Práctica nº3

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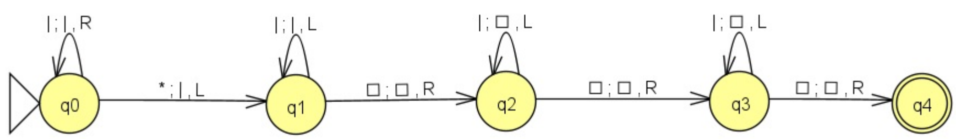
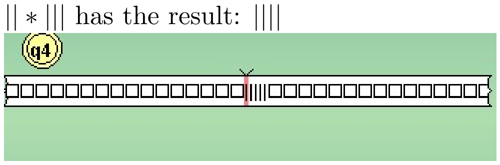
1 Define The TM solution of exercise 3.4 of the problem list and test its correct behaviour

The Turing Machine is the next one:

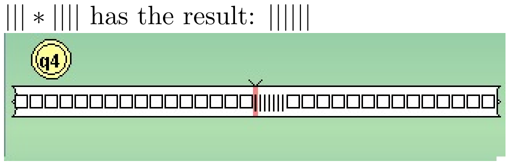


We see that if we introduce the next chains:

$|| * |||$ has the result: $||||$



$||| * ||||$ has the result: $||||||$



2 Define a recursive function for the sum of 3 values

When we apply recursivity to a primitive function, we have to define a function for the base chase g and a function for the recursive chase f. They are going to have as parameters in which step we are $m-1$, who is the result (previously calculated) of that step $h(n, m-1)$ and their parameters n. It should return the value of the function in m step.

Be $k \geq 0$ and functions:
$g : \mathbb{N}^k \to \mathbb{N}$
$h : \mathbb{N}^{k+2} \to \mathbb{N}$
If $f : \mathbb{N}^{k+1} \to \mathbb{N}$ is

$$f(n,m) = \begin{cases} g(n) & \text{if} \quad m = 0 \\ h(n, m-1, f(n, m-1)) & \text{if} \quad m > 0 \end{cases}$$

then f comes from g and h by primitive recursion.

Now we create sum_3 as a recursive function:
$g : \mathbb{N}^2 \to \mathbb{N}$
$h : \mathbb{N}^{2+2} \to \mathbb{N}$
$sum_3 : \mathbb{N}^{2+1} \to \mathbb{N}$
Then we have:
$g : \mathbb{N}^2 \to \mathbb{N}$
$h : \mathbb{N}^4 \to \mathbb{N}$
$sum_3 : \mathbb{N}^3 \to \mathbb{N}$

We take $g \equiv sum(x, y)$
And we take $h \equiv \sigma(\pi_4^4)$

We see that sum is another recursive function. As we have seen in class:
$sum(n) = \langle \pi_1^1 | \sigma(\pi_3^3) \rangle (n)$

Our final function sum_3 is:
$sum_3(n) = \langle \langle \pi_1^1 | \sigma(\pi_3^3) \rangle | \sigma(\pi_4^4) \rangle (n)$

Lets test how it operates using *Octave*:

`>> evalrecfunction('suma_3', 2, 5, 3)`

$suma_3(2,5,3)$

$<<\pi^1_1|\sigma(\pi^3_3)>|\sigma(\pi^4_4)>(2,5,3)$

$<<\pi^1_1|\sigma(\pi^3_3)>|\sigma(\pi^4_4)>(2,5,2)$

$<<\pi^1_1|\sigma(\pi^3_3)>|\sigma(\pi^4_4)>(2,5,1)$

$<<\pi^1_1|\sigma(\pi^3_3)>|\sigma(\pi^4_4)>(2,5,0)$

$<\pi^1_1|\sigma(\pi^3_3)>(2,5)$

$<\pi^1_1|\sigma(\pi^3_3)>(2,4)$

$<\pi^1_1|\sigma(\pi^3_3)>(2,3)$

$<\pi^1_1|\sigma(\pi^3_3)>(2,2)$

$<\pi^1_1|\sigma(\pi^3_3)>(2,1)$

$<\pi^1_1|\sigma(\pi^3_3)>(2,0)$

$\pi^1_1(2) = 2$

$\sigma(\pi^3_3)(2,0,2)$

$\pi^3_3(2,0,2) = 2$

$\sigma(2) = 3$

$\sigma(\pi^3_3)(2,1,3)$

$\pi^3_3(2,1,3) = 3$

$\sigma(3) = 4$

$\sigma(\pi^3_3)(2,2,4)$

$\pi^3_3(2,2,4) = 4$

$\sigma(4) = 5$

$\sigma(\pi^3_3)(2,3,5)$

$\pi^3_3(2,3,5) = 5$

$\sigma(5) = 6$

$\sigma(\pi^3_3)(2,4,6)$

$\pi^3_3(2,4,6) = 6$

$\sigma(6) = 7$

$\sigma(\pi^4_4)(2,5,0,7)$

$\pi^4_4(2,5,0,7) = 7$

$\sigma(7) = 8$

$\sigma(\pi^4_4)(2,5,1,8)$

$\pi^4_4(2,5,1,8) = 8$

$\sigma(8) = 9$

$\sigma(\pi^4_4)(2,5,2,9)$

$\pi^4_4(2,5,2,9) = 9$

$\sigma(9) = 10$

`ans =  10`

`>>`

3 Implement a WHILE programm that computes the sum of 3 values. You must use an auxiliary variable that accumulates the result of the sum

$[Sum_3]$

while $X_2 \mathrel{!}= 0$ **do**
  $X_1 := X_1 + 1;$
  $X_2 := X_2 - 1;$
od
while $X_3 \mathrel{!}= 0$ **do**
  $X_1 := X_1 + 1;$
  $X_3 := X_3 - 1;$
od