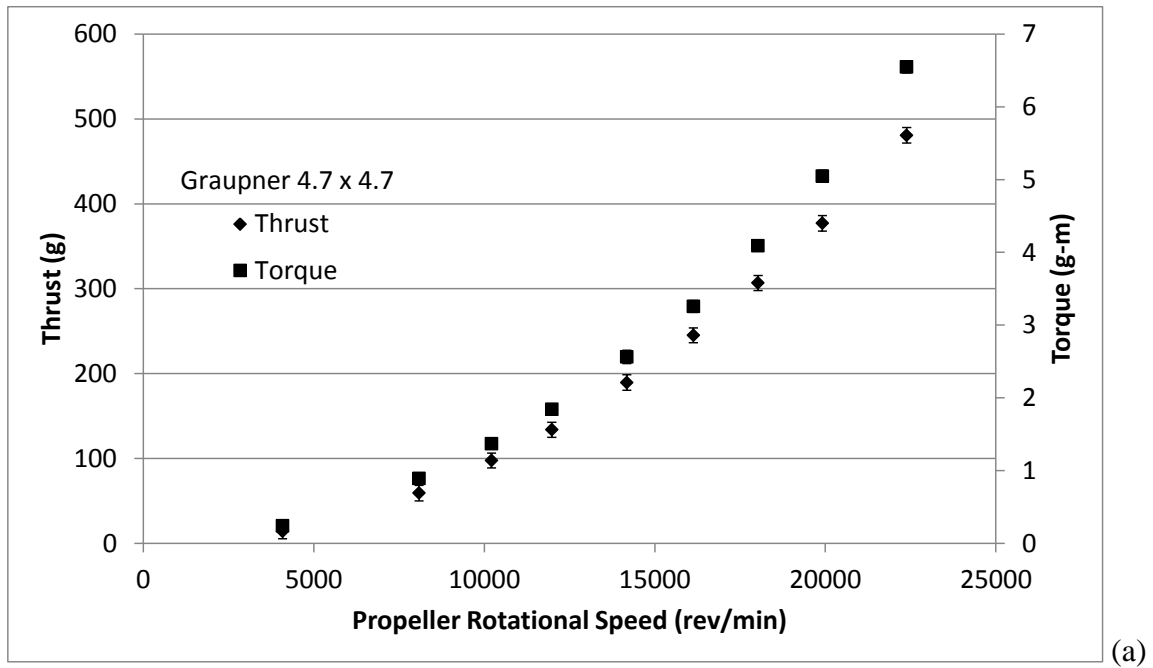


Figure 3: Typical Static Test Results (Graupner 4.7 × 4.7 inch Propeller): (a) Thrust and Torque versus Rotational Speed, (b) Coefficient of Thrust, Coefficient of Propeller Power and Total Propulsive Efficiency versus Rotational Speed.

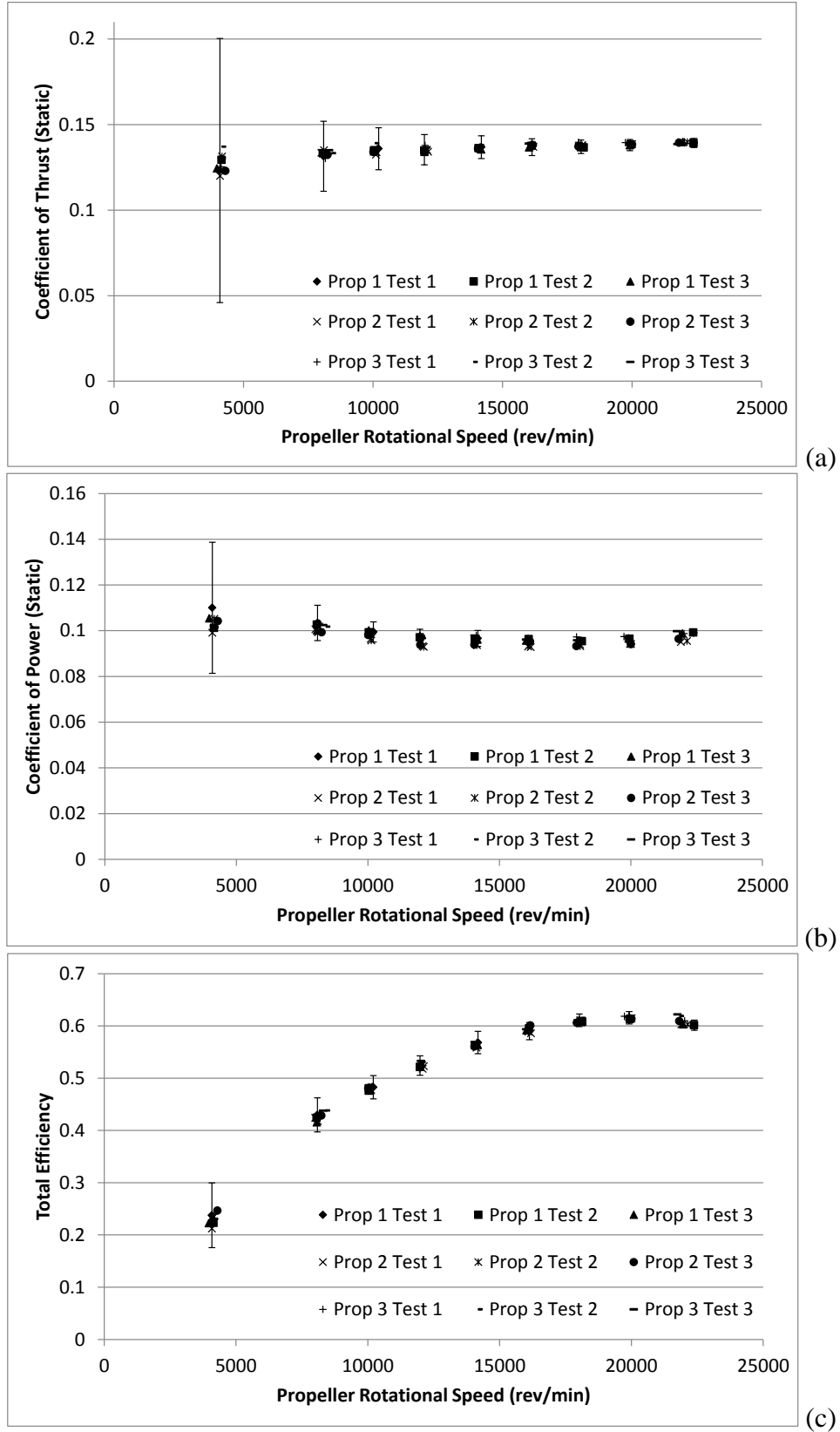


Figure 4: Comparison of Three Identical Propellers under Static Testing (Graupner 4.7 × 4.7): (a) Coefficient of Thrust, (b) Coefficient of Propeller Power, (c) Total Propulsive Efficiency.

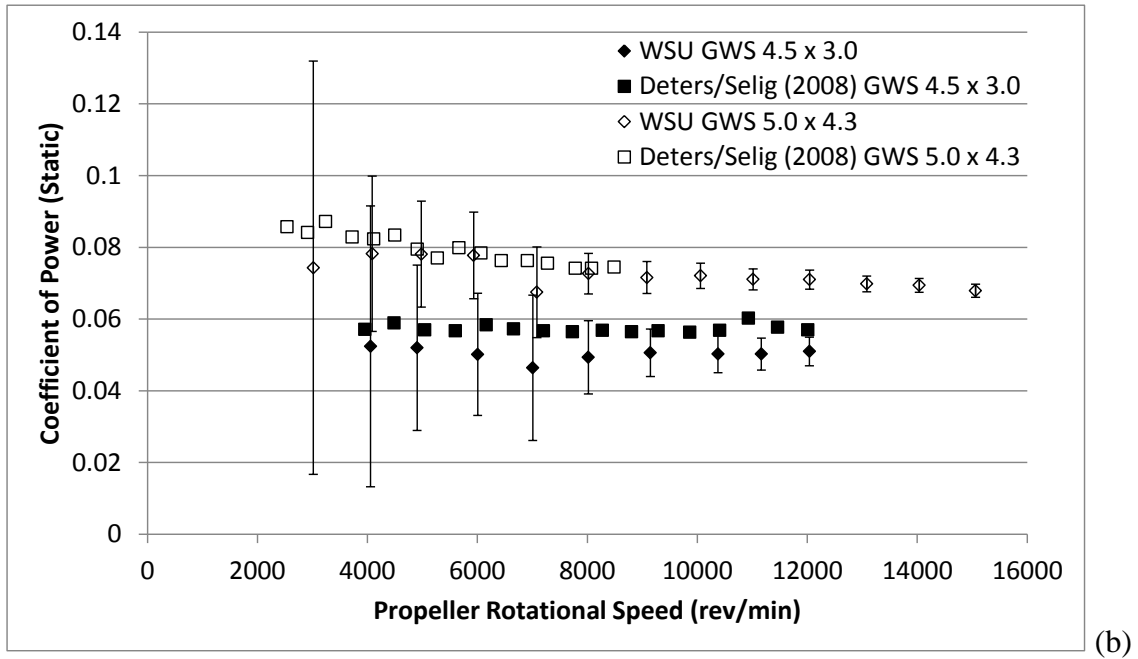
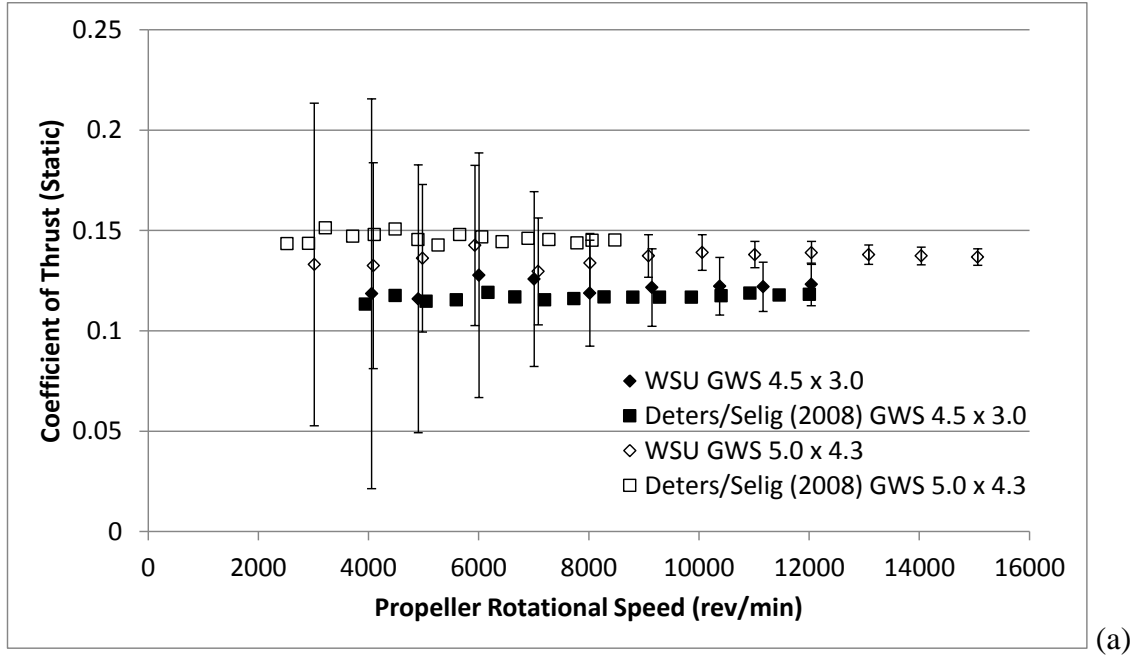


Figure 5: Comparison of the Present Static Results to Deters and Selig (2008) (GWS 4.5×3.0 and GWS 5.0×4.3 Propellers): (a) Coefficient of Thrust, (b) Coefficient of Propeller Power.

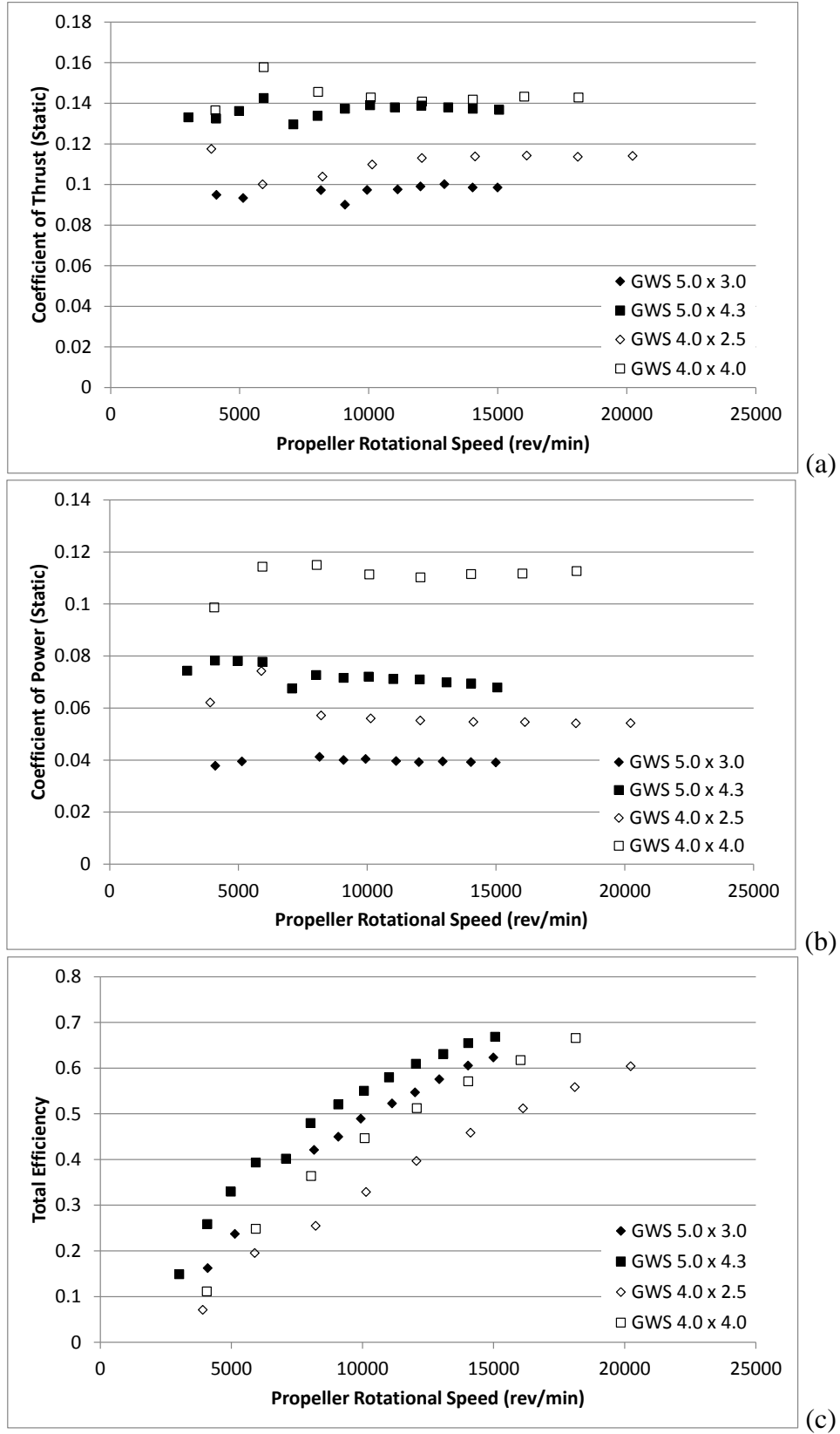


Figure 6: The Effect of Varying Pitch while holding Diameter Constant (Static Testing): (a) Coefficient of Thrust, (b) Coefficient of Propeller Power, (c) Total Propulsive Efficiency.

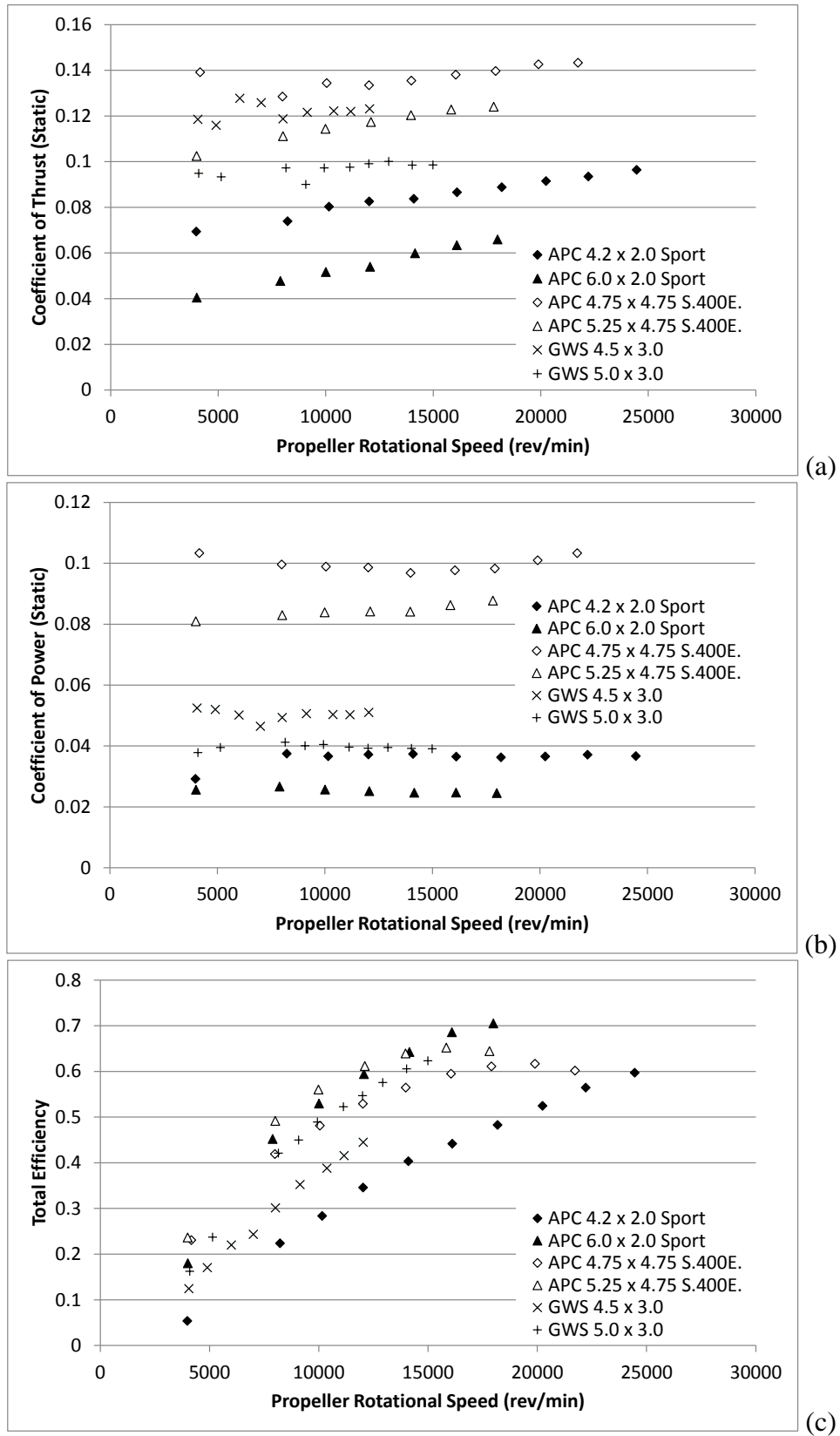
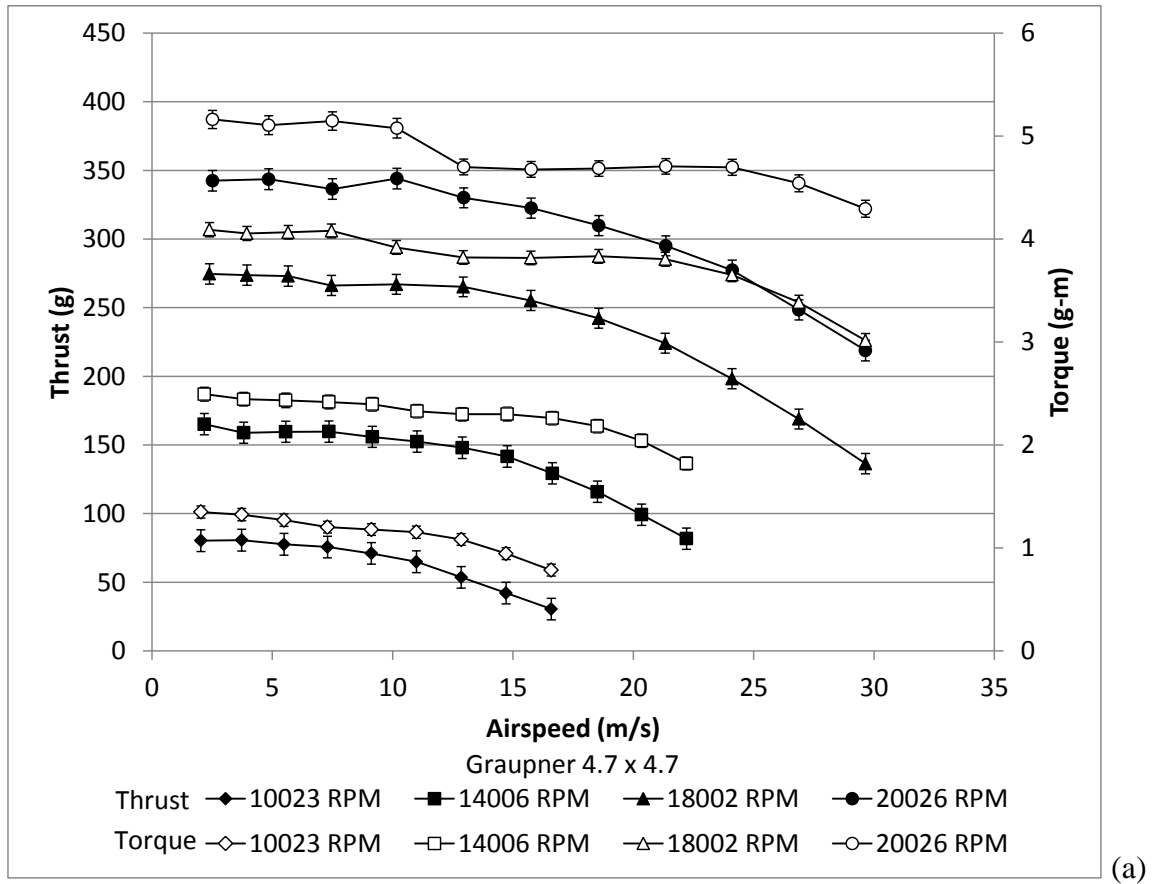
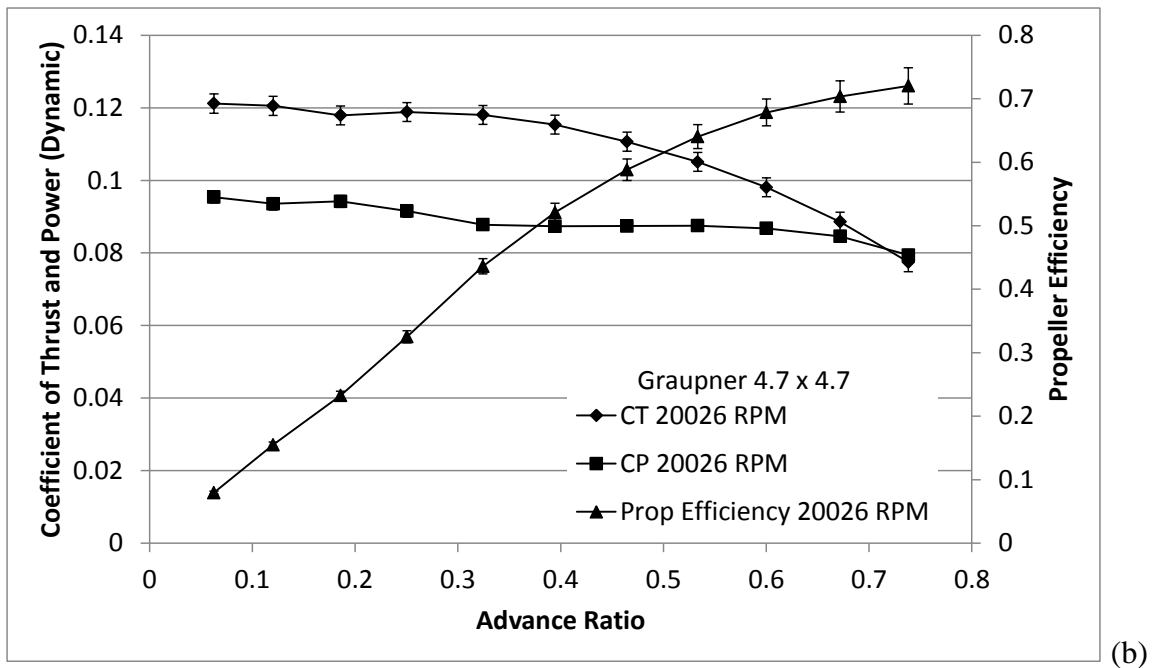


Figure 7: The Effect of Varying Diameter while holding Pitch Constant (Static Testing): (a) Coefficient of Thrust, (b) Coefficient of Propeller Power, (c) Total Propulsive Efficiency.



(a)



(b)

Figure 8: Typical Dynamic Test Results (Graupner 4.7 × 4.7 inch Propeller): (a) Thrust and Torque versus Airspeed for Various Rotational Speeds, (b) Coefficient of Thrust, Coefficient of Propeller Power and Propeller Efficiency versus Advance Ratio.

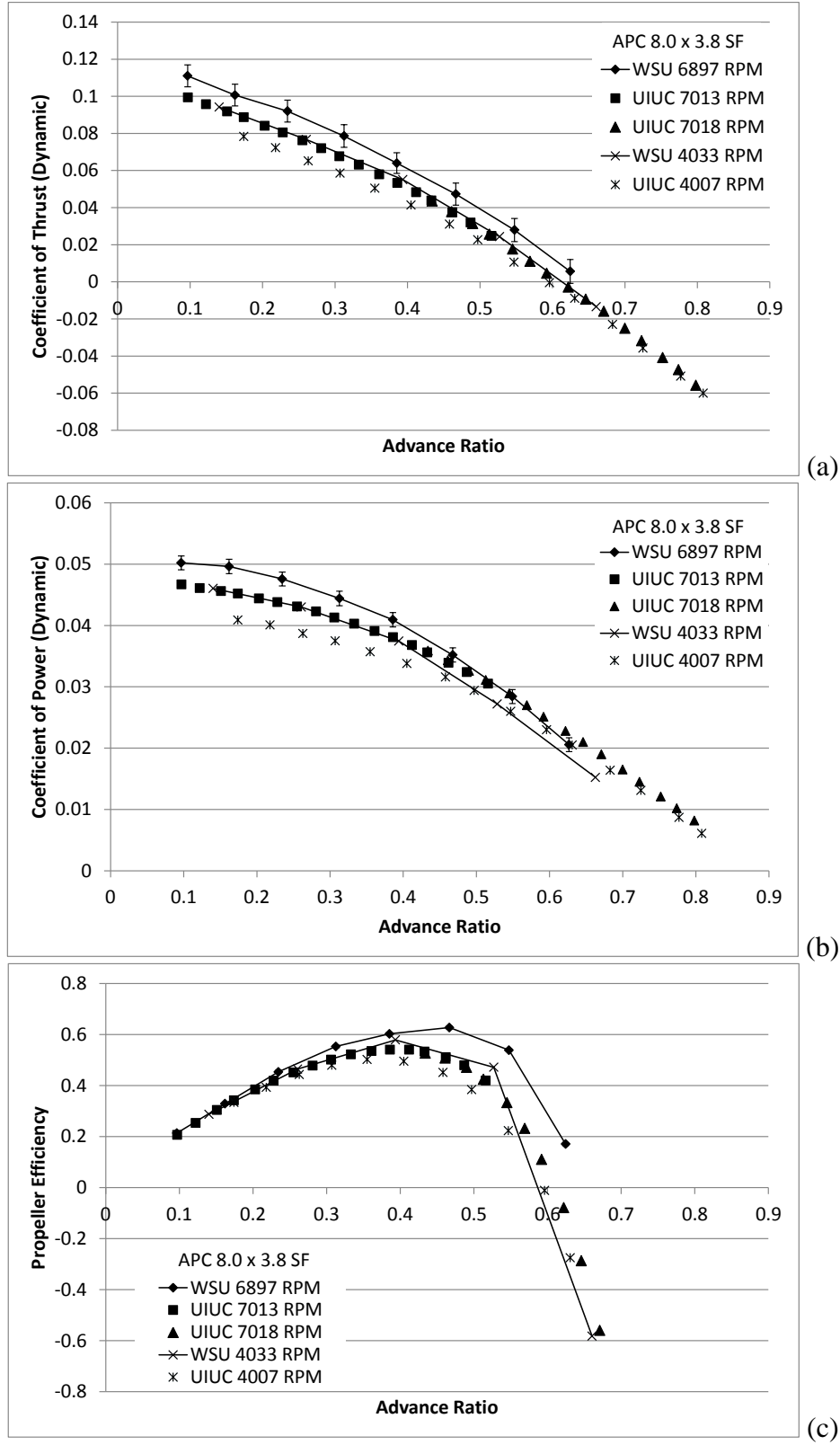


Figure 9: Comparison of Present Dynamic Results to Selig (2012) (APC 8.0 x 3.8 SF): (a) Coefficient of Thrust, (b) Coefficient of Propeller Power, (c) Propeller Efficiency.

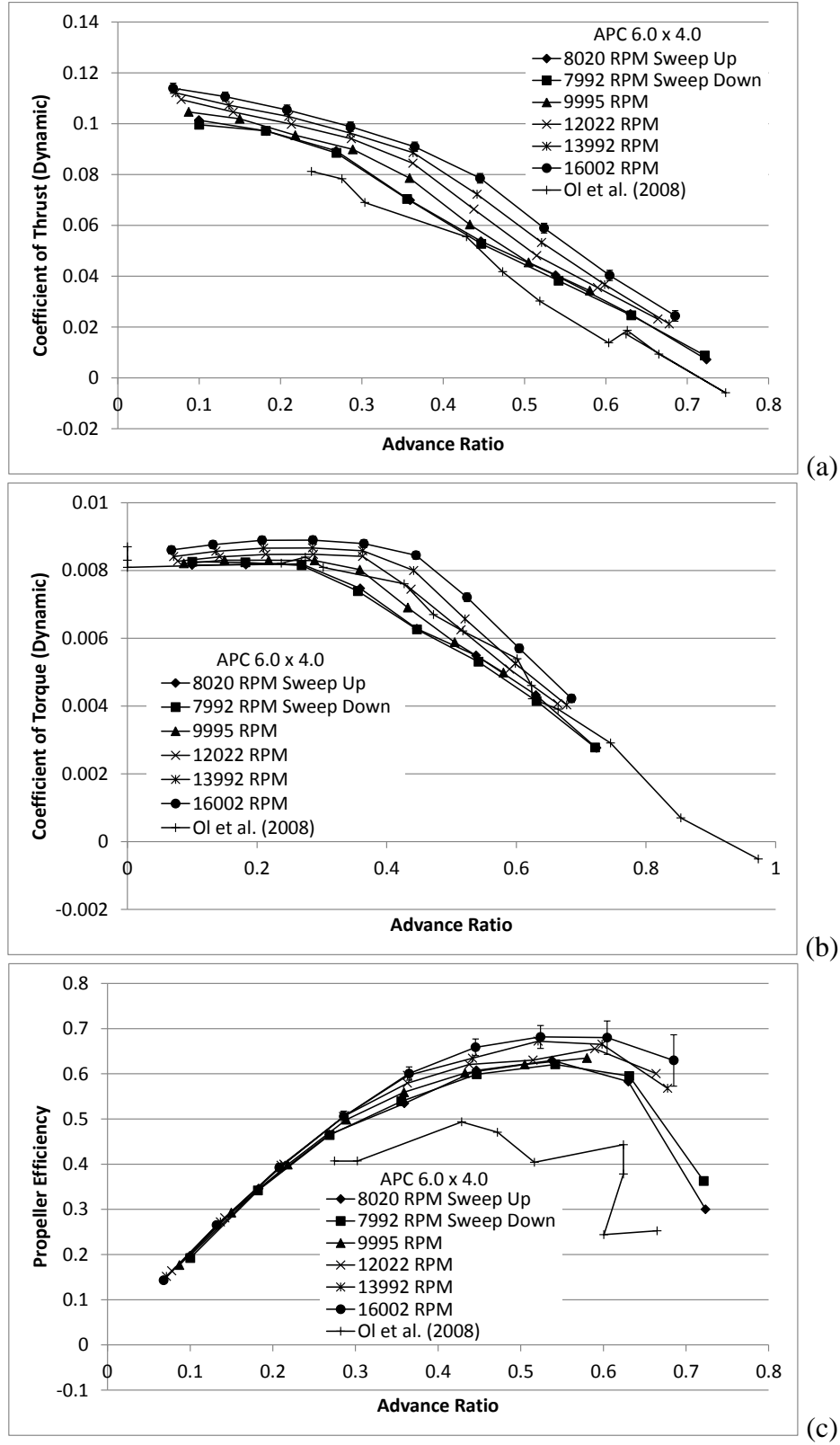


Figure 10: Comparison of Present Dynamic Results to Ol et al. (2008) (APC 6.0 \times 4.0): (a) Coefficient of Thrust, (b) Coefficient of Torque, (c) Propeller Efficiency.

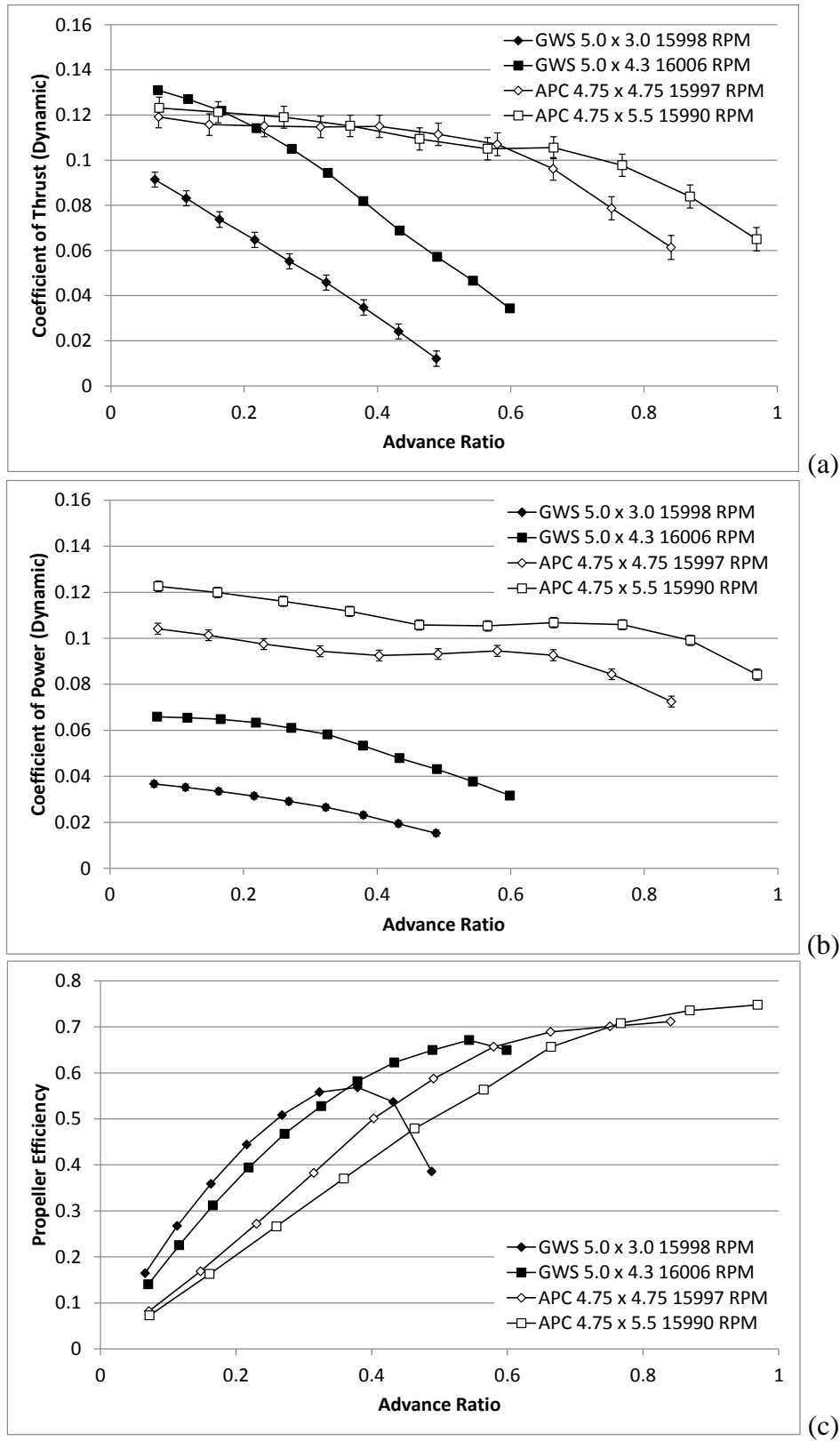


Figure 11: The Effect of Varying Propeller Pitch while holding Diameter Constant (Dynamic Testing): (a) Coefficient of Thrust, (b) Coefficient of Propeller Power, (c) Propeller Efficiency.

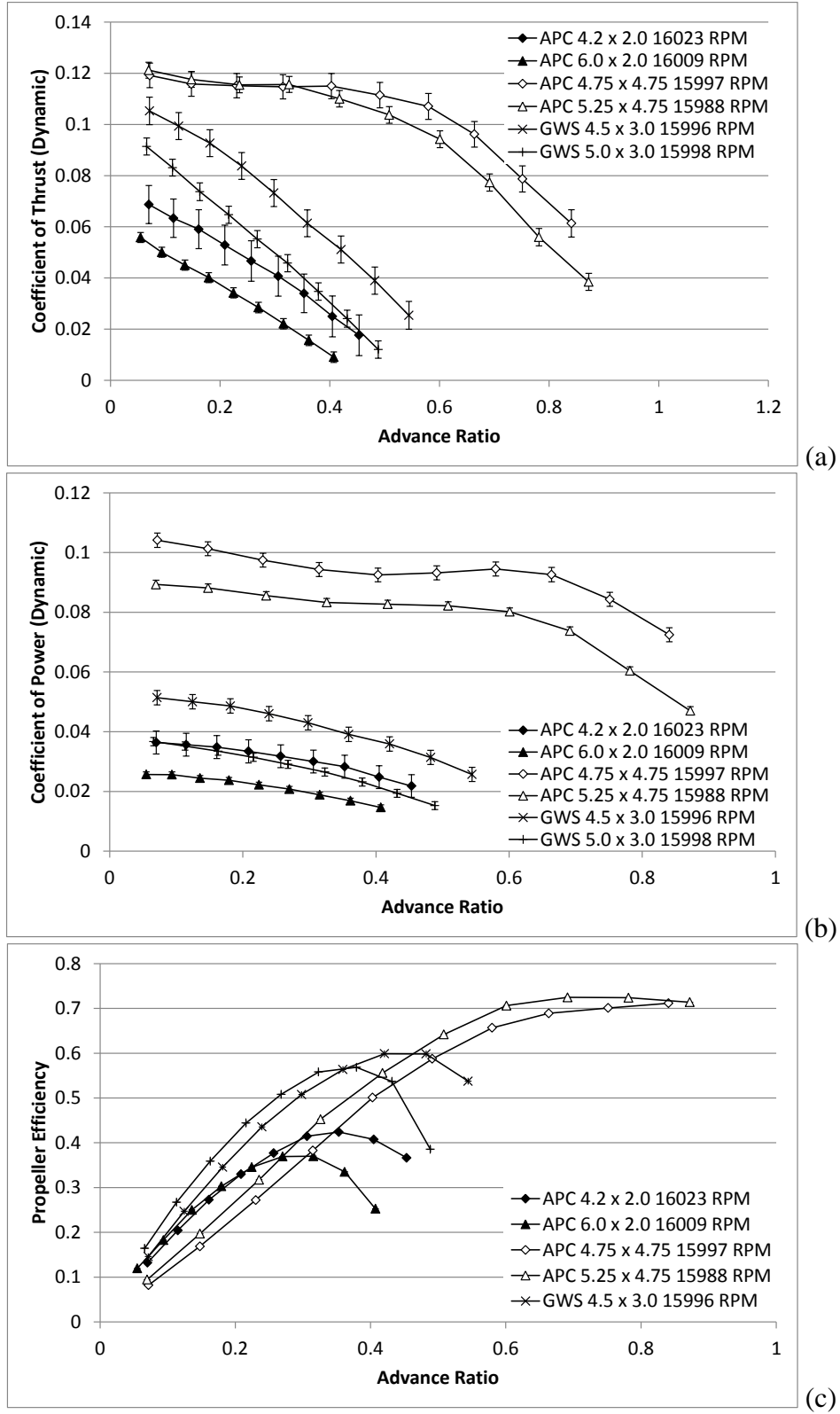


Figure 12: The Effect of Varying Propeller Diameter while holding Pitch Constant (Dynamic Testing): (a) Coefficient of Thrust, (b) Coefficient of Propeller Power, (c) Propeller Efficiency.

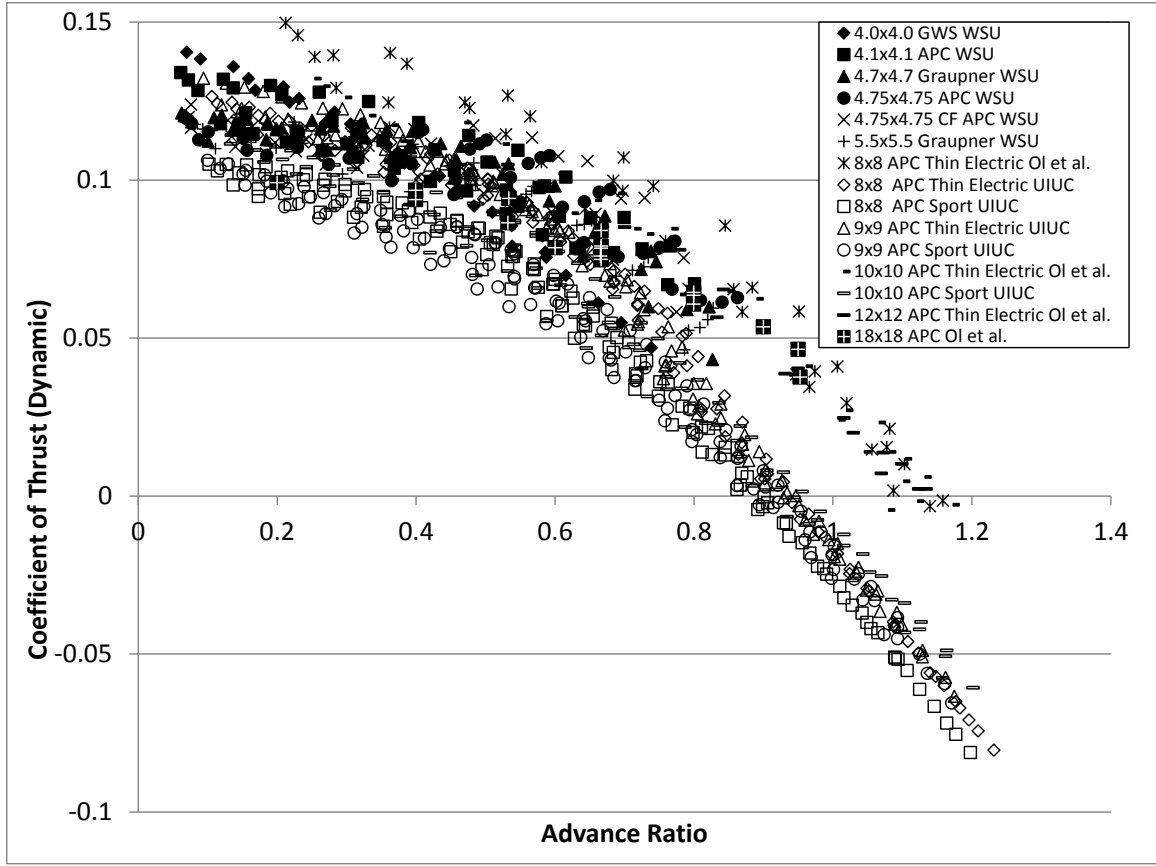


Figure 13: Coefficient of Thrust versus Advance Ratio for Square Propellers ($D/P = 1.0$) with Propeller Diameter Ranging from $4.0 \leq D \leq 18$ inches.

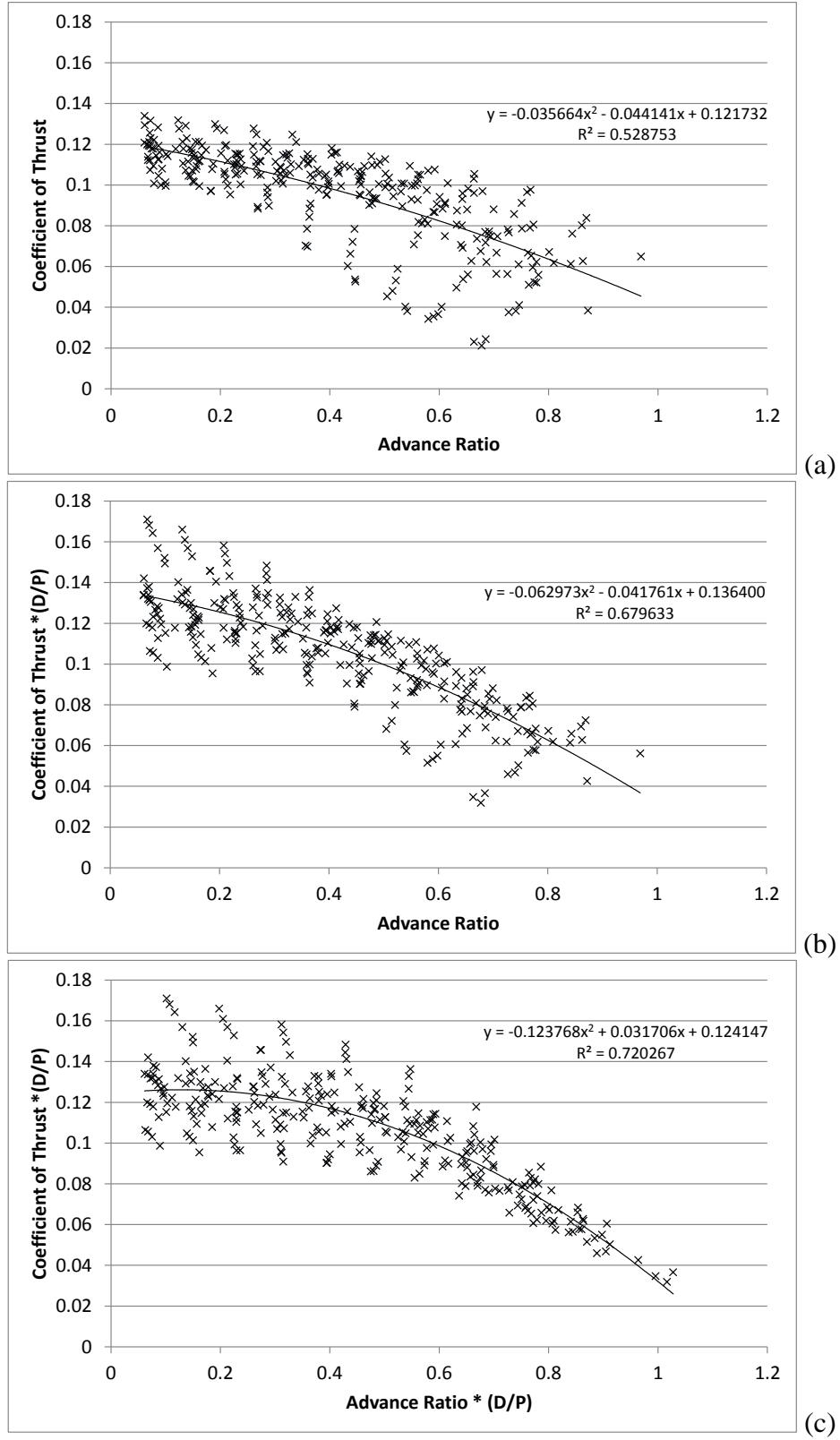


Figure 14: Coefficient of Thrust versus Advance Ratio for the APC Sport 400 Electric Propellers ($\Delta C_T \leq 20\%$): (a) Original Representation of C_T ; (b) C_T Modified by D/P , (c) C_T and J Modified by D/P .

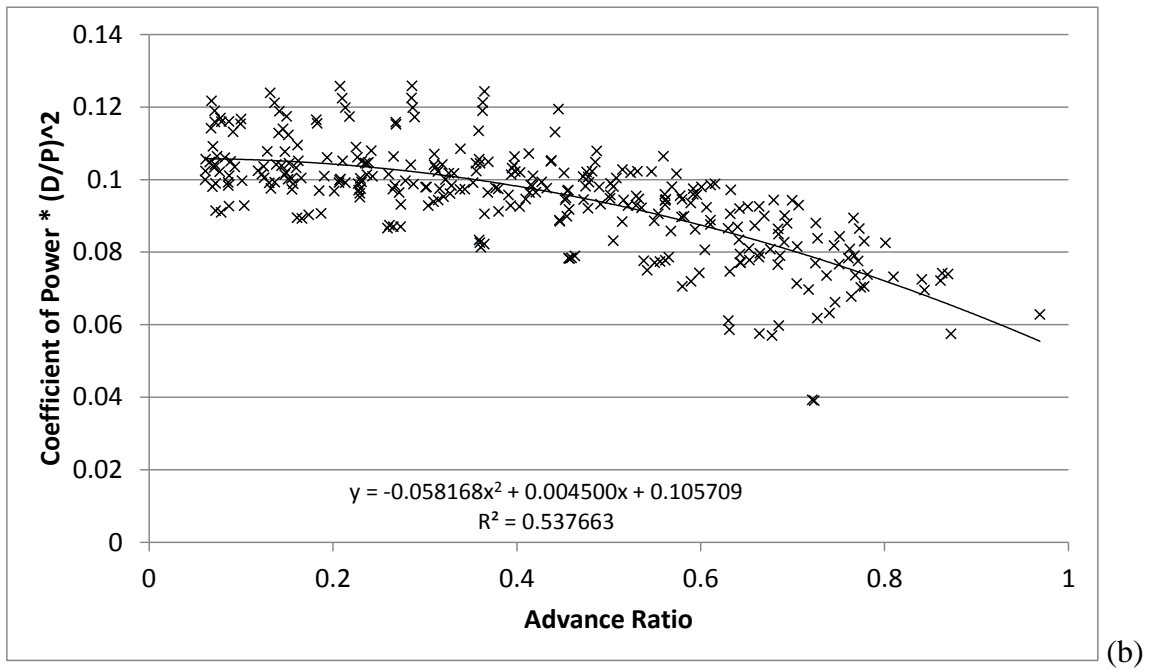
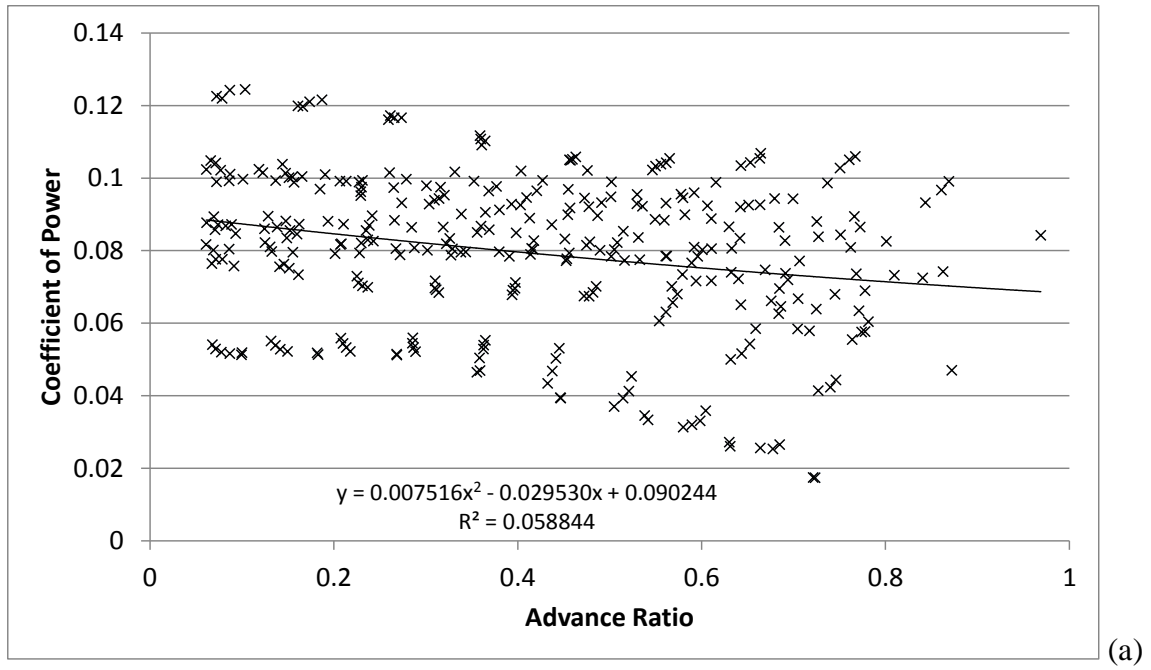


Figure 15: Coefficient of Propeller Power versus Advance Ratio for the APC Sport 400 Electric Propellers ($\Delta C_P \leq 20\%$): (a) Original Representation of C_P ; (b) C_P Modified by D/P .

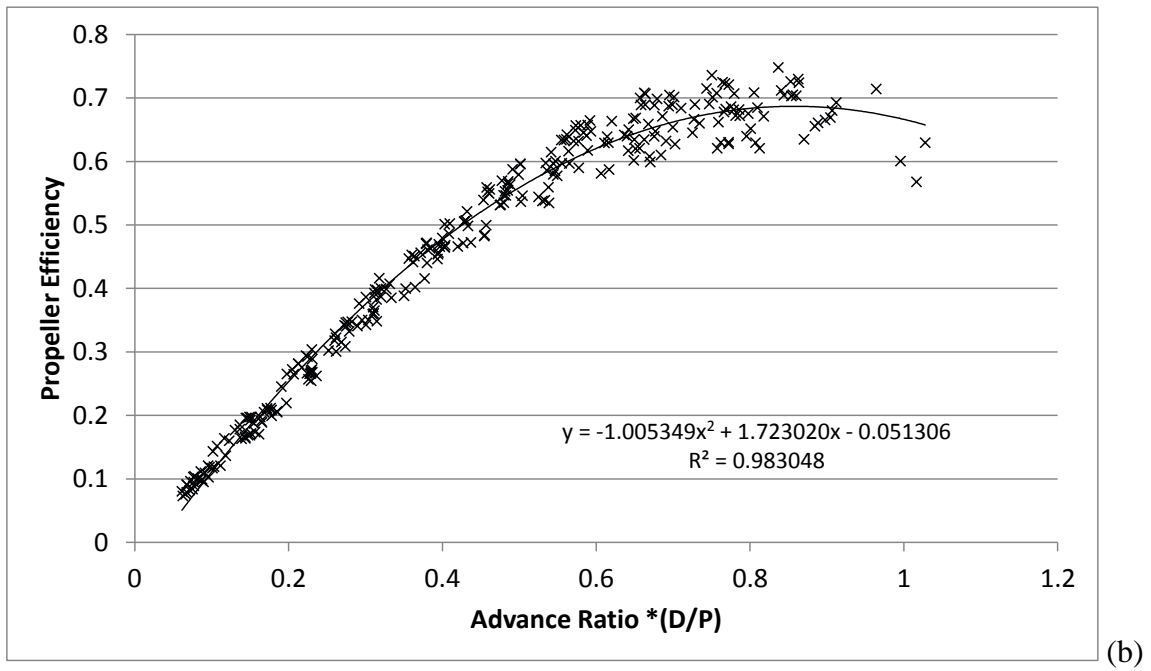
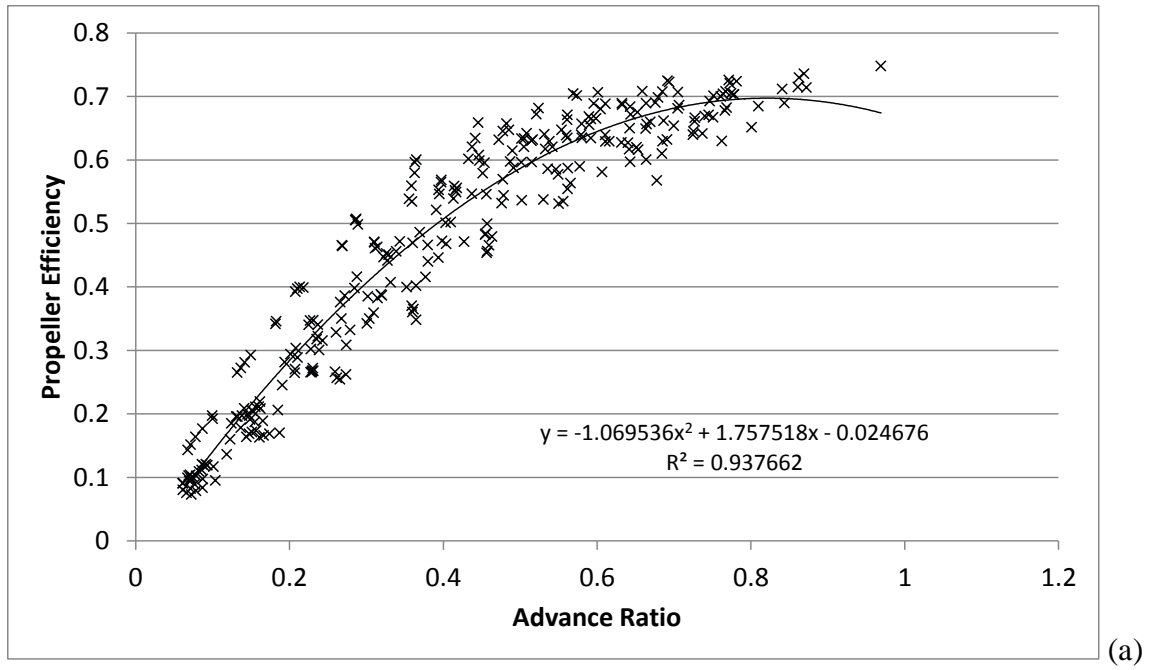


Figure 16: Propeller Efficiency versus Advance Ratio for the APC Sport 400 Electric Propellers ($\Delta\eta_P \leq 20\%$): (a) Original Representation of η_P ; (b) Advance Ratio Modified by D/P .

Table 1: Summary of Propellers Studied.

Manufacturer	Nominal D/P (in \times in)	Designation
APC	4.10×4.10	Speed 400 Electric
APC	4.20×2.00	Sport
APC	4.20×4.00	Free Flight
APC	4.50×4.10	Speed 400 Electric
APC	4.70×4.25	Speed 400 Electric
APC	4.75×4.75	Speed 400 Electric
APC	4.75×5.50	Speed 400 Electric
APC	$5.10 \times 4.50E$	Thin Electric
APC	5.25×4.75	Speed 400 Electric
APC	5.50×2.00	Free Flight
APC	5.50×4.50	Speed 400 Electric
APC	6.00×2.00	Sport
APC	$6.00 \times 4.00 E$	Speed 400 Electric
APC	8.00×3.8	Slow Flyer
Graupner	4.00×3.00	Cam Speed
Graupner	4.70×4.00	Cam Speed
Graupner	4.70×4.70	Cam Speed
Graupner	5.50×4.30	Cam Speed
Graupner	5.50×5.50	Cam Speed
GWS	4.00×2.50	
GWS	4.00×4.00	
GWS	4.50×3.00	
GWS	5.00×3.00	
GWS	5.00×4.30	

Table 2: Uncertainties of Primary Measurement Sensors and Calibration Sources.

Measurement	Sensor	Uncertainty
Thrust, T	Transducer Techniques LSP 1kg Load Cell	$\Delta T_{\text{cal}} = \pm 7.70 \text{ g}$
Torque, Q	Transducer Techniques RTS 25 oz-in Reaction Torque Sensor	$\Delta Q_{\text{cal}} = \pm 0.0498 \text{ g-m}$
Atmospheric Temperature, T_{atm}	Omega Type E Thermocouple	$\Delta T_{\text{atm,cal}} = \pm 0.0334 \text{ }^{\circ}\text{C}$
Calibration Mass	Ohaus Digital Scale	$\Delta m = \pm 1.00 \times 10^{-3} \text{ g}$
Propeller Diameter, D	Digital Vernier Calipers	$\Delta D = \pm 1.00 \times 10^{-5} \text{ m}$
Propeller Rotational Speed, n	Monarch Instruments Remote Optical Sensor (ROS) and ACT 3x Panel Tachometer	$\Delta n = \pm 1 \text{ RPM}$
Motor Voltage, V	National Instruments USB-4065 Digital Multi-Meter	$\Delta V = \pm 1.00 \times 10^{-3} \text{ V}$
Motor Current, I	CR Magnetics CR5210-30 Current Transducer	$\Delta I = \pm (1\% \times \text{Reading})$
Atmospheric Pressure, P_{abs}	Vaisala PTB110 Barometer	$\Delta P_{\text{atm}} = \pm 30.0 \text{ Pa}$
Pitot Tube Differential Pressure, P_{diff}	MKS 226A Differential Pressure Manometer	$\Delta P_{\text{diff}} = \pm (0.3\% \times \text{Reading})$

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