A virtual-decision-maker library considering personalities and dynamically changing preference structures for interactive multiobjective optimization

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***Abstract*—Interactive multiobjective optimization (IMO) methods aim at supporting human decision makers (DMs) to find their most preferred solutions in solving multiobjective optimization problems. Due to the subjectivity of human DMs, human fatigue, or other limiting factors, it is hard to design experiments involving human DMs to evaluate and compare IMO methods. In this paper, we propose a framework of a virtual-DM library consisting of a variety of virtual DMs which reflect characteristics of different types of human DMs. The virtual-DM library is used to replace human DMs to interact with IMO methods. The virtual DMs in the library can express different types of preference information and their most preferred solutions are known. When interacting with an IMO method, the library can select an appropriate virtual DM to provide preference information that the method asks for based on solutions offered by the method. Four types of hybrid virtual DMs are constructed to emulate human DMs with different personalities and dynamically changing preference structures. They can be used to test the ability of IMO methods to adapt to different human DMs and capture DMs' preferences. The usage of these four types of virtual DMs are demonstrated by comparing two IMO algorithms.**

I. Introduction

Multiobjective optimization problems (MOPs) which are ubiquitous in our social life often have several conflicting objectives to be optimized simultaneously [1], [2]. A general form of MOPs is as follows:

$$\begin{array}{ll} \text{minimize} & \mathbf{f}(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})\} \\ \text{subject to} & \mathbf{x} \in S \subset \mathbf{R}^n \end{array} \tag{1}$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T$ is the decision vector belonging to the feasible region S, and $\mathbf{f}(\mathbf{x})$ is the objective vector belonging to the objective space $\mathbf{R}^k$.

Because the objectives functions are contradictory, an MOP usually does not have a single optimal solution for all objectives but a set of Pareto optimal solutions called the Pareto set. The set of all Pareto optimal objective vectors is called the Pareto front (PF). The ultimate goal of multiobjective optimization is to aid the decision maker (DM) to find his/her most preferred solution (MPS) which most satisfies the DM among the Pareto set.

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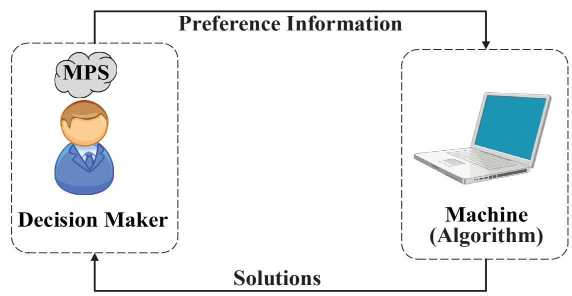

Fig. 1. The interaction process between the DM and the machine.

Depending on the stage in which the DM makes decisions in the solution process, multiobjective optimization methods can be divided into *a priori* methods, *a posteriori* methods, and *interactive* methods [3]–[5]. In *a priori* methods, after the DM specifying his/her global preference information like a scalar function, a Pareto optimal solution best satisfying the DM is found. This kind of method has low computational complexity. However, it is usually very difficult for the DM to provide global preference information when he/she knows little about the MOP.

In an *a posteriori* method, an approximation of the PF is found first. Then, the DM chooses the most preferred one among the approximation set. Up to now, numerous multiobjective evolutionary algorithms (MOEAs) have been proposed, aiming at finding a representative set of the PF in a single run by evolving a population [6]. The computational complexity is high since a lot of non-dominated solutions are needed to approximate the PF especially in the case of many-objective optimization.

In an interactive multiobjective optimization (IMO) method, the DM expresses preference information periodically to guide the search of the machine (algorithm) towards his/her region of interest (ROI). Compared to approximating the whole PF, approximating only the DM's ROI will dramatically reduce the computational complexity. Moreover, the DM can

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learn from the interaction process and adjust his/her preferences. The interaction process between the DM and the machine is shown in Fig. 1.

Once the DM's global preference information is known, his/her MPS is determined. Thus we can evaluate *a priori* methods by checking whether they converge to the DM's MPS. For *a posteriori* methods, both converging to the PF and covering the PF evenly are important. Some performance metrics like the inverted generational distance metric and the hypervolume metric have been developed to measure the convergence and diversity of MOEAs [7], [8].

Evaluating IMO methods is a hard nut because human DMs are subjective and their MPSs are unknown. Some literature has designed experiments involving human DMs to compare IMO methods [9]–[11]. Human DMs are asked to express their feelings about IMO methods such as ease of use, confidence, or degree of satisfaction on the final solution. The problems are that the results are subjective and it is difficult to repeat the experiment.

Some studies use virtual DMs to simulate and replace human DMs in the loop of the interaction process to evaluate or compare IMO methods [11]–[14]. Compared to experimenting involving human DMs, experimenting with virtual DMs is cheaper and less time consuming. Besides, sequential experiments can be conducted without considering human fatigue [15]. A common virtual DM is a value function (or utility function) which is a scalar function of all the objectives. The optimal solution of the value function (VF) in the feasible region serves as the DM's MPS. Hence, an IMO method can be evaluated by calculating the distance between its final solution and the MPS or the difference between the VF value of the final solution and the optimal VF value.

Commonly used VFs are often simple linear or nonlinear functions which are kept unaltered during the interaction process. In reality, a human DM's preferences are evolving since the DM is learning from the interaction process. A stationary VF may not be able to reflect the changes of the DM's preference structure. Therefore, it is necessary to construct dynamically changing VFs in order to better emulate human DMs. Besides, human DMs with different personalities may have different preference structures and thus make decisions differently. Therefore, taking the personality factor into account is also needful when constructing virtual DMs.

In this paper, we aim to construct a virtual-DM library which consists of a variety of types of virtual DMs. These virtual DMs can represent the characteristics of different human DMs. The library will replace human DMs to interact with IMO methods. Based on the solutions provided by an IMO method, the virtual-DM library can offer the method the type of preference information that it asks for. Virtual DMs which take human DMs' personalities and dynamically changing preference structures into consideration will be constructed.

The contributions of this paper are threefold:

• A framework of a virtual-DM library is proposed. They are supposed to assist the evaluation or comparison of IMO methods by replacing human DMs in the loop of the interaction process.

• Different virtual DMs which emulate human DMs with different personalities are constructed. They can be used to test the ability of IMO methods to adapt to different types of human DMs.

• Virtual DMs which change preference structures dynamically during the interaction process are constructed. They can evaluate the capability of IMO methods to follow and capture the preferences of DMs who dynamically change their preference structures.

The remainder of this paper is organized as follows. In Section II, a framework of the virtual-DM library is illustrated. In Section III, four types of virtual DMs which emulate human DMs with different personalities and dynamically changing preference structures are constructed. In Section IV, the virtual-DM library containing the four types of virtual DMs is used to compare two IMO algorithms. Conclusions are drawn in Section V.

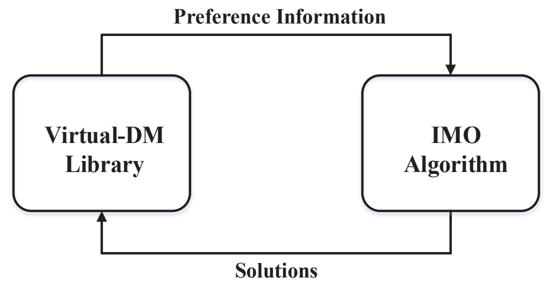


Fig. 2. The virtual-DM library instead of a human DM in the loop of the interaction process.

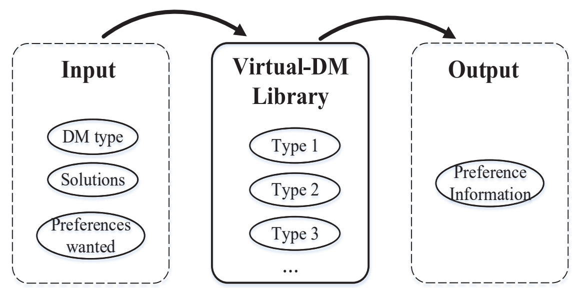


Fig. 3. The framework of the virtual-DM library with input and output.

II. Framework of the virtual-DM library

In order to compare IMO methods comprehensively and systematically, we propose a virtual-DM library which consists of many types of virtual DMs. These types of virtual DMs should be able to reflect different characteristics of human DMs and can evaluate performances of IMO methods from different aspects. Each virtual DM can take the form of a set of decision rules, a scalar function, etc. Its MPS is known and it can express different types of preference information, e.g, weights, reference points, tradeoffs, pairwise comparison of a set of solutions.

As shown in Fig. 2, the virtual-DM library is used to replace a human DM to interact with an IMO method. The framework of the virtual-DM library with input and output is shown in Fig. 3. An IMO method specifies the type of virtual DM and the type of preference information that it needs. Meanwhile, it provides one or more solutions for the virtual-DM library. Based on these inputs, the virtual-DM library can select a suitable virtual DM to offer the IMO method preferences on the provided solution(s).

For each type of virtual DM in the library, the convergence accuracy or the convergence speed (the number of interactions or the overall running time) of IMO methods can be evaluated. For measuring the convergence accuracy, all IMO algorithms can be terminated after the same predefined largest number of interactions to compare the distances from their final solutions to the MPS of the same virtual DM used. To compare the convergence speed, IMO algorithms can be set to stop until they have found a solution close to the MPS. More performances of IMO methods can be evaluated by using different but associated types of virtual DMs and comparing the results of IMO methods obtained in different cases.

III. Virtual DMs reflecting human DMs' personalities and dynamic preference structures

As stated in the previous section, different types of virtual DMs are needed in the virtual DM library to evaluate different performances of IMO methods. In this section, we construct four types of hybrid virtual DMs which emulate human DMs with different personalities and dynamically changing preference structures.

A. Virtual DMs considering personalities

Humans with different personalities may make decisions dissimilarly. A widely accepted personality taxonomy is a five factor model called "The Big Five" [16], [17]. Five dimensions of personalities are identified: neuroticism, extraversion, openness, agreeableness, and conscientiousness. Each dimension is made up of six trait facets [17].

Some of these trait facets will affect human DMs' decisions in IMO. For example, impulsive DMs are more likely to make mistakes than DMs with deliberation. Here we focus on the assertiveness and its opposite, i.e., the hesitation. An assertive DM is clear about what he/she wants and his/her MPS is specific. A hesitant DM's inner preference structure may vary within a range. Therefore, it is more accurate to say that he/she most prefers a region instead of a fixed point.

To simulate the assertiveness and hesitation of human DMs, we introduce determinacy and randomness into virtual DMs which take the form of VFs. A virtual DM representing assertive human DMs is formulated as a scalar function of a p-dimension parameter vector $\boldsymbol{\alpha}$ (the value of p depends on the specific form of the function) and the objective vector $\mathbf{f}$

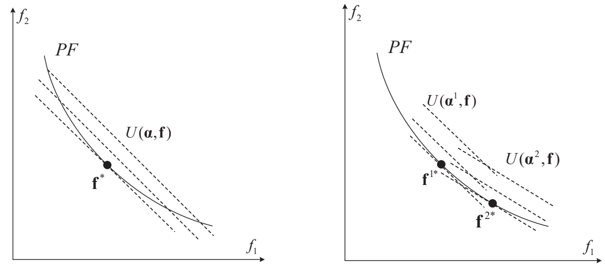
$$U = U(\boldsymbol{\alpha}, \mathbf{f}). \tag{2}$$

The function U is deterministic with the parameter vector $\boldsymbol{\alpha}$ being always the same. U can take various forms like a weighted sum of objectives $U = \alpha_1 f_1 + \alpha_2 f_2 + \cdots + \alpha_n f_n$ or a quadratic function $U = \alpha_1 f_1^2 + \alpha_2 f_2^2 + \cdots + \alpha_n f_n^2$.

For emulating hesitant DMs, we consider the parameter vector $\boldsymbol{\alpha}$ as a random vector which follows a certain distribution like multivariate normal distribution. A virtual DM with the parameter vector obeying multivariate normal distribution is as follows

$$U = U(\boldsymbol{\alpha}, \mathbf{f}),\ \boldsymbol{\alpha} \sim N_p(\boldsymbol{\mu}, \Sigma) \tag{3}$$

where $\boldsymbol{\mu}$ is a p-dimension vector and Σ is a p-order covariance matrix. Hesitant DMs can be used to test the robustness of IMO methods to preference fluctuations. Different distributions of the parameter vector can reflect different hesitation degree of a hesitant DM.



(a) (b)

Fig. 4. The contours and MPSs of two virtual DMs. (a) The virtual DM is deterministic and its MPS is specific. (b) The virtual DM is hesitant. The MPSs are different for different values of the random parameter vector.

Assume that the two virtual DMs $U(\boldsymbol{\alpha}, \mathbf{f}) = \alpha_1 f_1 + \alpha_2 f_2$ and $U(\boldsymbol{\alpha}, \mathbf{f}) = \alpha_1 f_1 + \alpha_2 f_2, \boldsymbol{\alpha} \sim N_2(\boldsymbol{\mu}, \Sigma)$ represent an assertive DM and a hesitant DM respectively for an MOP with 2 objectives. In order to demonstrate the two virtual DMs intuitively, the contours (dotted lines) of them and their MPSs ($\mathbf{f}^*$) are shown in Fig. 4. Fig. 4(a) shows that the MPS of an assertive DM is specific. In Fig. 4(b), the parameter vector of the virtual DM is a random vector, and when it takes different values, the MPS varies. This reflects that a hesitant DM does not have a specific MPS.

B. Virtual DMs with dynamically changing preference structures

Since a human DM can learn from an interaction process and adjust his/her preferences, his/her preference structure may vary during the interaction process. That is to say, we cannot use a stationary virtual DM to represent his/her preferences. Thus, we use the following dynamic function to emulate human DMs who dynamically change their preference structures

$$U(t) = U(\boldsymbol{\alpha}(t), \mathbf{f}) \tag{4}$$

where t is the interaction number. The parameter vector $\boldsymbol{\alpha}(t)$ is no longer kept the same, but varies with t.

In (4), the change of $U(t)$ is reflected by the change of $\boldsymbol{\alpha}(t)$. The functional form of $U(t)$ does not change. In fact, $U(t)$ can be designed to vary its functional form during the interaction process. For example, it can change from a simple linear function to a complicated nonlinear function. Virtual DMs with dynamically changing preference structures can be used to test the ability of the preference models of IMO methods to follow and capture a DM's preferences. Virtual DMs which change functional forms can also evaluate the applicability of IMO methods.

C. Four types of hybrid virtual DMs

According to the above virtual DM models, the following four types of hybrid virtual DMs can be summarized.

1) Deterministic & Stationary (DS): The formulation of this type of virtual DM is the same as (2). The parameter vector is deterministic and remains unchanged during the interaction process. The optimal solution (MPS) of this type of virtual DM can be obtained by minimizing (2) in the feasible region.

2) Deterministic & Dynamic (DD): This type of virtual DM is shown in (4). The parameter vector is deterministic but varies with the interaction number. We regard the optimal solution of the final form of $U(t)$ in (4) as the MPS of $U(t)$.

3) Stochastic & Stationary (SS): This kind of virtual DM does not vary with the interaction. Its parameter vector obeys a certain distribution as shown in (3). The optimal value of U in (3) is a function of $\boldsymbol{\alpha}$ with the form $U^*(\boldsymbol{\alpha})$. It is a random variable and its expectation $E[U^*(\boldsymbol{\alpha})]$ can be seen as the best value of U. Due to the complex mapping from $\boldsymbol{\alpha}$ to $U^*(\boldsymbol{\alpha})$, the expectation $E[U^*(\boldsymbol{\alpha})]$ may be hard to calculate. For simplicity, $U^*(\boldsymbol{\mu})$ can be regarded as an approximation of $E[U^*(\boldsymbol{\alpha})]$.

To evaluate an IMO method using this type of virtual DM, we can sample N parameter vectors to form N scalar functions and perform N independent experiments by using these functions respectively. The convergence of this IMO method can be evaluated by the difference from the mean value of the function values of the N final solutions to $E[U^*(\boldsymbol{\alpha})]$ or $U^*(\boldsymbol{\mu})$.

4) Stochastic & Dynamic (SD): This type of virtual DM is formulated as follows

$$U(t) = U(\boldsymbol{\alpha}(t), \mathbf{f}),\ \boldsymbol{\alpha}(t) \sim N_p(\boldsymbol{\mu}(t), \Sigma) \tag{5}$$

where $\boldsymbol{\alpha}(t)$ follows a multivariate normal distribution at each interaction. The p-dimension vector function $\boldsymbol{\mu}(t)$ in the distribution is changing with the interaction number t. The optimal value of this virtual DM can be set as the expectation $E[U^*(\boldsymbol{\alpha}(t))]$ or $U^*(\boldsymbol{\mu}(t))$ at the final interaction.

Given one or more solutions, the above four types of virtual DMs can provide preferences like tradeoffs at one solution or the comparison of multiple solutions. A tradeoff means sacrificing one objective to gain improvement of another one at a Pareto optimal solution. A common form of tradeoff is the indifference tradeoff (or marginal rate of substitution, MRS) which refers to the amount that one objective has to sacrifice to offset exactly one unit improvement of another one while the other objectives are kept the same. When the human DM is replaced by a function U, the vector of MRSs can be replaced by the gradient of U.

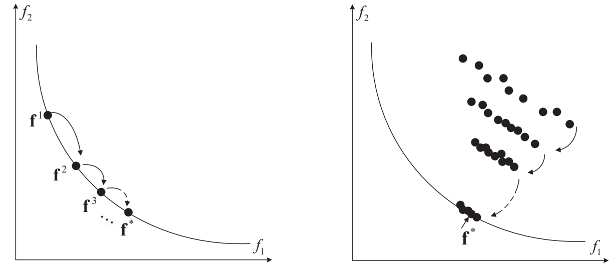
When provided with multiple solutions, the four types of virtual DMs can compare these solutions according to their function values. Suppose that a virtual DM U is to be minimized, it can make pairwise comparisons where for each pair of solutions $\mathbf{x}$ and $\mathbf{y}$, $\mathbf{x}$ is better than $\mathbf{y}$ if $U(\mathbf{x}) < U(\mathbf{y})$. By setting several function levels, U can also divide solutions into corresponding classes. In addition, U can pick out the best solution with the minimal function value.

IV. Experiments with the virtual-DM library

This section shows how the virtual-DM library containing the above four types of virtual DMs is used to compare two newly proposed IMO algorithms on two 2-objective benchmark MOPs: ZDT1 [18] and DTLZ1 [19].

A. Two IMO algorithms

The IMO algorithms used here are the tradeoff-based interactive multiobjective optimization method driven by evolutionary algorithms (T-IMO-EA) [20] and the interactive simple indicator-based evolutionary algorithm (I-SIBEA) [21].



(a) (b)

Fig. 5. The solutions found by the two algorithms in different interactions. (a) Solutions found by T-IMO-EA. (b) Solutions found by I-SIBEA.

In each interaction, T-IMO-EA transforms the MOP into a minimax problem and solves it by using an EA to find a Pareto optimal solution. At this solution, the DM is asked to provide MRSs with which a desirable direction is calculated. An appropriate step size along this direction is calculated to update the weighting vector with which a new minimax problem is formulated to find a new Pareto optimal solution. The solutions (in the objective space) found in different interactions are shown in Fig. 5(a).

I-SIBEA works differently with T-IMO-EA. The DM's preferences are used to guide the population of an MOEA towards the DM's ROI, as shown in Fig. 5(b). The DM can specify the number of interactions. When it is time to interact, the algorithm presents multiple non-dominated solutions to the DM who is expected to select preferred and non-preferred solutions from them. Based on the DMs preferences, a weight distribution function is defined to calculate the weighted hypervolume. Solutions are selected based on their contributions to the hypervolume.

B. Four virtual DMs

The following four virtual DMs are used to replace human DMs to interact with the above two IMO algorithms.

1) DS: U is a quadratic function of f_1 and f_2

$$\begin{aligned} &U_{DS}(\boldsymbol{\alpha}, \mathbf{f}) = \alpha_1 f_1^2 + \alpha_2 f_2^2, \\ &\boldsymbol{\alpha} = [\alpha_1, \alpha_2] = [3, 1]. \end{aligned} \tag{6}$$

2) DD:

$$\begin{aligned} &U_{DD}(\boldsymbol{\alpha}(t), \mathbf{f}) = \alpha_1(t) f_1^2 + \alpha_2(t) f_2^2, \\ &\boldsymbol{\alpha}(t) = \begin{cases} [2, 1] & t = 1 \\ [2.5, 1] & t = 2 \\ [3, 1] & t \geq 3. \end{cases} \end{aligned} \tag{7}$$

3) SS:

$$\begin{aligned} &U_{SS}(\boldsymbol{\alpha}, \mathbf{f}) = \alpha_1 f_1^2 + \alpha_2 f_2^2, \\ &\boldsymbol{\alpha} \sim N_2(\boldsymbol{\mu}, \Sigma), \boldsymbol{\mu} = [3, 1], \\ &\Sigma = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix} \end{aligned} \tag{8}$$

4) SD:

$$\begin{aligned} &U_{SD}(\boldsymbol{\alpha}(t), \mathbf{f}) = \alpha_1(t) f_1^2 + \alpha_2(t) f_2^2, \\ &\boldsymbol{\alpha}(t) \sim N_2(\boldsymbol{\mu}(t), \Sigma), \end{aligned} \tag{9}$$

$$\boldsymbol{\mu}(t) = \begin{cases} [2, 1] & t = 1 \\ [2.5, 1] & t = 2 \\ [3, 1] & t \geq 3, \end{cases}$$
$$\Sigma = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}.$$

Denote U^* as the minimal value of U in the feasible region. The values of U^*_{DS} on ZDT1 and DTLZ1 are 0.425517 and 0.187500, respectively. U^*_{DD} is set as the minimal value of U_{DD} when $\boldsymbol{\alpha}(t) = [3, 1]$. U^*_{SS} is set as the minimal value of U_{SS} when $\boldsymbol{\alpha} = \boldsymbol{\mu} = [3, 1]$. Similarly, U^*_{SD} is set as the minimal value of U_{SD} when $\boldsymbol{\alpha}(t) = \boldsymbol{\mu}(t) = [3, 1]$. Thus, the values of U^*_{DD}, U^*_{SS}, and U^*_{SD} are all the same as U^*_{DS}.

C. Experimental settings

For T-IMO-EA, the MRSs of the human DM are replaced by the gradient of the virtual DM U. T-IMO-EA is terminated when the intersection angle between the gradient of U and the normal vector of the tangent plane of the PF at the current solution is less than a small positive number (1° is used in this paper). For I-SIBEA, the virtual DM U takes the place of the human DM to select preferred and non-preferred solutions.

For each type of virtual DM and each benchmark MOP, both T-IMO-EA and I-SIBEA are run 20 times. For virtual DMs U_{SS} and U_{SD}, the parameter vector $\boldsymbol{\alpha}$ is generated randomly in each run.

The convergence performance of T-IMO-EA and I-SIBEA are compared by using the following error rate

$$ER = \frac{|U^{final} - U^*|}{U^*} \times \% \tag{10}$$

where U^{final} refers to the function value of the final solution found by T-IMO-EA or I-SIBEA.

The dimensions of the decision vectors of ZDT1 and DTLZ1 are 10 and 6 respectively. The population sizes NP and the maximum number of generations G_{max} used in the two algorithms are shown in Table I. Table I also shows two additional parameters used in T-IMO-EA. The initial weighting vector of T-IMO-EA is $W = [0.6, 0.4]^T$ for both ZDT1 and DTLZ1. Other parameters of the two algorithms are set the same as in [20], [21].

Remark: Both for the EA used in T-IMO-EA and the MOEA used in I-SIBEA, the values of NP and G_{max} affect the runtime and the convergence performance a lot. In our experiments, their settings in Table I aim to make the two algorithms converge as well as possible in similar runtime.

Since I-SIBEA needs the DM to specify the number of interactions, in order to compare the two algorithms with the same number of interactions, T-IMO-EA is run first, and its average number of interactions I_{mean} is calculated. The number of interactions of I-SIBEA is set as $\lceil I_{mean} \rceil$ where $\lceil \cdot \rceil$ means rounding up to the nearest number.

D. Experimental results and analysis

The experimental results on ZDT1 and DTLZ1 in terms of the four virtual DMs (DS, DD, SS, and SD) are shown in Tables II and III, respectively. ER_{max}, ER_{mean}, and ER_{min} refer to the the maximal, average, and minimal error rate in the 20 runs, respectively. T_{mean} refers to the average runtime of the 20 runs.

TABLE I. PARAMETERS USED IN THE TWO ALGORITHMS

		NP	G_{max}	m	δ_w
ZDT1	T-IMO-EA	100	500	2	0.005
	I-SIBEA	10	600	-	-
DTLZ1	T-IMO-EA	60	800	2	0.005
	I-SIBEA	10	500	-	-

The performance of T-IMO-EA and I-SIBEA on ZDT1 can be analysed according to Table II. For all the four virtual DMs, both T-IMO-EA and I-SIBEA can find solutions whose function values are within the range $[U^*, (1 \pm 2\%)U^*]$ (in the sense of average error rate) with similar runtime. This means that the two algorithms can adapt to the dynamical change of preference structures and find nearly optimal solutions of hesitant DMs. Both the value of ER_{mean} and the value of $ER_{max} - ER_{min}$ of T-IMO-EA are less than those of I-SIBEA, which reflects that T-IMO-EA is more robust than I-SIBEA in finding MPSs of virtual DMs.

From Table III, T-IMO-EA is able to find solutions whose function values are within the range $[U^*, (1 \pm 3\%)U^*]$ (in the sense of the mean error rate) on DTLZ1 for all the virtual DMs. Given the same number of interactions and similar runtime, the average error rate (ER_{mean}) of I-SIBEA is very large for each virtual DM, which means that I-SIBEA fails to find solutions close to the MPS in some runs. A larger population size can improve the performance of I-SIBEA, however, the runtime will increase.

From Table II and the results of T-IMO-EA in Table III, hesitant virtual DMs (SS and SD) bring larger maximal error rates and average error rates than deterministic virtual DMs (DS and DD), which is natural because a hesitant DM's MPS is not fixed. For different parameter vectors, the MPSs and their function values are different.

V. CONCLUSIONS

In this paper, we have proposed a framework of a virtual-DM library which is made up of various types of virtual DMs. The virtual-DM library can take the place of human DMs in interacting with IMO methods. Virtual DMs in the library can take different forms like decision rules or scalar functions. Based on the input of an IMO method, the virtual-DM library can select an appropriate virtual DM to provide corresponding preferences which will guide the search of the IMO method. The MPS of each virtual DM is known, thus each type of virtual DM can evaluate the performances of IMO methods such as accuracy of the final solution, convergence time, and the interaction number. Different types of virtual DMs can be combined to measure more performances of IMO methods.

Taking human DMs' personalities and preference structures with and without dynamic changes into consideration, four types of hybrid virtual DMs have been constructed. They can test not only the adaptability of IMO methods to different human DMs, but also the capacity of IMO methods to capture a DM's preferences when he/she changes his/her preference structure during the interaction process. The experiments have demonstrated how the four types of virtual DMs are designed and used to compare two IMO algorithms.

Future works include the following points:

1) building more types of virtual DMs considering other

TABLE II. RESULTS ON ZDT1 IN TERMS OF THE FOUR VIRTUAL DMS

		$ER_{max}(\%)$	$ER_{mean}(\%)$	$ER_{min}(\%)$	$ER_{max} - ER_{min}(\%)$	$\lceil I_{mean} \rceil$	$T_{mean}(s)$
DS	T-IMO-EA	0.0090	0.0079	0.0071	0.0019	3	20.7
	I-SIBEA	4.7388	0.9497	0.0001	4.7387	3	22.3
DD	T-IMO-EA	0.0027	0.0022	0.0016	0.0011	4	34.0
	I-SIBEA	9.2058	1.5042	0.0049	9.2009	4	30.1
SS	T-IMO-EA	10.5011	0.0953	0.0520	10.4491	4	27.2
	I-SIBEA	19.3990	1.9219	0.4753	18.9237	4	29.5
SD	T-IMO-EA	11.5623	1.4325	0.1526	11.4097	4	26.7
	I-SIBEA	18.2267	1.8274	0.0352	18.1915	4	26.7

TABLE III. RESULTS ON DTLZ1 IN TERMS OF THE FOUR VIRTUAL DMS

		$ER_{max}(\%)$	$ER_{mean}(\%)$	$ER_{min}(\%)$	$ER_{max} - ER_{min}(\%)$	$\lceil I_{mean} \rceil$	$T_{mean}(s)$
DS	T-IMO-EA	$< 10^{-7}$	$< 10^{-7}$	$< 10^{-7}$	$< 10^{-7}$	1	8.9
	I-SIBEA	6545.2	1005.0	6.7547	6538.4	1	10.9
DD	T-IMO-EA	0.29×10^{-3}	0.14×10^{-4}	$< 10^{-7}$	$< 0.3 \times 10^{-3}$	3	15.9
	I-SIBEA	12080	1181.8	3.9881	12076	3	16.5
SS	T-IMO-EA	16.7872	1.0025	0.2627	15.5245	2	9.3
	I-SIBEA	3742.6	838.34	0.5547	3742	2	17.0
SD	T-IMO-EA	17.9287	2.5470	0.0520	17.8767	3	16.3
	I-SIBEA	13202	1211.9	1.6409	13200	3	17.2

personalities and characters of human DMs.

2) incorporating various cognitive biases into virtual DMs.

3) developing performance metrics reflecting different capabilities of IMO methods.

4) designing experiments to compare more IMO methods.

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