Noise-tolerant Techniques for Decomposition-based Multi-objective Evolutionary Algorithms

Juan Li, Bin Xin, *Member, IEEE*, Jie Chen, *Senior Member, IEEE*, and Panos M. Pardalos

***Abstract*—Over the last decades, the decomposition-based multi-objective evolutionary algorithms (DMOEAs) have became one of the mainstreams for multi-objective optimization. However, there is not too much research on applying DMOEAs to uncertain problems till now. Usually the uncertainty is modeled as additive noise in the objective space, which is the case this paper concentrates on. This paper first carries out experiments to examine the impact of noisy environments on DMOEAs. Then four noise-handling techniques based upon the analyses of empirical results are proposed. Firstly, a Pareto-based nadir point estimation strategy is put forward to give a good normalization of each objective. Next, we introduce two adaptive sampling strategies that vary the number of samples used per solution based on the differences among neighboring solutions and their variance to control the trade-off between exploration and exploitation. Finally, a mixed objective evaluation strategy and a mixed repair mechanism are proposed to alleviate effects of noise and remedy the loss of diversity in the decision space, respectively. These features are embedded in two popular DMOEAs, i.e., MOEA/D and DMOEA-εC, and DMOEAs with these features are termed as noise-tolerant DMOEAs (NT-DMOEAs). NT-DMOEAs are compared with their various variants and four noise-tolerant multi-objective algorithms, including the improved NSGA-II, the classical algorithm BES, and the state-of-the-art algorithms MOP-EA and RTEA to show the superiority of proposed features on seventeen benchmark problems with different strength levels of noise. Experimental studies demonstrate that two NT-DMOEAs, especially NT-DMOEA-εC, show remarkable advantages over competitors on the majority of test instances.**

***Index Terms*—Decomposition-based multi-objective evolutionary algorithm, noisy optimization, nadir point estimation, adaptive sampling strategies, mixed objective evaluation, mixed repair mechanism.**

I. Introduction

In many practical applications, problems are not only characterized by multiple objectives to be optimized but also influenced by uncertainties as a consequence of uncontrollable factors. These problems to be considered are referred as uncertain multi-objective problems (UMOPs). A UMOP can be stated as:

$$\begin{aligned} \text{UMOP}: \ & \text{minimize} \quad \mathbf{F}(\mathbf{x},\alpha,\eta) = (f_1,...,f_m) \\ & \text{subject to} \quad \mathbf{x}\in\Omega, \alpha\in P, \eta\in U \end{aligned} \tag{1}$$

where $\mathbf{x} = (x_1,...,x_n)$ is the decision vector and belongs to the nonempty feasible set Ω. α represents the environment parameter, which is related to the problem itself and belongs to the parametric set P. η stands for the uncertain parameter and comes from the uncertain set U. η can be in any format such as random, fuzzy, stochastic, and so on. $\mathbf{F}:\Omega \to R^m$ consists of $m(\geq 2)$ uncertain objective functions $f_i : R^n \to R, i = 1,...,m$. Uncertainties exist in practice can be categorized into four classes: noise, robustness, fitness approximation, and dynamic fitness functions [1]. Generally, uncertainties are introduced into objective functions as additive noise, which is the case this paper pays attention to. This kind of UMOPs are also termed as noisy multi-objective problems.

Multi-objective evolutionary algorithms (MOEAs) are mainstream methods for tackling multi-objective problems (MOPs). Among various MOEAs, decomposition-based MOEAs (DMOEAs) are growing in popularity and have became major methodologies for approximating Pareto fronts (PFs) thanks to the success of the MOEA/D framework. However, the presence of noise in objective functions may affect the ability of algorithms to drive the search process toward the PF, since it is difficult to definitely determine the Pareto dominance relationship between two solutions. Thus it is necessary to design noise-tolerant MOEAs which can perform their actions notwithstanding the presence of noise. Evolutionary algorithms (EAs) are known to be inherently robust to low-level noise due to its distributed nature and nonreliance on gradient information. However, such a property may not extend well into MOEAs, since MOEAs require the evolutionary search process to maintain a set of nondominated solutions uniformly distributed along the PF [2]. There are quite a few studies on designing algorithms for UMOPs [2]–[9]. Fieldsend and Everson [8] put forward the Bayesian (1+1)-ES (BES) which assesses the probability of dominance and maintains a set of mutually nondominated solutions with a predefined probability. Goh et. al [2] proposed an MOEA with three robust features which include an experimental learning directed perturbation, a gene adaptation selection strategy, and a possibilistic archiving model based on the concept of possibility and necessity measures. Besides, a novel algorithm, named the rolling tide evolutionary algorithm (RTEA), was developed in [9]. RTEA progressively improves the accuracy

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of its estimated Pareto set, while simultaneously driving the population towards the true PF.

The most common strategy for noisy problems is sampling. There are many studies on dynamic adjusting sample size for single objective problems (SOPs) [10], [11]. However, the extension of these methods to MOPs is not straightforward. Only a few number of studies concern on the issue of adaptive sample size for MOPs [6], [12], [13]. For example, In [6], a linear relationship between the sample size of a trial solution and the fitness variance in its local neighborhood was employed for non-uniform sampling. Syberfeldt et al. [12] proposed a confidence-based dynamic sampling technique that varies the number of samples used per solution based on the amount of noise in combination with a user-defined confidence level. Most methods adopt the mean value of fitness samples as the fitness estimation of a trial solution to improve the accuracy of estimation. Rakshit and Konar [14], [15] proposed to determine the expected value of fitness samples on the basis of their distribution as the fitness measure of a trial solution.

Other noise-handling techniques in optimizing UMOPs include periodic re-evaluation of archived solutions [4], probabilistic Pareto ranking [16], possibilistic archiving [2], fitness inheritance [17], extended averaging scheme [18], and so on. In [4], Buche et al. proposed to modify the elite preservation scheme with the aim of reducing detrimental effects of outliers for noisy combustion processes. In particular, every solution is assigned a lifetime that is dependent on the fraction of the archive it dominates. Any archive solutions with expiring lifetime are reevaluated and added to the evolving population. In the subsequent archive updating procedure, expired solutions will not be considered. Hughes [16] demonstrated the possible deficiencies of the nondominated sorting approach [19] and then introduced a probabilistic Pareto ranking scheme to account for noisy and uncertain systems. Singh [18] proposed an extended averaging scheme to reduce the bias introduced by the small sample size in the optimization of groundwater remediation design. The extended averaging approach performs the averaging over all samples of identical individuals, which can be easily extended over different generations.

This paper firstly examines the performance of DMOEAs in noisy environments. It has been observed that impacts of noise on DMOEAs are different for benchmarks with different characteristics and different strength levels of noise. That is, DMOEAs tend to perform well for the majority of problems in the presence of low-level noise, and the evolutionary optimization process degenerates into a random search under increasing level of noise. While for some problems, the evolutionary optimization process is badly effected even by the presence of low-level noise. Based on the analyses of noise impacts on population dynamics of convergence and diversity, four noise-handling techniques including a Pareto-based nadir point estimation strategy, two adaptive sampling strategies, a mixed objective evaluation strategy, and a mixed repair mechanism are proposed and incorporated into two existing DMOEAs to improve their performances in the presence of noise. The differences between the proposed approach and existing methodologies are the way they deal with uncertainties. Features of the four techniques are summarized as follows.

1) The presence of noise leads to erroneous estimations of nadir points. Considering that Pareto-based MOEAs are featured with higher speed of convergence at the early stage of optimization than DMOEAs, we propose a Pareto-based nadir point estimation strategy (PNE) to give a good estimation of the nadir point at the initialization phase.
2) The majority of existing approaches for UMOPs employ fixed sample size or adjust the sample size according to the variance of a solution in order to eliminate the risk of promoting poor solutions to PFs. The development of DMOEAs bridges SOPs with MOPs, thus we extend two adaptive sampling strategies (ASs) designed for uncertain SOPs to UMOPs. The two methods vary the number of samples used per solution based on the differences among neighboring solutions and their variance. Specifically, the co-ranking adaptive sampling strategy allocates the sample size according to weighted rank values of the two factors. The optimal computing budget allocation (OCBA) strategy is based on the ratio between the two factors. They both can control the trade-off between exploration and exploitation of search space.
3) Most EAs estimate fitness of a trial solution by averaging a series of fitness samples. Unfortunately, the mean value is not always the most efficient estimation for an uncertain problem. A mixed objective evaluation method (MO) that varies the estimation of fitness between the mean and median values of fitness samples according to the estimated strength level of noise is proposed to alleviate detrimental effects of noise in an adaptive way.
4) Reproduction operators play an important role in algorithmic behaviors in the decision space. The commonly used truncation repair method will cause the loss of diversity in the decision space, which stagnates the evolutionary process. Thus we combine the truncation repair mechanism and a random repair mechanism together and propose a mixed repair mechanism (MR) to remedy the loss of diversity in the decision space adaptively.

The paper is structured as follows. Section II first provides a brief description of two popular DMOEAs, i.e., MOEA/D and DMOEA-εC. Then we examine the impacts of noise on performances of two DMOEAs. Based on the analyses of empirical results, Section III presents four noise-handling extensions for standard DMOEAs in noisy environments, which results in noise-tolerant DMOEAs (NT-DMOEAs). The preliminary experiments on parameters tuning are conducted in Section IV at first. Then Section IV examines the effectiveness of proposed features via comparing NT-DMOEAs with various variants and showing comparison results on NT-DMOEAs against four comparison algorithms on seventeen benchmark problems. Section V draws the conclusion and points out future research.

II. Preliminaries of Standard DMOEAs

This section presents a brief description of two DMOEAs, i.e., MOEA/D and DMOEA-εC, which will be used in the rest of this section to examine impacts of noise on DMOEAs.

A. DMOEAs: MOEA/D and DMOEA-εC

Decomposition is an efficient and prevailing strategy for MOPs. In DMOEAs, an MOP is decomposed into a number of scalar subproblems by using various scalarizing functions. The weighting method and the ε-constraint method are two classical generation methods in the field of mathematical programming and have been adopted for multi-objective optimization. Zhang et al. [20] adopted the weighting method as a scalarizing function and first proposed a multi-objective evolutionary algorithm based on decomposition (MOEA/D). Many variants of MOEA/D, such as MOEA/D-DRA [21], MOEA/D-AWA [22], and so on are proposed and have shown their good performances on solving MOPs. Inspired by MOEA/D, a new DMOEA, named DMOEA-εC, was proposed in [23] and demonstrated its superiority over a number of state-of-the-art MOEAs. Instead of the weighting method, DMOEA-εC adopts the ε-constraint method and decomposes an MOP into a series of scalar constrained optimization subproblems by assigning each subproblem with an upper bound vector for the first time. In essence, DMOEA-εC can be regarded an example of MOEA/D with a different decomposition method based on the ε-constraint method. In DMOEAs, all subproblems are optimized simultaneously by only using information from neighboring subproblems, which makes DMOEAs have lower computational complexity than Pareto-based and indicator-based MOEAs. Details of above two DMOEAs can be found in [20] and [23].

B. Impacts of Noise on DMOEAs

In this subsection, a brief description of benchmark problems, performance metrics, and information on experimental setup are provided at first. Then we examine impacts of noise on the dynamic of convergence and diversity over generations in DMOEAs and present motivations of this paper.

1) Benchmark Problems: Benchmark problems are used to reveal the capabilities and important characteristics of algorithms under evaluation. In the context of MOPs, researchers have identified several benchmarks with various characteristics. Among them, five ZDT test instances [24], two tri-objective DTLZ test instances [25], and ten UF test suites [26], including ZDT1-ZDT4, ZDT6, DTLZ2, DTLZ4, and UF1-UF10 are adopted in this paper. These test instances are used to examine the effectiveness of DMOEAs in converging and maintaining a diverse set of nondominated solutions under the influence of noise. In this study, noise is implemented as an additive perturbation on the objective value of each individual [16], i.e., $\tilde{f}_i(\mathbf{x},\alpha,\eta)=f_i(\mathbf{x},\alpha)+\eta_i, i=1,...,m$, where m stands for the number of objectives. η_i is the additive noise of ith objective function, which is often assumed to be normally distributed with zero mean and variance σ_i^2 [1]. σ_i^2 means the strength level of noise present in the ith objective function and is often represented as percentage of $|F_i^{\max}-F_i^{\min}|$, where $F_i^{\max}$ and $F_i^{\min}$ are the maximum and minimum of the ith objective function in the true PF [2]. $\tilde{f}$ and f denote the objective function with and without additive noise, respectively. Test functions of MOPs will be modified in the above form in order to include the influence of noise.

Ideally, MOEAs should work on the expected fitness function $E[\tilde{f}_i(\mathbf{x},\alpha,\eta)]$ and not be misled by the presence of noise. During optimization, the only measurable fitness value is a stochastic value $f_i(\mathbf{x},\alpha)+\eta_i$. Therefore, in practice, the expected fitness value $E[\tilde{f}_i(\mathbf{x},\alpha,\eta)]$ is often approximated by the averaged sum of a number of random samples: $\hat{f}_i(\mathbf{x},\alpha,\eta)=\frac{1}{n}\sum\limits_{i=1}^{n}(f_i(\mathbf{x},\alpha)+\eta_i), i=1,...,m$, where n denotes the sample size, and $\hat{f}$ is an unbiased estimation of f.

2) Performance Metrics: Performance metrics pertinent to the convergence and diversity play an important role when evaluating the quality of a set of obtained non-dominated solutions for MOPs. According to the law of large numbers, the mean value of sampled objective values converges to the noise-free objective function value when the number of samples become large enough for any noisy function. Thus true PFs of noisy and deterministic MOPs are the same, and thus performance metrics used in deterministic MOPs are still valid in noisy cases. Here two performance metrics, i.e., the averaged Hausdorff distance (Δ_p) [27] and the hypervolume (*HV*) [28] are employed.

The Δ_p metric measures the averaged Hausdorff distance between a set of uniformly distributed Pareto optimal points over the PF P^* and an approximation set P. It is composed of the generalized generational distance (GD_p) [29] and the generalized inverted generational distance (IGD_p) [30], which can be formulated as:

$$\Delta_p(P^*,P)=\max\{IGD_p(P^*,P),GD_p(P^*,P)\} \tag{2}$$

where $GD_p(P^*,P)=\left(\frac{1}{|P^*|}\sum_{x^*\in P^*}d(x^*,P)^p\right)^{\frac{1}{p}}$ and $IGD_p(P^*,P)=\left(\frac{1}{|P|}\sum_{x\in P}d(x,P^*)^p\right)^{\frac{1}{p}}$ are generalized *GD* and *IGD* metrics, respectively. $d(a,B)$ means the minimum Euclidean distance between the point a and any point in B. $|\cdot|$ denotes the cardinality of any set. Δ_p can detect outliers in candidate solutions. A smaller Δ_p indicates a better P.

The *HV* measures the size of the objective space dominated by solutions in P and bounded by the reference point $\mathbf{r}$. It is defined as:

$$HV(P,\mathbf{r})=VOL(\bigcup_{x\in P}[f_1(x),r_1]\times...\times[f_m(x),r_m]) \tag{3}$$

where $\mathbf{r}=(r_1,...,r_m)$ is a reference point in the objective space dominated by any Pareto optimal point, and $VOL(\cdot)$ denotes the Lebesgue measure. Mathematically, for each member in the nondominated set P, a hypercube $v=[f_1(x),r_1]\times...\times[f_m(x),r_m]$ is constructed with a reference point $\mathbf{r}=(r_1,...,r_m)$ and each member $[f_1(x),...,f_m(x)]$ as the diagonal vertices of the hypercube. A larger *HV* value implies a better P. Δ_p and *HV* both take into account diversity and convergence and provide general quality measures of the approximation set P.

[1]Additional numerical experiments on UMOPs with uniform noise are conducted to test the performances of comparison algorithms on non-Gaussian noise. Numerical results have concluded that no qualitative difference has been observed in the presence of Gaussian and non-Gaussian noise, which is consistent with the statement in [1].

3) Parameter Settings: Both MOEA/D and DMOEA-εC are adopted in this section to examine impacts of noise on DMOEAs. They employ a fixed-size population and an archive which stores non-dominated solutions along the evolution. The archive is updated at each cycle, i.e., a candidate solution will be added into the archive if it is not dominated by any member in the archive. Likewise, any archive member dominated by this solution will be removed from the archive. When the predetermined archive size is reached, a truncation process based on the crowding distance [19] is used to eliminate the most crowded archive member. Besides, the dynamic resource allocation strategy [21] is added into both MOEA/D and DMOEA-εC. Other implementations adopted here of two algorithms are the same as those in [23].

Experiments are conducted at noise strength levels of $\sigma^2 \in \{0.1\%, 0.2\%, 0.5\%, 1\%, 5\%, 10\%, 20\%\}$ in order to study impacts of noise on DMOEAs. For a fair comparison, the choice of parameters are the same for both MOEA/D and DMOEA-εC. Specifically, the population size is set to $N = 100$ for ZDT problems. As to the DTLZ problems, due to the differences in algorithmic frameworks, N is set to 351 and 324 for MOEA/D and DMOEA-εC, respectively. For UF instances, the population size N is set to 600 and 1,000 for bi-objective and tri-objective instances, respectively. The size of archive is set as $S = N$. Besides, the DE operator and Gaussian mutation are used in solving ZDT and DTLZ test problems, and the DE operator and polynomial mutation are adopted for UF instances [23]. In order to eliminate effects of noise, the uniform sampling strategy is adopted, and the number of samples is set $n = 5$ evenly for each subproblem at each generation. Both algorithms terminate when the number of evaluations reaches the maximum number. Based on the parameter settings in deterministic cases, the maximum number of function evaluations are set as 250,000, 375,000, and 1500,000 for ZDT, DTLZ, and UF instances, respectively [23]. Finally, each algorithm is executed 30 times independently on each instance.

Two performance metrics, i.e., Δ_p and *HV* are employed to evaluate the performance of all comparison algorithms. With the purpose of calculating the Δ_p metric value, P^* is chosen to be a set of 500 uniformly distributed points along the true PF for ZDT problems, and 1,024 points for DTLZ instances. As to bi-objective UF, a set of 1,000 uniformly distributed points along the true PF are chosen as P^* except that 21 uniformly distributed points are chosen as P^* for UF5. For tri-objective UF test problems, P^* is chosen to be a set of 10,000 uniformly distributed points along the true PF [23]. Besides, in order to compute the *HV* metric value, the reference point is set as 1.1 times the true nadir point.

4) Empirical Results of Noise Impacts on DMOEAs: Figs.1-2 illustrate distributions of the final populations in the objective space with the minimum Δ_p metric value within 30 runs found by MOEA/D and DMOEA-εC on four test problems under the influence of 0.5%, 5%, and 20% noise levels, respectively. Both MOEA/D and DMOEA-εC obtain solutions uniformly spreaded along the true PF on the ZDT2 and UF2 test instances with 0.5% noise level except UF2 on which MOEA/D can only cover a part of the true PF. As to the tri-objective DTLZ2 and UF10 test instances with 0.5% noise level, DMOEA-εC performs better than MOEA/D and obtains final populations that well cover the whole PF but with bad uniformity. According to the observation of Nissen and Propach [31], population-based EAs are inherently robust in single objective optimization under a low-level noise. It is also confirmed from Figs.1-2 that DMOEAs are capable of obtaining satisfactory solutions on some test instances under the influence of low-level noise. When it comes to the high-level noise, i.e., 5% and 20% noise levels, both MOEA/D and DMOEA-εC cannot achieve approximations with both good convergence and diversity on the ZDT2 and UF2 test instances. Besides, for DTLZ2 and UF10 with high-level noise, final solutions obtained via MOEA/D and DMOEA-εC can hardly approximate and cover the whole PF very well.

In order to further examine algorithmic behaviors in the objective space, objective values during the search process are recorded. The evolution of objective values with the number of iterations obtained via DMOEA-εC on DTLZ2 with 0.1% and 20% noise levels are presented in Fig.3. The traces of objective values are sufficient to demonstrate impacts of noise on multiple objective values during the evolutionary process. It can be seen from Fig.3 that the introduction of high-level noise degrades the performance of DMOEA-εC on DTLZ2. Besides, it should be noted that DMOEA-εC stagnates in the later stage of evolution on DTLZ2 with 0.1% and 0.2% noise levels. This fact has been stated in [2] that basic EAs suffer from degenerate convergence properties and face the problem of maintaining a diverse solution set under the influence of noise.

Generally, when using the sampling strategy to deal with noisy problems, the mean value over several samples is regarded as the fitness estimation to eliminate detrimental effects induced by noise. However, other statistics can also be adopted as criteria when evaluating a solution in noisy environments. We conduct experiments on all test instances to see the differences of final populations obtained via DMOEAs with the mean criterion and with the median criterion. Due to lack of space, results obtained via MOEA/D on three instances with different noise levels are just illustrated in Fig.4. It shows that MOEA/D with mean and median criteria perform similarly when the noise level is low. When the noise level increases, MOEA/D with the mean criterion tends to obtain solutions with good convergence. However, solutions found by MOEA/D with the median criterion show good diversity. Based on these observations, it can be concluded that mean and median criteria have their advantages when tackling noisy problems. It is natural to have an idea that combining them together and obtaining a mixed criterion which performs well on all noise levels.

As stated above, DMOEAs stagnate at the later evolution stage because of diversity loss. Reproduction operators are the primary source of new solutions and should be responsible for the loss of diversity in the decision space which will lead to the loss of diversity in the objective space. We notice that the truncation repair mechanism is commonly used when solutions exceed search boundaries. Truncation repairing means each element of a solution that exceed the search space will be

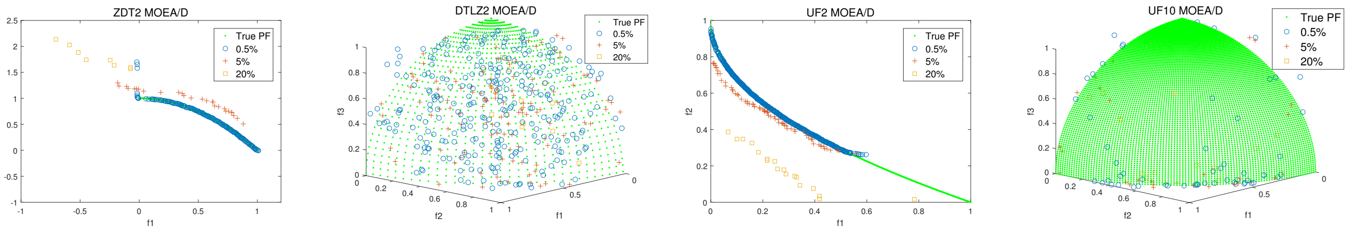

Fig. 1. Final populations in the objective space with the minimum Δ_p metric value within 30 runs obtained by MOEA/D on four test problems under the influence of 0.5%, 5%, and 20% noise levels

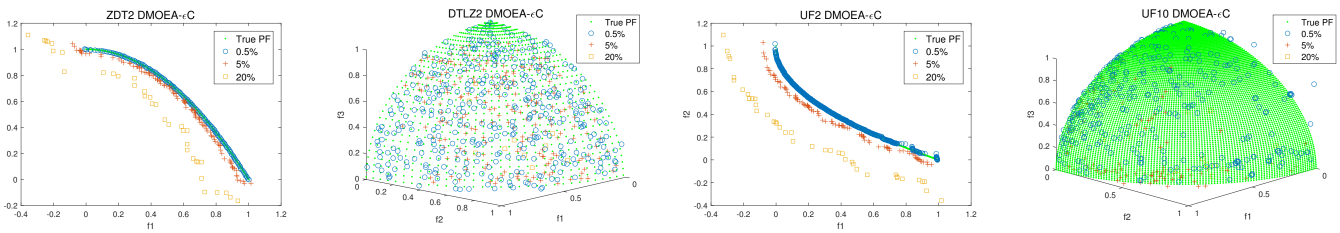

Fig. 2. Final populations in the objective space with the minimum Δ_p metric value within 30 runs obtained by DMOEA-εC on four test problems under the influence of 0.5%, 5%, and 20% noise levels

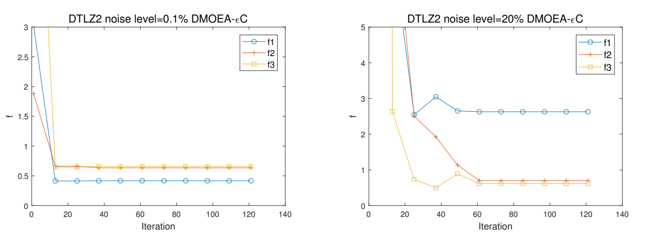

Fig. 3. Traces of three objective values obtained via DMOEA-εC for DTLZ2 under the influence of 0.1% and 20% noise levels

truncated back to the edge of its domain. Such a fixed form of repair mechanism isn't conducive to the diversity in the decision space, and the effect of losing diversity is more outstanding when handling noisy problems. Random repair methods mean each element of solution that exceed the search space will be replaced with a random feasible value. We try to replace the truncation repair method with a random one and test performances of DMOEAs with the random repair mechanism on all test problems. Detailed results obtained via DMOEAs with the random repair mechanism on all test instances with different noise levels are omitted due to the limited space. It is concluded that usage of the random repair mechanism will enhance the diversity of decision space which will also lead to more diverse final populations. Based on these observations, it is natural to have an idea that combining the two repair mechanisms together and proposing a mixed repair mechanism which performs well on all levels of noise. The rationality and superiority of the mixed criteria and the mixed repair mechanism will be shown in Section IV-B.

III. Noise-tolerant DMOEAs (NT-DMOEAs)

From observations in the previous section, it is confirmed that the performance of DMOEAs deteriorates sharply at high level noise. This section extends and improves standard DMOEAs by four noise-handling features based upon the analyses of population dynamics under the influence of noise.

A. Pareto-based Nadir Point Estimation Strategy (PNE)

Ideal points and nadir points have important impacts on determining the search range in the objective space of DMOEAs. Imprecise estimations may mislead the search direction of evolutionary process and result in approximations of only parts of PFs. Nadir points are much harder to obtain than ideal points. Several approaches have been proposed to calculate nadir points [32], [33]. In standard DMOEAs, ideal points and nadir points are replaced by the minimum value of each objective function in the population and the maximum value of each objective function in the external archive, respectively. Besides they will be updated iteratively. However, this procedure is not effective enough in noisy environments.

Pareto-based and decomposition-based MOEAs are two prevailing approaches for MOPs which have their own advantages and disadvantages [34]. Specifically, Pareto-based MOEAs are featured with fast convergence at the early stage of optimization. Along with the searching process of Pareto-based MOEAs, the majority of solutions in evolving population become non-dominated and the concept of Pareto dominance cannot distinguish these solutions. Then diversity maintaining mechanisms, instead of Pareto dominance relationships, become the main power that pushes evolving population towards the true PF, which slows down the convergence at the late stage of optimization. DMOEAs do not suffer from the ineffectiveness of Pareto dominance relationship but need information of ideal and nadir points [23], [35]. Thus it is reasonable to obtain an estimation of the nadir point by taking the advantage of Pareto-based MOEAs at the early stage of optimization. With the guidance of a good estimated nadir point, DMOEAs are adopted to evolve the population towards the true PF with a quick convergence speed. By combining strengths of two

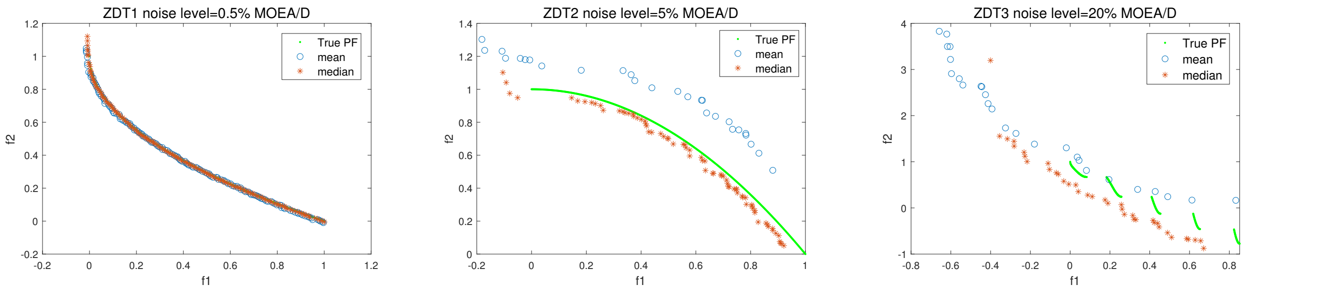

Fig. 4. Final populations in the objective space with the minimum Δ_p metric value within 30 runs obtained by MOEA/D using the mean and median values as objectives estimation on three instances under the influence of different noise levels

MOEAs, not only a good estimation of the nadir point, but also a well initialized population can be obtained. Here NSGA-II [19] is selected as a representative of Pareto-based MOEAs.

B. Adaptive Sampling Strategies (ASs)

One common approach for uncertain problems is to perform sampling for individuals, otherwise the evolutionary selection process may become unstable. A crucial aspect here is to find the best trade-off between the number of solutions evaluated and the number of samplings for each solution. The larger the number of solutions evaluated, the more the search space can be explored and the greater the probability of finding its optimum will be. However, at the same time, sampling of solutions is necessary in order to prevent the search from being misdirected by noise. It is not wise to equally sample all candidate solutions with different qualities, especially when a small number of samplings are allowed. Many researches on adaptively adjusting sample size of each solution have been done on uncertain SOPs. These techniques cannot be directly extended to UMOPs since expected solutions of UMOPs are a set of solutions with good convergence and diversity. However, the development of DMOEAs bridges SOPs with MOPs, and thus some sample size adjustment strategies designed for uncertain SOPs can be extended to UMOPs. Here, we introduce two typical adaptive sampling strategies, i.e., the co-ranking allocation strategy [11] and the optimal computing budget allocation (OCBA) strategy [36], [37].

In DMOEAs, an MOP is converted into a series of scalar subproblems, and each subproblem is associated with a solution in the population and a set of neighboring solutions which can be regarded as the population of each subproblem. The neighborhood relations among these subproblems are defined based on the Euclidean distance between their weight vectors or upper bound vectors ε^i $(i = 1, ..., N)$, where N is the number of subproblems. The neighborhood of the ith subproblem consists of all subproblems with the weight vectors or upper bound vectors from the neighborhood of ε^i. In the proposed noise-tolerant DMOEAs, the concept of neighborhood is used for two main purposes including the update of neighboring solutions and the update of samples. Thus we define two types of neighborhoods for each subproblem. The neighborhoods of the ith subproblem for updating solutions and updating samples are denoted as $B_1(i)$ and $B_2(i)$, respectively.

In both adaptive sampling strategies, each subproblem has a predefined sample size and samples will be allocated among its neighboring solutions. It should be noted that since a subproblem falls within neighbourhoods of several subproblems, the final sample size of each subproblem is determined after adaptive sampling strategies are done for all subproblems. Then the estimated objective value of a solution is given based on the adaptive sample size and the mixed objective evaluation strategy which will be presented below.

1) Co-ranking Adaptive Sampling Strategy: In order to ensure a high probability to find the best solution, a larger sample size is adopted for individuals with higher estimated variance. Besides, with the purpose of distinguishing two solutions, a larger number of samplings should be conducted on two solutions with similar performance. The difference between two solutions is measured with the absolute value of the difference between two estimated objective values as described in the line 7 of **Algorithm 1**. The estimated objective value of a solution can be the mean, median or mixed criterion value over a number of sampled objective values. Thus a synthetical evaluation of each solution is conducted by a weighted aggregation of rank values of the variance and the difference value between neighboring solutions. Different weighted coefficients α mean different aggregation forms. For example, $\alpha = 1$ means allocating sample size completely depends on the variance value of each solution. However, $\alpha = 0$ stands for the adaptive sample size is only related to the difference between two solutions. Here, we emphasize two factors equally and adopt $\alpha = 0.5$.

2) OCBA Strategy: The OCBA technique proposed by Chen et al. [36], [37] is used to optimally choose the number of simulations for all of the solutions in order to maximize simulation efficiency with a given computing budget. Here the OCBA technique is applied to UMOPs to allocate limited sampling budgets among solutions to provide reliable evaluation and identification of good individuals. Details of the OCBA sampling strategy are illustrated in **Algorithm 2**. Similar to the co-ranking sampling strategy, the variance value and the difference value are two main components in the OCBA strategy. However, the OCBA strategy uses them in a different way and does not need an extra coefficient α. Readers can refer to [36], [37] for details of the OCBA technique.

Algorithm 1 Co-ranking allocation strategy

Input: The predefined sample size for each subproblem k, a weighting coefficient α, and N subproblems.
Output: Sample size for N subproblems, i.e., $q_1, ..., q_N$.

```
Initialize q_i = 0, i = 1, ..., N.
for i = 1 to N do
    Get the neighborhood of updating samples for the ith subproblem, i.e., B_2(i).
    while B_2(i) is nonempty do
        Randomly select an index j ∈ B_2(i).
        Calculate the variance of jth subproblem var_j.
        Calculate the difference between the two subproblems dis_j = |ḡ^i − ḡ^j|, where ḡ^i and ḡ^j are estimated objective values of the ith and jth subproblems.
        B_2(i) = B_2(i)/ {j}.
    end while
    Sort {var_1, ..., var_|B_2(i)|} in a descending order and obtain the corresponding rank value of each subproblem rank_j^v, j ∈ B_2(i).
    Sort {dis_1, ..., dis_|B_2(i)|} in a descending order and obtain the corresponding rank value of each subproblem rank_j^d, j ∈ B_2(i).
    for j = 1 to |B_2(i)| do
        rank_j = α · rank_j^v + (1 − α) · rank_j^d. // rank_j is the overall rank value of the jth subproblem.
    end for
    Initialize count = 0.
    while count ≤ k do
        s = argmin_{l=1,...,|B_2(i)|}(rank_1, ..., rank_|B_2(i)|).
        q_s = q_s + 1; count = count + 1.
        B_2(i) = B_2(i)/{s}.
    end while
end for
```

Algorithm 2 OCBA strategy

Input: The sample size for each subproblem k and N subproblems.
Output: Sample size for N subproblems, i.e., $q_1, ..., q_N$.

```
Initialize q_i = 0, i = 1, ..., N.
for i = 1 to N do
    Get the neighborhood of updating samples for the ith subproblem, i.e., B_2(i).
    while B_2(i) is nonempty do
        Randomly select an index j ∈ B_2(i).
        Calculate the variance of jth subproblem var_j.
        Calculate the difference between the two subproblems dis_j = |ḡ^i − ḡ^j|, where ḡ^i and ḡ^j are estimated objective values of the ith and jth subproblems.
        B_2(i) = B_2(i)/ {j}.
    end while
    Use the OCBA technique to compute the sample size k_j (j = 1, ..., |B_2(i)|) for each neighboring solution of ith subproblem.
    for j = 1 to |B_2(i)| do
        q_j = q_j + k_j.
    end for
end for
```

C. Remedy for the Loss of Diversity

It has been observed in Section II-B that DMOEAs suffer from degenerate convergence properties and face the problem of maintaining diversity of population under the influence of noise. Besides, Section II-B also reveals that the usage of the median value as the objective estimation and the random repair mechanism both can improve the diversity of evolving population. Since DMOEAs show similar performances on test instances with noise level smaller than 1%, between 1% and 10%, and larger than or equal to 10%, seven levels of noise are classified into three categories. The three categories can be regarded as low-, medium-, and high-level noise, which is rational in practice. Actually, the strength levels of noise cannot be known in advance and are usually estimated by variance values of individuals [14]. Specifically, the estimated noise level in the ith objective function is defined as follows:

$$\hat{\sigma_i^2} = \sum_{j=1}^{N} \sigma_{ij}^2 \Big/ \left(N \cdot |\widehat{F_i^{\max}} - \widehat{F_i^{\min}}|\right) \tag{4}$$

where σ_{ij}^2 represents the variance value in the ith objective function of the jth solution, and N is the number of solutions in the population. $\widehat{F_i^{\max}}$ and $\widehat{F_i^{\min}}$ denote the maximum and minimum value of the ith objective function in the population, respectively. Besides, no prior information is required during estimating the noise level.

1) Mixed Objective Evaluation Strategy (MO): As can be seen from Fig.4, DMOEAs with the mean criterion show better convergence performance, and DMOEAs adopting the median criterion maintain better diversity of evolving population. Results on different test problems display high similarity. We define a parameter, denoted as r_{median}, to represent the percentage of the median value in the fitness estimation of a solution. In other words, the fitness estimation of a solution is the sum of $r_{median} \cdot 100\%$ of the median value of samples and $(1 - r_{median}) \cdot 100\%$ of the mean value of samples. The parameter r_{median} varies according to the following equation:

$$r_{median} = \begin{cases} 0 & \hat{\sigma^2} < 1\% \\ r_1 + \Delta_1 \cdot \frac{\hat{\sigma^2}}{\max(\hat{\sigma^2})} & 1\% \le \hat{\sigma^2} < 10\% \\ r_2 + \Delta_2 \cdot \frac{\hat{\sigma^2}}{\max(\hat{\sigma^2})} & \hat{\sigma^2} \ge 10\% \end{cases} \tag{5}$$

where $\hat{\sigma^2}$ means the averaged estimated noise level of all objective functions, i.e., $\hat{\sigma^2} = \sum_{i=1}^{m} \hat{\sigma_i^2} \Big/ m$, where m is the number of objectives. $\max(\hat{\sigma^2})$ returns the maximum of estimated noise levels found so far. r_i and Δ_i $(i = 1, 2)$ are random values in the interval $[0, 1]$.

When the estimated noise level is low (i.e., lower than 1%), DMOEAs with mean and median criteria perform similarly. Since the mean value is easier to compute, the mean value is regarded as the final fitness estimation. When the estimated noise level increases, we integrate $r_i\% \sim (r_i+\Delta_i)\%$ $(i = 1, 2)$ of the median value into the objective estimation. Based on empirical studies, these parameters are set as $r_1 = 0.3$, $r_2 = 0.1$, and $\Delta_i = 0.2$ $(i = 1, 2)$. Specifically, when the estimated noise level is medium (i.e., falls between 1% and

TABLE I
SUMMARY OF NOTATIONS USED IN THE DESCRIPTION OF NT-DMOEAS

N	The number of weight vectors or upper bound vectors (the same as the population size)
n	Predefined sample size for each subproblem
k_1	Predefined sample size for a newly generated solution
T_1	Neighborhood size for the update of solutions
T_2	Neighborhood size for the update of samples
δ	Probability of selecting mate solutions from its neighborhood
$NFE\_preprocess$	The number of function evaluations used by Pareto-based MOEAs
$DRA\_interval$	Iteration interval of utilizing the dynamic resource allocation strategy
NFE	Maximum number of function evaluations
$rand$	A randomly distributed value in the interval $[0, 1]$

10%), 30%$\sim$50% of the median value of a series of samples is integrated into the objective estimation. This attempts to enhance the diversity of population and ensure convergence at the same time. When the estimated noise level is high (i.e., higher than or equal to 10%), the final fitness estimation consists of 70%$\sim$90% of the mean value, since it is easy to maintain the diversity in the presence of high-level noise.

2) Mixed repair mechanism (MR): As mentioned before, it is obvious that the commonly used truncation repair mechanism gives rise to the loss of diversity in the decision space, which also results in losing diversity in the objective space. The proposed mixed repair mechanism adopts the truncation repair mechanism and the random repair mechanism simultaneously. Similar to the mixed objective evaluation strategy, a parameter which represents the percentage of infeasible variables that use the random repair mechanism is defined and denoted as r_{random}. Thus, when repairing an infeasible solution, $\lceil r_{random} \cdot 100\% \rceil$ of infeasible variables will be repaired by the random repair mechanism, where $\lceil \cdot \rceil$ returns the nearest integer in the direction of positive infinity. The remaining infeasible variables will be repaired via the truncation repair mechanism. The parameter r_{random} changes according to the following equation:

$$r_{random} = \begin{cases} 0 & \hat{\sigma^2} < 1\% \\ r_3 + \Delta_3 \cdot \frac{\hat{\sigma^2}}{\max(\hat{\sigma^2})} & 1\% \le \hat{\sigma^2} < 10\% \\ r_4 + \Delta_4 \cdot \frac{\hat{\sigma^2}}{\max(\hat{\sigma^2})} & \hat{\sigma^2} \ge 10\% \end{cases} \quad (6)$$

Similar to the previous subsection, these parameters are given as $r_3 = 0.5$, $r_4 = 0.3$, and $\Delta_i = 0.2$ $(i = 3, 4)$ according to empirical results. Specifically, the truncation repair mechanism is still adopted when the estimated noise is small since low-level noise has little effects on the performance of DMOEAs on UMOPs. When the estimated noise level is medium, we apply the random repair mechanism on 50%$\sim$70% of infeasible variables. Finally, when the estimated noise level is high, only 30%$\sim$50% infeasible variables will be repaired via the random repair mechanism.

Given the above, the proposed mixed objective evaluation strategy and the mixed repair mechanism are tradeoffs between two evaluation criteria and two repair mechanisms, respectively. They adaptively strike a balance between convergence and diversity from the aspects of objective space and decision space, respectively.

D. Implementation

These proposed techniques are incorporated into DMOEAs, and the resulting noise-tolerant DMOEAs are named NT-DMOEAs. Two representative DMOEAs, i.e., MOEA/D and DMOEA-εC, are taken as examples. MOEA/D and DMOEA-εC with these noise-tolerant features are named as NT-MOEA/D and NT-DMOEA-εC, respectively. The notations used in the NT-DMOEAs are given in Table I. The algorithmic description of NT-DMOEAs is presented in **Algorithm 3**.

IV. NUMERICAL EXPERIMENTS

This section first conducts preliminary experiments of algorithmic performances on parameter settings and then performs two types of numerical experiments. The first one is to compare relative performances of the individual extensions embedded in NT-DMOEAs, and the second one is to compare NT-DMOEAs with other noise-tolerant algorithms. Similar to Section II-B, seven different noise strength levels of $\sigma^2 \in \{0.1\%, 0.2\%, 0.5\%, 1\%, 5\%, 10\%, 20\%\}$ are applied to all test instances. Due to the differences in algorithmic frameworks, N is set to 351, 324, and 300 for the DTLZ instances for MOEA/D, DMOEA-εC, and the remaining algorithms, respectively. The neighborhood sizes for the update of solutions and the update of samples are set as $T_1 = \lfloor 10\% \cdot N \rfloor$ and $T_2 = \lfloor 10\% \cdot N \rfloor$, respectively. The sample size of each subproblem is set as $n = 5$, and the number of samples for each new solution is set as $k_1 = 2$ based on the parameters tuning which will be presented in the following. Moreover, other parameters are the same as those in Section II-B. All comparison algorithms terminate when the number of evaluations reaches the maximum number.

A. Parameters Tuning

In this section, the parameters analyses of NT-DMOEAs are deeply researched. Specifically, effects of the parameters T_1, T_2, n, and k_1 on the performances of NT-DMOEAs are investigated.

1) Effects of T_1 and T_2: In NT-DMOEAs, the neighbourhood sizes T_1 and T_2 are used for the purposes of updating neighboring solutions and updating samples, respectively. We investigate how NT-DMOEA-εC is influenced by the combination of two parameters. We take ZDT2 and UF2 as examples

Algorithm 3 Framework of NT-DMOEAs

Input: A UMOP and related parameters.
Output: An external archive population EP.

for i=1 to N **do**
  Set two types of neighborhoods for the ith subproblem, i.e, $B_(i)$ and $B_2(i)$.
end for
Initialize samples of noise randomly.
Randomly initialize the evolving population $P = \{x^1, ..., x^N\}$; apply the mixed objective evaluation strategy to calculate sampled objective values of each subproblems; initialize the ideal point z^*.
Set $gen = 0$ and $nfe = N$.
while $nfe \leq NFE\_preprocess$ **do**
  Apply NSGA-II to obtain the population P'.
  $nfe = nfe + N \cdot n$; $gen = gen + 1$.
end while
Extract nondominated individuals from P' and denote the set of them as EP; estimate the nadir point z^{nad} and take P' as the initial population for the following DMOEAs.
while $nfe \leq (NFE - NFE\_preprocess)$ **do**
  if gen is a multiple of $DRA\_interval$ **then**
    Update the indices of the subproblems I that will be processed in next generation by applying the dynamic resource allocation scheme [21].
  end if
  for $i \in I$ **do**
    $$P = \begin{cases} B_1(i), & if\ rand < \delta \\ \{1, 2, ..., N\}, & otherwise \end{cases}$$
    Reproduction: select parent individuals from P' randomly and then apply certain reproduction operator to generate a new solution y and reevaluate the new solution k_1 times.
    Repair: if y is infeasible, use the mixed repair mechanism to repair it.
    Using adaptive sampling strategies (**Algorithm 1** or **Algorithm 2**) to allocate the remaining $(n - k_1)$ samples among neighboring solutions in $B_2(i)$.
    Apply the mixed objective evaluation strategy to update objective estimation of each neighboring solution; $nfe = nfe + n$.
    Update the approximated ideal point z^*.
    Compare y with neighboring solutions in $B_1(i)$ and update these neighboring solutions by using the objective values directly or the feasibility rule.
    Update the external archive EP and prune it by using the crowding distance approach.
    Update the approximated nadir point z^{nad}.
  end for
  $gen = gen + 1$.
end while

and test different settings of T_1 and T_2 in the implementation of NT-DMOEA-εC. Different T_1 and T_2 values are both set as $\lfloor (3\%, 5\%, 10\%, 30\%, 50\%) \cdot N \rfloor$. All the other parameters are kept the same as described before, and 30 independent runs have been conducted for each configuration on these test instances.

Fig.5 shows the mean Δ_p metric values across all combinaitons of T_1 and T_2 values on the selected test problems. As shown in Fig.5, NT-DMOEA-εC performs well with a wide range of T_1 and T_2 values on both ZDT2 and UF2 instances except ZDT2 with 10% noise level on which a large T_1 performs better. For the majority of test problems, $T_1 = \lfloor 10\% \cdot N \rfloor$ and $T_2 = \lfloor 10\% \cdot N \rfloor$ are the best parameter settings.

2) Effects of n and k_1: n and k_1 are major parameters in the sampling strategy, and they decide the sample size for each solution and the number of samples for each newly generated solution, respectively. Similarly, we take ZDT2 and UF2 as examples and study effects of different settings of n and k_1 in NT-MOEA/D. Different n and k_1 values are set as $\{3, 4, 5, 7, 9\}$ and $\{2, 3, 4, 5, 6\}$, respectively. Fig.6 displays the mean Δ_p metric values across all combinations of n and k_1 values on the selected test problems. It should be noted that for the combination that $k_1 > n$, we set $k_1 = n$.

As can be seen in Fig.6, NT-MOEA/D performs well with $n \in \{3, 4, 5\}$ for ZDT2 with 1% and 10% noise levels and UF2 with 10% noise level. As to UF2 with 1% noise level, NT-MOEA/D performs well when $n \in \{4, 5\}$. It can be claimed that $n = 5$ is a good setting for the majority of test instances. Generally, a larger n is good for the stability of the proposed algorithm, while a smaller n gives a high probability of detecting new solutions.

Fig.6 reveals that NT-MOEA/D with $k_1 \in \{2, 3, 4, 5, 6\}$ values shows very different performances on both problems, especially on UF2 with 10% noise level. In order to allocate more samples among neighborhoods based on the information of current population, we set $k_1 = 2$ with which NT-MOEA/D exhibits competitive performances on both test instances with various levels of noise.

B. Comparisons with Various Variants

In this section, the effects of proposed noise-handling techniques are deeply analyzed on all test problems with different strength levels of noise. First, effects of the Pareto-based nadir point estimation strategy (PNE) and two adaptive sampling strategies (ASs) are examined. Then effects of the mixed objective evaluation strategy (MO) and the mixed repair mechanism (MR) are further investigated.

1) Effects of the Pareto-based Nadir Point Estimation Strategy (PNE) and Adaptive Sampling Strategies (ASs): As mentioned above, the Pareto-based nadir point estimation strategy make takes a full advantage of the high speed of convergence at the early stage of optimization of the Pareto-based MOEAs. Then does this nadir point estimation strategy indeed enhance the performances of standard DMOEAs (SDMOEAs) [2]? In or-

[2]SDMOEAs mean standard DMOEAs adopting the uniform sampling strategy, the mean value of samples as the objective estimation, and the truncation repair mechanism.

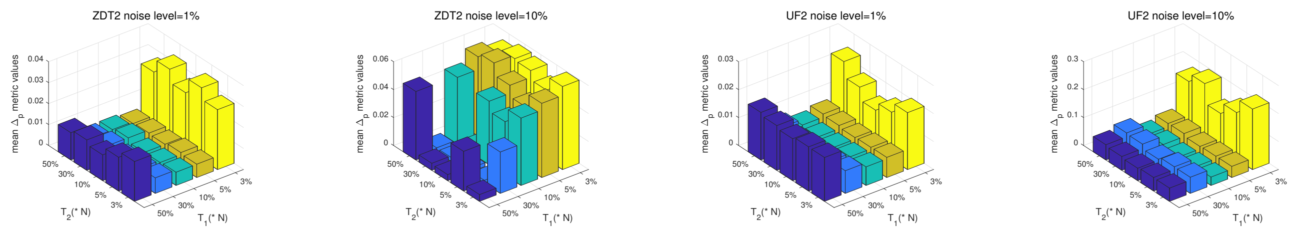

Fig. 5. Mean Δ_p values within 30 runs versus values of T_1 and T_2 in NT-DMOEA-εC for ZDT2 and UF2 test problems with 1% and 10% noise levels

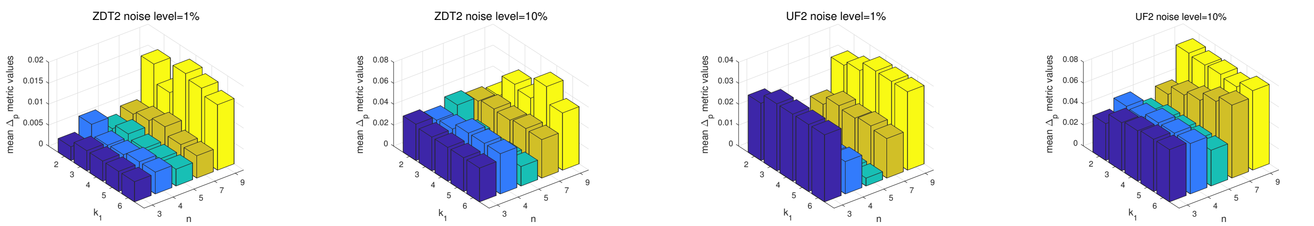

Fig. 6. Mean Δ_p values within 30 runs versus values of n and k_1 in NT-MOEA/D for ZDT2 and UF2 test problems with 1% and 10% noise levels

der to answer this question, a variant of SDMOEAs, denoted as SDMOEAs-PNE is developed for comparing with SDMOEAs. Furthermore, do two types of adaptive sampling strategies, i.e., co-ranking and OCBA strategies, play an important role in SDMOEAs? Similarly, two SDMOEAs variants, denoted as SDMOEAs-PNE-CR and SDMOEAs-PNE-OCBA are designed to answer this question. Detailed descriptions of these variants are given in the following.

SDMOEAs-PNE: Different from SDMOEAs, the process of estimating the nadir point is replaced with the Pareto-based approaches.

SDMOEAs-PNE-CR: Different from SDMOEAs-PNE, the number of samples is adaptively allocated according to the co-ranking allocation strategy (**Algorithm 1**).

SDMOEAs-PNE-OCBA: Different from SDMOEAs-PNE, the number of samples is allocated via the OCBA strategy (**Algorithm 2**).

All three variants are experimentally compared with SDMOEAs on all test instances with seven different strength levels of noise. The mean rank values in terms of the Δ_p metric over 30 independent runs of SDMOEA-εC and its variants on four test instances with seven different strength levels of noise are shown in Table II. The bold data in the table are the best mean rank values for each instance. Besides, the last row of each table summarizes the overall performance, i.e., the mean ranks of all compared algorithms on all test instances with seven different strength levels of noise in terms of the Δ_p metric values.

As can be seen in Table II, in terms of Δ_p metric values, SDMOEA-εC-PNE shows significant advantages over SDMOEA-εC on all test problems. The performances of SDMOEA-εC-PNE with two types of adaptive sampling strategies, i.e., SDMOEA-εC-PNE-CR and SDMOEA-εC-PNE-OCBA are better than that of SDMOEA-εC and SDMOEA-εC-PNE. Besides, SDMOEA-εC-PNE-CR shows superiority over SDMOEA-εC-PNE-OCBA on all test problems in terms of Δ_p metrics. As to MOEA/D, similar results

TABLE II
OVERALL PERFORMANCES OF SDMOEA-εC AND ITS VARIANTS ABOUT PNE AND TWO ASS OVER 30 INDEPENDENT RUNS ON FOUR TEST INSTANCES WITH SEVEN DIFFERENT STRENGTH LEVELS OF NOISE IN TERMS OF Δ_p METRICS

Instance	SDMOEA-εC	-PNE	-PNE-CR	-PNE-OCBA
ZDT2	3.856	2.571	**1.429**	2.143
DTLZ2	4.000	2.857	**1.000**	2.143
UF2	3.571	3.143	**1.428**	1.857
UF10	3.571	3.143	**1.429**	1.857
Mean Rank	3.714	2.950	**1.420**	1.706

can be obtained with respect to Δ_p metrics. Specifically, SMOEA/D-PNE with two types of adaptive sampling strategies outperform SMOEA/D-PNE whose performance is better than SMOEA/D. Among two adaptive sampling strategies, the co-ranking allocation strategy shows better performance than the OCBA strategy. Results in this subsection highlight the effectiveness of the Pareto-based nadir point estimation strategy and two adaptive sampling strategies, especially the co-ranking allocation strategy, embedded in both DMOEA-εC and MOEA/D with respect to Δ_p metric values. Since SDMOEAs with the Pareto-based nadir point estimation strategy and the co-ranking adaptive sampling strategy perform best among their variants, we take them as baselines for the further investigation and denote them as SDMOEAs for short.

2) Effects of the Mixed Objective Estimation Strategy (MO) and the Mixed Repair Mechanism (MR): Experiments in this subsection are conducted in a similar manner as in the previous subsection with an aim to compare SDMOEAs and its variants and show superiority of the proposed mixed objective evaluation strategy and the mixed repair mechanism. What's more, since the proposed two mechanisms are applicable to NSGA-II, similar experiments are performed to verify the effectiveness of two strategies in NSGA-II. In order to further investigate the superiority of two mixed objective strategies,

TABLE III
OVERALL PERFORMANCES OF SMOEA/D AND ITS VARIANTS ABOUT MO AND MR OVER 30 INDEPENDENT RUNS ON FOUR TEST INSTANCES WITH SEVEN DIFFERENT STRENGTH LEVELS OF NOISE IN TERMS OF Δ_p METRICS

Instance	SMOEA/D	-ME	-MO	-RR	-MR	-MM
ZDT2	5.143	3.714	2.571	4.143	3.286	**2.143**
DTLZ2	5.571	4.714	3.143	3.143	2.429	**2.000**
UF2	5.286	3.857	2.000	5.000	3.143	**1.857**
UF10	5.429	3.429	2.429	4.143	3.714	**1.857**
Mean Rank	5.170	3.781	2.840	3.781	3.025	**2.076**

TABLE IV
OVERALL PERFORMANCES OF NSGA-II AND ITS VARIANTS ABOUT MO AND MR OVER 30 INDEPENDENT RUNS ON FOUR TEST INSTANCES WITH SEVEN DIFFERENT STRENGTH LEVELS OF NOISE IN TERMS OF Δ_p METRICS

Instance	NSGA-II	-ME	-MO	-RR	-MR	-MM
ZDT2	5.561	3.143	2.429	5.143	3.143	**1.571**
DTLZ2	5.426	3.286	2.714	4.714	3.286	**1.571**
UF2	5.286	2.857	2.429	5.143	3.286	**2.000**
UF10	5.857	2.857	2.571	4.714	3.143	**1.714**
Mean Rank	5.478	3.076	2.555	5.059	3.017	**1.807**

we develop the following variants.

-ME: Different from SDMOEAs and NSGA-II, the median value is regarded as the objective estimation.

-MO: In this variant, the mixed objective evaluation is adopted as the objective value.

-RR: Different from SDMOEAs and NSGA-II, the truncation repair mechanism is replaced with the random repair method after an infeasible solution is generated.

-MR: In this variant, the mixed repair mechanism is adopted after an infeasible solution is generated.

-MM: in this variant, the mixed objective evaluation is regarded as the objective value and the mixed repair mechanism is adopted after an infeasible solution is generated.

All five variants are experimentally compared with SDMOEAs and NSGA-II on all test instances with seven different strength levels of noise. Tables III-IV display the mean rank values in terms of the Δ_p metric over 30 independent runs of SMOEA/D and NSGA-II and its variants on four test instances, respectively. The bold data in each table are the best mean rank values for each instance. The last row of each table summarizes the overall performance, i.e., the mean ranks of all compared algorithms on all test instances with seven different strength levels of noise in terms of the Δ_p metric.

As can be seen in Tables III-IV, in terms of Δ_p metric values, SMOEA/D-MO shows a significant advantage over SMOEA/D and SMOEA/D-ME on all test problems. SMOEA/D-MR outperforms SMOEA/D and SMOEA/D-RR on all test instances in terms of Δ_p metric values. Besides, it is observed that SMOEA/D-MM performs best among all variants on all test problems in terms of Δ_p metrics. As to the NSGA-II, similar results can be obtained with respect to the Δ_p metric. Specifically, NSGA-II-MO and NSGA-II-MR both show superiority over NSGA-II, and they outperform NSGA-II-ME and NSGA-II-RR, respectively. Also, NSGA-II-MM shows the best performance compared with other variants on all test problems in terms of Δ_p metrics. Tables III-IV demonstrate that DMOEAs-MM, denoted as NT-DMOEAs in the following, performs better than all the other variants on all test instances in terms of the Δ_p metric values. Besides, NSGA-II-MM shows advantages over its variants on all test instances, and it will be adopted as a competitor in the following comparison experiments. Therefore, the effectiveness of the mixed objective evaluation strategy and the mixed repair mechanism are confirmed experimentally.

C. Comparisons with Other Noise-tolerant Algorithms

In order to examine the effectiveness of NT-DMOEAs including NT-DMOEA-εC and NT-MOEA/D, a comparative study with NSGA-II-MM, BES [8], MOP-EA [12] [3], and RTEA [9] is carried out based upon all benchmark problems with seven different strength levels of noise.

Table V exhibits the mean rank values in terms of both Δ_p and *HV* metrics over 30 independent runs for each test instance with seven different strength levels of noise in order to have a global view of performance of all algorithms. The bold data in the table are the best mean rank values for each instance. Besides, the last row of Table V summarizes the overall performance i.e., the mean ranks of five algorithms on all instances in terms of both metric values.

As can be seen in Table V, in terms of *HV* metric values, NT-DMOEA-εC and NT-MOEA/D show significant advantages over comparison algorithms on the majority of seventeen test instances except DTLZ4, UF9, and UF10 on which RTEA shows better or similar performance. As to the remaining algorithms, RTEA outperforms other algorithms remarkably. What's more, NSGA-II-MM and MOP-EA perform similarly on the majority of test problems and exhibit obvious advantages over BES. To be specific, NT-DMOEA-εC has the lowest rank value on ZDT2, ZDT3, and ZDT6 test problems. RTEA and NT-DMOEA-εC have the same rank values and show superiority over other algorithms on ZDT1. As to ZDT2 and ZDT4, NT-DMOEA-εC and NT-MOEA/D both have the lowest rank value, which means the best performance among all comparison algorithms. For UF1, UF5, UF7, and UF8 test problems, NT-MOEA/D performs better than NT-DMOEA-εC and RTEA. As to UF2, UF3, UF4, and UF6 instances, NT-DMOEA-εC shows better rank values over NT-MOEA/D and RTEA. Similar results can be obtained in terms of Δ_p metric values according to statistical results in Table V. That is, NT-DMOEA-εC, NT-MOEA/D, and RTEA outperforms other comparison algorithms on all seventeen test instances. NT-DMOEA-εC and NT-MOEA/D have smaller rank values than RTEA on the majority of test instances. To summarize, as can be seen in Table V, NT-DMOEA-εC and NT-MOEA/D perform better than other comparison algorithms on all test problems. NT-DMOEA-εC achieves the best *HV* and Δ_p

[3]The algorithm of Syberfeldt et al. [12] is designed to cope with variable noise. We follow the practice of [12] and do not incorporate the surrogate which is designed for expensive real-world problems. We label the algorithm MOP-EA here.

TABLE V
OVERALL PERFORMANCES OF SIX ALGORITHMS OVER 30 INDEPENDENT RUNS ON SEVENTEEN INSTANCES WITH SEVEN DIFFERENT STRENGTH LEVELS OF NOISE IN TERMS OF Δ_p AND HV METRICS

Instance	NT-DMOEA-εC		NT-MOEA/D		NSGA-II-MM		BES		MOP-EA		RTEA	
	Δ_p	HV	Δ_p	HV	Δ_p	HV	Δ_p	HV	Δ_p	HV	Δ_p	HV
ZDT1	**1.857**	**2.000**	2.000	2.143	4.714	4.857	5.571	5.429	4.571	4.571	2.286	**2.000**
ZDT2	**2.000**	**1.857**	2.143	2.429	4.429	4.429	5.714	5.571	4.429	4.429	2.286	2.286
ZDT3	**1.714**	**1.571**	1.857	2.143	4.857	4.857	5.429	5.571	4.714	4.571	2.429	2.286
ZDT4	**2.000**	**1.857**	**2.000**	**1.857**	4.571	4.429	5.286	5.429	5.000	5.000	2.143	2.429
ZDT6	**1.714**	**1.714**	2.143	2.429	4.714	4.429	5.571	5.714	4.714	4.857	2.143	1.857
DTLZ2	2.000	**2.000**	**1.857**	**2.000**	4.857	5.000	4.857	5.000	5.143	4.143	2.286	2.857
DTLZ4	**1.714**	**1.857**	2.286	2.429	4.429	4.571	5.571	5.714	5.000	4.714	2.000	1.714
UF1	2.143	1.857	**1.714**	**1.714**	4.714	4.571	5.286	5.429	4.857	4.714	2.286	2.714
UF2	**1.714**	**1.714**	2.000	1.857	4.714	4.714	5.286	5.286	4.714	4.714	2.571	2.714
UF3	**1.857**	**1.714**	2.286	2.429	4.571	4.429	5.429	5.714	4.857	4.429	2.000	2.286
UF4	**1.571**	**1.714**	2.143	1.857	4.714	4.857	5.571	5.429	4.714	4.714	2.286	2.429
UF5	2.143	2.000	**2.000**	**1.857**	4.857	4.857	4.857	4.857	4.857	5.143	2.286	2.286
UF6	**1.857**	**1.857**	**1.857**	2.143	4.714	4.571	5.571	5.571	4.571	4.857	2.429	2.000
UF7	2.286	2.143	**2.143**	**2.000**	4.857	4.714	4.857	5.000	4.714	4.571	2.143	2.571
UF8	**1.714**	1.857	1.857	**1.714**	4.714	4.857	5.714	5.857	4.571	4.286	2.429	2.429
UF9	**2.000**	2.286	2.429	2.429	4.714	4.571	5.429	5.571	4.286	4.000	2.143	**2.143**
UF10	**2.000**	**2.286**	2.286	**2.286**	4.714	4.571	5.000	5.143	4.714	4.143	2.286	**2.286**
Mean Rank	**1.899**		2.050		4.681		5.391		4.647		4.572	

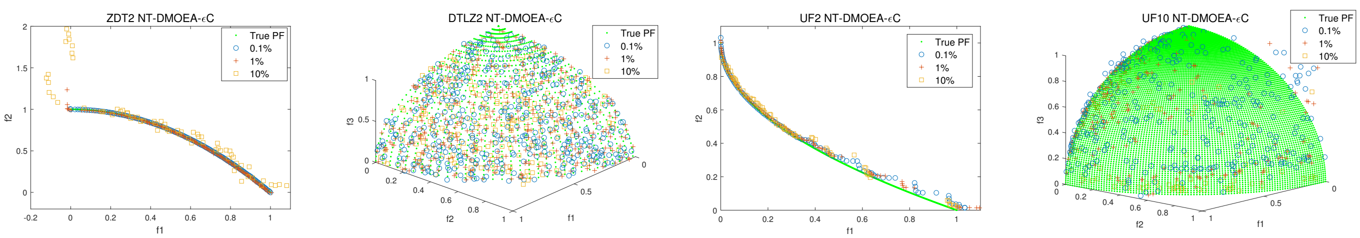

Fig. 7. Final populations in the objective space with the minimum Δ_p metric value within 30 runs obtained by NT-DMOEA-εC on four test problems with 0.1%, 1%, and 10% noise levels

metric values over others on the majority of test instances except DTLZ4, UF9, and UF10 on which RTEA exhibits better or similar performances.

The last row of Table V summarizes these statistical results and reveals the overall rank of the six algorithms, that is, NT-DMOEA-εC, NT-MOEA/D, RTEA, MOP-EA, NSGA-II-MM, and BES according to the mean rank values of both Δ_p and HV metric values. It indicates that NT-DMOEA-εC and NT-MOEA/D have advantages over comparison algorithms on all test instances and NT-DMOEA-εC shows the best performance on these test problems in terms of two metrics. The superiority of NT-DMOEA-εC and NT-MOEA/D can be attributed to the Pareto-based nadir point estimation strategy, two adaptive sampling strategies, the mixed objective estimation strategy, and the mixed repair mechanism.

Fig.7 shows the distribution of the final solutions with the minimum Δ_p value of four test instances with 0.1%, 1%, and 10% noise levels within 30 runs found by NT-DMOEA-εC. It is visually evident that for ZDT and bi-objective UF with 0.1% noise level, the final population obtained by NT-DMOEA-εC can cover the whole PFs very well and spread uniformly. NT-DMOEA-εC also shows good convergence and obtains solutions with good diversity on ZDT and bi-objective UF with 1% noise level. For ZDT and bi-objective UF with 10% noise level, final solutions obtained by NT-DMOEA-εC do not approximate the PFs very well but spread widely along the PFs. As to the DTLZ and tri-objective UF instances with noise, even the 0.1% noise level has a significant impact on final solutions obtained via NT-DMOEA-εC. Specifically, NT-DMOEA-εC can only find solutions which are not approximated nearly enough to the true PF but spread widely along the true PF. For DTLZ and tri-objective UF with 1% and 10% noise levels, both convergence and diversity of final solutions obtained via NT-DMOEA-εC deteriorates. In summary, Fig.7 shows that NT-DMOEA-εC can achieve the approximations with both good convergence and diversity for most of ZDT and bi-objective UF test instances and can find solutions with good diversity for DTLZ and tri-objective UF test instances.

V. CONCLUSION

Uncertainties widely exist in real-world problems which are mostly with multiple conflicting objectives. Usually the uncertainty is modeled as additive noise in the objective space. The decomposition-based multi-objective evolutionary algorithms (DMOEAs) are important approaches for multi-objective optimization and have not been applied to uncertain problems extensively. In order to design effective noise-tolerant DMOEAs (NT-DMOEAs) for uncertain problems, this paper has conducted extensive studies to examine the impact of noise on DMOEAs, particularly for the population dynamics of convergence and diversity. Based on observations of noise impacts on DMOEAs, four noise-handling techniques have been proposed and embedded in two popular DMOEAs, i.e., MOEA/D and DMOEA-εC. The proposed NT-DMOEAs utilize the composite benefits of four major extensions.

Specifically, the Pareto-based nadir point estimation strategy makes full use of the high speed of convergence at the early stage of optimization of the Pareto-based MOEAs to improve the accuracy of nadir point estimation. Two adaptive sampling strategies vary the number of samples used per solution based on the differences among neighboring solutions and their variance to control the trade-off between search space exploration and exploitation. Then the mixed objective evaluation strategy varies the estimation of an uncertain objective function between the mean and median values of fitness samples adaptively according to the estimated strength level of noise to alleviate effects of noise. Finally, the mixed repair mechanism adaptively combines the truncation repair mechanism and a random repair mechanism together to adaptively remedy the loss of diversity in the decision space.

Two NT-DMOEAs have been compared with their various variants and four noise-tolerant algorithms to show the superiority of proposed features on seventeen test instances with different strength levels of noise. A systematical experimental study has shown that two NT-DMOEAs significantly outperform their variants on the majority of test instances with different strength levels of noise. It also has shown that two NT-DMOEAs, especially NT-DMOEA-εC, exhibit competitive or superior performance in terms of proximity and diversity for the majority of test instances.

Many real-world problems suffer from noisy disturbances. Under such circumstances, standard DMOEAs are inadequate to provide Pareto optimal solutions for these noisy problems, which calls the necessity of noise-tolerant algorithms. Actually, it is usually difficult to give a good representation of the noise in real environments, thus how to estimate the noise accurately is an important task when handling noisy optimization problems. The most common way of estimating noise is to sample candidate solutions multiple times and measure the strength level of noise based on the variance value of a series of samples. This method is simple and straightforward, but not effective enough when the characteristics of noise are difficult to describe. What's more, there are many different forms of noise whose characteristics change as a function of location (in the design or objective space), or which alter during the course of an optimisation [9]. This poses new challenges to the uncertain optimization. It will be our future work to propose an effective way of giving a good representation of the noise and estimating the strength level of noise accurately. Furthermore, new mechanisms that can deal with other types of uncertainties will be investigated. The performances of proposed NT-DMOEAs will also be tested on more complex real-world problems.

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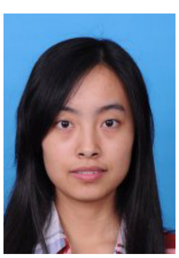

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