7)

a) $f(x) = x^2$; $f'(x) = 2x$; $f''(x) = 2$

$$f'(x) = \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$$

$$f'(x^2) = \frac{-(x+2h)^2 + 4(x+h)^2 - 3x^2}{2h} = \frac{-(x^2+4xh+4h^2) + 4(x^2+2xh+h^2) - 3x^2}{2h}$$

$$f'(x^2) = \frac{\cancel{-x^2} - 4xh \cancel{-4h^2} + \cancel{4x^2} + 8xh \cancel{+4h^2} - \cancel{3x^2}}{2h} = \frac{4xh}{2h} = 2x \checkmark$$

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

$$f''(x^2) = \frac{(x+h)^2 - 2x^2 + (x-h)^2}{h^2} = \frac{\cancel{x^2} \cancel{+2xh} + \cancel{-2x^2} + \cancel{x^2} \cancel{-2hx}}{h^2} = \frac{2h^2}{h^2} = 2 \checkmark$$

Metodos Computacionales

7) $f(x) = \sin(x)$; $f'(x) = \cos(x)$; $f''(x) = -\sin(x)$

$$f'(x) = \frac{-\sin(x+2h) + 4\sin(x+h) - 3\sin(x)}{2h} = \frac{-\sin(x)\cos(2h) - \cos(x)\sin(2h) + 4[\sin(x)\cos(h) + \cos(x)\sin(h)] - 3\sin(x)}{2h}$$

$$= \frac{-\sin(x)\,[1 - 2\sin^2(h)] - \cos(x)\cdot 2\sin(h)\cos(h) + 4\sin(x)\cos(h) + 4\cos(x)\sin(h) - 3\sin(x)}{2h}$$

$$= -\frac{4\sin(x)}{2h} + \frac{2\sin^2(h)\sin(x)}{2h} - \frac{2\cos(x)\sin(h)\cos(h)}{2h} + \frac{4\sin(x)\cos(h)}{2h} + \frac{4\cos(x)\sin(h)}{2h}$$

$$= \frac{4\sin(x)(\cos(h) - 1)}{2h} + \frac{2\sin^2(h)\sin(x)}{2h} - \frac{\cancel{2}\cos(x)\sin(h)\cos(h)}{\cancel{2}h} + \frac{\overset{2}{\cancel{4}}\cos(x)\sin(h)}{\cancel{2}h}$$

$$= \lim_{h\to 0} \frac{4\sin(x)\cdot \overset{\to 0}{-2\sin^2(h)}}{2h} + \lim_{h\to 0}\frac{\overset{\to 0}{2\sin^2(h)\sin(x)}}{2h} - \cos(x)\cdot\lim_{h\to 0}\frac{\overset{\to 1}{\sin(h)\cos(h)}}{h} + 2\cos(x)\cdot\lim_{h\to 0}\frac{\overset{\to 1}{\sin(h)}}{h}$$

$$= -\cos(x) + 2\cos(x)$$

$$f'(\sin(x)) = \cos(x)$$

$$f''(x) = \frac{\sin(x+h) - 2\sin(x) + \sin(x-h)}{h^2} = \frac{\sin(x)\cos(h) + \cancel{\cos(x)\sin(h)} + \sin(x)\cos(h) - \cancel{\cos(x)\sin(h)} - 2\sin(x)}{h^2}$$

$$= \frac{2\sin(x)\cos(h) - 2\sin(x)}{h^2} = \frac{2\sin(x)(\cos(h) - 1)}{h^2} = \frac{2\sin(x)(\cos^2(h) - 1)}{h^2(\cos(h) + 1)}$$

$$= \frac{2\sin(x)}{\cos(h)+1}\cdot\frac{-\sin^2(h)}{h^2} = \lim_{h\to 0}\frac{2\sin(x)}{\cos(h)+1}\cdot\lim_{h\to 0} -\frac{\sin^2(h)}{h^2}$$

$$= \frac{2\sin(x)}{2}\cdot -1 = -\sin(x)$$