

1 calcule la matriz inversa con el método de Gauss-Jordan.

$$A_1 = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 1 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 2 & 4 & 6 & 1 \\ 4 & 7 & 5 & -6 \\ 2 & 5 & 18 & 10 \\ 6 & 12 & 38 & 16 \end{bmatrix}$$

Método inversa = descomposición LU

$$A = \left(\begin{array}{ccc|cc} 1 & 3 & 4 & 1 & 0 \\ 2 & 1 & 3 & 0 & 1 \\ 4 & 2 & 1 & 0 & 0 \end{array} \right) \xrightarrow{\text{Método Gauss-Jordan}}$$

$F_2 \rightarrow F_2 - 2F_1$ $F_3 \rightarrow F_3 - 4F_1$

$$\left(\begin{array}{ccc|cc} 1 & 3 & 4 & 1 & 0 \\ 0 & -5 & -5 & -1 & 1 \\ 4 & 2 & 1 & 0 & 0 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{ccc|cc} 1 & 3 & 4 & 1 & 0 \\ 0 & 1 & 1 & -1 & 1 \\ 4 & 2 & 1 & 0 & 0 \end{array} \right)$$

$F_3 \rightarrow F_3 - 4F_2$ $F_3 \rightarrow F_3 + 10F_2$

$$\left(\begin{array}{ccc|cc} 1 & 3 & 4 & 1 & 0 \\ 0 & 1 & 1 & -1 & 1 \\ 0 & -10 & -15 & -11 & 1 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{ccc|cc} 1 & 3 & 4 & 1 & 0 \\ 0 & 1 & 1 & -1 & 1 \\ 0 & 1 & -5 & 9 & 1 \end{array} \right)$$

$F_3 \rightarrow F_3 - 2F_2$ $F_2 \rightarrow F_2 - F_3$

$$\left(\begin{array}{ccc|cc} 1 & 3 & 4 & 1 & 0 \\ 0 & 1 & 1 & -1 & 1 \\ 0 & 1 & -5 & 9 & 1 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{ccc|cc} 1 & 3 & 4 & 1 & 0 \\ 0 & 1 & 0 & -2 & 1 \\ 0 & 0 & 1 & 9 & 1 \end{array} \right)$$

$F_1 \rightarrow F_1 - 3F_2$ $F_1 \rightarrow F_1 - 4F_3$

$$\left(\begin{array}{ccc|cc} 1 & 0 & 9 & -1/5 & -7/5 & 7/5 \\ 0 & 1 & 0 & 9/5 & -7/5 & 7/5 \\ 0 & 0 & 1 & 0 & 9/5 & -7/5 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{ccc|cc} 1 & 0 & 0 & -1/5 & -7/5 & 7/5 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array} \right)$$

$F_1 \rightarrow F_1 - 3F_3$ $F_1 \rightarrow F_1 - 4F_3$

$$\left(\begin{array}{ccc|cc} 1 & 0 & 9 & -1/5 & -7/5 & 7/5 \\ 0 & 1 & 0 & 9/5 & -7/5 & 7/5 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{ccc|cc} 1 & 0 & 0 & -1/5 & -7/5 & 7/5 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array} \right)$$

$A_2 = \left(\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 0 \\ 0 & 1 & 4 & 0 & 1 \\ 5 & 6 & 0 & 0 & 1 \end{array} \right) //$

$F_3 \rightarrow F_3 - 5F_1$ $F_2 \rightarrow F_2 + 4F_1$

$$\left(\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 0 \\ 0 & 1 & 4 & 0 & 1 \\ 0 & 4 & -15 & -25 & 1 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 0 \\ 0 & 1 & 4 & 0 & 1 \\ 0 & 0 & -15 & -15 & 1 \end{array} \right)$$

$F_2 \rightarrow F_2 - 4F_3$ $F_1 \rightarrow F_1 - 5F_3$

$$\left(\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 0 \\ 0 & 1 & 4 & 0 & 1 \\ 0 & 0 & -15 & -15 & 1 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 0 \\ 0 & 1 & 4 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right)$$

$F_1 \rightarrow F_1 - 2F_2$ $T_1 = F_1 - 2F_2$

$$\left(\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 0 \\ 0 & 1 & 4 & 0 & 1 \\ 0 & 0 & 0 & -2 & -1 \end{array} \right) //$$

$A_3 = \left(\begin{array}{ccc|cc} 4 & 2 & 1 & 1 & 0 \\ 2 & 1 & 3 & 0 & 1 \\ 1 & 3 & 4 & 0 & 1 \end{array} \right)$

$F_1 = 1/4 \cdot F_1$ $F_3 \rightarrow F_3 - F_1$

$$\left(\begin{array}{ccc|cc} 1 & 1/2 & 1/4 & 1/4 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 1 & 3 & 4 & 0 & 1 & 0 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{ccc|cc} 1 & 1/2 & 1/4 & 1/4 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 0 & 9/2 & 11/4 & -1/4 & 0 & 0 \end{array} \right)$$

$F_2 \rightarrow F_2 - 2F_1$ $F_2 \leftrightarrow F_3$

$$\left(\begin{array}{ccc|cc} 1 & 1/2 & 1/4 & 1/4 & 0 & 0 \\ 0 & 9/2 & 11/4 & -1/4 & 0 & 1 \\ 0 & 9/2 & 11/4 & -1/4 & 0 & 0 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{ccc|cc} 1 & 1/2 & 1/4 & 1/4 & 0 & 0 \\ 0 & 1 & 1 & -1/4 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$F_3 \rightarrow 2/5 \cdot F_3$ $F_2 \rightarrow 2/5 \cdot F_2$

$$\left(\begin{array}{ccc|cc} 1 & 1/2 & 1/4 & 1/4 & 0 & 0 \\ 0 & 1 & 1 & -1/4 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{ccc|cc} 1 & 1/2 & 1/4 & 1/4 & 0 & 0 \\ 0 & 1 & 1 & -1/4 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$F_2 \rightarrow F_2 - 3/2 \cdot F_3$ $F_1 \rightarrow F_1 - 1/4 \cdot F_3$

$$\left(\begin{array}{ccc|cc} 1 & 1/2 & 1/4 & 1/4 & 0 & 0 \\ 0 & 1 & 1 & -1/4 & 0 & 1 \\ 0 & 0 & 1 & -1/5 & 3/2 & 0 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{ccc|cc} 1 & 1/2 & 1/4 & 1/4 & 0 & 0 \\ 0 & 1 & 1 & -1/4 & 0 & 1 \\ 0 & 0 & 1 & -1/5 & 3/2 & 0 \end{array} \right)$$

$F_1 \rightarrow F_1 - 1/2 \cdot F_2$ $F_1 \rightarrow F_1 - 1/2 \cdot F_2$

$$\left(\begin{array}{ccc|cc} 1 & 0 & 0 & 1/5 & 1/5 & -1/5 \\ 0 & 1 & 1 & -1/4 & 0 & 1 \\ 0 & 0 & 1 & -1/5 & 3/2 & 0 \end{array} \right) //$$

$A_4 = \left(\begin{array}{cccc|cc} 2 & 4 & 6 & 1 & 0 & 0 \\ 4 & 7 & 5 & -6 & 0 & 0 \\ 2 & 5 & 18 & 10 & 0 & 0 \\ 6 & 12 & 38 & 16 & 0 & 0 \end{array} \right)$

$F_1 \rightarrow 1/2 \cdot F_1$ $F_2 \rightarrow F_2 - 4F_1$

$$\left(\begin{array}{cccc|cc} 1 & 2 & 3 & 0 & 0 & 0 \\ 4 & 7 & 5 & -6 & 0 & 0 \\ 2 & 5 & 18 & 10 & 0 & 0 \\ 6 & 12 & 38 & 16 & 0 & 0 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{cccc|cc} 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 3 & 2 & -3 & 0 & 0 \\ 2 & 5 & 18 & 10 & 0 & 0 \\ 6 & 12 & 38 & 16 & 0 & 0 \end{array} \right)$$

$F_3 \rightarrow -2F_1 - F_2$ $F_4 \rightarrow F_4 - 6F_1$

$$\left(\begin{array}{cccc|cc} 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 3 & 2 & -3 & 0 & 0 \\ 2 & 5 & 18 & 10 & 0 & 0 \\ 6 & 12 & 38 & 16 & 0 & 0 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{cccc|cc} 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 2 & 5 & 18 & 10 & 0 & 0 \\ 6 & 12 & 38 & 16 & 0 & 0 \end{array} \right)$$

$F_2 \rightarrow -1/2 \cdot F_2$ $F_3 \rightarrow F_3 - 1/2 \cdot F_2$

$$\left(\begin{array}{cccc|cc} 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 2 & 5 & 18 & 10 & 0 & 0 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{cccc|cc} 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 15 & 10 & 0 & 0 \end{array} \right)$$

$F_2 \rightarrow -5/3 \cdot F_2$ $F_4 \rightarrow F_4 - 20 \cdot F_2$

$$\left(\begin{array}{cccc|cc} 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 15 & 10 & 0 & 0 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{cccc|cc} 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1/3 & 2/3 & 0 \end{array} \right)$$

$F_4 \rightarrow 1/4 \cdot F_4$ $F_3 \rightarrow F_3 - 5F_4$

$$\left(\begin{array}{cccc|cc} 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1/3 & 2/3 & 0 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{cccc|cc} 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1/3 & 2/3 & 0 \end{array} \right)$$

$F_2 \rightarrow F_2 - 8F_4$ $F_1 \rightarrow F_1 - 1/2 \cdot F_4$

$$\left(\begin{array}{cccc|cc} 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1/3 & 2/3 & 0 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{cccc|cc} 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1/3 & 2/3 & 0 \end{array} \right)$$

$F_2 \rightarrow F_2 - FF_3$ $F_1 \rightarrow F_1 - 3F_3$

$$\left(\begin{array}{cccc|cc} 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1/3 & 2/3 & 0 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{cccc|cc} 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1/3 & 2/3 & 0 \end{array} \right)$$

$F_1 \rightarrow F_1 - 2F_2$

$$\left(\begin{array}{cccc|cc} 1 & 0 & 0 & 0 & 16/5 & -7/5 \\ 0 & 1 & 0 & 0 & -2/5 & 8/15 \\ 0 & 0 & 1 & 0 & -4/5 & 13/45 \\ 0 & 0 & 0 & 1 & 1 & -4/9 \end{array} \right) //$$

2 calcule la descomposición LU.

Decomposition LU

$$A = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 3 \\ 0 & 2 & 1 \end{pmatrix}$$

$$r_2 \leftarrow r_2 - 2r_1 \quad r_3 \leftarrow r_3 - 4r_1$$

$$\begin{pmatrix} 1 & 3 & 4 \\ 0 & -5 & -5 \\ 0 & 2 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 3 & 4 \\ 0 & 5 & 5 \\ 0 & 0 & -5 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 3 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & -5 \end{pmatrix}$$

$$A_{23} = \begin{pmatrix} 1 & 3 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & -5 \end{pmatrix}$$

$$r_3 \leftarrow r_3 + r_2$$

$$r_2 \leftarrow r_2 - r_3 \quad r_3 \leftarrow r_3 - r_2$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -5 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A_{23} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$

$$r_2 \leftarrow r_2 - r_1 \quad r_3 \leftarrow r_3 - 4r_1$$

$$r_3 \leftarrow r_3 - 4r_2$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A_{23} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$r_2 \leftarrow r_2 - 2r_1 \quad r_3 \leftarrow r_3 - 4r_1$$

$$r_3 \leftarrow r_3 - r_2$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$r_3 \leftarrow r_3 - r_2$$

$$r_3 \leftarrow r_3 - r_1$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$r_3 \leftarrow r_3 - r_1$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$