A tri-level optimization model for facility location-protection problem considering design and redesign decisions under disruption

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Abstract

Purpose: Fortification-interdiction models provide system designers with a broader perspective to identify and protect vital components. Based on this concept, we examine how disruptions impact critical supply systems in this paper and propose the best possible protection strategies based on three levels of decision-makers. This paper aims to investigate location and fortification decisions in the first level. Moreover, a redesign problem is presented in the third level to locate backup facilities and reallocate undisrupted facilities following the realization of the disruptive agent decisions at the second level.

Design/methodology/approach: To address this problem, we develop a tri-level planner-attacker-defender optimization model. The model minimizes investment and demand satisfaction costs and alleviates maximal post-disruption costs. While decisions are decentralized at different levels, we develop an integrated solution algorithm to solve the model using the column-and-constraint generation method.

Findings: The model and the solution approach are tested on a real supply system consisting of several hospitals and demand areas in a region in Iran. Results indicate that incorporating redesign decisions at the third level reduces maximum disruption costs.

Originality: The paper makes the following contributions: presenting a novel tri-level optimization model to formulate facility location and interdiction problems simultaneously, considering corrective measures at the third level to reconfigure the system after interdiction, creating a resilient supply system that can fulfill all demands after disruptions, employing a nested column-and-constraint generation (C&CG) method to solve the model.

Keywords: tri-level programming, facility fortification-interdiction, facility location, column-and-constraint generation

1. Introduction and literature review

Supply system design has long been an area of interest for scholars and planners. A supply system consists of a number of facilities and demand points, where the facilities provide goods and services to the demand points. Among them, power transmission lines, telecommunication systems, water supply lines, and emergency services, such as hospitals, fire stations, and police stations, are considered critical infrastructures. In light of the fact that these systems serve as the backbone of the primary supply chain, any threat to their functionality would have catastrophic consequences. To ensure system reliability against disruptions caused by natural or man-made disasters, planners identify vulnerable facilities and invest in the fortification of vital infrastructures. If identified prior to disruption, critical systems can be effectively protected by allocating protection resources. Fortification-interdiction models are widely regarded as a prominent approach to analyzing supply system vulnerabilities in light of an intelligent adversary aiming to disrupt critical facilities and impose maximal costs on the supply system. In a system planner's view, these models can identify facilities that are most likely to be attacked so that they can be protected to minimize maximum disruption costs (Aksen and Aras 2012). This problem was addressed for the first time in (Wollmer 1964). After that, several studies have been conducted to investigate the impact of man-made or natural disasters on critical components in different areas. Studies in this field fall into two major categories: arc interdictions and facility interdictions. (Wood 1993) and (Cormican, Morton et al. 1998) are pioneers in the network interdiction domain, while the latter presents a more realistic interdiction model, employing a two-stage stochastic programming formulation. It was the very first time that the concept of formulating interdiction decisions as binary variables in mathematical programming was introduced.

According to recent studies, network interdiction problems are programmed as bi-level and tri-level models, in which defenders and attackers are considered as decision-makers at different levels (Israeli and Wood 2002, Sayed, Wang et al. 2019, Nadizadeh and Sabzevari Zadeh 2021, Nemati, Latify et al. 2021, Sarhadi, Tulett et al. 2022). These studies aim to design fortified networks, i.e. graphs of nodes and arcs, by protecting arcs while an attacker at a different level removes arcs to impose maximum costs on the defender. One of the most evolved areas of network interdiction is the design of resilient power grid networks—one of the nations' most critical infrastructures—in response to intentional and natural failures. In this regard, the most recent and extensive research has been conducted by (Fang and Sansavini 2017, Sayed, Wang et al. 2019, Nemati, Latify et al. 2021, Rocchetta 2022). (Fang and Sansavini 2017, Nemati, Latify et al. 2021) present a tri-level optimization model to formulate restorative actions for an electrical grid network under intentional attacks. In addition, a novel approach based on spectral clustering is employed by (Rocchetta 2022) to assess the impact of failures on the power grid performance and measure components' vulnerability. Moreover, (Hasanzad and Rastegar 2022) study the connection between natural gas transmission networks and electrical power grids to assess their vulnerability to disruptions.

Thus far, most studies have focused on arc interdiction models, while facility interdiction has received less attention from scholars (Zhang, Zheng et al. 2016). In general, facility interdiction problems are developed based on the p-median and maximal covering problem by (Church, Scaparra et al. 2004, Church and Scaparra 2007, Scaparra and Church 2008, Liberatore, Scaparra et al. 2011, Aksen and Aras 2012, Aksen, Aras et al. 2013, Aliakbarian, Dehghanian et al. 2015, Mahmoodjanloo, Parvasi et al. 2016). As a pioneer, (Church, Scaparra et al. 2004) investigated a facility interdiction problem and introduced the r-interdiction median model (RIM). Here, the attacker seeks to eliminate r facilities to maximize the total demand satisfaction costs. The main purpose is to minimize the maximum costs incurred by the attacker by protecting the facilities targeted by the attacker. Subsequent studies (Church and Scaparra 2007, Scaparra and Church 2008) formulate protection strategies as binary decision variables besides interdiction variables in a single-level model. This problem is known as the r-interdiction median problem with fortification (RIMF), which has been used as a reference model in subsequent research such as (Li and Savachkin 2013).

Former studies consider fortification-interdiction problems as a single-level model where interdiction and protection decisions are taken from the system planner or attacker's perspective. Nevertheless, most recent studies have been developed based on multi-level programming models. Bi-level interdiction models have been introduced by (Scaparra and Church

2008, Losada, Scaparra et al. 2012, Aksen, Aras et al. 2013). As a result of considering a decision-maker at a different level who intends to maximize failure costs, the system planner (defender) can identify critical facilities and fortify them.

Many studies have been conducted on formulating interdiction problems under uncertain conditions. (Liberatore, Scaparra et al. 2011, Li, Li et al. 2021, Li, Li et al. 2022) have employed two-stage stochastic programming to model uncertainty in attacker strategies and the extent of disruptive actions. Another novel model is proposed by (Alisan, Ghorbanzadeh et al. 2020). They utilize r-interdiction and p-median models separately in two steps to identify critical shelters and harden them against natural disasters. Another version of interdiction problems is partial interdiction, in which facilities are not eliminated completely, and according to the extent of the attack, they can partially serve demands (Zhang, Zheng et al. 2016, Forghani, Dehghanian et al. 2020). Partial interdiction in problems is characterized by the attacker's stochastic decisions and the probability of a loss in facility capacities.

Several studies have been expanded into tri-level optimization models to formulate post-attack measures (Mahmoodjanloo, Parvasi et al. 2016, Akbari-Jafarabadi, Tavakkoli-Moghaddam et al. 2017, Hesam Sadati, Aksen et al. 2020, Hasanzad and Rastegar 2022). Due to the complexity of tri-level models, particularly those with binary variables at different levels, most of studies have used inexact solution methods such as heuristics, metaheuristics, and enumeration approaches to solve them. However, (Fang and Sansavini 2017, Ghorbani-Renani, González et al. 2021, Hasanzad and Rastegar 2022) develop decomposition approaches based on column-and-constraint and benders methods.

As discussed earlier, few studies have addressed facility interdiction problems in the literature. Therefore, this paper investigates a facility location-allocation problem while addressing protection and redesign decisions in response to a disruptive agent referred to as an attacker. This study is based on (Zeng and Zhao 2013) and (Fang and Sansavini 2017). Nevertheless, it is the first time a tri-level model has been proposed for facility location-interdiction problems with binary variables at three levels. This paper differs from previous studies in the following ways: 1. a tri-level planner-attacker-defender model is proposed to formulate both the facility location and interdiction problem; 2. To design a reliable system, in addition to facility location and protection decisions in the first level, a redesign problem is developed in the third level; 3. In response to disruptions, installing new backup facilities and capacity transshipment are formulated for the first time in the proposed model; 4. A nested solution approach based on the column-and-constraint generation (C&CG) method is proposed to solve the proposed model.

This paper is outlined as follows. Section 2 presents a mixed-integer nonlinear tri-level programming model to formulate the interactions between players in the planner-attacker-defender problem. The solution approach and computational results are illustrated in Section 3 and Section 4, respectively. In Section 5, we discuss conclusions and directions for future research.

2. The tri-level planner-attacker-defender (PAD) optimization model

Facility location-allocation problems have been thoroughly studied; however, designing a reliable system while addressing strategic decisions has not been investigated. For this reason, in this study, a tri-level PA with three decision-makers is developed: (a) a planner with the objective of minimizing investment costs and maximal costs imposed by an intelligent disruptive agent (attacker); (b) an attacker aiming to maximize system redesign costs by destroying facilities following the realization of planner decisions; and (c) a defender in the third-level who takes corrective measures in response to worst-case disruption scenarios. The planner, attacker, and defender interactions are formulated as a static Stackelberg game. The disruptive agent, the first follower at the second level, following the planner's decisions—the leader at the first level—decides to interdict the facilities to cause maximum failures. In response, the defender, as the second follower, redesigns the supply system at the third level to minimize post-disruption costs and fulfill all demands. These interactions are formulated as a mixed-integer nonlinear tri-level min-max-min model. The model is applied to a real supply system consisting of a set of hospitals and demand nodes in a region in Iran. According to the model, the following assumptions are made:

- Fortified facilities are immune to interdiction and cannot be disrupted by the attacker.
- In the event of disruptions, backup facilities are set up and protected from attacks.
- There are constraints on the number of fortified and interdicted facilities.
- Interdicted facilities will be completely out of use and cannot be allocated to any demand nodes.
- The opening and protection costs vary from one facility to another according to their locations
- Due to the limited capacity of the main and backup facilities, multiple allocations are allowed; each demand node can be assigned to more than one facility.
- In the proposed model, the planner makes prior-disruption location-allocation decisions at the first level. Moreover, the defender makes post-disruption location-allocation decisions at the third level.

The proposed tri-level PAD model uses the following entities:

Indices and sets:

L_P	Set of main facility locations
L_N	Set of new backup facility location
L	Set of all facilities ($L = L_P \cup L_N$)
K	Demand nodes
i, j	Index used for facility locations

k	Index used for demand nodes

Parameters:

FC_j	Fixed cost of running facility $j \in J$
PC_j	Fixed cost of protecting facility $j \in J$
TC	Traveling cost per unit demand and distance between the facility and demand node
FW_i	Distance between demand point $i \in I$ and facility $j \in J$
EC_i	Transshipment and consolidation cost per capacity $i \in I$
CAP_j	Capacity of facility $j \in J$
d_{ki}	Distance between demand node $k \in K$ and facility $j \in J$
h_k	Demand for node $k \in K$
N_{att}	Maximum number of disruptive events
N_{def}	Maximum fortified facilities
α	Weighting factor for transportation costs
β	Weighting factor for disruption costs

Decision variables:

Y_j	Binary variable equals 1 if facility $j \in L_p$ is opened, being 0 otherwise
X_{kj}	Percent of demand point k allocated to facility $j \in L_p$ before disruption
Z_j	Binary variable equals 1 if facility $j \in L_p$ is fortified, being 0 otherwise
S_j	Binary variable equals 1 if facility $j \in L_p$ is disrupted, being 0 otherwise
U_{ki}	Percent of demand point k allocated to facility $i \in L$ after disruption
W_i	Binary variable equals 1 if new backup facility $i \in L_N$ is opened, being 0 otherwise
V_{ji}	Binary variable equals 1 if the capacity of facility $j \in L_P$ is transferred to facility i $(i \in L, j \neq i)$ or remains open $(j = i)$, being 0 otherwise

To formulate facility location and fortification-interdiction problem, we propose a tri-level planner-attacker-defender model as follows:

First-level

$$Z_{def} = \underset{y,z,x}{Min} \sum_{j \in L_p} FC_j . Y_j + \sum_{j \in L_p} PC_j . Z_j + \alpha (TC \times \sum_{k \in K} \sum_{j \in L_p} h_k . d_{jk} . X_{jk}) + \beta Z_{att} \tag{1a}$$

Subject to:

$$\sum_{j \in L_p} X_{kj} = 1 \qquad \forall k \in K \tag{1b}$$

$$\sum_{k \in K} h_k . X_{kj} \leq CAP_j . Y_j \qquad \forall j \in L_p \tag{1c}$$

$$M \times \sum_{j' : d_{kj'} \leq d_{kj}} X_{kj'} \geq Y_j \qquad \forall j \in L_p, k \in K \tag{1d}$$

$$Z_j \leq Y_j \qquad \forall j \in L_p \tag{1e}$$

$$\sum_{j \in L_p} Z_j \leq N_{def} \tag{1f}$$

$$Y_j, Z_j \in \{0,1\} \qquad \forall j \in L_p \tag{1g}$$

$$X_{kj} \geq 0 \qquad \forall j \in L_p, k \in K \tag{1h}$$

Second-level

$$Z_{att}(y, z, x) = \underset{s}{Max}\, Z_{def} \tag{2a}$$

Subject to:

$$\sum_{j \in L_p} S_j \leq N_{att} \tag{2b}$$

$$S_j \leq Y_j \qquad \forall j \in L_p \tag{2c}$$

$$S_j \in \{0,1\} \qquad \forall j \in L_p \tag{2d}$$

Third-level

$$Z_{def}(y,z,x,s)=\underset{u,v,w}{M\text{in}}\,\alpha(TC\times\sum_{k\in K}\sum_{i\in L}U_{ki}.d_{ik}.h_k)+\sum_{i\in L}EC_i\times\sum_{\substack{j\in L_p\\ j\neq i}}CAP_j.V_{ji} \tag{3a}$$

$$+\sum_{i\in L_N}FW_i.W_i$$

Subject to:

$$\sum_{i\in L}U_{ki}=1 \qquad \forall k\in K \tag{3b}$$

$$\sum_{j\in L_p}V_{ji}\leq\left|L_p\right|W_i \qquad \forall i\in L_N \tag{3c}$$

$$\sum_{j\in L_p}V_{ji}\leq\left|L_p\right|Y_i \qquad \forall i\in L_p \tag{3d}$$

$$\sum_{i\in L}V_{ji}\leq Y_j \qquad \forall j\in L_p \tag{3e}$$

$$\sum_{k\in K}h_k.U_{ki}\leq\sum_{j\in L_p}CAP_j.V_{ji}.(1-S_i.(1-Z_i)) \qquad \forall i\in L_p \tag{3f}$$

$$\sum_{k\in K}h_k.U_{ki}\leq\sum_{j\in L_p}CAP_j.V_{ji}.S_j.(1-Z_j) \qquad \forall i\in L_N \tag{3g}$$

$$V_{ji}\in\{0,1\} \qquad \forall j\in L_p,\, j\in L \tag{3h}$$

$$W_i\in\{0,1\} \qquad \forall i\in L_N \tag{3i}$$

$$U_{ki}\geq 0 \qquad \forall k\in K, i\in L_N \tag{3j}$$

The tri-level optimization model includes three decision-makers at different levels, each of whom plays their role following the realization of the decisions at the upper level. In the first level, the system planner problem (1a) to (1h) formulates decisions regarding the supply system design. The second level (2a) to (2d) models the disruptive agent's behavior when planning investments are realized; This model is known as the attacker problem. Following the worst-case disruption scenarios, the third level (3a) to (3j) takes redesign decisions to respond to the attacker's actions. In the following subsections, we discuss each level's objective functions and constraints in more detail.

2.1 The First-level problem

The first-level model is called the planner problem, who decides where to locate and open facilities and allocate them to the nearest demand nodes based on their capacities. Furthermore, it identifies the number of potential facilities to be fortified to minimize investment costs and maximal post-attack costs. The objective function (1a) consists of fixed costs of opening main facilities, fortification costs, traveling costs for demand satisfaction before disruptions, and the maximum post-attack costs. Constraint (1b) ensures that the demand for each node is satisfied. Equation (1c) expresses the capacity constraint on facilities while it assures that demand nodes can only be assigned to opened facilities. In addition, constraint (1c) with the binary variable Y_j, together with constraint (1d), are closest assignment (CA) constraints, enforcing the allocation of each demand node to the nearest opened facilities. The CA constraints are derived from (Teixeira and Antunes 2008) and guarantee that each demand node is assigned to the closest established facilities. More precisely, for any demand node k and facility j, if the facility j is opened ($Y_j = 1$), then due to constraint (1d)—together with constraint (1c)—demand node k is allocated to the closest opened facilities. Furthermore, they limit the amount of demand that can be satisfied by a facility to its capacity. The CA constraints have no effect if facility j is not established ($Y_j = 0$). Constraint (1e) enforces that a facility can only be protected if it is established ($Y_j = 1$). Constraint (1f) bounds the number of protected facilities due to limited protection budgets. Expression (1g) is the binary constraint on the planner's facility location and fortification decisions. Constraint (1h) is the nonnegative boundary for allocation decisions.

2.2 The Second-level problem

The attacker's behavior aimed at interdicting facilities is formulated in the second level (2a)-(2d). $Z_{att}(y, z, x)$ represents that given the planner's decisions, the variables $Y_j, Z_j, and\, X_{kj}$ derived at the first level are parameterized as input to the attacker's problem. In this regard, the attacker's decision to interdict a facility (S_j) depends on the planner's decisions (Y_j). Constraint (2c) ensures that if facility j is opened ($Y_j = 1$), then it can be attacked ($S_j = 1$), otherwise it cannot be disrupted ($S_j = 0$). By eliminating facilities, the attacker's objective is to maximize system performance loss, which is calculated by maximizing the defender's problem $Z_{def}(y, z, x, s)$ at the third level. The maximum number of disrupted facilities is enforced by constraint (2b). Constraint (2d) reflects the binary notation of interdiction variables.

2.3 The Third-level problem

The defender's reactions to the attacker are characterized by the optimization model $Z_{def}(y, z, x, s)$ in the third level (3a) to (3j). Third-level decisions such as opening backup facilities, transferring facility capacities, and reallocating decisions following the attack rely on second-level interdiction decisions and first-level planning strategies. Therefore, term $Z_{def}(y, z, x, s)$ is subject to the attacker's decision variables S_j and to the planner problem's variables Y_j, Z_j, X_{kj}. Given the planner and attacker's actions, the defender's problem is formulated as a model (3a)-(3j) to minimize total response costs after interdiction. Expression (3a) is the defender's objective function to minimize the total costs. The term comprises fixed costs of opening backup facilities, traveling costs of demand satisfaction after disruption, and capacity transshipment and consolidation costs. Constraint (3b) requires that every demand node be satisfied after the attack, which necessitates opening backup facilities. Logic constraint (3c) prevents existing facilities' capacity from being transferred to a new backup facility if it has not yet been established. Constraint (3d) assures that the capacity of existing facilities cannot be transferred to a closed facility. Constraint (3e) enforces the logic that for each existing facility, its capacity can be transferred or consolidated ($V_{ji} \neq 0$) only if the facility has been established ($Y_j = 1$) in the first-level planning phase. This constraint underlines at the same time the connection between first-level and third-level decisions. Based on the constraint (3f), it is only possible to transfer or consolidate the capacity of an existing facility (V_{ji}) after an attack, if it has been either protected ($Z_i = 1$) or not interdicted ($S_i = 0$). Furthermore, it ensures that the total demand served by the existing facility cannot exceed the installed capacity. Constraint (3f), backed by constraint (3e), highlights the effects of the planner and attacker's actions on the defender's response to the attacks. More exactly, if facility j has been established ($Y_j = 1$) but not protected and disrupted ($Z_i = 0 \; and \; S_i = 1$)—required by the defender's objective function to minimize redesign costs,—it is eliminated and cannot be used in the redesign phase ($V_{ji} = 0$). Constraint (3g)—together with constraints (3c) and (3e)—prevents any existing facility capacity that is either protected or not interdicted from being transferred to the opened backup facilities. Otherwise, the existing facility's capacity can be shifted to a backup facility that can fulfill demands up to its capacity. The third-level minimization model automatically assigns demand nodes to the closest facilities; therefore, CA constraints are no longer necessary here. Constraints (3h) to (3j) state third-level variables' binary and continuous notation, respectively.

3. Solution approach

The planner-attacker-defender problem (1)-(3) is formulated as a mixed-integer nonlinear tri-level model. Multi-level programming models that contain discrete variables, such as binary variables in different levels, are more complicated. To solve them, solution methods such as Karush-Kuhn-Tucher (KKT) conditions or duality principles present an equivalent single-level model for them. These solution methods gradually reconstruct the upper-level problem using the dual problem of the lower-level model. However, these methods cannot be used due to binary variables in the second and third levels of the proposed model (Thiele, Terry et al. 2009).

This paper adopts a solution methodology based on the column-and-constraint generation (C&CG) method. This method adds cutting planes containing lower-level recourse variables as recourse constraints to the upper-level problem (Zeng and Zhao 2013). This method is developed based on the primal cuts containing primal variables, which are planning, attack, and redesign decisions in this model. We use this method to develop a nested C&CG algorithm to solve the tri-level optimization model (1)-(3). As shown in Figure 1, the model is decomposed into two layers; each consists of a master problem (MP) and a subproblem (SP). The inner-layer algorithm, which is the outer-layer SP (6), iteratively exchanges its optimal solution S_j, which is the worst-case disruption scenario from the planner's view, and outer-layer MP's (12) planning decisions (Z_j, Y_j) until the two-layer algorithm converges to an optimal solution. The outer-layer SP is a bi-level max-min problem that can be formulated as a max-min-min model by separating defender's binary and continuous decision variables. Moreover, the outer-layer SP (7) is decomposed into an inner-layer MP (9) and SP (11) (Fang and Sansavini 2017). They interact with each other through a cutting plane algorithm that iteratively trades primal attacker strategies S_j and redesign variables V_{ji}, W_i until an optimal solution is obtained (Zeng and Zhao 2013). As illustrated, the nested algorithm uses the C&CG method in each layer to solve inner-layer and out-layer problems sequentially.

Section 3.2 presents the tri-level model (1)-(3) in an abstract form to simplify the formulation and concentrate on the solution methodology.

Figure 1. An overview of the nested C&CG algorithm

3.1 Linearization

The nonlinearity of terms (3c) and (3d), formed by the products of multiple binary variables, is removed by replacing them with constraints (4c) and (4d):

$$f^1_i = S_i.(1-Z_i) \qquad \forall i \in L_p \qquad (4a)$$

$$f^2_{ji} = V_{ji}.(1-S_i.(1-Z_i)) = V_{ji}.(1-f^1_i) \qquad \forall j,i \in L_p \qquad (4b)$$

$$f^1_i \in \{0,1\}:\ f^1_i \le S_i,\ f^1_i \le (1-Z_i),\ f^1_i \ge S_i - Z_i \qquad \forall i \in L_p \qquad (4c)$$

$$f^2_{ji} \in \{0,1\}:\ f^2_{ji} \le V_{ji},\ f^2_{ji} \le 1-f^1_i,\ f^2_{ji} \ge V_{ji} - f^1_i \qquad \forall j,i \in L_p \qquad (4d)$$

3.2 Abstract Mathematical formulation

In order to focus on developing the solution method, we simplify the model formulation and present it in a compact form. The idea of the abstract formulation is derived from (Fang and Sansavini 2017). In the following sections, we first introduce the abstract model notations for the tri-level model (1)-(3). Then, we present abstract forms of outer-layer and inner-layer models and outline the C&CG algorithm for solving them iteratively until to achieve the optimal solution.

The abstract formulation of the PAD model is as follows:

Abstract model notations:

δ	Variable representing planner's binary decision variables Y_j, Z_j
x	Variable representing planner's continuous decision variables X_{ij}
s	Variable representing attacker's binary decision variables S_j
$\wedge$	Set of possible attacker's strategies
$\phi(\delta, s)$	Feasible defender's decisions after planner's and attacker's decisions are made
v	Variable representing defender's binary decision variables V_{ji}, W_i
u	Variable representing defender's continuous decision variables U_{ij}

$$\min_{\delta,x} a\delta + bx + \max_{s\in\wedge} \min_{v,u\in\phi(\delta,s)} cv + eu \qquad (5a)$$

$$st: F\delta + Gx \le g_1 \qquad (5b)$$

$$(\delta, x) \in \{0,1\}^{m_1} \times \mathbb{R}_+^{m_2} \qquad (5c)$$

$$\wedge = \left\{ s \in \left\{0,1\right\}^{m_3} : Hs \le g_2 - L\delta \right\} \tag{5d}$$

$$\phi(\delta,s) = \left\{ (v,u) \in \left\{0,1\right\}^{m_4} \times \mathbb{R}_+^{m_5} : Rv + Nu \le g_3 - P\delta - Ts \right\} \tag{5e}$$

In the above abstract form, expression (5a) represents the min-max-min objective function of the PAD optimization model, i.e. (1a), (2a), and (3a). Constraints (5b) and (5c) address the planner's problem (1b) to (1h), constraint (5d) corresponds to the attacker's problem (2b) to (2d), and constraint (5e) refers to the defender's problem (3b) to (3j). Coefficients of variables in the planner and defender's objective functions are indicated by a,b,c,e , and F,G,H,L,R,N,P,T represent coefficients of variables in the constraints. Dimensions of variables δ, x, s, v, u are specified by m_1, m_2, m_3, m_4 , respectively. Constraints' right-hand side parameters are defined by g_1, g_2, g_3 .

3.3 Bi-level outer-layer SP converted to an inner-layer C&CG algorithm

The outer-layer SP is a bi-level programming model consisting of two decision makers, the attacker in the first level and the defender in the second level. To solve the outer bi-level model, we decompose it into an MP and SP and then develop an inner-layer algorithm based on the C&CG method to achieve optimal solutions for the outer-layer SP.

The abstract form of the bi-level out-layer SP is presented in (6):

$$\mathcal{H}(\delta^*) = \max_{s \in \wedge} \min_{v,u \in \phi(\delta^*,s)} cv + du \tag{6}$$

Bi-level max-min model (6) includes binary variables (s,v) in the first and second levels, respectively, besides continuous variables u in the second level. δ^* indicates the realization of the first-level binary planning decisions, and we observe that for a given δ^* , $Z_{def}(\delta^*, s)$ is feasible. Therefore, $\phi_v(\delta^*, s)$ is a finite set that generates corresponding countable solutions for the binary variables of the defender's problem v^k . The set $\phi_u(\delta^*, s, v^k) = \left\{ u^k \in \mathbb{R}_+^{m_5} : Nu^k \le g_3 - P\delta^* - Ts - Rv^k \right\}$ enforces the feasibility of continuous variables of the defender's problem u for any solution of δ^*, s and v^k .

$$\mathcal{H}(\delta^*) = \max_{s \in \wedge} \min_{v \in \phi_v(\delta^*,s)} \min_{u \in \phi_u(\delta^*,s,v)} cv + du \tag{7}$$

As a result, the binary variable v and continuous variable u of model (7) can be separated and reformulated as model (8a) to (8c) (Zeng and Zhao 2013):

$$\mathcal{H}(\delta^*) = \max_{s,u} \eta \tag{8a}$$

$$\eta \le \min \left\{ du^k : Nu^k \le g_3 - P\delta^* - Ts - Rv^k, u^k \in \mathbb{R}_+^{m_5} \right\} \qquad k = 1,2,3,......,K \tag{8b}$$

$$s \in \wedge = \left\{ \{0,1\}^{m_3} : Hs \le g_2 - L\delta^* \right\} \qquad (8c)$$

The right-hand side of constraints (8b) is linear and can be replaced by its equivalent KKT conditions, which ensure the optimality of the model. By doing so, it is possible to transform the bi-level SP (7) into a single-level model (9) as follows:

$$\mathcal{H}(\delta^*) = \max_{s,u} \eta \qquad (9a)$$

$$\eta \le cv^k + du^k \qquad k = 1, 2, 3, ..., K \qquad (9b)$$

$$Nu^k \le g_3 - P\delta^* - Ts - Rv^k \qquad k = 1, 2, 3, ..., K \qquad (9c)$$

$$N^T \lambda^k \le d^T \qquad k = 1, 2, 3, ..., K \qquad (9d)$$

$$\lambda^k (Nu^k - g_3 + P\delta^* + Ts + Rv^k) = 0 \qquad k = 1, 2, 3, ..., K \qquad (9e)$$

$$u^k (N^T \lambda^k - d^T) = 0 \qquad k = 1, 2, 3, ..., K \qquad (9f)$$

$$u^k \ge 0 \qquad k = 1, 2, 3, ..., K \qquad (9g)$$

$$\lambda^k \ge 0 \qquad k = 1, 2, 3, ..., K \qquad (9h)$$

$$s \in \wedge = \left\{ \{0,1\}^{m_3} : Hs \le g_2 - L\delta^* \right\} \qquad k = 1, 2, 3, ..., K \qquad (9i)$$

λ^k are the dual variables of constraints $Mu^k \le g_3 - P\delta^* - Ts - Rv^k$ in (8b). Constraints (9c) and (9g), and (9d) and (9h) ensure primal and dual feasibility of (8b), respectively. The complementary slackness conditions (9e) and (9f) ensure that primal and dual values are the same. Obviously, these constraints are nonlinear, and the "Big-M" method is used to remove this nonlinearity. For example, this method results in the linearization of constraint (9e) as follows (Fortuny-Amat and McCarl 1981):

$$u^k \le (1 - q^k)M \qquad (10a)$$

$$(N^T \lambda^k - d^T) \le q^k M \qquad (10b)$$

Where q^k represents a binary variable for each iteration and M is a large positive constant. Expression (9i) refers to (8c), and both represent the attacker’s problem variables and constraints (2b) to (2d).

In the outer-layer SP $\mathcal{H}(\delta^*)$ (6), it is not realistic to enumerate all of the defender’s binary variables v and incorporate associated variables and constraints. Furthermore, only a small set of these variables and constraints can significantly lead us to a better solution (Fang and Sansavini 2017). Thus, to incorporate only the effective values of the defender’s binary variables, the C&CG

method is applied to solve the model (Zeng and Zhao 2013). In this view, problem $\mathcal{H}(\delta^*)$ (6) can be rephrased into the problem $\widetilde{\mathcal{H}}(\delta^*)$. $\widetilde{\mathcal{H}}(\delta^*)$ is the inner-layer MP, and incorporate just the values of defender's binary variables v^{k*} that produce better solutions by adding cutting planes (9b) in every iteration. In other words, cutting planes in form of constraints (9b) together with the objective function (9a) guarantee to incorporate the solutions that maximize the objective value and consequently remove nonoptimal solutions. Assuming that index k represents the number of iterations, and given the solutions of v^{k*} for each iteration ($k = 1, 2, 3, ..., K$), the model (9) is solved iteratively by adding constraints (9b) to (9i).

Since the inner-layer MP $\widetilde{\mathcal{H}}(\delta^*)$ is the relaxed model of $\mathcal{H}(\delta^*)$, its solutions generate an upper bound for the inner-layer algorithm, i.e. outer-layer SP (6). On the other hand, in every iteration, feasible solutions of v^k to model (6) are obtained by solving $\min_{v,u \in \phi(\delta^*, s^*)} cv + du$ where s^* is any feasible solution of s acting as the inner-layer SP. In each iteration, the lower bound for the inner-layer algorithm is obtained by solving model (11) to optimality. Moreover, the solution of v^k obtained by solving model (11) is fixed on its value v^{k*}, and along with its associated variables and constraints, it is added to the model (9) iteratively until the lower and upper bounds of the inner-layer algorithm converge.

$$\min_{v,u \in \phi(\delta^*, s^*)} cv + du \tag{11a}$$

$$s.t : \phi(\delta^*, s^*) = \left\{ (v,u) \in \{0,1\}^{m_4} \times \mathbb{R}_+^{m_5} : Rv + Nu \leq g_3 - P\delta^* - Ts^* \right\} \tag{11b}$$

The inner-layer MP (9) and SP (11) are both mixed integer linear optimization models that can be solved by optimization solution methods such as branch-and-cut and the appropriate solver, CPLEX. The C&CG method and its convergence properties have been studied by (Zeng and Zhao 2013), and they provide the convergence proof for the developed algorithm.

3.4 Outer-layer C&CG algorithm

The output of the inner-layer algorithm, also called the outer-layer SP, is the optimal solution of the attacker's decision variables S^i in each iteration ($i = 1, 2, 3, ..., I$). Adopting the same strategy, the tri-level model (5) is reformulated to model (12). This means that in each iteration, given the outer-layer SP's optimal solutions (S^{i*}), model (12) is solved. The following model (12) represents the outer-layer MP of the PAD problem (5), a mixed-integer linear programming model. In this model, the outer-layer MP and SP are connected by variable ρ and corresponding cutting planes (12c). Moreover, variable ρ allows the model to partially enumerate those feasible solutions of S^{i*} that minimize the objective value (12a)—as (12) in this case is a minimization problem. Additionally, constraints (12c) and (12d) remove nonoptimal solutions, i.e. v^i, u^i.

The abstract form of the outer-layer MP is presented as follows:

$$\min_{\delta,x,v^i,u^i} a\delta + bx + \rho \tag{12a}$$

$$st: F\delta + Gx \le g_1 \tag{12b}$$

$$\rho \ge dv^i + eu^i \qquad i = 1,2,3,......,I \tag{12c}$$

$$Rv^i + Nu^i + P\delta \le g_3 - Ts^{i*} \qquad i = 1,2,3,......,I \tag{12d}$$

$$Hs^{i*} + L\delta \le g_2 \qquad i = 1,2,3,......,I \tag{12e}$$

$$(\delta, x) \in \{0,1\}^{m_1} \times \mathbb{R}_+^{m_2} \tag{12f}$$

$$(v^i, u^i) \in \{0,1\}^{m_4} \times \mathbb{R}_+^{m_5} \qquad i = 1,2,3,......,I \tag{12g}$$

Where S^{i*} is the optimal solution of the attacker's problem obtained by solving the inner-layer algorithm which is illustrated in Section 3.3. The outer-layer MP variables include binary planning variables δ, i.e. facility location and protection decisions, continuous facility allocation decisions x, redesign decisions v^i, u^i following the attack, and the variable ρ, which interconnects the MP and the SP in each iteration.

In order to solve the tri-level model (1)-(3), it is decomposed into an outer-layer MP (12) and outer-layer SP (6). By iteratively exchanging the optimal value of outer-layer SP variables $s^*(S_j^*)$ as inputs to the MP, and reciprocally MP variables $\delta^*(Y_j^*, Z_j^*)$ as an input to the inner-layer algorithm, the PAD model is optimally solved. In each iteration, the outer-layer MP provides lower bounds, and the optimal value of the outer-layer SP provides upper bounds for the outer-layer algorithm. The two models continue to interact until the lower and upper bounds converge, at which point the optimal solution is obtained.

4. Computational Results

In this section, to test the application of the proposed model and solution algorithm, we study a real supply system consisting of 10 hospitals, 30 demand areas, and 20 potential locations for backup facilities in a region in Iran. Figure 2 shows the general overview of our case study. This study addresses a supply system configuration problem in which we aim to locate emergency facilities, i.e. hospitals, as close to demand areas as possible. The best locations and number of facilities to be opened and protected to serve all demands are among the decision variables of the model. Meanwhile, there is an intelligent disruptive agent in the system called the attacker. This agent seeks to maximize the distance between hospitals and demand areas by destroying hospitals.

Figure 2. A schematic diagram of the investigated supply system

In response to the attacker, we incorporate preventive measures such as fortifying facilities in the planning phase. In addition, we plan post-attack responses, such as installing backup facilities and redesigning the remaining system to minimize the total cost. Further information regarding our case study, such as its parameters and location, is confidential and cannot be disclosed. The tri-level optimization model reformulated by the two-layer C&CG algorithm is solved by CPLEX 12.10 on a PC with an Intel i5-8350U and a quad-core processor running at 1.90GHz 8 GB of RAM in a Windows 10 environment. The optimality tolerance is set to 10^{2} for the inner-layer algorithm and 10^{3} for the outer-layer algorithm.

The upper and lower bounds of the solution algorithm are shown in Figure 3. In Table I, given the limited budget for the number of protected and interdicted facilities, the optimal solution enforces opening facilities (2, 3, 4, 5, 7, 8, 9, 10). Moreover, the planner, attacker, and defender strategies are iteratively evaluated until the algorithm converges on the global optimal solution.

Figure 3. Two-layer C&CG algorithm and its convergence

Table I. Optimal solutions of the tri-level model using C&CG algorithm

Upon examining the planner's and defender's behavior, we observe that the planner does not use all of its protection budgets, i.e. the maximum number of protected facilities; instead, new backup facilities are installed in the response phase. Since it is more expensive to fortify existing facilities than to establish backup facilities after disruptions, the model does not utilize all budgets to protect facilities. We investigate the differences in post-attack costs by tightening and relaxing the protection budget constraint (1f) under various scenarios: A. protection budget has not been fully used, B. protection budget has been fully used. The results are shown in Table II for each protection and interdiction strategy.

Table II. Planner versus attacker strategies

Table II demonstrates that the attacker's budget N_{att} impacts investment costs and the number of protected and opened backup facilities. We observe that the attacker uses all interdiction budgets to incur maximum costs on the planner, resulting in higher investment and post-attack costs. Nevertheless, the planner does not utilize the entire budget for protection but compensates by opening backup facilities after the attack. As a result of forcing the model to use the entire protection budget, the investment costs increase while post-attack costs decrease.

In this section, we conduct various analyses of the proposed model to provide broader perspectives of its performance under different conditions. As mentioned, the planner and attacker's behavior is influenced by various combinations of protection and interdiction budgets. In this regard, Figures 4 and 5 show how investment costs and post-attack costs vary under different fortification-interdiction scenarios. Figure 4 illustrates the investment costs regarding protection budgets and the number of attacks. Investment costs increase if the planner is restricted to use all of the protection budgets. Meanwhile, under a specific protection budget N_{def} =6, investment costs rise accordingly as the number of interdictions (disruptions) increases. Furthermore, Figure 5 illustrates how protection budgets impact the maximum post-attack costs.

It explains that fortifying facilities with all protection budgets substantially reduces post-attack costs.

Figure 4. protection budget versus investment costs

Figure 5. protection budget versus post-attack costs

The effect of an increase in the protection budget on post-attack costs is shown in Figure 6. To this end, we consider the model for a particular interdiction budget (N_{att} =6) under different protection strategies, i.e. N_{def} = 2, 3, 4, 5, 6. Figure 6 demonstrates that post-attack costs decrease as the protection budget increases.

Figure 6. protection strategy versus post-attack cost

At the third level, the defender takes corrective measures to counteract the attacker's disruptive actions and attempts to satisfy all demands. More precisely, reallocating the remaining system to the demand nodes is among the defender's decisions in the response phase. To explain the necessity of developing the fortification-interdiction model to a tri-level model, we compare post-attack costs regarding traveling costs under different formulations: (i) a tri-level model with redesign decisions, (ii) a classical bi-level fortification-interdiction model presented by (Scaparra and Church 2008). The results of our analysis are shown in Figure 7. Figure 7 depicts that by adopting a tri-level model with redesign decisions at the third level, we gain 18% savings in traveling costs compared to the bi-level interdiction model without redesign decisions. This difference is a considerable amount for critical infrastructure system design problems.

Figure 7. Tri-level versus bi-level model

To analyze the performance of the tri-level model under different interdiction strategies, we consider N_{att}=2, 3, 4, 5, 6. As shown in Figure 8, under a particular protection strategy N_{def} =3, investment costs increase as the number of interdictions increases. The analysis concludes that to prevent most disruptive actions, the planner and defender should invest more in fortifying facilities and installing backup facilities.

Figure 8. Interdiction budget versus investment costs

We conduct sensitivity analyses on the weighting factor and measure the maximum traveling cost after the attack to investigate the importance of serving demands based on their distance from facilities. Figure 9 illustrates that the maximum traveling cost after interdiction decreases as the weighting factor increases. More precisely, the weighting factor forces the minimization model to give traveling costs a higher priority. Weighting factor α specifies the importance of traveling costs, so an increase in α results in lower post-attack traveling costs. In contrast, as shown in Figure 10, as the weighting factor increases, the model sacrifices minimizing investment costs to lower traveling costs, resulting in higher investment costs. Figure 10 depicts the trade-offs between the investment and traveling costs as the weighting factor α increases.

Figure 9. weighting factor versus maximal traveling cost

Figure 10. weighting factor versus maximal traveling and investment cost

5. Conclusions and Recommendations for Future Research

Facility location-allocation problems have been extensively investigated by scholars and supply system planners in the literature. Nevertheless, designing a reliable integrated system of critical facilities has not been the forethought of recent studies. Incorporating protection strategies while designing a supply system enhances the efficiency of critical infrastructures in case of man-made or natural disasters. To address this problem, in this paper, we proposed a tri-level planner-attacker-defender (PAD) optimization model to design a reliable supply system. To identify critical facilities, we considered a hypothetical attacker aiming to interdict vital facilities of a supply system. Then, we intended to fortify them and take corrective measures such as installing backup facilities and redesigning the remaining system to minimize the maximal imposed cost after interdiction. Furthermore, we adopted a nested C&CG algorithm to solve the proposed model. Our sensitivity analyses on various parameters, including interdiction and protection budgets, maximal traveling costs, investment costs, as well as redesign decisions, revealed the performance of the proposed tri-level model. It revealed the efficiency of the nested C&CG algorithm to obtain exact optimal solutions for the studied supply system. In addition, by developing a tri-level model and incorporating redesign decisions, we reduced the maximal traveling cost compared to a classical fortification-interdiction model. While the proposed model and solution approach is effective, there are some limitations that can serve as a basis for future research. Firstly, developing a practical solution method for solving large-size multi-level interdiction problems has remained challenging. Second, novel studies can be conducted on facility interdiction problems while interdiction can simultaneously occur along multiple routes between demand nodes and serving facilities. An integrated facility-arc interdiction location-routing problem can be investigated in future studies.

References

Akbari-Jafarabadi, M., R. Tavakkoli-Moghaddam, M. Mahmoodjanloo and Y. Rahimi (2017). "A tri-level r-interdiction median model for a facility location problem under imminent attack." Computers & Industrial Engineering **114**: 151-165.

Aksen, D. and N. Aras (2012). "A bilevel fixed charge location model for facilities under imminent attack." Computers & Operations Research **39**(7): 1364-1381.

Aksen, D., N. Aras and N. Piyade (2013). "A Bilevel p-median model for the planning and protection of critical facilities." Journal of Heuristics **19**(2): 373-398.

Aliakbarian, N., F. Dehghanian and M. Salari (2015). "A bi-level programming model for protection of hierarchical facilities under imminent attacks." Computers & Operations Research **64**: 210-224.

Alisan, O., M. Ghorbanzadeh, M. B. Ulak, A. Kocatepe, E. E. Ozguven, M. Horner and W. Huang (2020). "Extending interdiction and median models to identify critical hurricane shelters." International Journal of Disaster Risk Reduction **43**: 101380.

Church, R. L. and M. P. Scaparra (2007). "Protecting Critical Assets: The r-Interdiction Median Problem with Fortification." Geographical Analysis **39**(2): 129-146.

Church, R. L., M. P. Scaparra and R. S. Middleton (2004). "Identifying Critical Infrastructure: The Median and Covering Facility Interdiction Problems." Annals of the Association of American Geographers **94**(3): 491-502.

Cormican, K. J., D. P. Morton and R. K. Wood (1998). "Stochastic Network Interdiction." Operations Research **46**(2): 184-197.

Fang, Y. and G. Sansavini (2017). "Optimizing power system investments and resilience against attacks." Reliability Engineering & System Safety **159**: 161-173.

Forghani, A., F. Dehghanian, M. Salari and Y. Ghiami (2020). "A bi-level model and solution methods for partial interdiction problem on capacitated hierarchical facilities." Computers & Operations Research **114**: 104831.

Fortuny-Amat, J. and B. McCarl (1981). "A Representation and Economic Interpretation of a Two-Level Programming Problem." Journal of the Operational Research Society **32**(9): 783-792.

Ghorbani-Renani, N., A. D. González and K. Barker (2021). "A decomposition approach for solving tri-level defender-attacker-defender problems." Computers & Industrial Engineering **153**: 107085.

Hasanzad, F. and H. Rastegar (2022). "Application of optimal hardening for improving resilience of integrated power and natural gas system in case of earthquake." Reliability Engineering & System Safety **223**: 108476.

Hesam Sadati, M. E., D. Aksen and N. Aras (2020). "A trilevel r-interdiction selective multi-depot vehicle routing problem with depot protection." Computers & Operations Research **123**: 104996.

Israeli, E. and R. K. Wood (2002). "Shortest-path network interdiction." Networks **40**.

Li, Q., M. Li, Z. Gong, Y. Tian and R. Zhang (2022). "Locating and protecting interdependent facilities to hedge against multiple non-cooperative limited choice attackers." Reliability Engineering & System Safety **223**: 108440.

Li, Q., M. Li, R. Zhang and J. Gan (2021). "A stochastic bilevel model for facility location-protection problem with the most likely interdiction strategy." Reliability Engineering and System Safety **216**.

Li, Q. and A. Savachkin (2013). "A heuristic approach to the design of fortified distribution networks." Transportation Research Part E: Logistics and Transportation Review **50**: 138-148.

Liberatore, F., M. P. Scaparra and M. S. Daskin (2011). "Analysis of facility protection strategies against an uncertain number of attacks: The stochastic R-interdiction median problem with fortification." Computers & Operations Research **38**(1): 357-366.

Losada, C., M. P. Scaparra and J. R. O'Hanley (2012). "Optimizing system resilience: A facility protection model with recovery time." European Journal of Operational Research **217**(3): 519-530.

Mahmoodjanloo, M., S. P. Parvasi and R. Ramezanian (2016). "A tri-level covering fortification model for facility protection against disturbance in r-interdiction median problem." Computers & Industrial Engineering **102**: 219-232.

Nadizadeh, A. and A. Sabzevari Zadeh (2021). "A bi-level model and memetic algorithm for arc interdiction location-routing problem." Computational and Applied Mathematics **40**(3): 100.

Nemati, H., M. A. Latify and G. R. Yousefi (2021). "Tri-level coordinated transmission and electrical energy storage systems expansion planning under physical intentional attacks." Journal of Energy Storage **42**: 103095.

Rocchetta, R. (2022). "Enhancing the resilience of critical infrastructures: Statistical analysis of power grid spectral clustering and post-contingency vulnerability metrics." Renewable and Sustainable Energy Reviews **159**: 112185.

Sarhadi, H., D. M. Tulett and M. Verma (2022). "A tri-level mixed-integer program for the optimal fortification of a rail intermodal terminal network." International Journal of Operational Research **43**(1-2): 65-95.

Sayed, A. R., C. Wang and T. Bi (2019). "Resilient operational strategies for power systems considering the interactions with natural gas systems." Applied Energy **241**: 548-566.

Scaparra, M. P. and R. L. Church (2008). "A bilevel mixed-integer program for critical infrastructure protection planning." Computers & Operations Research **35**(6): 1905-1923.

Scaparra, M. P. and R. L. Church (2008). "An exact solution approach for the interdiction median problem with fortification." European Journal of Operational Research **189**(1): 76-92.

Teixeira, J. C. and A. P. Antunes (2008). "A hierarchical location model for public facility planning." European Journal of Operational Research **185**(1): 92-104.

Thiele, A., T. Terry and M. Epelman (2009). "Robust linear optimization with recourse." Rapport technique: 4-37.

Wollmer, R. (1964). "Removing Arcs from a Network." Operations Research **12**(6): 934-940.

Wood, R. K. (1993). "Deterministic network interdiction." Mathematical and Computer Modelling **17**(2): 1-18.

Zeng, B. and L. Zhao (2013). "Solving two-stage robust optimization problems using a column-and-constraint generation method." Operations Research Letters **41**(5): 457-461.

Zhang, X., Z. Zheng, S. Zhang and W. Du (2016). "Partial interdiction median models for multi-sourcing supply systems." The International Journal of Advanced Manufacturing Technology **84**(1): 165-181.

Table I. Optimal solutions of the tri-level model using C&CG algorithm

Iteration	Protected Facilities	Disrupted Facilities	Backup Facilities	Maximal imposed costs ($\times 10^6$) (IRR)
1	-	2, 3	15, 17, 18	68443
2	2, 3, 10	5, 7, 9	2, 13 ,18	67322
3	2, 3, 5	7, 8, 9	3, 18, 19	67019
4	2, 3, 9	4, 5 ,8	15, 18, 19	66980
5	2, 3, 5, 9	4, 7, 9	2, 4, 18	66578
9	2, 3, 7, 9	5, 8, 9	3, 17, 18	65469
10	2, 3, 7, 9	4, 5, 10	2, 3, 18	64567
11	2, 3, 7, 9	5, 8, 10	2, 17, 18	63568

Table II. Planner versus attacker strategies

N_{def}	N_{att}	Strategy	Investment Cost ($\times 10^9$) (IRR)	Post-attack Cost ($\times 10^6$) (IRR)	Opened Facilities	Protected Facilities	Backup Facilities	C&CG (CPU time sec.)
3	4	A	69960	44937	2-3-5-7-8-9	2-3	7-18	123.89
		B	81280	31159	1-2-3-7-9-10	2-3-7	18	132
	6	A	112180	72840	2-3-5-7-9-10	2-7	3-14-17-18-19	182.87
		B	151223	61159	1-2-3-7-9-10	2-3-7	3-18-19	229.04
4	2	A	46880	23450	2-3-4-7-8-9-10	2	3-18	150.89
		B	57640	13467	1-2-3-7-8-9-10	2-3-7-9	-	398.81
	4	A	69220	45978	2-3-5-7-8-9	2-3	17-18	418.62
		B	93180	32145	2-3-5-7-8-9-10	2-3-7-9	14	561.79
6	2	A	46880	23450	2-3-4-7-8-9-10	2	3-18	125.44
		B	57640	13467	2-3-7-8-9-10	2-3-7-8-9-10	-	781.09
	4	A	79873	53156	2-3-5-7-8-9	2-3	17-18	390.21
		B	100940	43456	2-3-7-8-9-10	2-3-7-8-9-10	-	450.65
	5	A	81280	67895	1-2-3-7-9-10	2-3-7	2-18-19	689.32
		B	160260	45124	1-2-3-7-8-9-10	2-3-7-8-9-10	-	799.12
	6	A	198792	41089	2-3-4-7-8-9-10	2-3-9	3-14-18-19	971.99
		B	287966	37129	2-3-4-7-8-9-10	2-3-7-8-9-10		1089.0

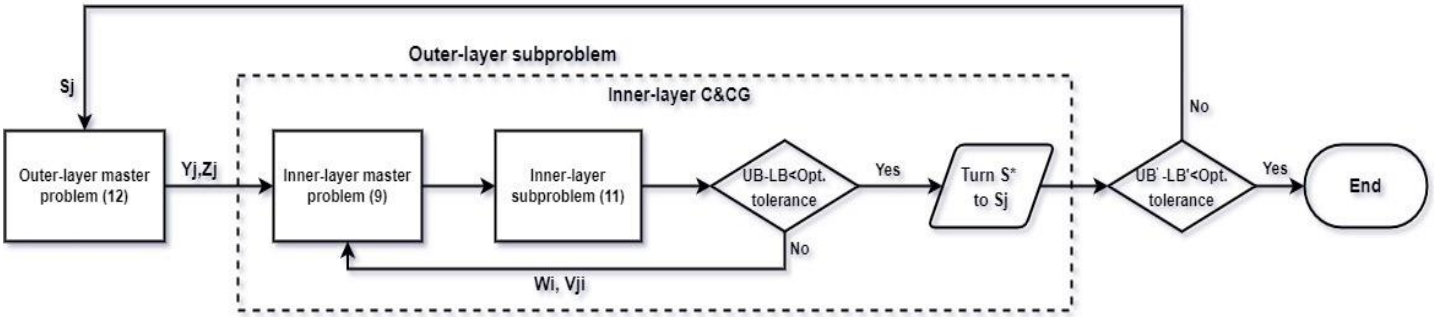

Figure 1. An overview of the nested C&CG algorithm

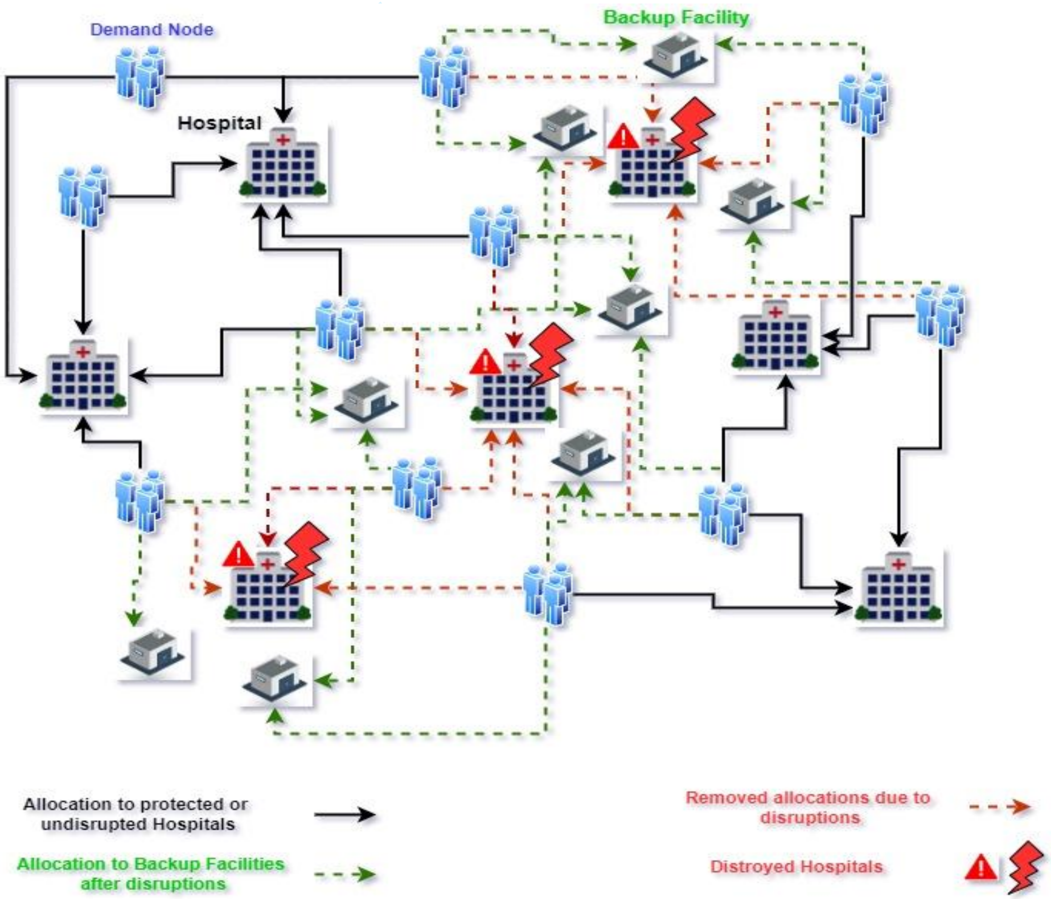
Demand Node
Backup Facility
Hospital
Allocation to protected or undisrupted Hospitals
Removed allocations due to disruptions
Allocation to Backup Facilities after disruptions
Distroyed Hospitals

Figure 2. A schematic diagram of the investigated supply system

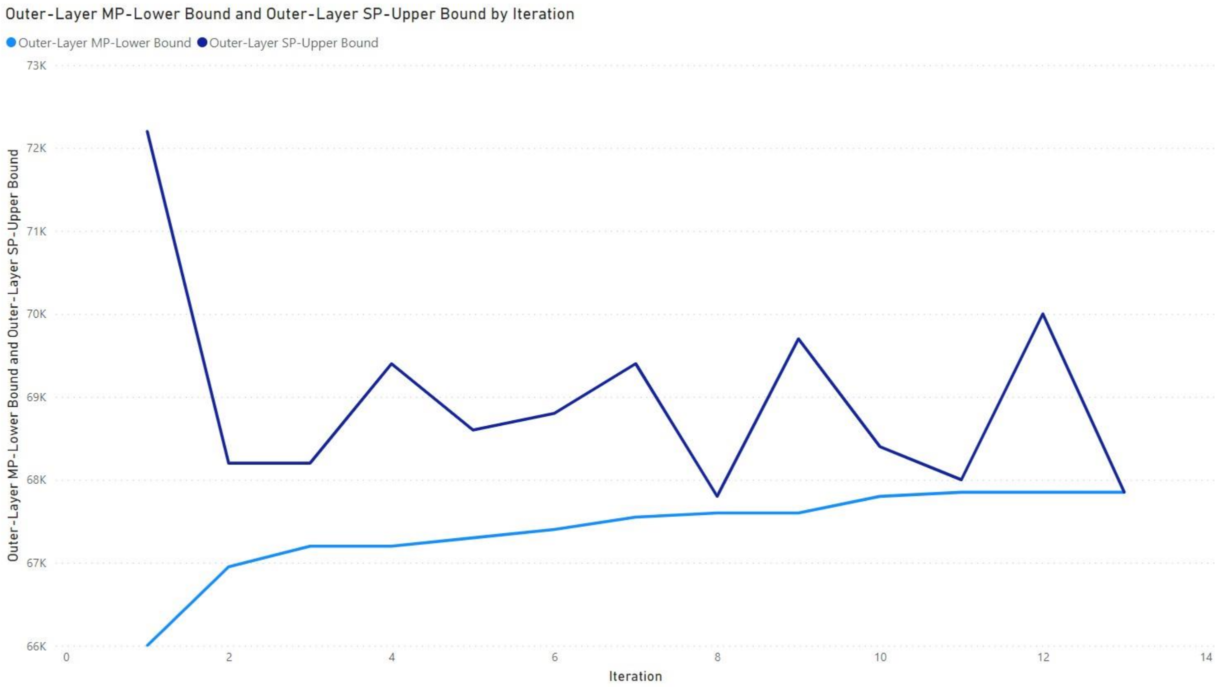

Figure 3. Two-layer C&CG algorithm and its convergence

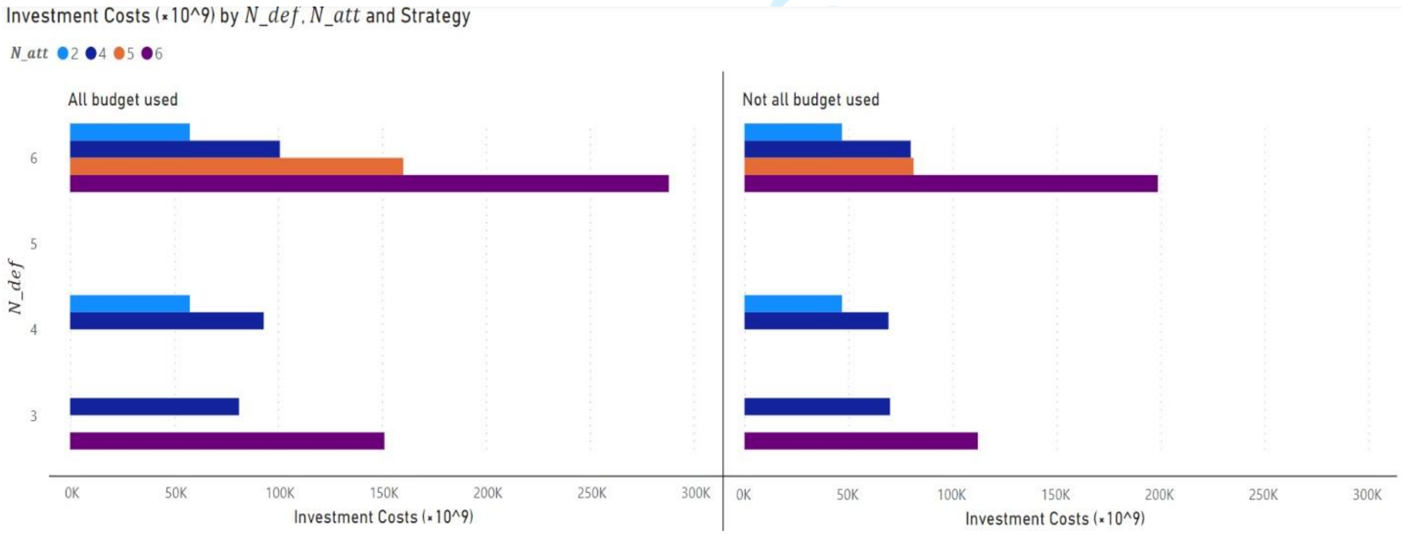

Figure 4. protection budget versus investment costs

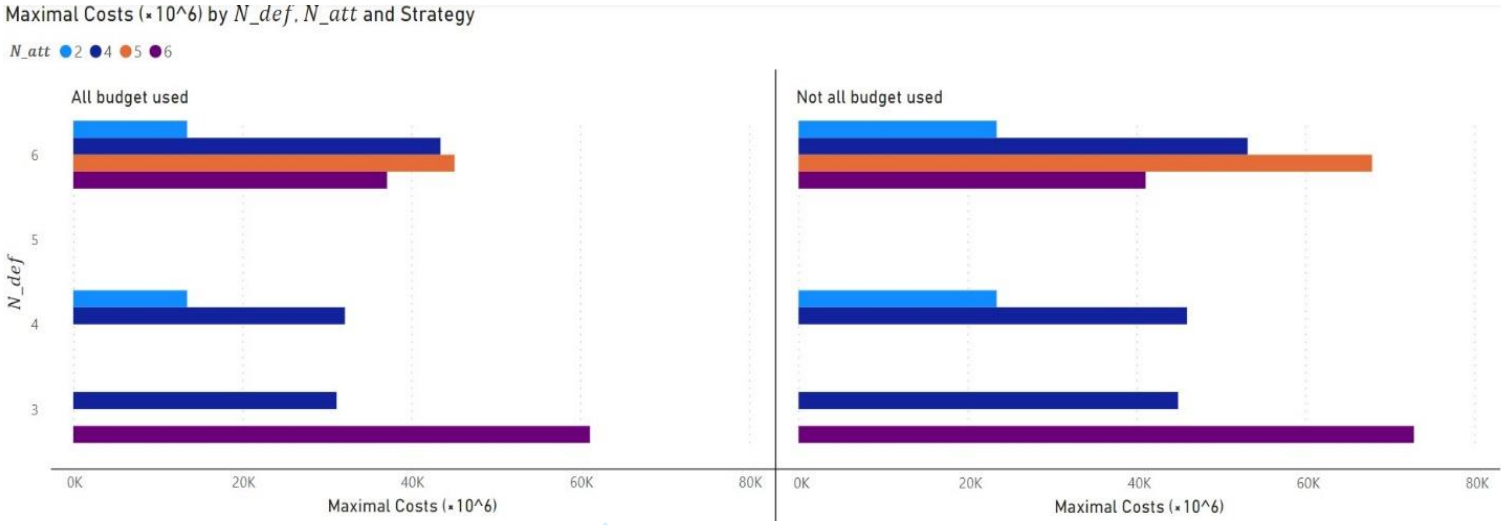

Figure 5. protection budget versus post-attack costs

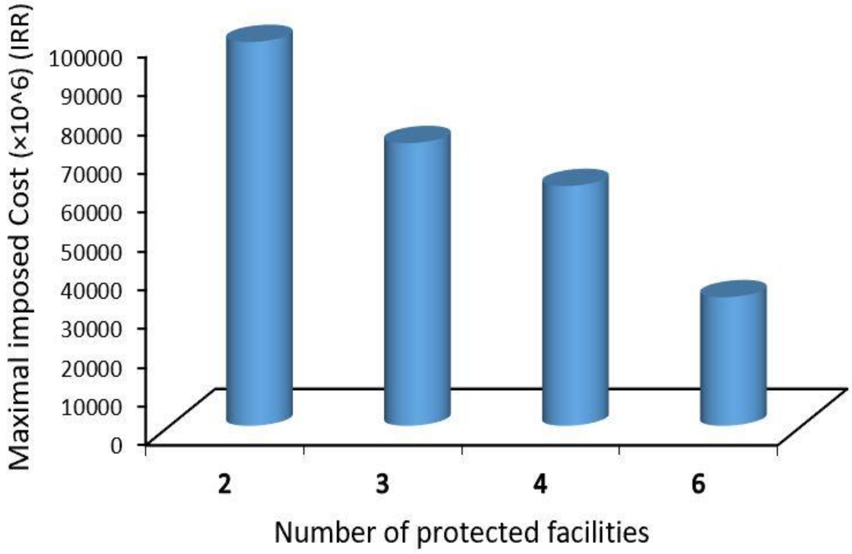

Figure 6. protection strategy versus post-attack cost

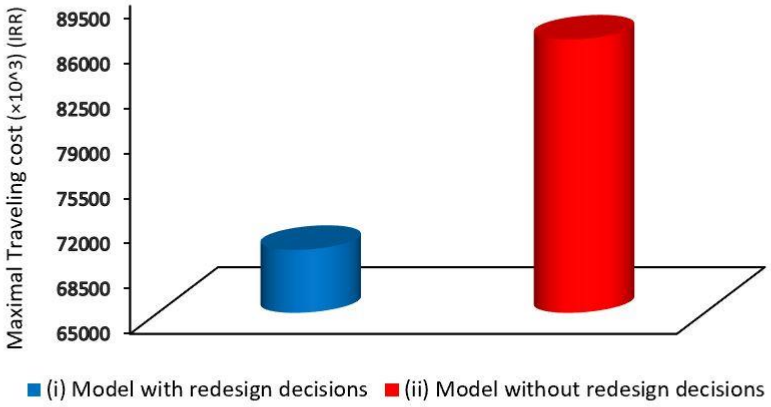

Figure 7. Tri-level versus bi-level model

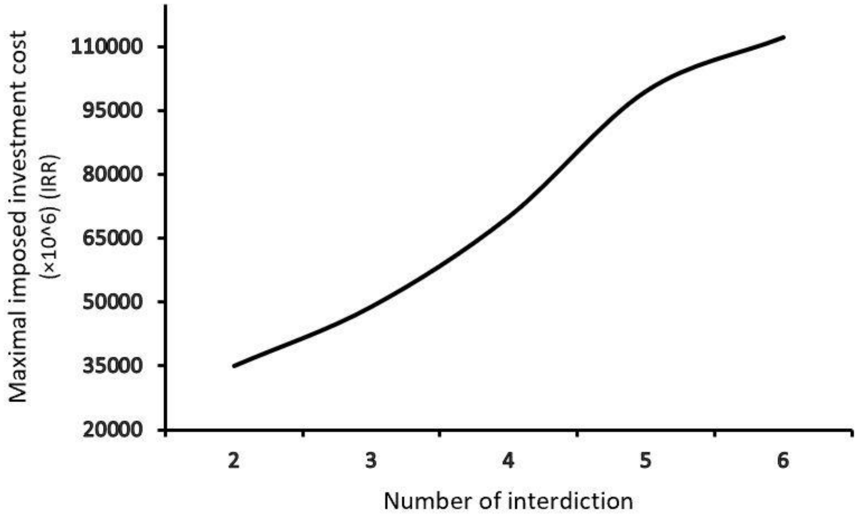

Figure 8. Interdiction budget versus investment costs

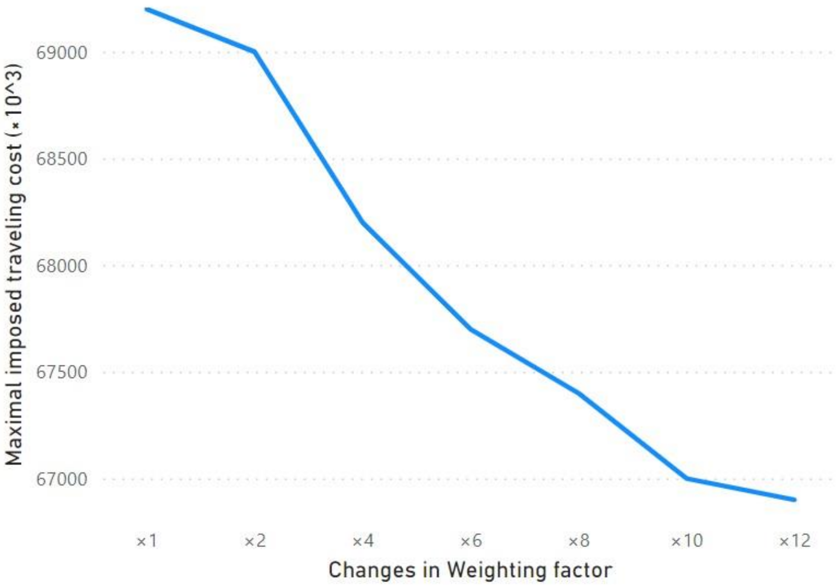

Figure 9. weighting factor versus maximal traveling cost

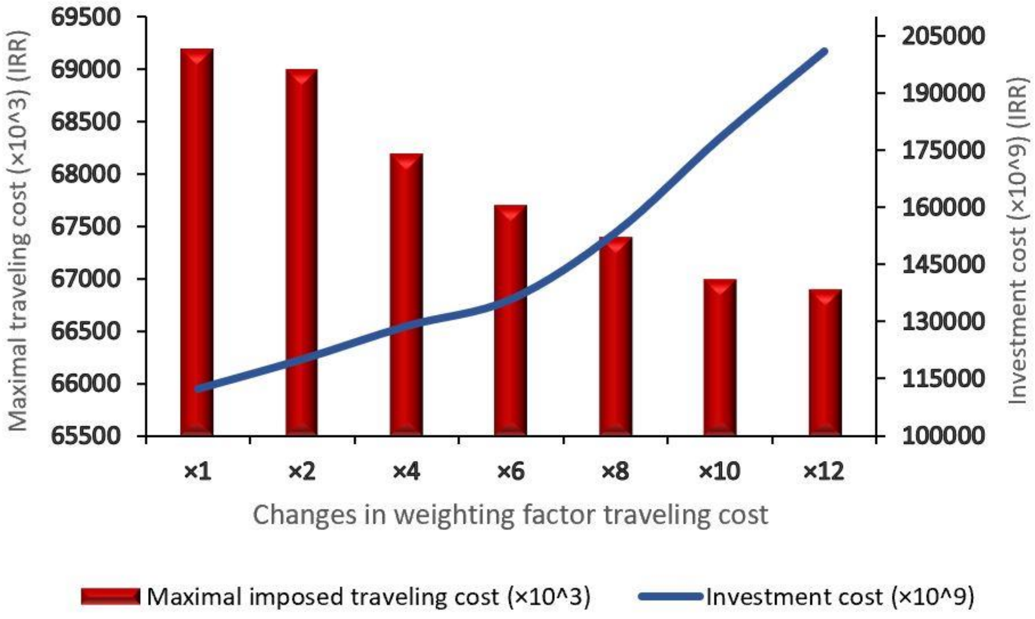

Figure 10. weighting factor versus maximal traveling and investment cost