# Solving the Uncertain Multi-objective Multi-stage Weapon Target Assignment Problem via MOEA/D-AWA

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***Abstract*— The weapon target assignment (WTA) problem is a fundamental problem arising in defense-related applications of operations research. And the multi-stage weapon target assignment (MWTA) problem is the basis of dynamic weapon target assignment (DWTA) problems which commonly exist in practice. The MWTA problem considered in this paper is with uncertainties, namely the uncertain MWTA (UMWTA) problem, and is formulated into a multi-objective constrained combinatorial optimization problem with two competing objectives. Apart from maximizing damage to hostile targets, this paper follows the principle of minimizing ammunition consumption under the assumption that each element of the kill probability matrix follows four different probability distributions. In order to tackle the two challenges, i.e., multi-objective and the uncertainty, the multi-objective evolutionary algorithm based on decomposition with adaptive weight adjustment (MOEA/D-AWA) and the Max-Min robust operator are adopted to solve the problem efficiently. Then comparison studies between the MOEA/D-AWA and a single objective solver used for a relaxed formulation on solving both certain and uncertain instances of two different scaled MWTA problems which include four uncertain scenarios are conducted. Numerical results show that MOEA/D-AWA outperforms the single objective solver on solving both certain and uncertain multi-objective MWTA problems discussed in this paper. Comparisons between the results of the certain and uncertain formulation also indicate the necessity of the robust formulation of practical problems.**

***Keywords—multi-stage weapon target assignment (MWTA); multi-objective constrained optimization problem; uncertain optimization; Max-Min robust operator, multi-objective evolutionary algorithm based on decomposition with adaptive weight adjustment (MOEA/D-AWA); combinatorial optimization***

## I. INTRODUCTION

The weapon target assignment (WTA) problem is a fundamental problem arising in defense-related applications of military operations research [1], which deals with how to obtain a weapon-target scheme or a set of weapon-target schemes that meet decision makers' operational goals regarding the combating effects and expenditures. The WTA problem is a classical constrained combinatorial optimization problem and has been proved to be NP-complete [2] which means any enumeration-based solver faces exponential computational complexity as the problem size increases. Hosein et al. grouped the WTA problem into two categories: the static WTA (SWTA) problem and the dynamic one (DWTA) [3-5]. In the SWTA problem, all weapons engage with targets in a single stage. On the contrary, DWTA is a multistage problem where some weapons engage with targets at one stage, and then the strategy for the next stage is decided based on the assessment of the former stage. The DWTA problem is a global decision-making process, which takes the whole defense effects through all stages into account, incorporates the concept of time window and is much more complicated than the SWTA one. In real combat situations, after making decisions at one stage, there will be a damage assessment during which a number of new targets may appear or some old targets may exit [6]. Following with that, a new decision making process of the next stage will be triggered. This process is the same as the previous, except that the computational complexity decreases as the reduction of the numbers of weapons and targets. Thus, there is a cyclic computation: "Decision Making → Damage assessment → Decision Making" in an actual DWTA process. The multi-stage weapon target assignment (MWTA) problem falls between the SWTA and DWTA one, which also takes time windows into account but does not possess the dynamic process like DWTA. In a word, multi-stage WTA problems lay the foundation of dynamic ones.

According to different combat situations and missions, WTA problems can be categorized into two types: the target-based and the asset-based. The aim of the target-based WTA problem is to maximize the expectation of the damage of the targets, while the goal of the asset-based one is to minimize the expected loss of protected assets. There is no essential distinction between the two types, and the target-based model can be regarded as a special case of the asset-based one. The target-based one is taken into consideration in the following. In an actual combat situation, a proper WTA scheme should not only satisfy certain operational requirements, such as maximizing the expected damage of targets or minimizing the expected loss of protected assets, but also minimizing the operational cost. Since the two goals conflicts in the sense that more ammunition cost, better operational effect, the WTA problem in practice is a multi-objective optimization problem (MOP).

Traditional research on WTA problems, no matter in the single objective [7-9] or multi-objective [10] formulation, concerns on the deterministic case in which parameters in the

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model are precisely known in advance. However, uncertainties widely exist in military applications such as planning and scheduling. And in the past years, research on the issue of uncertainty is a hot topic. Various decision-making in military applications should be made in dynamic, distributed, and uncertain environments. Due to the uncertainties that some of parameters may have, effectivenesses of solutions generated from the deterministic case may be deteriorated. It is therefore conceivable that as a parameter takes values different from the deterministic nominal one, the optimal solution found using the deterministic nominal data may be no longer optimal or even infeasible for the model using the changed one. This observation raises the natural question of designing solution approaches that are immune to parameters' uncertainties, namely the robust approach. The WTA problem features a whole spectrum of uncertainties, such as the number and type of targets in the battle space, their positions and the probabilities of weapons to destroy targets (e.g., probability of kill) [11]. In this paper, we address the uncertainties in the weapons' probability of kill and propose a multi-objective formulation of the uncertain MWTA (UMWTA) problems. This kind of uncertainty is quite in common and varies with the battle situation under consideration. Similar to our previous research on the MWTA problem [10], this paper also models it as a two-objective constrained one and solves it via MOEA/D-AWA (MOEA/D with adaptive weight adjustment) [12], an improved version of the famous MOEA/D (multi-objective evolutionary algorithm based on decomposition) [13]. A distinct advantage of MOEA/D-AWA is that it is able to handle the discontinuous and long tail Pareto front (PF) and provide a well-distributed PF. Therefore, human decision-makers can have a global view of candidate desirable MWTA schemes. Contributions of this paper are as follows:

• Most studies which model WTA problems as MOPs concentrate on SWTA problems instead of the MWTA ones studied in this paper.

• Most studies considered MWTA problems as deterministic, while uncertainties widely and definitely exist in actual situations. Optimums for deterministic problems may be unsuitable or even infeasible for uncertain ones. From the point of the actual application, uncertain MWTA problems should be paid attention to and solved by effective approaches, which is of great importance. This paper models the uncertain MWTA problem as a Max-Min form which can give a conservative but reliable assignment decision or a set of such decisions.

• As to the optimizers, MOEA/D and its variants are very popular in the field of MOP solving. But previous research mostly benchmarked these algorithms on a series of continuous instances, some specific combinatorial optimization problems, such as multi-objective traveling salesman problems (MOTSPs) and SWTA problems. Besides, by noticing the fact that PF of the MWTA problem is discontinuous, and may have long-tail effects in some cases, the improved MOEA/D with adaptive weight vector adjustment (MOEA/D-AWA) proposed by Yutao Qi et al. [12] is adopted here for solving the UMWTA problems.

The rest of this paper is organized as follows. In Section II, the deterministic mathematical formulations of the multi-objective MWTA problem and its robust counterpart are given. Section III gives algorithmic descriptions of the MOEA/D-AWA which is adopted to solve the uncertain multi-objective combinatorial problem. Section IV presents the parameter settings of two different-scaled certain and uncertain MWTA problems and experimental results. Section V concludes the paper and points out future work.

## II. Description and Mathematical Formulation of UMWTA Problems

In this section, the mathematical descriptions of multi-objective UMWTA problems including deterministic and uncertain formulations are presented. And in the uncertain case, the weapon kill probabilities are assumed to be uncertain. Firstly, the formulation of the deterministic MWTA problem which will be used in subsequent description of the formulation of the UMWAT one is given.

### A. Description of the Deterministic Multi-Objective MWTA Problems

Models of MWTA problems depend on many factors, e.g., defense strategies and features of targets and weapons. The scenario considered in this paper is delineated as follows: At certain time, the defender detects $T$ hostile targets and has $W$ weapons to intercept targets. Besides, before these offensive targets break through the defense and escape, there are at most $S$ stages available for the defender to use its own weapons to hit the targets. The above-mentioned combat scenario is very common, e.g., in air-defense-oriented naval group combating. Given a set of targets and available weapons, one must find the optimal assignment of weapons to targets, such that, for example, the damage to the targets is maximized, or the cost of operations is minimized. The MWTA problem is formulated as a two-objective problem from the perspective of target-based models under the above-described assumption. The generic deterministic formulation of the two-objective MWTA problem is as follows:

$$\max \quad D_t(X^t) = \sum_{j=1}^{T(t)} v_j \Big(1 - \prod_{s=t}^{S} \prod_{i=1}^{W(t)} (1 - p_{ij}(s))^{x_{ij}(s)}\Big) \tag{1}$$

$$\min \quad C_{ammu} = \sum_{s=1}^{S} \sum_{j=1}^{T} \sum_{i=1}^{W} \beta_i u_{ij}(s) x_{ij}(s) \tag{2}$$

$$\text{s.t.} \quad \sum_{i=1}^{W} x_{ij}(t) \le m_j \quad \forall j \in I_j, \forall t \in I_t \tag{3}$$

$$\sum_{j=1}^{T} x_{ij}(t) \le n_i \quad \forall i \in I_i, \forall t \in I_t \tag{4}$$

$$\sum_{j=1}^{T} \sum_{t=1}^{S} x_{ij}(t) \le N_i \quad \forall i \in I_i \tag{5}$$

$$x_{ij}(t) \le f_{ij}(t) \quad \forall i \in I_i, \forall j \in I_j, \forall t \in I_t \tag{6}$$

$$I_i = \{1,2,...,W\}; I_j = \{1,2,...,T\}; I_t = \{1,2,...,S\}$$

where $D_t(X^t)$ in (1) is the expected damage at stage $t$; $C_{ammu}$ in (2) is the overall ammunition consumption through all stages. $X^t = [X_t, X_{t+1}, ..., X_S]$ with $X_t = [x_{ij}(t)]_{W \times T}$ is the decision matrix at stage $t$, and $x_{ij}(t)$ is a binary decision variable taking a value of one (i.e., $x_{ij}(t) = 1$) if weapon $i$ is assigned to target $j$ at stage $t$, or zero (i.e., $x_{ij}(t) = 0$) otherwise. $W(t)$ and $T(t)$ represent the remaining number of weapons and targets at stage $t$, respectively ($W(1) = W, T(1) = T$). $v_j$ means the threat value of target $j$. $p_{ij}(s)$ denotes the probability that weapon $i$ destroys target $j$ at stage $s$, which is also called the kill probability. $u_{ij}(s)$ denotes the ammunition consumption when weapon $i$ is allocated to target $j$ at stage $s$, $\beta_i$ represents the unit economic cost of the ammunition that weapon $i$ consumes. If all weapons are the same, $\beta_i$ are assumed to be constant (e.g., one unit in this paper).

Constraints (3)-(5) are resource constraints. Constraint (3) means at each stage at most $m_j$ weapons can be used to destroy target $j$, which limits the ammunition cost for each target at each stage. The value of $m_j$ usually depends on the performance of available weapons and the tactical preference of commanders. Constraint (4) reflects the capability of weapons of firing at multiple targets at the same time. To be more precise, weapon $i$ can fire at most $n_i$ targets at the same time. Actually, most weapons can fire only one target, while for special cases, e.g., artillery-based defense systems, the value of $n_i$ may be larger than two. In these special cases, these weapons can be regarded as $n_i$ independent weapons, so it is assumed that $n_i = 1, \forall i \in I_i$. Constraint (5) indicates the amount of available ammunitions of weapon $i$. Constraint (6) is the engagement feasibility constraint which is an important feature of DWTA against SWTA, since it considers the influences of time windows on the engagement feasibility of weapons. $f_{ij}(t)$ is a binary variable. If weapon $i$ cannot shoot target $j$ at stage $s$ for various reasons (e.g., the target is beyond the shooting range of the weapon), then $f_{ij}(t) = 0$, otherwise $f_{ij}(t) = 1$. Due to the time-dependent property of the engagement feasibility, the feasibility matrix should be updated after each stage. Besides, the update process also increases the complexity of DWTA and the difficulty of generating feasible solutions.

According to the theory of shooting and performances of weapons, the kill probability and ammunition consumption matrices at any stage s, denoted by $P(s)$ and $U(s)$ respectively, can both be obtained. The two objectives are conflicting in the sense that when assigning more weapons to a target, the expected damage of targets will be higher, but this, in turn, will result in higher overall ammunition consumptions. Hence, the aim of two-objective optimization is to find an acceptable trade-off between operational effect and cost.

### B. *Description of the Roubst Multi-objective MWTA Problems*

Traditional MWTA problems assume that parameters in the model are precisely known in advance and equal to some deterministic nominal values which may be given based on a set of historical experience. Therefore it is referred to as deterministic models. The deterministic model, however, does not take into account the influence of uncertain parameters on the feasibility and quality of the optimal solutions to the model. Actually during an execution, uncertainties widely exist in combat environments and can induce different kinds of failures, such as the failure to protect own assets, the mission incompleteness (e.g., missing a target) and the inaccurate attack (e.g. false target attack) [11]. Therefore, in the modern combat situation with uncertainties, robust decision-making procedures are necessary. Such procedures must take into account uncertain factors, and generate decisions that are not only effective on average (in other words, have good "expected" performance) but also safe enough under the whole range of possible scenarios [14]. In this regard, robust optimization in military applications might be suitable.

In this paper, we investigate the uncertain version of the MWTA problem with the uncertain kill probability matrix, and propose a two-objective model for the UMWTA problem. To take into account influences of uncertain kill probabilities during the assignment, the traditional Max-Min operator which is a typical robust concept is adopted. For a maximization problem, the Max-Min operator means to optimize the worst case. Under the Max-Min robust approach which might be too conservative but effective, we are willing to accept suboptimal solutions in order to ensure that these solutions remain feasible and near optimal when parameters are changed.

In this section we present the two-objective formulation of the UMWTA problem. The uncertainties are introduced into the model by assuming that probabilities $p_{ij} = p_{ij}(\xi)$ are stochastic and dependent on a random parameter $\xi$ which may relate to battle situations, weather conditions, and so on. In the following, we model the stochastic behavior of probabilities $p_{ij}(\xi)$ using various distributions. That is, kill probabilities $p_{ij}(\xi)$ follow the following two common probability distributions [15]:

- Uniform distributions: $p_{ij} \sim U((1-\alpha)\cdot p_{ij}, (1+\alpha)\cdot p_{ij})$
- Normal distributions: $p_{ij} \sim N(p_{ij}, \alpha \cdot p_{ij})$

In any case, the central tendency of a distribution always corresponds to the deterministic kill probability $p_{ij}$ for the instance under consideration. The parameter $\alpha$ is used to tune the degree of a deviation of the kill probability. In the following, two $\alpha$-values will be considered: $\alpha \in \{0.1, 0.2\}$. For instance, there is a deviation of ±10% or ±20% for a uniform distribution, alternatively. Such a distribution may be constructed, for example, by utilizing historical observations of weapons' efficiency in different environments, or by using simulated data, experts' opinions etc. Note that different probability distributions of $p_{ij}$ represent different uncertain situations discussed in the previous section. In fact, the Max-Min robust optimization considered here is based on the consideration of so-called worst-case optimality, i.e., the solution should keep optimality in the worst case. In this sense, robust multi-objective MWTA optimization problems can be formulated as:

$$\max \quad RD_t(X^t) = \min_{k=1,\dots,K}\{\sum_{j=1}^{T(t)} v_j (1 - \prod_{s=t}^{S} \prod_{i=1}^{W(t)} (1 - p_{ij}(s,\xi_k))^{x_{ij}(s)})\} \tag{7}$$

$$\min \quad C_{ammu} = \sum_{s=1}^{S} \sum_{j=1}^{T} \sum_{i=1}^{W} \beta_i u_{ij}(s) x_{ij}(s)$$

s.t. (3), (4), (5), and (6)

where $RD_t(X^t)$ in (7) is the minimum expected damage to targets over $K$ sampled uncertain weapon kill probability matrices $P(s,\xi_k)(k=1,\dots,K)$. Except for the objective of minimizing the minimum expected damage of targets in the model, the decision variables and other deterministic parameters in the UMWTA models are identical to their deterministic predecessor.

## III. THE MULTI-OBJECTIVE MWTA SOLVERS

Numerous classical intelligent optimization algorithms have been applied to solve WTA problems whether static or dynamic, single-objective or multi-objective. Recently, MOEA/D (Multi-Objective Evolutionary Algorithm based on Decomposition) in which the decomposition idea is applied instead of the classical Pareto dominance relation has achieved a great success in the field of evolutionary multi-objective optimization. It decomposes an MOP into a set of scalar subproblems using uniformly distributed aggregation weight vectors and optimizes them concurrently. Generally, the uniformity of weight vectors in MOEA/D can ensure the diversity of the Pareto optimal solutions based on the assumption that the PF is close to the hyper-plane in the objective space. However, the basic assumption might be violated in the case that the PF of the target MOP is complex, i.e. the PF is discontinuous or has a shape of sharp peak and long tail. Therefore, some studies have been done to refine the weight vectors in MOEA/D [12, 16-19].

An improved MOEA/D with the adaptive weight vector adjustment (MOEA/D-AWA) proposed by Yutao Qi et al. [12] is one of such research, in which an adaptive weight vector adjustment (AWA) strategy is introduced to obtain the uniformly distributed PF of the target MOP. It is natural to have an uniformly distributed PF by adopting an AWA strategy to regulate the distribution of weight vectors periodically. Firstly, the AWA strategy removes subproblems located in crowded regions whose crowded degrees are measured by the vicinity distance which evaluates the sparsity level of a solution among current population. Next, the elite population is deployed for helping add new subproblems into the real sparse regions of the complex PF rather than the discontinuous parts which are pseudo sparse regions. If an elite individual is located in a sparse region of the evolving population, it will be recalled into the evolving population and a new weight vector will be generated and added to the subproblem set. MOEA/D-AWA has the same framework as the version of MOEA/D with the dynamic resource allocation strategy (MOEA/D-DRA) [20]. In this version, a utility function is defined and computed for each subproblem. Computational efforts are distributed to each of the subproblems based on their utility function values. The major difference between MOEA/D and the suggested MOEA/D-AWA lies on the periodically update of the weight vectors during the search procedure.

Based on our working paper, experimental results indicate that MOEA/D-AWA outperforms MOEA/D on solving MWTA problems whose PF is discontinuous, uniform and may be long tail in some study cases. So in the following, the MOEA/D-AWA with the decimal coding is adopted to solve the deterministic and uncertain multi-objective MWTA problems. Similar to our previous work applying MOEA/D on solving deterministic MWTA problems [10], the decimal encoding, two point crossover, one point mutation and the random repair mechanism are also used in MOEA/D-AWA. The only difference between the deterministic and the uncertain MWTA problem is the way that evaluates the first objective which contains uncertain parameters. Readers can refer to [12], [13] and [20] for a deeply understanding of MOEA/D-DRA and MOEA/D-AWA. The pseudocode of MOEA/D-AWA is summarized as follows:

**Input**: A stopping criterion, parameters

**Output**: Pareto set, Pareto front

**Step1**. Initialization

**Step2**. Dynamic computing resources allocation (DRA)

**Step3**. Evolution

**Step3.1**. selecting mating pool

**Step 3.2**. reproduction: crossover and mutation followed with a repair mechanism

**Step3.3**. update: update ideal point and neighborhood solutions

**Step4**. Adaptive weight adjustment

If certain condition is satisfied, adaptively adjust the weight vectors as follows:

**Step4.1**. Update the external population EP by the new generation of offspring according to the vicinity distance based non-dominated sorting.

**Step4.2**. Delete the overcrowded subproblems.

**Step4.3**. Add new subproblems into spare regions.

**Step4.4**. Rebuilt neighborhood of each solution.

**Step5**. Stopping criteria: If the stopping criterion is met, stop; else gen:=gen+1, go to **Step2**.

Since there is no protocol fully adapted to evaluate the effectiveness of multi-objective optimization methods for uncertain problems exists by now, we choose to find the solution whose operational cost is minimized while satisfying certain operational requirement. That is to say, among all non-dominated solutions, the solution whose damage to targets is larger than or equal to a predefined value and operational cost is minimized will be regarded as the final optimal solution for further comparison. Therefore, we adopt a single objective setup, where the total cost of operations is minimized, while satisfying constraints (3)-(6) and an additional constraint on damage accomplishment of targets. The following constraints (8) and (9) correspond to the certain and uncertain case, respectively, in which $d$ and $d(\xi)$ are the corresponding predefined operational requirements.

$$D_t(X^t) = \sum_{j=1}^{T(t)} v_j \Big(1 - \prod_{s=t}^{S} \prod_{i=1}^{W(t)} (1 - p_{ij}(s))^{x_{ij}(s)}\Big) \ge d \tag{8}$$

$$RD_t(X^t) = \min_{k=1,\ldots,K} \Big\{\sum_{j=1}^{T(t)} v_j \Big(1 - \prod_{s=t}^{S} \prod_{i=1}^{W(t)} (1 - p_{ij}(s,\xi_k))^{x_{ij}(s)}\Big\} \ge d(\xi) \tag{9}$$

For a fair comparison, experiments on single objective UMWTA problems also use the decimal coding. Besides, the same recombination operators and random repair mechanism are also conducted. Since constraints (3)-(5) and (8)/(9) are contradicted with each other in the sense that the decrease of the assignments benefits the satisfaction of constraints (1)-(3) while is adverse to the constraint (8)/(9) , it is difficult to design an efficient and effective repair approach to handle the two kinds of constraints at the same time. So the constraint (8)/(9) is relaxed during the evolutionary process and the same evolutionary operators used in the MOEA/D-AWA are adopted. Specifically, initialize a set of feasible solutions and perform recombination operators followed by the random repair mechanism on constraint (3)-(6) without considering the constraint (8)/(9) in deterministic and uncertain situations, respectively. Then select individuals that satisfy constraint (8)/(9) (satisfaction of constraints (3)-(5) is already guaranteed during the recombination operators) in the pool of parents and offspring, denote as *Fea_Pop* whose size is *Fea_N*. If *Fea_N* is larger than or equal to the size of population *N*, select *N* individuals with best fitness value from *Fea_Pop*; Otherwise, *Fea_Pop* feasible individuals combined with $N - Fea\_N$ individuals with the largest value of damage to targets in the remaining of the pool are regarded as the offspring. Finally the process is repeated as a common GA until a stop criterion is satisfied. More details of the recombination operators and repair mechanism during applying MOEA/D on solving the deterministic MWTA problems can be found in [10].

## IV. NUMERICAL EXPERIMENTS

### A. Parameter Settings

Since the actual data is difficult to obtain, in this section, two instances with different problem scales, i.e. small and medium cases, are randomly generated. The number of weapons/targets/stages in small and medium-scaled instances are 3/5/3 and 20/12/5, respectively. The value vector of targets comes from [21]. For each instance, the deterministic weapon kill probability matrix $P(s)$ and the ammunition consumption matrix $U(s)$ are randomly generated within a given range related to the problem scale, which are exactly the same as [10]. Then in order to generate uncertain instances based on the deterministic instances, four probability distributions that the weapon kill probability may follow can be applied over the deterministic data. Specifically, each element of the uncertain weapon kill probability matrix $P(s,\xi)$, namely $p_{ij}(s,\xi)$, is sampled from the abovementioned four different and typical distributions: $U((1-\alpha)\cdot p_{ij},(1+\alpha)\cdot p_{ij}),(\alpha=0.1,0.2)$ and $N(p_{ij},\alpha\cdot p_{ij}),(\alpha=0.1,0.2)$. Again, in any case, the central tendency of the distribution always corresponds to the deterministic kill probability $p_{ij}$ of the instance under consideration and $\alpha$ represents the degree of the deviation of the kill probability. For the convenience of result display, denote the uncertain scenarios whose uncertain parameters $p_{ij}(s,\xi)$ follow the uniform distribution with $\alpha = 0.1, 0.2$ as *U1* and *U2*, respectively. Similarly, the uncertain cases whose uncertain parameters $p_{ij}(s,\xi)$ follow the normal distribution with $\alpha = 0.1, 0.2$ are shorten for *N1* and *N2*, respectively. Parameters in constraints including $m_j$, $n_i$ and $N_i$ are deterministic and are assigned the same values in [10]. The predefined values of damage to targets $d(\xi)$ vary with different uncertain scenarios. To get a reasonable value of $d(\xi)$ for each uncertain scenario under each problem scale, we randomly generate 1000 solutions satisfying constraints (1)-(4) and then calculate their robust damage to targets via $RD_t(X^t)$ using the corresponding uncertain weapon kill probability matrix $P(s,\xi)$. The value of $d(\xi)$ for every uncertain scenario can be obtained ahead of the optimization based on the 70% of the maximum value of the 1000 uncertain damage values. Besides, the crossover and mutation rates are set to 0.1 and 0.9, respectively. All the other parameters for each instance are given in TABLE I:

TABLE I. PARAMETER SETTINGS OF THE MOEA/D-AWA FOR INSTANCES OF THREE SCALES ON MWTA PROBLEMS

| Instance | $n_i$ | $m_j$ | $N_i$ | $d$ in *U1* | $d$ in *U2* | $d$ in *N1* | $d$ in *N2* | pop | gen |
|---|---|---|---|---|---|---|---|---|---|
| Small | 1 | 1 | 2 | 2.0479 | 1.6949 | 1.8413 | 1.4298 | 100 | 100 |
| Medium | 1 | 2 | 2 | 8.8791 | 8.5030 | 8.46629 | 7.97524 | 100 | 200 |

### B. Comparison Results

To demonstrate the effectiveness of the multi-objective optimizer on solving the UMWTA problems, we test it on two different scaled instances, and each instance contains four different uncertain scenarios whose kill probabilities follow different distributions. For each kind of distribution, *K* sampled points are randomly sampled via the Monte Carlo method, which means *K* function evaluations need to be performed for each solution per run for each instance. Based on the statistic knowledge, the formulation of *K* is shown as follows and is named the minimized sample size which represents the minimum sample times that are needed to obtain results satisfying given confidence level:

$$K \approx \frac{{z_{\alpha/2}}^2 \sigma^2}{E^2}$$

where $z_{\alpha/2}$ is the reliable coefficience, and when the confidence level is 95%, $z_{\alpha/2} = 1.96$; $\sigma^2$ is the sample variance which measures the deviation between samples and the mean value of the whole. The larger $\sigma^2$ is, the more samples needed; *E* represents the sample error which is related to the mean value and is taken as 10% of the mean value. Thus the sample size for *U1, U2, N1* and *N2* can be roughly set as 5, 10, 40 and 80, respectively.

Firstly, comparison studies are done on each uncertain scenario under each problem instance to show the superiority of MOEA/D-AWA on solving UMWTA problems. As stated earlier, the best solution whose operational cost is minimized while satisfying that damage to targets is larger than or equal to the predefined value of certain uncertain scenario under certain problem instance among all non-dominated solutions is selected as the final optimal solution for the further comparison. By now, the single objective solver adopting the same GA framework with MOEA/D-AWA except for the relaxation of the additional constraint (robust_b') (denoted as SOP_relax in the following) can be compared with the MOP solver fairly.

TABLES II-III provide statistical results of the optimal operational cost and corresponding robust damage to targets with respect to each robust model for the small and medium-scaled cases, respectively. Note that the statistical results including the best, average and standard deviation values are over 20 independent trials to smooth out random characteristics in simulations conducted. Besides the above performance measures, the non-parametric Wilcoxon signed ranks test [22] is adopted to compare the performance of the proposed algorithm with the other algorithm statistically. It is used for finding a significant difference between the behaviors of two algorithms. The level of significance is assigned as 0.05, which indicates that if the p-value is smaller than 0.05, there is a significant difference between the two algorithms. For more details about the test computations refer to [22].

TABLE II. RESULTS UNDER FOUR UNCERTAIN SCENARIOS OF THE SMALL-SCALED INSTANCE

| Robust Small Case | | | MOEA/D-AWA | SOP_relax |
|---|---|---|---|---|
| *U1* (2.0479) | operation cost | best | 26 | **25** |
| | | average | 26 | **25.9** |
| | | standard deviation | 0 | 0.567646 |
| | damage to targets | corresponding damage | 2.1693 | 2.0583 |
| | | average | 2.1693 | 2.11209 |
| | | standard deviation | 0 | 0.050957 |
| | p-value | | 0.564 | |
| *U2* (1.6949) | operation cost | best | **21** | 21 |
| | | average | **21** | 21.6 |
| | | standard deviation | 0 | 0.699206 |
| | damage to targets | corresponding damage | 1.6957 | 1.6957 |
| | | average | 1.6957 | 1.70055 |
| | | standard deviation | 0 | 0.006415 |
| | p-value | | 0.034 | |
| *N1* (1.8413) | operation cost | best | **25** | 25 |
| | | average | **25** | 25.2 |
| | | standard deviation | 0 | 0.421637 |
| | damage to targets | corresponding damage | 1.8549 | 1.8549 |
| | | average | 1.8549 | 1.85781 |
| | | standard deviation | 0 | 0.016564 |
| | p-value | | 0.157 | |
| *N2* (1.4298) | operation cost | best | **23** | 23 |
| | | average | **23** | 24.1 |
| | | standard deviation | 0 | 0.567646 |
| | damage to targets | corresponding damage | 1.4455 | 1.4455 |
| | | average | 1.4455 | 1.50204 |
| | | standard deviation | 0 | 0.029286 |
| | p-value | | 0.005 | |

TABLE III. RESULTS UNDER FOUR UNCERTAIN SCENARIOS OF THE MEDIUM-SCALED INSTANCE

| Robust Medium Case | | | MOEA/D-AWA | SOP_relax |
|---|---|---|---|---|
| *U1* (8.8791) | operational cost | best | **116** | 156 |
| | | average | **120.1** | 160.5 |
| | | standard deviation | 3.665151 | 2.877113 |
| | damage to targets | corresponding damage | 8.9388 | 8.8874 |
| | | average | 8.94691 | 9.033222 |
| | | standard deviation | 0.041233 | 0.182281 |
| | p-value | | 0.005 | |
| *U2* (8.503) | operational cost | best | **113** | 153 |
| | | average | **117.8** | 160.7 |
| | | standard deviation | 4.131182 | 4.423423 |
| | damage to targets | corresponding damage | 8.5499 | 8.6549 |
| | | average | 8.56039 | 8.85168 |
| | | standard deviation | 0.056809 | 0.24178 |
| | p-value | | 0.005 | |
| *N1* (8.46629) | operational cost | best | **112** | 156 |
| | | average | **118** | 163.6 |
| | | standard deviation | 3.829708 | 3.50238 |
| | damage to targets | corresponding damage | 8.4907 | 8.589 |
| | | average | 8.51576 | 8.60733 |
| | | standard deviation | 0.059153 | 0.115367 |
| | p-value | | 0.005 | |
| *N2* (7.97524) | operational cost | best | **116** | 153 |
| | | average | **120.7** | 160.3 |
| | | standard deviation | 3.713339 | 4.522782 |
| | damage to targets | corresponding damage | 8.0086 | 8.0235 |
| | | average | 8.06944 | 8.11174 |
| | | standard deviation | 0.07843 | 0.09577 |
| | p-value | | 0.005 | |

Take TABLE II as an illustration. First of all, the number in bracket of the first column for each scenario is the predefined value of the damage to targets. And both the two approaches can find feasible solutions since each value of the damage to targets for each method is larger than or equal to the corresponding predefined value. Next, the performance of the MOEA/D-AWA is investigated by comparing the objective value (i.e., operational cost) between MOEA/D-AWA and SOP_relax, and better values are highlighted in boldface. Results of the corresponding two columns indicate that MOEA/D-AWA can find the solution with better objective value for almost every uncertain situation, which also gives the evidence on the effectiveness of the multi-objective optimizer. It's worth noting that the MOP approach based on MOEA/D-AWA not only can give a better solution in the sense of single objective, but also can give a wide range of solutions which are non-dominated with each other. Optimizers can select one or a set of solutions from the candidate pool according to its own preference. Similar analysis can be done on the medium cases. The bold numbers in the last two columns also mean the algorithm in that column performs better than the other one. Besides, it is clear that the p-values are mostly smaller than 0.05, which reveals that the MOP approach outperforms the SOP one significantly. Thus from the robust experimental results shown in TABLES II-III, we can have the conclusion that the MOP approach shows a better performance than the SOP_relax in large degree.

Since the convergent processes are similar for different uncertain scenarios under certain scaled problem, for the sake of brevity, the fitness curves by generations only for the *U1* scenario in each instance are shown in Fig.1. Evidently, in our simulation, those curves all converge to a fixed value, which shows the convergent process.

TABLES IV-V are the statistical results of the deterministic models. Similar conclusions can be obtained among the four approaches for the deterministic model as shown in TABLE IV- V that the MOP approach shows better performance than the SOP one significantly in the deterministic model. In addition, for the purpose of demonstrating the necessity of robust optimization models, the robust damage to targets corresponding to the uncertain kill probability of the deterministic best solution (lies on the bottom line of each uncertain scenario) is displayed in TABLES IV- V. The grey block means the corresponding

TABLE IV. RESULTS UNDER FOUR UNCERTAIN SCENARIOS AND THE VALUE OF ROBUST DAMAGE TO TARGETS CORRESPONDING TO EACH BEST SOLUTION IN EACH UNCERTAIN SCENARIO OF THE SMALL-SCALED INSTANCE

| Deterministic Small Case | | | MOEA/D-AWA | SOP_relax |
|---|---|---|---|---|
| *U1* (2.0479) | operational cost | best | **24** | 25 |
| | | average | **24** | 25.8 |
| | | standard deviation | 0 | 0.421637 |
| | damage to targets | corresponding damage | 2.07 | 2.0871 |
| | | average | 2.07 | 2.10909 |
| | | standard deviation | 0 | 0.023753 |
| | corresponding robust damage | | 2.0295 | 2.0287 |
| | p-value | | 0.003 | |
| *U2* (1.6949) | operational cost | best | **21** | 21 |
| | | average | **21** | 21.3 |
| | | standard deviation | 0 | 0.674949 |
| | damage to targets | corresponding damage | 1.8468 | 1.7036 |
| | | average | 1.8468 | 1.75347 |
| | | standard deviation | 0 | 0.089539 |
| | corresponding robust damage | | 1.6957 | 1.5554 |
| | p-value | | 0.180 | |
| *N1* (1.8413) | operational cost | best | **21** | 21 |
| | | average | **21** | 22.1 |
| | | standard deviation | 0 | 0.567646 |
| | damage to targets | corresponding damage | 1.8468 | 1.8468 |
| | | average | 1.8468 | 1.88041 |
| | | standard deviation | 0 | 0.041983 |
| | corresponding robust damage | | 1.5915 | 1.5915 |
| | p-value | | 0.005 | |
| *N2* (1.4298) | operational cost | best | **17** | 17 |
| | | average | **17** | 17.4 |
| | | standard deviation | 0 | 0.516398 |
| | damage to targets | corresponding damage | 1.4435 | 1.4436 |
| | | average | 1.4435 | 1.45621 |
| | | standard deviation | 0 | 0.027007 |
| | corresponding robust damage | | 0.8926 | 0.8926 |
| | p-value | | 0.046 | |

TABLE V. RESULTS UNDER FOUR UNCERTAIN SCENARIOS AND THE VALUE OF ROBUST DAMAGE TO TARGETS CORRESPONDING TO EACH BEST SOLUTION IN EACH UNCERTAIN SCENARIO OF THE MEDIUM-SCALED INSTANCE

| Deterministic Medium Case | | | MOEA/D-AWA | SOP_relax |
|---|---|---|---|---|
| *U1* (8.8791) | operational cost | best | **113** | 154 |
| | | average | **119.1** | 160.5 |
| | | standard deviation | 4.148628 | 4.813176 |
| | damage to targets | corresponding damage | 8.8811 | 9.3095 |
| | | average | 8.9328 | 9.12142 |
| | | standard deviation | 0.038735 | 0.191531 |
| | corresponding robust damage | | 8.6921 | 9.2254 |
| | p-value | | 0.005 | |
| *U2* (8.503) | operational cost | best | **111** | 151 |
| | | average | **114** | 157.1 |
| | | standard deviation | 4.346135 | 3.754997 |
| | damage to targets | corresponding damage | 8.5112 | 8.5862 |
| | | average | 8.57458 | 8.74407 |
| | | standard deviation | 0.059996 | 0.253752 |
| | corresponding robust damage | | 8.1623 | 8.4974 |
| | p-value | | 0.005 | |
| *N1* (8.46629) | operational cost | best | **110** | 147 |
| | | average | **111** | 154.2 |
| | | standard deviation | 2.581989 | 4.184628 |
| | damage to targets | corresponding damage | 8.5461 | 8.6369 |
| | | average | 8.55011 | 8.73914 |
| | | standard deviation | 0.055233 | 0.234542 |
| | corresponding robust damage | | 8.0647 | 8.0949 |
| | p-value | | 0.005 | |
| *N2* (7.97524) | operational cost | best | **107** | 142 |
| | | average | **105.2** | 144.7 |
| | | standard deviation | 1.032796 | 4.922736 |
| | damage to targets | corresponding damage | 7.9964 | 9.0118 |
| | | average | 8.02356 | 8.30671 |
| | | standard deviation | 0.060809 | 0.302408 |
| | corresponding robust damage | | 6.8321 | 7.0327 |
| | p-value | | 0.005 | |

deterministic optimum is still feasible in the corresponding robust situation. For instance, the number 1.6957 in the TABLE IV means that the optimum obtained by MOEA/D-AWA in deterministic situation is still feasible in the *U2* uncertain situation for the small-scaled problem. But as whole, this kind of deterministic solutions exist rarely, which manifests the fact that the optimal assignment obtained in the deterministic MWTA problem can be very poor or even infeasible for the corresponding uncertain counterpart with uncertain variables in the kill probability. This kind of comparison shows the efficiency and superiority of the multi-objective approach, and the necessity of the robust MWTA optimization.

## V. CONCLUSION AND FUTURE WORKS

In this paper, the uncertain multi-stage weapon target assignment (UMWTA) problem, which is the basis of DWTA problems in practice is considered and formulated into a robust multi-objective optimization problem. Apart from maximizing damage to hostile targets, this paper follows the principle of minimizing ammunition consumption under the assumption that each element of the kill probability matrix follows four different probability distributions from a practical standpoint. Then the multi-objective evolutionary algorithm based on decomposition with adaptive weight adjustment (MOEA/D-AWA) and the Max-Min robust operator are adopted to handle the multi-objective and uncertainties, and achieve efficient problem solving. To give a fair comparison on solving multi-objective UMWTA problems, a single objective solver for the relaxed formulation is depicted and comparisons on both certain and uncertain instances of two different scaled MWTA problems are conducted. Numerical results show that MOEA/D-AWA outperforms the single objective solver in the sense of the given single objective setup on solving both uncertain and deterministic multi-objective MWTA problems discussed in this paper. Besides, the typical objective value curves for the *U1* scenario for each instance all converge to a fixed value, which shows the convergent process. Finally, comparisons between the uncertain and deterministic results also indicate the necessity of the robust formulation of practical problems.

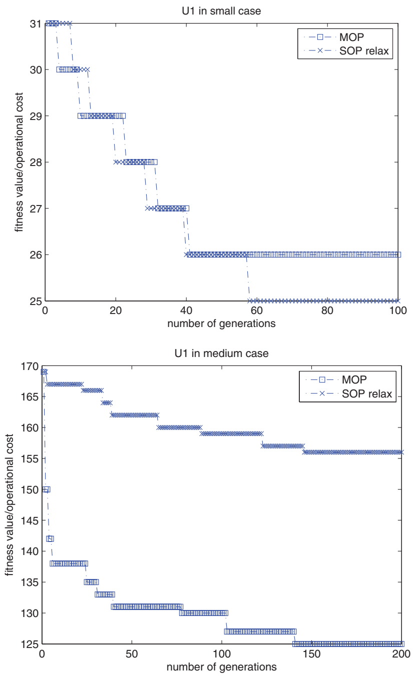


Fig. 1 Convergence curves of various algorithms for the *U1* uncertain scenario under the small and medium-scaled instances

In the paper, we model the UMWTA problems via a Max-Min formulation which is effective but a slightly conservative. It will be our future work to formulate the UMWTA problems as other types of robust models and find corresponding efficient solvers. Besides, many other practical uncertainties such as the number and types of targets in the battle space and their positions can be taken into consideration in the future research.

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