



Optimized micro-hydro power plants layout design using messy genetic algorithms

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ABSTRACT

Micro Hydro-Power Plants (MHPP) represent a powerful and effective solution to address the problem of energy poverty in rural remote areas, with the advantage of preserving the natural resources and minimizing the impact on the environment. Nevertheless, the lack of resources and qualified manpower usually constitutes a big obstacle to its adequate application, generally translating into sub-optimal generation systems with poor levels of efficiency. Therefore, the study and development of expert, simple and efficient strategies to assist the design of these installations is of especial relevance. This work proposes a design methodology based on a tailored messy evolutionary computational approach, with the objective of finding the most suitable layout of MHPP, considering several constraints derived from a minimal power supply requirement, the maximum flow usage, and the physical feasibility of the plant in accordance with the real terrain profile. This profile is built on the basis of a discrete topographic survey, by means of a shape-preserving interpolation, which permits the application of a continuous variable-length Messy Genetic Algorithm (MGA). The optimization problem is then formulated in both single-objective (cost minimization) and multi-objective (cost minimization and power supply maximization) modes, including the study of the Pareto dominance. The algorithm is applied to a real scenario in a remote community in Honduras, obtaining a 56.96% of cost reduction with respect to previous works.

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1. Introduction

Universal access to clean and affordable energy represents one of the biggest challenges that humanity must face in the coming years (Nejat, Jomehzadeh, Taheri, Gohari, & Majid, 2015). Despite the increasing rate of industrialization and the improvements in the use and generation of electricity, in 2017 there were still 1.06 billion people who lacked access to electricity, and up to 2.6 billion who depended on unreliable energy sources, such as fuel and biomass, to meet their basic needs, according to the World Data Bank (International Energy Agency, 2017). What is worse, this situation is not expected to vary noticeably (IEA et al., 2010). This problem is especially aggravated in rural areas (27% of the population do not have access to electricity) with respect to urban areas, which tend to be more electrified (4% of the population without access to electricity).

In this context, Renewable Energy Sources (RES) can make an important contribution to covering basic energy needs with a minimal impact (OECD, 2016) on the environment. These energy

sources have demonstrated to be especially suitable for supplying remote and isolated areas (Kanase-Patil, Saini, & Sharma, 2010; Bugaje, 2006), where accessing the centralized grid is either not possible or too expensive. On this basis, a wide range of RES technologies (such as photo-voltaic (Goel & Sharma, 2017), wind (Nandi & Ghosh, 2009) and bio-mass (Shahzad, Zahid, ur Rashid, Rehan, & Ali, 2017) systems) have been proposed in the literature (Goel & Sharma, 2017; Nandi & Ghosh, 2009; Shahzad et al., 2017). Among the different alternatives, the use of Micro-hydro Power Plants (MHPPs) has been proven to constitute an excellent way to deal with energy poverty (Jawahar & Michael, 2017; Paish, 2002) in remote isolated areas (Sahoo, 2016; Saheb-Koussa, Haddadi, & Belhamel, 2009; Mohammed, Mokhtar, Bashir, & Saidur, 2013), given their reliability, ease of use and low installation and maintenance costs (ESMAP, 2017).

Nowadays, hydro-power energy represents the most used RES around the world to generate electrical power for human consumption. Its popularity is such that it provides 19% of the planet's electricity (Sachdev, Akella, & Kumar, 2015) and has one of the highest efficiency rates (Kaldellis & Kavadias, 2000). Modern hydro-power technology makes this energy competitive not only for mass energy supply, but also for the supply of small isolated

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areas (Izadyar, Ong, Chong, & Leong, 2016; Elbatran, Yaakob, Ahmed, & Shabara, 2015). Furthermore, Micro Hydro Power Plants (MHPPs) constitute a usual solution to supply small off-grid communities, given their simplicity (see Fig. 1), low cost and high robustness (Mandelli, Barbieri, Mereu, & Colombo, 2016). It is also relevant to note the extremely low environmental impact of these installations, given the lack of large structures involved.

This paper proposes an evolutionary computational approach (Goldberg, Deb, & Korb, 1989) to address the problem of determining the most suitable layout of an MHPP, in terms of cost, power capacity, flow usage, and feasibility. The use of hydro-power constitutes an area of high relevance that has motivated an important research effort in the literature (Mishra, Singal, & Khatod, 2011; Elbatran, Yaakob, Ahmed, & Shabara, 2015; Bozorg Haddad, Moradi-Jalal, & Marino, 2011). Nevertheless, the majority of these works focus on developing general guidelines and decision criteria, while the development of specific strategies and algorithms to optimize the design of MHPPs in rural areas is still a matter of study. The main difficulties lie in the challenge of generalizing the problem to provide a global strategy. This must be done without the need of assuming strong simplifications, which might imply deviations that are especially significant for MHPP in low resources contexts. Therefore, the motivation of this work is the development of a design system that can be used with a continuous accurate approximation of the real terrain profile. Consequently, without making strong simplifications in the design methodology. This work proposes an evolutionary approach to determine the positions of the dam and the powerhouse, together with the layout of the penstock and its diameter, in accordance to a set of performance criteria.

The proposed approach is based on a Messy Genetic Algorithm (MGA) that makes use of variable length chromosomes to work with such a continuous terrain profile. To this end, a set of tailored genetic operators are developed. This way, the algorithm makes it possible to determine the optimal layout of the plant. Notice that this is a clear contribution with respect to previous works that fixed the possible positions of the mentioned configuration parameters (Tapia, Reina, & Millán, 2019; Tapia, Millán, & Gómez-Estern, 2018).

To provide a deep analysis of the potential of the MHPP, the problem is studied in both single-objective and multi-objective mode based on Pareto dominance, resulting in a better understanding of the influence of the different parameters. Moreover, a

set of topographical data from a real location is used to evaluate the performance of the proposed approach.

The main contributions of this paper are:

- Development of a tailored messy genetic algorithm for finding the most suitable layout of an MHPP under real conditions. The proposed approach makes the decision making easier, with respect to the emplacement of the main components of the installation, this is, the location of the dam and the powerhouse, the pipe diameter, and the number of elbows.
- Validation of the algorithm in real scenarios, resulting in a reduction of the installation cost of MHPP by more than 50 % with respect to previous works, where the terrain domain was considered in a discrete form.

Nevertheless, as it will be discussed later, there are two main limitations of the proposed approach: the first is the two-dimensional formulation of the approach, which builds on the assumption of a negligible curvature of the river path. The second one is the requirement of an experimental topographic survey, in order to define the height profile of the terrain.

The rest of the paper is organized as follows: first, some works related to the proposed approach are reviewed in Section 2. A general overview of the problem is presented in Section 3, in terms of which the optimization problem is defined. In Section 4, the MGA is presented, being it used in a real scenario application. Finally, an extensive analysis of the results is summarized in Section 6, together with the concluding remarks and the further work.

2. Related work

The optimization of MHPPs has been a matter of extensive study during the last decades (Gingold, 1981; Marliansyah, Putri, Khootama, & Hermansyah, 2018; Iqbal, Azam, Naeem, Khwaja, & Anpalagan, 2014; Banos et al., 2011). A detailed review of the current state of the art in computational optimization methods applied to RES can be found in Banos et al. (2011), where the latest research advances in the field are summarized. Also, an especially relevant work can be found in Iqbal et al. (2014), where the authors present a review of different optimization methods for the deployment and operation of different RES generation units. Given the complexity of the problem (Singal, Saini, & Raghuvanshi, 2010; Elbatran et al., 2015), the approaches proposed in the literature are generally focused on a specific aspect of the problem, such as determining efficient operation strategies (Mohamad, Mokhlis, & Ping, 2011; Kishor, Saini, & Singh, 2007) or determining the optimal design parameters (Anagnostopoulos & Papantonis, 2007b).

These optimization problems have traditionally been addressed analytically, for which general simplifications are usually required. In Alexander and Giddens (2008), the authors present an approach to optimize penstocks in MHPPs with the objective of minimizing the cost of the supplied energy. The river profile is approximated by a constant slope. In Leon and Zhu (2014) a dimensional analysis is developed to determine the optimal flow discharge and optimal penstock diameter in order to assist the design of action hydro-power plants. The proposed approach pursues the water usage minimization and is based on the geometric and hydraulic characteristics of the penstock, the total hydraulic head and the desired power production. In addition, a set of dimensionless relationships between the relevant design variables are derived, which makes it possible to determine the optimal water flow rate and penstock diameter. In Basso and Botter (2012), the authors propose an analytical framework to determine the performance and profitability of an MHPP considering flow duration curves and environmental requirements. The proposed approach is validated in a real case,

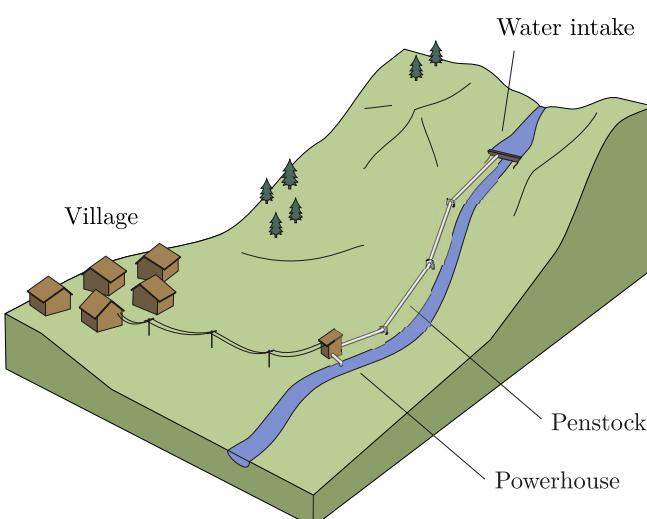


Fig. 1. Scheme of an MHPP.

evidencing the potential of the method as a design tool for practical applications. In a more practical way, authors in [Yildiz and Vrugt \(2019\)](#) present a toolbox to simulate the energy production, cost and profit of an MHPP in terms of the Flow Duration Curves (FDC) and design variables. On this basis, an optimization module is also presented. This can determine optimal decision variables related to the type of turbine, its configuration and the design flow. Similarly, in [Anagnostopoulos and Papantoni \(2007a\)](#) a numerical method to determine the optimal sizing of an MHPP is developed. The algorithm is based on a model that simulates the plant operation during a year, evaluating the performance (production and cost). A parametric study is first performed to evaluate the effects of the different factors, being a stochastic evolutionary algorithm implemented to the optimization process. Furthermore, some authors have demonstrated the capability of these approaches to evaluate the potential of existing MHPPs and proposed improvement redesigns ([Aslan, Arslan, & Yasar, 2008](#); [Marliansyah et al., 2018](#)). These works focus on providing basic design guidelines regarding the main variables related to the size and power generation, but in general the particular terrain morphology is not considered, what might lead to differences between the predicted and the real performances. On the contrary, this work pursues the development of a design tool that considers the topography of the environment of the plant, providing the most suitable utilization of emplacement of its components and a better knowledge about its potential.

Although traditional optimization approaches such as Linear Programming (LP) ([Yoo, 2009](#)), Integer Linear Programming (ILP) ([Tapia et al., 2018](#)) or Mixed-Integer Non-Linear Programming (MINLP) ([Yoo, 2009](#)) have been proven useful to address these problems, meta-heuristic algorithms are gaining relevance in the field. An illustrative example of this can be found in [Eteram, Karami, Mousavi, Farzin, and Kisi \(2018\)](#), where three modern evolutionary algorithms used to optimizing reservoir operation and water supply are studied. In [Bozorg Haddad et al. \(2011\)](#) the authors propose a strategy for the optimal design, control and operation of MHPPs by using a Honey Bee Mating Optimization (HBMO) algorithm, being the annual benefits and operation costs obtained. This algorithm determines the turbine type, the number of units, and the penstock diameter, while scheduling the operation that results in the maximum benefit for a given set of river inflow histograms. However, the authors do not consider the terrain profile of the system location. Similar to the present work, but taking into account this last issue, the MHPP layout is optimized in [Tapia et al. \(2019\)](#), where the authors develop a Genetic Algorithm (GA) to find the most adequate locations for the different parts of the plant, including powerhouse, dam and penstock layout. The main objective is to achieve a certain power rate with minimal cost and satisfying a set of constraints related to flow usage and feasibility of the layout for a certain terrain profile.

This work proposes the use of a Messy Genetic Algorithm (MGA) ([Goldberg et al., 1989](#)) to determine the optimal location of the different elements of the MHPP along the terrain. The optimization problem is formulated in term of the minimization of the cost, adding generated power, flow usage, and physical feasibility as constraints. In order to improve the approach proposed in [Tapia et al. \(2019\)](#), where the terrain profile is considered to be composed of a number of discrete points from a topographic survey, in this paper the survey is used to provide a continuous approximation of the real terrain by means of a shape-preserving interpolation, being the optimization problem reformulated in a continuous form. To solve this problem, the MGA developed considers a variable chromosome length. To verify the benefits of this approach, the proposed algorithm is used to optimize an MHPP in a real emplacement.

3. Problem statement

In general, an MHPP exploits the natural height difference of a natural water course to transform potential energy into electrical energy, by means of a mechanical interaction between the water flow and the rotor of a turbine. For this, the water is diverted from its natural course by means of a long pipe, named penstock. This element is responsible of driving the water downhill to the powerhouse (see [Fig. 1](#)), where the water flow is driven into the generation system. This is composed of a Pelton turbine, which transforms the energy of the water into mechanical energy, and a generator, which transforms this last into electrical energy. Although it is clear that both higher values of gross height and water flow lead to higher power rates, this relation is strongly conditioned by the dissipation effects due to friction through the pipe, which depends on the length and diameter of the penstock, as will be explained in Section 3.1. For these reasons, finding the most suitable layout of an MHPP at a certain irregular terrain results in a complex and challenging problem.

The traditional design procedure of an MHPP in rural areas is generally based on the following steps:

1. Measurement of available flow rate, Q .
2. Measurement of available gross height difference, H_g .
3. Estimation of the power generation, P .
4. Decision making of intake and powerhouse locations.
5. Sizing of the equipment.

The measurement of the available flow rate and gross height has the purpose of estimating the suitability of the emplacement, and to do that traditional measurements techniques are employed. Gauging weirs are the most extended tool to measure water flow ([Streamflow Herschy, 2014](#)), while topographic maps are typically used to estimate the gross head. With these parameters, a gross assessment of the generated power is done, and the viability of the MHPP, together with its capability of providing the required power supply are evaluated. The precise location of the powerhouse and the dam is determined on the basis of experience of the local technicians and thumb rules ([Thake, 2000](#)), being the layout of the penstock decided during its deployment. Although this methodology have provided successful development of MHPP over the years, the approach proposed in this paper pursues a better utilization of the resources without compromising either the resources or the simplicity of the methodology.

First, a realistic non-linear model of the MHPP is developed, on the basis of which the optimization algorithm will be formulated.

3.1. Model of the MHPP

The main variables required to evaluate the performance of an MHPP are the gross height, H_g , the length of the penstock, L , and the water flow rate, Q . In terms of these variables, the generated power, P , and the overall cost of the plant, C , can be defined.

3.1.1. Generated power

The obtainable power, P , can be expressed as

$$P = \gamma Q h \eta, \quad (1)$$

where γ is the specific weight of water, η represents the efficiency of the energy transformation in the turbine and generator, and h is introduced to define the height of the water at the entrance of the turbine. It is relevant to note that, due to the effects of friction dissipation through the penstock, h is noticeable lower than H_g . Using h_L to denote the friction loss, it can be written:

$$h = H_g - h_L. \quad (2)$$

Assuming an action turbine (e.g. a Pelton wheel), the height h can be expressed as the kinetic energy of the water at the entrance of the turbine (as it is projected in the form of an atmospheric jet):

$$h = \frac{1}{2g} v_{jet}^2, \quad (3)$$

being v_{jet} the speed of the water jet. From water incompressibility, the speed of the jet, v_{jet} , can be written in terms of the volumetric flow, Q , and the cross-sectional area of the jet, S_{jet} . Approximating this last variable in terms of the area of the nozzle, S_{noz} , through an experimental coefficient c_D (Thake, 2000), it yields

$$v_{jet} = \frac{Q}{c_D S_{noz}}. \quad (4)$$

Moreover, a simple expression for the height loss, h_L , can be obtained from Green (2008), yielding

$$h_L = k_p \frac{L}{D_p^5} Q^2, \quad (5)$$

where the influence of the length L and diameter of the penstock D_p is clear. The expression (4) can be substituted in (3) to obtain an expression of h in terms of the flow Q :

$$h = \frac{1}{2g} \frac{1}{c_D^2 S_{noz}^2} Q^2. \quad (6)$$

This last expression, together with (5), can be introduced in (2), and after isolating Q , the following expression is obtained:

$$Q = \left[\frac{H_g}{\frac{1}{2g c_D^2 S_{noz}^2} + \frac{k_p}{D_p^5} L} \right]^{\frac{1}{2}} \quad (7)$$

Finally, (7) and (6) can be introduced in (1) to obtain a general expression for the power P in terms of the geometrical parameters H_g and L as

$$P = \frac{\eta \rho}{2c_D^2 S_{noz}^2} \left[\frac{H_g}{\frac{1}{2g c_D^2 S_{noz}^2} + \frac{k_p}{D_p^5} L} \right]^{\frac{3}{2}}. \quad (8)$$

It can be seen that, for a given equipment (pipes, turbine and generator), given a location of the dam and the turbine and the layout of the penstock, both power generation and water flow are completely determined.

3.1.2. Cost of the plant

In the context of supplying remote and isolated rural areas with off-grid energy systems, the cost of the installation is considered as the most severe limitation (Holland, 1989), and thus its minimization will be considered as the main objective function in a Single Objective (SO) mode. In addition, the maximization of the generated power will be also considered as an objective function in a Multi-Objective (MO) mode of the problem.

The overall cost of an MHPP can be calculated as the sum of the costs related to the generation equipment, the penstock, the electrical system, and the civil work (this covers the installation of the powerhouse and the dam). Nevertheless, some considerations simplify the calculation of the overall plant cost. First, the cost of the generation equipment is not expected to vary, as its sizing is defined by the objective power. Therefore, this term is not considered in the optimization problem. In addition, the powerhouse and the dam does not depend on the layout of the MHPP, as they have the same function and size, and thus there is no need of including these costs in the objective function. In what regards the civil works, as they are normally assumed to be a contribution from the community involved in these projects, its associated cost will

not be considered to be minimized. Finally, it can be considered that the distance from the village to the river does not vary significantly, and thus the cost on the electrical system is not considered. With these assumptions, the objective function can be defined as the cost of the penstock, which is typically proportional to the total length, L , and inversely proportional to the squared diameter, D_p^2 (Alexander & Giddens, 2008). In addition, as the penstock is modeled as a set of rectilinear connected pipe lengths, it has been considered relevant to include in the cost a term depending on the number of elbows, n_c , including a parameter λ_c defined as the length-equivalent cost of a single elbow, has been introduced. Consequently, the objective function can be finally written as

$$C = c_L D_p^2 (L + \lambda_c n_c), \quad (9)$$

where c_L is a suitable constant to express the cost in the appropriate units.

3.2. Definition of the terrain

The proposed model stems from an N -discretization of the terrain, which is considered to be obtained by means of a topographic survey. This discretization consists of a set of N spatial points in the form of

$$\left\{ \tilde{s}_i, \tilde{z}_i \right\}, \quad \forall i = 1 \dots N, \quad (10)$$

where the terms \tilde{s}_i and \tilde{z}_i represent the coordinates of the 2D-development of the river profile, as represented in Fig. 2. The two-dimensional approximation of the river profile is supported by several assumptions. First, due to the mountainous nature of the remote studied areas, the water courses are generally upper-course rivers with low or negligible curvature. Secondly, as the available power levels are generally low, the required penstocks are short enough to neglect the improvement of shorten the layout along curves. Finally, when construction requirements and accessibility are considered, the deployment of the penstock along the river course is generally the preferred option, as it provides a clearer terrain, with easy access for both people and material transportation, avoiding ground-related complications.

In terms of the discretization (10), a continuous function $\varphi = \varphi(s)$ is defined by interpolating the points $(\tilde{s}_i, \tilde{z}_i)$. The proposed interpolation method is based on a Piece-wise Cubic

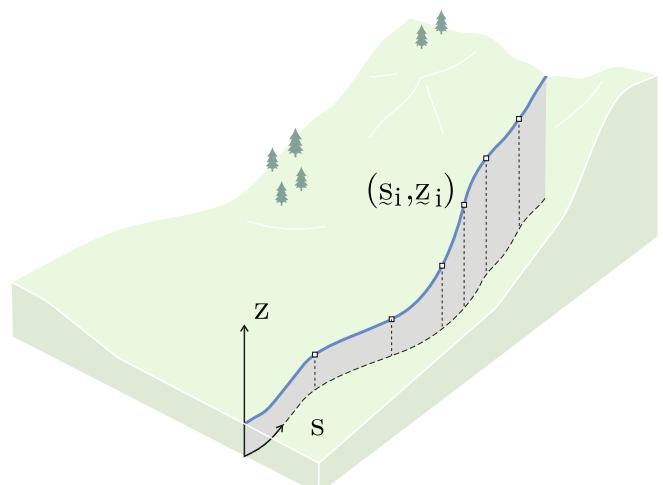


Fig. 2. Approximation of the river profile by means of a topographic discrete survey.

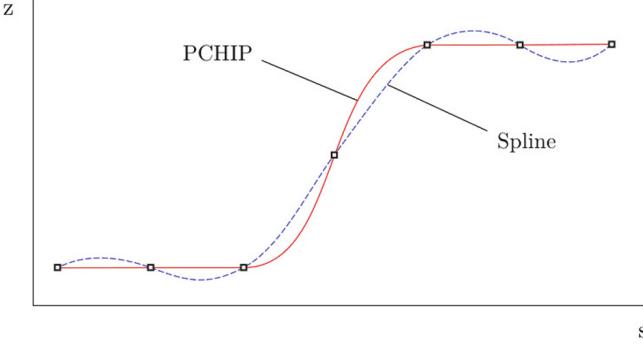


Fig. 3. Comparison between Spline and PCHIP interpolation methods. It can be seen that the PCHIP improves the shape preservation, avoiding the formation of local maxima and minima.

Hermite Interpolating Polynomial (PCHIP) (Fritsch & Carlson, 1980; Dougherty, Edelman, & Hyman, 1989). The advantage of this approach with respect to the traditional cubic spline interpolation lies in its monotonic nature (an illustrative example is shown in Fig. 3 for a better understanding), which meets the natural monotonicity of a river profile, where there are neither oscillations nor bumps, as the water flows by means of a decreasing height layout. The function $\varphi(s)$ is formulated as

$$\varphi(s) = a_i s^3 + b_i s^2 + c_i s + d_i, \quad \tilde{s}_i \leq s \leq \tilde{s}_{i+1}, \quad (11)$$

for which the constants $\{a, b, c, d\}_i$ constants must be calculated for each $i = 1 \dots N - 1$. A detailed description of this calculation can be found in Fritsch and Carlson (1980). The function (11) will be considered as an accurate approximation of the terrain profile, in order to optimize the MHPP layout.

3.3. MHPP layout

Following the approach proposed in Tapia et al. (2019), each possible penstock layout is modeled as a linear piece-wise function $\Gamma(s)$, which is defined by means of a set of n nodes that represent either pipe elbows, where two straight lengths are connected (internal nodes), or the powerhouse and dam locations (first and last

node, respectively). In other words, the penstock $\Gamma(s)$ consists on the linear interpolation of the n selected points $(x_i, \varphi(x_i))$, this is

$$\Gamma(s) = e_j s + f_j, \quad x_j \leq s < x_{j+1}, \quad (12)$$

where the coefficients e_j and f_j can be determined by linearly interpolating the points (s_j, z_j) and (s_{j+1}, z_{j+1}) . Thus, to identify each solution, an array Δ , containing its n nodes, is proposed:

$$\Delta = [x_1, x_2, x_3, \dots, x_n], \quad (13)$$

where every x_i represents the position along the s -domain where a node is located. It is clear that the values of x_i must be inside the domain of the river, this is

$$\tilde{s}_1 \leq x_i \leq \tilde{s}_N, i = 1 \dots n. \quad (14)$$

For simplicity, the values of x_i in Δ are assumed to be in ascending order, in such a way that

$$x_i \leq x_{i+1}, i = 1 \dots n - 1. \quad (15)$$

It is easy to see that the size of the solution, n , which represents the total number of nodes of the penstock, is restricted to be an integer higher than 2 (note that the trivial case of a single node does not represent a penstock). To illustrate the model proposed, an example is shown in Fig. 4, where a set of $N = 6$ geographical points have been used to define the river profile and an arbitrary solution Δ with $n = 3$ nodes has been represented.

To formulate the problem, it is necessary to express the main variables, H_g and L , in terms of the solutions (13). Assuming a certain solution Δ with n elements, it is clear that the gross head, H_g , can be calculated as the difference between the height of the highest and the lowest of its nodes:

$$H_g = \Gamma(x_n) - \Gamma(x_1) = \varphi(x_n) - \varphi(x_1). \quad (16)$$

With respect to the length, L , it can be easily calculated as the sum of the $n - 1$ intervals that defines the penstock. Using the Euclidean distance, this can be written as

$$L = \sum_{i=1}^{n-1} \left[(x_{i+1} - x_i)^2 + (\Gamma(x_{i+1}) - \Gamma(x_i))^2 \right]^{\frac{1}{2}}. \quad (17)$$

Finally, the expressions for the obtainable power P , extracted flow rate Q and cost C , can be obtained by introducing (16) and (17), into (8), (7) and (9), respectively.

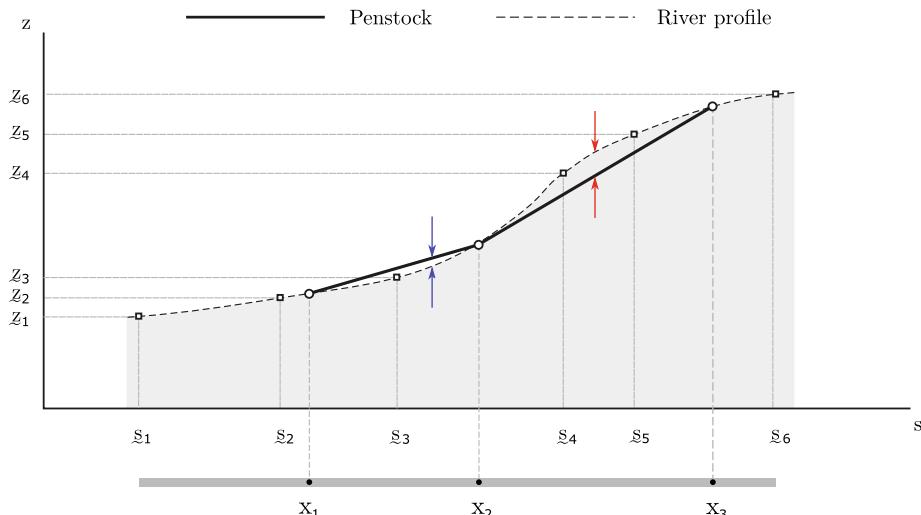


Fig. 4. Representation of an example set of $N = 6$ geographic data points (squares), the interpolated river profile $\varphi(s)$, a solution Δ with $n = 3$ nodes (circles), and the correspondent penstock, $\Gamma(s)$. Note the formation of positive (highlighted in blue) and negative (highlighted in red) gaps between the penstock and the terrain. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

3.4. Problem formulation

The problem proposed in this paper consists of finding the most suitable MHPP layout. To evaluate the suitability of a solution, two different objective functions are proposed:

- Minimization of the overall cost, C .
- Maximization of the power generation, P .

Regarding the requirements that must be fulfilled, the following constraints are proposed:

- Power constraint: A constraint is introduced to guarantee that the power generation of the plant reaches a certain minimal value.
- Flow constraint: To avoid the environmental impact, only a certain fraction of the natural flow rate is considered allowed to be extracted.
- Feasibility constraint: A set of constraints must be introduced to ensure the physical feasibility of the solution. These constraints will be further elaborated in the next section.

3.4.1. Power constraint

This constraint is introduced to guarantee that the developed MHPP covers a certain basic supply. To this end, an estimation of the basic needs (those are, illumination for homes, community household appliances and other devices) is made, in terms of which a value P_{min} is defined. Then, the constraint can be written as

$$P \geq P_{min}, \quad (18)$$

where the expressions of the power (8), together with expressions (16) and (17) can be introduced, leading to

$$\frac{\Gamma(x_n) - \Gamma(x_1)}{\frac{1}{2g_D^2 S_{noz}^2} + \frac{k_p}{D_p^5} \sum_{i=1}^{n-1} [(x_{i+1} - x_i)^2 + (\Gamma(x_{i+1}) - \Gamma(x_i))^2]^{\frac{1}{2}}} \geq \left[2C_D^2 S_{noz}^2 \frac{P_{min}}{\eta\rho} \right]^{\frac{2}{3}}$$

3.4.2. Flow rate constraint

The available water flow rate of the river, Q_{river} , constitutes itself the most important limitation to its exploitation. Following this, a parameter κ is introduced to define the maximum portion of flow, which is considered acceptable to be extracted at maximum, in order to avoid harming the natural environment. This basic constraint can be formulated as

$$Q \leq \kappa Q_{river}. \quad (19)$$

Substituting the expression of the flow (7), together with expressions (16) and (17), the following can be obtained.

$$\frac{\Gamma(x_n) - \Gamma(x_1)}{\frac{1}{2g_D^2 S_{noz}^2} + \frac{k_p}{D_p^5} \sum_{i=1}^{n-1} [(x_{i+1} - x_i)^2 + (\Gamma(x_{i+1}) - \Gamma(x_i))^2]^{\frac{1}{2}}} \leq \kappa^2 Q_{river}^2.$$

3.4.3. Feasibility constraint

To guarantee the physical feasibility of a solution Δ , the extension of the necessary supports and excavations must be admissible. To model this, a maximum height of the supports, ϵ_{sup} , and a maximum depth of excavations, ϵ_{exc} , are imposed. On this basis, a solution is considered physically feasible if the supports and excavation limits are enough to bridge all the gaps between the penstock layout, $\Gamma(s)$, and the terrain profile, $\varphi(s)$. This can be generalized as

$$\begin{cases} \Gamma(s) - \varphi(s) \leq \epsilon_{sup} \\ \varphi(s) - \Gamma(s) \leq \epsilon_{exc} \end{cases} \forall s \in [x_1, x_n] \quad (20)$$

It is clear that, although the constraints (20) must be met for every point inside the penstock domain, it is enough to check the points where the differences $\Gamma(s) - \varphi(s)$ and $\varphi(s) - \Gamma(s)$ are maximum, which can be analytically determined as the explicit expressions of $\Gamma(s)$ and $\varphi(s)$ are known. Note that, as both $\Gamma(s)$ and $\varphi(s)$ are piece-wise functions, the constraints of (20) have different expressions along the domain, and for this reason, these constraints must be checked in each of the segments that result from substituting (11) and (12) in (20).

4. Evolutionary computational approach

The use of evolutionary algorithms has established itself as a powerful strategy to address complex optimization problems where the use of deterministic approaches is not feasible (Holland, 1984). Although a wide variety of algorithms can be grouped under this designation, Genetic Algorithms (GA) have received a special attention in the literature (Reina, Camp, Munjal, & Toral, 2018; Arzamendia, Gregor, Reina, & Toral, 2017; Gutiérrez-Reina, Sharma, You, & Toral, 2018), due to their capability of dealing with complex non-linear engineering problems.

GA are, essentially, search algorithms based on the mechanics of nature and natural genetics. To this purpose, the potential solutions (individuals) are encoded in a structure (chromosome), composed of a set of genes representing the independent variables of the problem. A population of individuals evolves through generations by combining solutions with randomized and structured changes. Through the creation of new offspring, the individuals adapt to the optimization landscape according to the survival of the fitness principle of the Darwinian theory. The higher the fitness of an individual is, the better its adaptation to the optimization landscape. The offspring is created by means of three genetic operators: selection, crossover and mutation. The selection is an elitist operation that consists of selecting the parents that will participate in the crossover and mutation operations. The crossover consists of combining the genetic information of two different individuals to create other two new individuals. With respect to the mutation, it modifies the genetic information of a single individual to generate a new one. Both crossover and mutation are probabilistic operations. With a proper tuning of these operations, GAs achieve good exploration and exploitation capabilities within the search landscape of the optimization problem. In this work, each individual represents a possible layout of the MHPP. The problem is addressed in both single-objective (SO) and multi-objective (MO) modes. In SO mode, the objective function consists of minimizing the cost of the plant. In the MO mode, two different objectives, such as minimizing the cost and maximizing the generated power, are simultaneously considered. In this last case, the optimization is achieved by using the Pareto dominance-based technique, like the NSGA-II (Deb, Pratap, Agarwal, & Meyarivan, 2002).

In this paper, we propose a Messy Genetic Algorithm (MGA). MGAs process variable-length strings that may be either under- or over-specified with respect to the problem being solved (Goldberg et al., 1989). In contrast to classical genetic algorithm implementations, they allow to vary the length of the chromosomes along the evolution of the genetic algorithm. MGAs were proposed back in the 80s (Goldberg et al., 1989) and, since then, they have not attracted much attention from the research community. The main reason is that the majority of optimization problems has a fixed number of decision variables. However, in the presented problem of designing MHPP, we demonstrate that working with MGAs is suitable for selecting the number of elbows among pipes (number of decision variables) and their positions (layout of the MHPP) since the optimal number is unknown.

4.1. Single-objective optimization problem

Several implementations of single-objective GAs have been proposed in the literature. In this work, a messy $\mu+\lambda$ scheme (Ter-Sarkisov & Marsland, 2011), shown in Algorithm 1, is used. This scheme begins with creating a random initial population P_i , which is evaluated. Then, the offspring μ is created by using crossover and mutation operations, with probabilities of p_{cx} and p_{mut} , respectively. Next, the evaluation of the new offspring takes place, being the new population, λ , selected from this offspring and the previous generation, P_g . As the offspring competes with their parents to be selected for the next generation, this guarantees a good level of elitism (Ter-Sarkisov & Marsland, 2011).

Algorithm 1 GA $\mu+\lambda$

- 1 Create initial population P_i ;
- 2 Evaluate P_i ;
- 3 $P_g = P_i$;
- 4 **while** stop == False **do**
- 5 Parents' selection;
- 6 Create offspring μ (crossover p_{cx} and mutation p_{mut});
- 7 Evaluate μ ;
- 8 Select new population λ ($\mu + P_g$);
- 9 $P_g = \lambda$;
- 10 **end**

Once the algorithm finishes, being a certain number of generations the general stop criterion, the resulting population contains the best solutions for the optimization problem.

4.1.1. Individual representation

Each individual represents a possible MHPP layout. Thus, the chromosome of each individual consists of a list containing the n nodes of the penstock (13). Additionally, the diameter of the penstock, D_p , is embedded in the chromosome by means of an additional gen, as represented in Fig. 5. Within this, for a chromosome with length S , the number of nodes of the given penstock is $n = S - 1$ or $n = S$, depending on whether the penstock diameter is considered as an optimization variable or not, respectively.

Although the initial population is generated randomly, a tailored generation scheme is proposed in order to provide feasible solutions in the initial step, favoring a higher diversity of individuals during the first generations of the GA. The proposed scheme, detailed in Algorithm 2, consists on selecting a diameter D_p and the powerhouse location x_1 by means of uniform distributions. Additional nodes x_i are successively appended along the s -domain (see Fig. 6) until the solution verifies the power constraint (3.4.1). To guarantee that the feasibility constraints (20) are met, the appended nodes x_i are chosen consecutively from a dense discretization of the profile $\varphi(s)$. This refinement is made in such a way that the average height difference between two adjacent points is less than the more restrictive limit form ϵ_{exc} and ϵ_{sup} divided by a refining parameter n_f that acts as a safety coefficient.

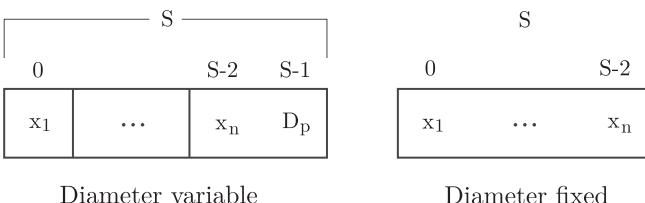


Fig. 5. Scheme of the individual representation.

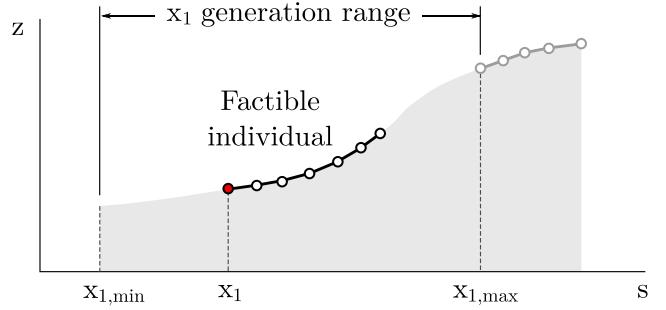


Fig. 6. Scheme of the individual generation. Note the existence of a maximum value of x_1 to guarantee that an admissible individual can be created.

Algorithm 2 Generation of the initial population

- 1 Refine the domain
- 2 $k = 1$;
- 3 **while** $k < N_p$ **do**
- 4 Generate D_k ;
- 5 Calculate $x_{1,max}$;
- 6 **repeat**
- 7 Append next point to individual
- 8 Calculate P_k
- 9 **until** $P_k \geq P_{min}$ and $Q_i \leq \kappa Q_{river}$;
- 10 Append D_k to individual;
- 11 **end**

The diameter D_p is generated by means of a random uniform distribution

$$D \sim \mathcal{U}(D_{min}, D_{max}) \quad (21)$$

With respect to the location of the powerhouse, x_1 , it is generated by means a random uniform distribution

$$x_1 \sim \mathcal{U}(x_{1,min}, x_{1,max}), \quad (22)$$

where $x_{1,min}$ is defined as \tilde{s}_1 . Note that there is a maximum value for x , named $x_{1,max}$, for which the admissible individual can be generated.

By using this generation tool, the possibility of generating unfeasible solutions is minimized, permitting a better exploration during the first generations of the MGA. A similar strategy has been used in previous approaches (Tapia et al., 2019) with good results.

4.1.2. Fitness function

The fitness function is given by the cost of the plant, described in (9): the lower the cost, the better the solution is. Nevertheless, as invalid solutions must be discarded in order not to participate in the following generations of the GA, death penalty is used to penalize invalid individuals. As a result, the fitness of a solution is calculated as

$$\begin{cases} \text{if } \text{solution valid} & F = (9), \\ \text{else} & F = \infty. \end{cases} \quad (23)$$

4.1.3. Genetic operators

In this work, the tournament selection mechanism has been used, since it provides suitable results (Luke, 2009). In each tournament, a number of individuals are randomly selected, and compete each other to be chosen as parents. The best one is then selected as one of the parents to be used in crossover and mutation operations (Luke, 2009). A tournament size of three individuals has been

chosen, as it has demonstrated a good performance for the majority of the problems.

A tailored blend crossover technique has been used (Takahashi & Kita, 2001). The main idea is combine the genetic information of two selected parents. Since the selected parents can have different length, the shortest one should be enlarged in order to carry out the blend operation. The parameter α determines the similarity of the children with respect to their parents. We select a value of α of 0.5 since we consider that both parents should have the same influence of their children. This approach is shown in detail in Algorithm 3.

Algorithm 3 Tailored blend crossover algorithm

```

1 Ind1, Ind2: the selected parents;
2 L1, L2: lengths of chromosomes;
3 Add zeros to chromosome min(L1, L2);
4 for i = 0; i<=max(L1, L2)-1; i++ do;
5    $\gamma = 1 + 2 \alpha * \text{random}() - \alpha;$ 
6   Ind1[i] = (1- $\gamma$ )*Ind1[i] +  $\gamma$ *Ind2[i];
7   Ind2[i] =  $\gamma$ *Ind1[i] + (1 -  $\gamma$ )*Ind2[i];
8 end

```

Regarding the mutation operator, the Algorithm 4 shows the proposed approach. First, the value of p_m determines whether an individual is mutated or not. Two mutation schemes are performed to the genes of a mutated individual. First, each gene can be removed from the individual with a probability p_{cut} , which translates into shortening the length of the individual. It is important to recall that the last gene of the chromosome represents the diameter, and therefore it cannot be removed from the solution. Then, the genes of the individual go through a Gaussian mutation (see Fig. 7) according to p_{gen} , which is the probability to mutate the gene i .

Algorithm 4 Mutation algorithm

```

1 if p <=  $p_m$  then
2   if p <=  $p_{cut}$  then
3     Remove a gene randomly from 0 to length-2;
4   end
5   for i = 0; i<=length-1; i++ do
6     if p <=  $p_{gen}$  then
7       Gaussian mutation of gene i;
8     end
9   end
10 end

```

In the tailored Gaussian mutation approach, each gene or variable x_i can change according to a Gaussian distribution with mean and standard definition defined as

$$\mu_i = x_i \quad (24a)$$

$$\sigma_i = \sigma x_i. \quad (24b)$$

It is relevant to note that, with the proposed scheme, the algorithm tends to gradually reduce the size of the individuals, which leads to a reduction in the cost associated to both length and nodes. In Section 5, a thorough analysis of the influence of the parameters of the mutation algorithm is carried out. Please note that, given the generation scheme proposed, the initial individuals are expected to have a high length, far from its optimal. This will be verified within the numerical results in Section 5.4.

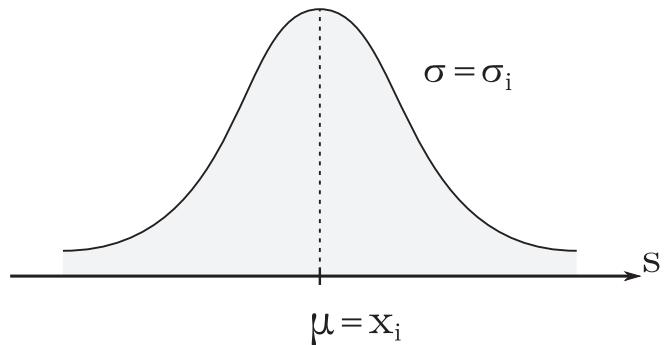


Fig. 7. Scheme of the Gaussian mutation.

4.2. Multi-objective optimization problem

In this case, the objective consists of a simultaneous minimization of the cost of the MHPP, C , and a maximization of the power generation, P . The MO algorithm used is the NSGA-II (Deb et al., 2002). This algorithm, which is based on the Pareto dominance, has demonstrated to provide a good performance with a wide range of engineering optimization problems (Sayyad & Ammar, 2013; Yusoff, Ngadiman, & Zain, 2011; Jeyadevi, Baskar, Babulal, & Iruthayarajan, 2011; Gutiérrez-Reina et al., 2018; Deb et al., 2007). The Pareto dominance establishes that a solution dominates another iff it is strictly superior in all considered objectives. Therefore, the aim of the proposed algorithm is to find all non-dominated solutions, forming the so-called Pareto front. For this optimization problem, the Pareto front will be a 2D curve in the Power-Cost space.

In Algorithm 5, the implemented MO mode MGA is shown. The fundamental differences with respect to the SO mode are:

1. The evaluation of the individuals must be computed for both fitness functions, power, and cost.
2. The selection mechanism is based on the Pareto dominance.

At each iteration the Pareto front is updated, so at the end of the algorithm the resulting Pareto front will include all non-dominated solutions found through the different generations of the MGA.

Algorithm 5 GA based on NSGA-II

```

1 Create initial population  $P_i$ ;
2 Evaluate  $P_i$ ;
3  $P_g = P_i$ ;
4 while stop == False do;
5   Parents' selection;
6   Create offspring  $\mu$  (crossover  $p_{cx}$  and mutation  $p_{mut}$ );
7   Evaluate  $\mu$ ;
8   Calculate dominance;
9   Update Pareto front;
10 Select new population based on dominance  $\lambda (\mu + P_g)$ ;
11  $P_g = \lambda$ ;
12 end

```

4.2.1. Individual representation

The individual representation is the same considered for the SO case, explained in Section 4.1.1.

4.2.2. Fitness function

In this case, the fitness function of each individual consists of a tuple with two different components, one for each of the



Fig. 8. Aerial view of the studied river profile (black) and a small tributary (white).

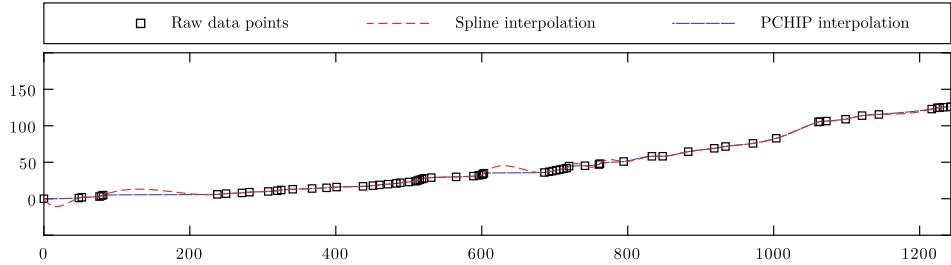


Fig. 9. Topographic data points (squares), Cubic Spline interpolation (red dashed line) and PCHIP interpolation (blue dashed line). It can be seen that the PCHIP interpolation provides a better shape preservation, avoiding the formation of local maximums and minimums along the course of the river. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

objectives. Death penalty is also considered in this problem to penalize invalid solutions, which must be employed for both objectives. Therefore, the fitness of each individual will be calculated as

$$\begin{cases} \text{if } \text{solution valid} & F = (9), (8) \\ \text{else} & F = \infty, \infty. \end{cases} \quad (24c)$$

4.2.3. Genetic operators

The crossover and mutation schemes used in Section 4.1.3 are employed.

5. Simulation results

This section shows the application of the proposed evolutionary optimization algorithm to a real use case in the Department of Santa Bárbara, in Honduras. The remote community chosen it has recently been considered by the local government to be supplied with an MHPP. A satellite image of the river is shown from an aerial view in Fig. 8.

5.1. Scenario settings

The proposed discretization consists in a set of $N = 67$ topographic points in the form of (10), obtained via aerial topographic survey. The points, together with the proposed shape-preserving interpolation, are represented in Fig. 9 for a better understanding.

For this river profile, the design of an MHPP is proposed to supply the community with a basic power need established in 8 kW. The flow of the river is 50 l/s, with an extraction allowance of 50 %. In addition, an equivalent cost of $\lambda_c = 50$ m is considered for the installation of the pipe elbows. The characteristics of the terrain are such that supports and excavations up to 1.5 m are considered admissible. The parameters associated to these constraints are summarized in Table 1.

5.2. Genetic algorithm settings

The design of the MHPP is addressed by means of a GA, by using the generation, crossover and mutation rules proposed in Section. The simulator¹. Table 2 contains the main configuration parameters of the MGA implementations.

5.3. Case-studies proposal

For this problem, a set of three cases, summarized on Table 3, are proposed:

Case-study 1 – Variable diameter, single-objective: The problem is formulated in the form of a cost-minimization problem (single-objective mode), with the objective function formulated as minimizing the cost of the MHPP. The location of the powerhouse and the dam, the pipe diameter, and the distribution and number of the pipe elbows are the design variables. This case has been used for tuning the configuration parameters of the proposed evolutionary approach, these are p_{cx} , p_m , σ , and p_{cut} .

Case-study 2 – Fixed diameter, single-objective: This second case is similar to the previous one except that a fixed penstock diameter of 20 cm is considered. The main purpose of this case-study is to compare the results of the proposed approach with that of previous work (Tapia et al., 2018), where the problem was formulated as an ILP. In Tapia et al., 2018 a diameter of 20 cm was used, since the formulation required a fixed penstock diameter. Consequently and for the sake of comparison, the same value has been used in this case-yearstudy.

Case-study 3 – Variable diameter, multi-objective: The last case is formulated on the same basis of case-study 1 but considering a multi-objective mode, where the cost of the plant is minimized and the generated power P is maximized.

¹ The code is available in D. (2019) has been developed using Python and DEAP (Fortin, Rainville, Gardner, Parizeau, & Gagné, 2012)

Table 1
Parameters of the example problem.

Parameter	Value	Unit
P_{min}	8	kW
Q_{river}	50	L/s
κ	0.5	–
λ_c	50	m
ϵ_{exc}	1.5	m
ϵ_{sup}	1.5	m

Table 2
Parameters of the MGA.

Parameter	Value
λ	2000
μ	2000
Individuals (MO)	2000
Generations	200
Selection	Tournament size = 3 (single-objective) NSGA-II (multi-objective)
Crossover	Tailored Blend Crossover $p_{cx} = [0.4, 0.5, 0.6, 0.7, 0.8]$
Mutation	Gaussian mutation $p_m = [0.6, 0.5, 0.4, 0.3, 0.2]$ $\sigma = [0.01, 0.05, 0.10, 0.15]$ $p_{cut} = [1, 0.8, 0.6]$
Number of trials	30
Generation	Refinement $n_f = 2$ Powerhouse location range $x_{1,min} = \tilde{s}_1, x_{1,max}$ is determined for each diameter Diameter range $D_{min} = 1$ cm, $D_{max} = 33$ cm

Table 3
Description of the case-studies.

	Objective function	Pipe diameter
Case-study 1	min C (17)	Variable
Case-study 2	min C (9)	Fixed
Case-study 3	min C (9), max P (8)	Variable

5.4. Results

5.4.1. Case-study 1 – Variable diameter, single-objective

In this case-study, an equivalent cost $\lambda_c = 50$ m has been considered. In order to determine the most suitable parameters of the GA, the influence of the crossover and mutation probabilities, p_{cx} and

p_{mut} , the mutation displacement coefficient, σ , and the cut probability, p_{cut} , has been analyzed in this case study. Table 4 summarizes the results obtained using different values of p_{cx} and p_{mut} , using $\sigma = 0.10$ and $p_{cut} = 1$. Then, using the combination that provides the best individual, the influence of σ is evaluated. The results are shown in Table 5. Using the most suitable parameters obtained in Table 5, the influence of p_{cut} is evaluated. The results are summarized in Table 6.

In addition, the best individual obtained is represented in Fig. 11. Please note that the highest probability of cutting a gene (this is $p_{cut}=1$) provides the best results. Although this is an unusually high value for a mutation probability, it demonstrates the goodness of this operator given the high length of the initial solutions.

The combination of parameters summarized in Table 7 has been proven as the most suitable. The evolution of both fitness function and individuals length are shown in Fig. 10 (a) and (b), respectively. It can be seen that convergence is reached after the first 100 generations with slight improvements until generation 150, resulting in an optimal individual length of 5. Also, note that in Fig. 10 it is demonstrated that a shorter length of the individuals does not necessarily guarantee a lower cost.

This case study is solved in Tapia et al. (2019) by means of a discrete approach, and thus the results can be directly compared (see Table 8). In sight of this comparison, the following comments can be made. First, the results evidence that the continuous approach proposed in this paper provides an important improvement of the solution, providing a reduction of the 56.96% of the cost while reaching the power required. In addition, it is important to note that all the proposed parameters (see Tables 4–6) provided better solutions than the obtained with the discrete approach. Also, given the irregular distribution of the raw data points, the improvements of using a continuous approach were expected. Proof of this is that the power generated satisfies the minimal supply constraint without exceeding it, which is not possible if the pipe elbows are restricted to be located in the discrete data points (solution from Tapia et al. (2019)) exceeds the requested power by a 22.5%. Finally, the same applies to the continuous consideration of the penstock diameter, as in Tapia et al. (2019) it was limited to a set of discrete values.

5.4.2. Case-study 2 – Fixed diameter

For this case-study, the parameters summarized in Table 7 have been used. The results are summarized in Table 9, and the best solution obtained is represented in Fig. 12. In addition, the comparison of these results with the results obtained by means of a discrete approach GA (Tapia et al., 2018) in Table 10.

Table 4

Results of case-study 1 for different crossover and mutation probabilities, using $\sigma= 0.1$ and $p_{cut} = 1$.

GA Parameters					
p_{cx}	0.40	0.50	0.60	0.70	0.80
p_{mut}	0.60	0.50	0.40	0.30	0.20
Final population fitness					
Mean (c.u.)	9.001	8.874	9.142	9.239	8.729
Std. dev. (c.u.)	0.644	0.415	0.877	0.952	2.538
Best individual					
Gross h. (m)	87.331	88.518	89.323	89.118	88.519
Flow r. (L/s)	13.696	13.696	13.696	13.696	13.701
Power (kW)	8.001	8.000	8.000	8.000	8.009
Length (m)	538.726	545.282	553.371	552.970	548.847
Elbows	6	6	6	6	6
Pipe dia. (cm)	9.91	9.83	9.79	9.81	9.85
Cost (c.u.)	8.244	8.167	8.177	8.200	8.233

Table 5Results of case-study 1 for different value of mutation displacement, using $p_{cut} = 1$.

GA Parameters					
p_{cx}	0.5				
p_{mut}	0.5				
σ	0.01	0.05	0.10	0.15	
Final population fitness					
Mean (c.u.)	8.748	8.626	8.874	9.313	
Std. dev. (c.u.)	0.418	0.318	0.415	0.873	
Best individual					
Gross height (m)	89.224	79.259	88.518	88.767	
Flow rate (L/s)	13.696	13.745	13.696	13.723	
Power (kW)	8.000	8.087	8.000	8.055	
Length (m)	552.396	484.078	545.282	550.788	
Elbows	6	4	6	6	
Pipe diameter (cm)	9.79	10.78	9.83	9.86	
Cost (c.u.)	8.176	7.953	8.167	8.277	

Table 6Results of case-study 1 for different value of probability p_{cut} .

GA Parameters			
p_{cx}	0.5		
p_{mut}	0.5		
σ	0.05		
p_{cut}	1.0	0.8	0.6
Final population fitness			
Mean (c.u.)	8.623	8.794	8.795
Std. dev. (c.u.)	0.318	0.669	0.715
Best individual			
Gross height (m)	79.259	89.213	88.993
Flow rate (L/s)	13.745	13.696	13.696
Power (kW)	8.087	8.000	8.000
Length (m)	484.078	552.486	550.982
Elbows	4	8	6
Pipe diameter (cm)	10.78	9.80	9.81
Cost (c.u.)	7.953	9.1391	8.187

Table 7

Most suitable parameters of the MGA.

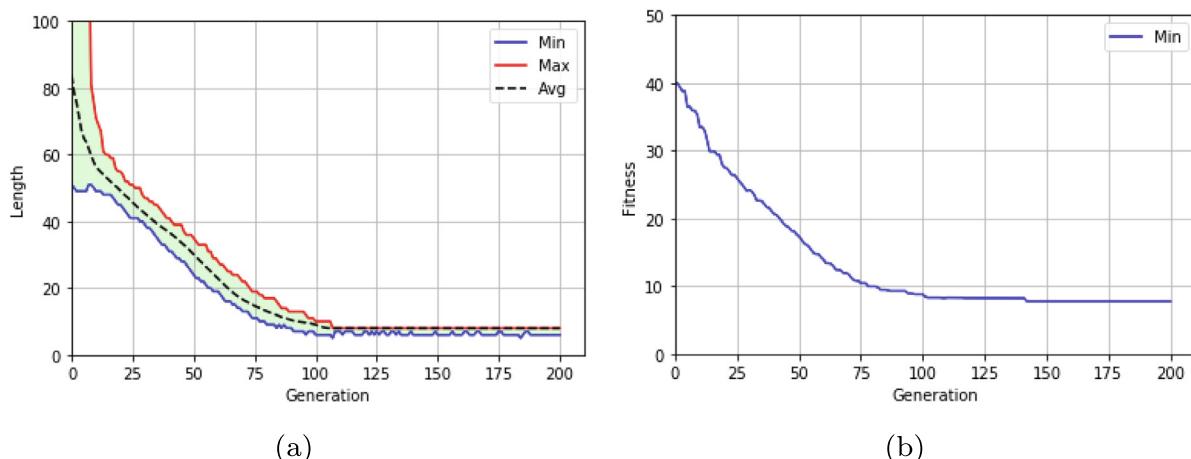
MGA parameters	Optimal value
p_{cx}	0.50
p_{mut}	0.50
σ	0.05
p_{cut}	1.00

It can be seen that a higher value of the diameter imply a shorter length of the penstock. Nevertheless, given the quadratic dependency of the cost with the diameter in (9), the obtained costs are noticeably higher.

5.4.3. Case-study 3 – Variable diameter, multi-objective

This case is presented to discuss the adaptability of the multi-objective approach to this problem. The influence of the parameters can be studied in depth by solving the optimization problem from Case 1 considering a two-objective optimization algorithm. This MO mode, which is formulated combining the economic objective of minimizing the cost while maximizing the generated power, provides the Pareto front (Reina, Ciobanu, Toral, & Dobre, 2016). The results are shown in Fig. 13, where the Pareto front is represented.

Observing Fig. 13, a few comments can be made. First, the Pareto front clearly evidences the competitive nature of the two objectives, as the generated power cannot be increased without compromising the cost, and the opposite. Second, two different parts can be observed in the Pareto front. It can be seen that the solutions with a low value of power and cost show an approximately linear tendency, which has been zoomed and represented by a dashed line in Fig. 14 for a better understanding. This represents the capacity of the solutions of increasing the generated power by extending the domain of the layout along the river profile. The slope of this region can be estimated, resulting in 2.667 c.

**Fig. 10.** Evolution of individual's length (a) and best fitness value (b) through the generations.

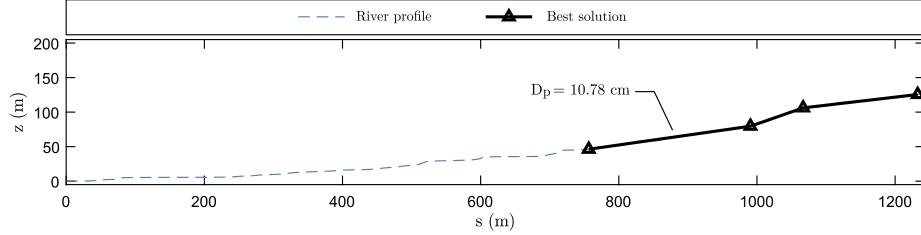


Fig. 11. Best solution obtained in case 1.

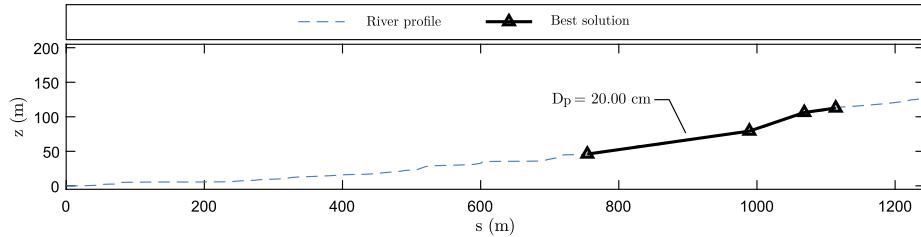


Fig. 12. Best solution obtained in case 2.

Table 8

Comparison of the results obtained for Case study 1 with the results obtained using discrete approach GA ([Tapia et al., 2018](#)).

MGA	Discrete approach GA (Tapia et al., 2019)
Gross height (m)	79.259
Flow rate (L/s)	13.745
Power (kW)	8.087
Length (m)	484.078
Elbows	4
Pipe diameter (cm)	10.78
Cost (c.u.)	7.953
	77.756
	14.654
	9.800
	471.740
	5
	16.00
	18.477

Table 10

Comparison of results for case-study 2, with the discrete approach GA ([Tapia et al., 2019](#)) and ILP ([Tapia et al., 2018](#)).

MGA	Discr. GA (Tapia et al., 2019)	ILP (Tapia et al., 2018)
Gross height (m)	66.658	67.367
Flow rate (L/s)	13.696	13.766
Power (kW)	8.000	8.123
Length (m)	366.857	389.567
Elbows	4	5
Cost (c.u.)	22.674	25.583

Table 9
Results of case-study 2, with $D_p=20$ cm.

GA Parameters	
p_{cx}	0.50
p_{mut}	0.50
σ	0.05
p_{cut}	1
	Final population
Mean cost (c.u.)	24.503
Std dev. cost (c.u.)	0.498
Best individual	
Gross height (m)	66.658
Flow rate (L/s)	13.696
Length (m)	366.857
Number of elbows	4
Power (kW)	8.000
Diameter (cm)	20
Cost	22.674

With respect to the asymptotic behavior of the Pareto front, for solutions with high values of power and cost, the abrupt change of the linear tendency can be easily explained observing the morphology of the illustrative solutions (a)-(d) in [Fig. 13](#). These points represent solutions where the river domain is saturated, and thus, the increases of power can only be achieved by increasing the penstock diameter D_p , with the consequent increase of cost, given the dependency (9).

5.5. Application of the proposed mutation algorithm in other meta-heuristic algorithms

To verify the benefits of the proposed GA, the problem defined in Case 1 is addressed by using a Simulated Annealing (SA) algorithm ([Kirkpatrick, Gelatt, & Vecchi, 1983](#); [Leitold, Vathy-Fogarassy, & Abonyi, 2018](#)). The SA algorithm is a trajectory-based algorithm and uses probabilistic modifications (mutation) of an initial solution to guide it towards the global optimization points. Since the strength of this algorithm relies in the mutation capabilities, it is a good approach to measure the goodness of the proposed mutation scheme. The results of the SA algorithm are summarized in [Table 11](#), and demonstrate that the proposed mutation scheme is suitable for the target optimization problem.

u./kW, which represents the marginal cost of increasing 1 kW the generated power. In relative terms, increasing 10% the generated power requires increasing the cost 26.76 %. It is relevant to note that the linear tendency observed in [Fig. 14](#) is more likely to dominate the Pareto front in river profiles with a low variability of the slope. For this reason, this parameter is proposed as an indicator of the potential of the emplacement.

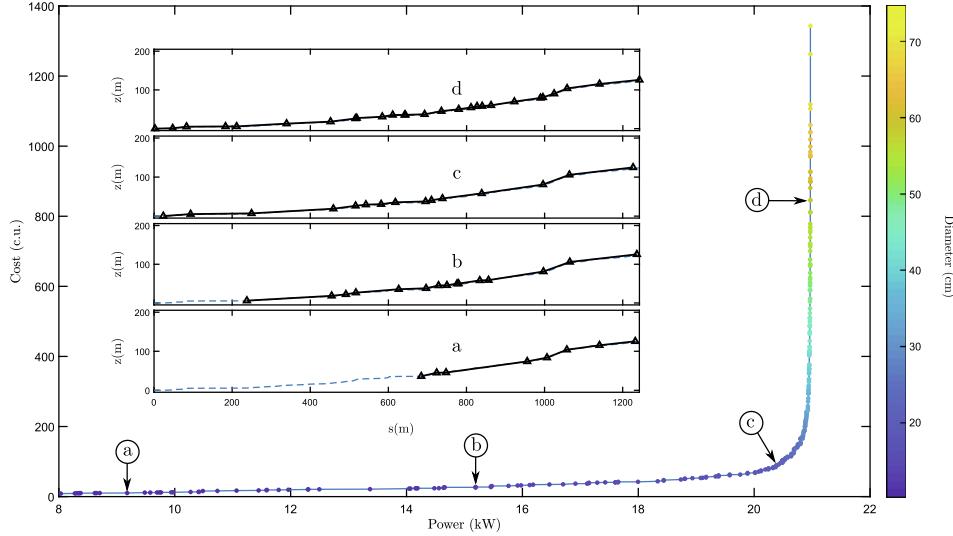


Fig. 13. Representation of the Pareto front. The color of each solution represents the penstock diameter. The layout (dark line) and nodes (triangles) corresponding to four representative solutions are plotted.

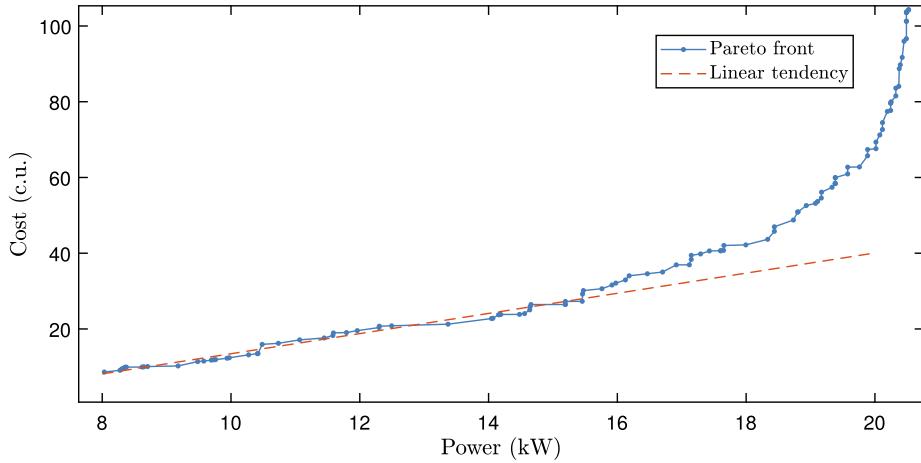


Fig. 14. Representation of the lineal tendency of the Pareto front.

Table 11
Results using SA.

	SA parameters
p_{cx}	0.50
p_{mut}	0.50
σ	0.05
p_{cut}	1.00
Mean cost (c.u.)	45.427
Std. dev. (c.u.)	31.261
Best individual	
Gross height (m)	69.911
Flow rate (L/s)	13.703
Power (kW)	8.013
Length (m)	391.654
Number of elbows	6
Pipe diameter (cm)	13.242
Cost (c.u.)	11.251

6. Conclusions

6.1. Summary of the results

In this work, a continuous MGA is developed to optimize an MHPP installation, with power supply, water flow rate, and

physical feasibility constraints, being the real height profile of the emplacement considered as the domain. In view of the obtained results, the following conclusions can be drawn:

- The influence of the MGA parameters (p_{cx} , p_{mut} , σ and p_{cut}) has been evaluated for the problem of minimizing the cost and the combination with better performance has been determined as $p_{cx} = 0.50$, $p_{mut} = 0.50$, $\sigma = 0.05$, $p_{cut} = 1.0$
- The application of the MGA to minimize of the MHPP cost has demonstrated to be particularly effective, providing much better solutions than the discrete approach. In particular a cost reduction of 56.96% is obtained.
- The use of a continuous approach allows to circumvent the problems derived from the irregular distribution of the raw geographical data points, which strongly conditioned the feasible solutions. This is illustrated by the reduced margin with which the power constraint is satisfied (exceeded in 1.1%), in comparison with the solutions that the discrete approach provides (exceeded in 22.5%).
- The MGA has also been applied to the optimization problem without considering the penstock diameter as an optimization variable, being it introduced as a constant. This approach led

- to shorter penstocks (the best solution achieved a 24.22% length reduction with respect to the solution with the optimized diameter) but noticeably more expensive (285% higher).
- In all the studied cases, the convergence is reached in the first 100 generations, with small improvements until the 200th generation.
 - The optimization problem has been addressed in a multi-objective mode, both minimizing the cost and maximizing the generated power. By determining the Pareto front, a marginal rate of substitution has been determined as 2.667 c.u. per each additional kW, allowing the understanding of how variances in the cost can affect the performance of the plant, and thus, providing a better understanding of the potential of the emplacement.
 - The problem has also been solved using a SA algorithm, which has been formulated by means of the mutation scheme proposed for the MGA. Despite of the high variability of the individuals of the last generations, the best solution obtained results in a cost only a 2.95% higher than the solution from MGA. This closeness of the results demonstrates the goodness of the proposed mutation scheme to address this problem.
 - The algorithm has been tested and validated by means of the definition of three different scenarios in a real use case.

6.2. Limitations of the approach and further work

The proposed MGA has demonstrated a good performance and provided good results for the addressed optimization problem. Nevertheless, given the assumptions that have been taken along its formulation, some comments must be considered regarding its limitations.

First of all, this development is based on a two-dimensional approach of the problem. For this reason, the approach is only reliable in those applications where river curvature is low, in such a way that cutting through rough terrain would not provide any advantage in terms of pipe length. However, the low power requirements associated to typical micro-hydro applications translates into short penstocks, supporting the previous consideration. Furthermore, the low-curvature hypothesis is generally acceptable for upper-course rivers, where there is no formation of meanders.

Furthermore, the use of this approach specifically requires a topographic analysis of the terrain, which can constitute an important issue for its application in developing areas. Nevertheless, an approximation of the river profile can be made by using traditional methods such as Abney levels or topographic maps. It is relevant to note that, although the resulting low-sampled areas represent a severe issue in the discrete formulation of this problem (Tapia et al., 2019), the consideration of a continuous approach has demonstrated a good tolerance to this problem.

Further work includes the extension of this approach to a 3D formulation. The potential benefits of this extension would not just permit its application to rivers with a high sinuosity, but also would allow the consideration additional variables of relevance, such as the location of the loads (the supplied village/s) and the cost of the required electrical grid.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

A. Tapia: Conceptualization, Methodology, Validation, Writing - original draft. **D.G. Reina:** Conceptualization, Methodology, Software, Validation, Writing - review & editing, Funding acquisition.

P. Millán: Conceptualization, Methodology, Writing - review & editing, Funding acquisition.

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