Objectives

- State-feedback control
- Using Arduino to control the rotary pendulum of the QUBE-Servo 2.
- To deepen in reachability and observability concepts.

1 Pre-lab: Inverted Pendulum

In this Pre-lab, we will make a program using Mathematica to control the inverted pendulum. We will simulate the behavior of the controller in a real implementation.

The car with an inverted pendulum, shown below, is "bumped" with an impulse force, F . It is assumed that the motion takes place in a vertical plane, see Fig. 1.

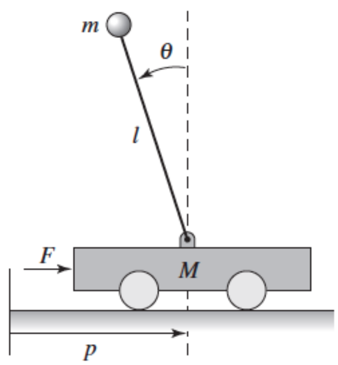

Figure 1: Car and pendulum

The dynamics of the system are given by:

$$(M+m)\ddot{p} - ml\cos(\theta)\ddot{\theta} = -c\dot{p} - ml\sin(\theta)\dot{\theta}^2 + F$$
$$(J+ml^2)\ddot{\theta} - ml\cos(\theta)\ddot{p} = \gamma\dot{\theta} + mgl\sin(\theta)$$

For simplicity, we take

$$c = \gamma = 0$$

. Linearizing around the equilibrium point $x_e = (p, 0, 0, 0)$, the dynamics matrix and the control matrix are

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{m^2l^2g}{\mu} & 0 & 0 \\ 0 & \frac{M_tmgl}{\mu} & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \frac{J_t}{\mu} \\ \frac{lm}{\mu} \end{bmatrix}$$

where $\mu = M_tJ_t - m^2l^2$, $M_t = M + m$ and $J_t = J + ml^2$.

[1] This laboratory take some material from the Quanser Inc workbooks. And this laboratory represent the 4% of the student's final grade

1. Find the reachability matrix. Is the system reachable?
2. Find the observability matrix. Is the system observable?
3. Calculate the value of k_r such that $y_e = r$.
4. Design a state estimator.
5. Design a feedback controller.
6. Simulate the real implementation of your feedback controller, you can suppose that the sample time is $1\ ms$.

2 Lab: Rotary pedulum

The rotary pendulum model is shown in Figure 2. The rotary arm pivot is attached to the QUBE-Servo 2 system and is actuated. The arm has a length of L_r, a moment of inertia of J_r, and its angle θ increases positively when it rotates counter-clockwise (CCW). The servo (and thus the arm) should turn in the CCW direction when the control voltage is positive ($V_m > 0$).

The pendulum link is connected to the end of the rotary arm. It has a total length of L_p and it center of mass is at $L_p/2$. The moment of inertia about its center of mass is J_p. The inverted pendulum angle α is zero when it is hanging downward and increases positively when rotated CCW.

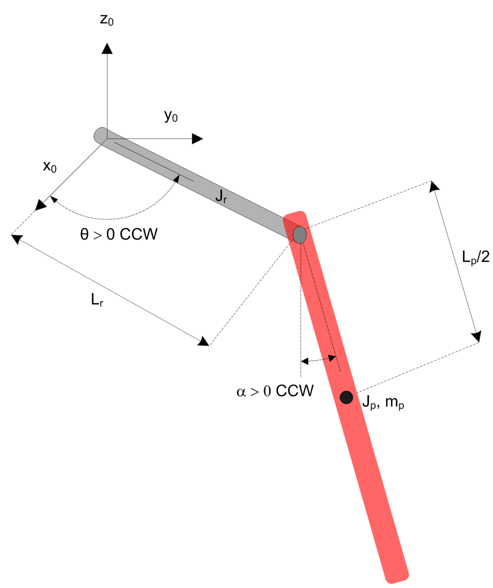

Figure 2: Rotary inverted pendulum model

Balancing is a common control task. You must find a control strategy that balance the pendulum in the upright position while maintaining a desired position of the arm. Based on the model in Lab 2, perform the following analysis

1. (0.5 point) Near of the upright position, show the open-loop response of the system and calculate the eigenvalues of A. Is stable, asymptotically stable or unstable?
2. (0.25 points) Show that the system is reachable based on the reachability matrix. Validate your analysis using Mathematica.

3. (0.25 points) According to your intuition, is the system observable? Validate your analysis using Mathematica.

4. (0.5 points) If the system is observable, design a state estimator. Are the estimated states a good approximation? If not, explain why there might be discrepancies.

5. (0.5 points) If the system is controllable, design a feedback controller. Make a detailed description of the parameters (e.g., poles) used in the design of the controller. Show the output of the system simulating with initial conditions.

6. (1 points) Simulate a real implementation of your controller, taking into account the sample time, the limit values of the input and the constraints in the movement of the rotary pendulum. If your controller is not able to control the system in a real implementation, try modifying some parameters of your design. Justify your criteria to choose the new design parameters. Simulate your new feedback controller with different initial conditions.

7. (0.5 point) Analyze the performing of your final controller. Under which conditions it has a proper operating? Is it possible to improve it? If your implementation is not capable to control the rotary pendulum explain why.

8. (1.0 points) Based on your simulations, implement a controller for the rotary pendulum of the QUBE-Servo 2 using Arduino.

Make sure your report is not too long, but concise. Its length should not exceed 5 pages. You must send the Mathematica notebook (0.5 points).