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Performance Study of different Model Reference Adaptive Control Techniques applied to a DC Motor for Speed Control

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Abstract— **The objective of this work is to apply Model Reference Adaptive Control (MRAC) using Massachusetts Institute of Technology (MIT) rule and MRAC using Lyapunov method to control the speed of a Direct Current (DC) motor system. The speed control of DC motor is one of the widely used industrial controls due its specific characteristics. Different values of adaptation gains are taken for comparative analysis in MATLAB Simulink environment. A detail comparative performance analysis has been stated with different MRAC strategies applied to the DC motor system.**

Keywords—Adaptation Gain, Adaptive Control, Lyapunov Stability Theory, MIT rule, MRAC, Performance indices

I. INTRODUCTION

A control system is actually an interconnection between various physical components which form a system configuration that provides a desired response. Adaptive Control is a strategy that adjusts the controller in automatic way, for achieving or maintaining a desired level of system performance whenever the parameters of the plant dynamic model are not known and/or change with time [1].

In automatic industrial process control, Proportional Integral Derivative (PID) controller is extensively used as it is simple and has good performance. For the plant having fixed parameters, adjusting gains of the PID controller desired output performance can be achieved. However the performance of the system using PID is not acceptable owing to low bandwidth, dependency of error on plant for its parameter variation and noise sensitivity [2], [3], [4]. Hence adaptive control technique is used when the plant parameters are not accurately known. In this methodology, an additional loop is situated over and above the ordinary feedback loop, is required to reduce the difference between the output of plant and reference model [2]. In this paper a comparative study is being discussed on MRAC using MIT rule and MRAC using Lyapunov method, on speed control of a DC motor.

II. MODEL REFERENCE ADAPTIVE CONTROL

Due to change of environment condition, the parameters of a system are not accurately known, so the response of this system using ordinary feedback loop turns inaccurate, under this circumstances adaptive control is mostly used. The MRAC adjustment mechanism can be performed by gradient method commonly known as MIT rule or by using a stability theory commonly known as Lyapunov method or by augmented error [1]. The MRAC using MIT rule does not guarantee about the stability of the system all the time, while the MRAC using Lyapunov method determines stability of the system [4].

In MRAC technique, a reference model as designed by the designer is selected which describes the desired response. The difference between the output of plant and reference model, is applied to adjust parameters of the controller in such a way that plant is enforced to behave like the reference model, in this situation error tends to zero [3]. The maintenance of stability of the overall system is also a important factor that must be decided during the operation of the design of the controller [4], [5].

The MRAC comprised of four blocks i) Reference Model ii) Plant iii) Controller iv) Adaptive mechanism, which adjusts the controller to drive tracking error towards zero (Fig. 1), [2], [3], [6]. To design MRAC, tracking error $e(t)$, is described as the deviation of plant output from the reference model.

$$e(t) = y(t) - y_m(t) \tag{1}$$

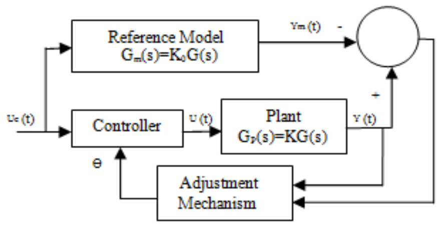

Fig. 1. Model Reference Adaptive Control

III. DC MOTOR

The DC motor is extensively used in industrial control application such as medical, automobile and aircraft applications etc. It has some specific characteristics such as, it is smooth, efficient, provides high starting torque, immense speed control range and also has a property of quick reversal.

The transfer function of a DC Motor is given by, [7], [8]

$$G(s) = \frac{\dot{\theta}(s)}{E_a(s)} = \frac{K_T}{(R_a + sL_a)(Js + f_0) + K_T K_b} \tag{2}$$

TABLE I. PARAMETERS OF DC MOTOR

	Physical Parameter	Value
1	Moment of inertia of motor and load as referred to motor shaft, J	0.5 Kg-m^2
2	Viscous friction coefficient in rotating parts of motor and load as referred to motor shaft, f_0	0.5Nm/rad/s
3	Motor torque constant, K_T	1 Nm/A
4	Back emf constant, K_b	1 V-s/rad
5	Inductance of armature winding, L_a	1 Henry
6	Resistance of armature, R_a	1 Ohm

IV. Performance Indices

Performance indices namely Integral of the Absolute magnitude of Error (IAE), Integral Square Error (ISE), Integral Time Absolute Error (ITAE) and Integral Time Square Error (ITSE) are the most important parameters for inspecting accuracy, sensitivity, selectivity of adaptive control system. This system desires performance index that is a function of variable system parameters. When the performance index achieves an extremum value, generally minimum value, then the system turns into optimal control system[7].

The mathematical notations of the above indices are as follows:

$$IAE \ = \int_0^{\propto} |e(t)|dt \quad (3)$$

$$ISE \ = \int_0^{\propto} [e(t)]^2 dt \quad (4)$$

$$ITAE = \int_0^{t} t|e(t)|dt \quad (5)$$

$$ITSE = \int_0^{\propto} t[e(t)]^2 dt \quad (6)$$

V. Performance Analysis

By applying different adaptive strategies the performance analyses of a DC motor are illustrated as follows:

A. MRAC using MIT Rule

The MIT rule or Gradient Method was exposed by the Instrumentation laboratory at Massachusetts Institute of Technology and initially this rule was utilized for designing autopilot system for aircrafts. Now a day, this method is implemented for constructing a controller for any practical system.

Herein, a cost function $J(\theta)$ is created from the tracking error as expressed in Eq. (1). θ is basically a adjustable parameter which is adjusted inside the controller and commonly known as control parameter.

The selection of $J(\theta)$ will later decided how the parameters are updated. The cost function is given as,[2], [9]

$$J(\theta) = \frac{e^2}{2} \quad (7)$$

As per the MIT rule, $\dot{\theta}$ is directly proportional to the negative gradient of $J(\theta)$, which is given by

$$\frac{d\theta}{dt} = -\gamma\frac{\partial J}{\partial\theta} = -\gamma e\frac{\partial e}{\partial\theta} \quad (8)$$

Here, $\frac{\partial e}{\partial\theta}$ is known as sensitivity derivative that determines how θ will be updated for the given system. The relationship between change in θ and $J(\theta)$ is called MIT rule. Hither γ is a positive quantity which represents the adaptation gain of the controller [2],[3],[4],[9]

The choice of cost function is arbitrary. For cost function $J(\theta) = |e|$, the adjustment rule turns

$$\frac{d\theta}{dt} = \gamma\frac{\partial e}{\partial\theta} sign\,(e) \quad (9)$$

where, $sign = +1,\ e > 0$
$\quad = \ 0,\ e = 0$
$\quad = -1,\ e < 0$

Suppose, the plant is linear with transfer function $G_p(s) = KG(s)$, assuming K is unknown and $G(s)$ is the transfer function of the system.

The desired goal is to design a controller so that the plant tracks the reference model $G_m(s)$, where, $G_m(s) = K_0G(s)$ and K_0 is known parameter.

Define control law $U(t) = \theta U_c(t)$ (10)

From Eq. (1),

$$E(s) = KG(s)U(s) - G_m(s)U_c(s)$$
$$= KG(s)\theta U_c(s) - K_0G(s)U_c(s) \quad (11)$$

By taking partial derivative,

$$\frac{\partial E(s)}{\partial\theta} = KG(s)U_c(s) = \frac{K}{K_0}Y_m(s) \quad (12)$$

From Eq. (8) and Eq. (12)

$$\frac{d\theta}{dt} = -\gamma\frac{\partial e}{\partial\theta} = -\gamma e\frac{K}{K_0}y_m = -\gamma' e y_m \quad (13)$$

Eq. (13) gives the law for adjusting the parameter θ. Selection of proper value of γ' results stability of the system.

In the given DC motor, K_0 for reference model also K for plant are selected as 1.5 and 1 respectively.

Hence the transfer function of reference model is obtained as $G_m(s) = \frac{1.5}{0.5s^2 + s + 1.5}$ and transfer function of plant becomes $G_p(s) = \frac{1}{0.5s^2 + s + 1.5}$ [Table. I], [8].

By choosing adaptation gain having the values 0.5, 1, 2, the response of the plant along with the reference model is plotted (Fig. 2).

It is observed, for high value of adaptation gain the system responses fast having high value of overshoots, but for low value of adaptation gain the system responses slow having low value of overshoot. However above certain limit system performance turns extremely poor as indicated by the plots and values of different performance indices, Figs. (3), (4), (5), (6), [Table. II], [8], [10], [11], [12].

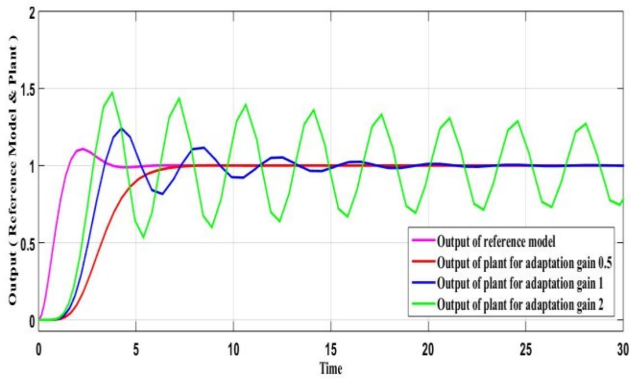

Fig. 2. Tracking performance with MIT Rule

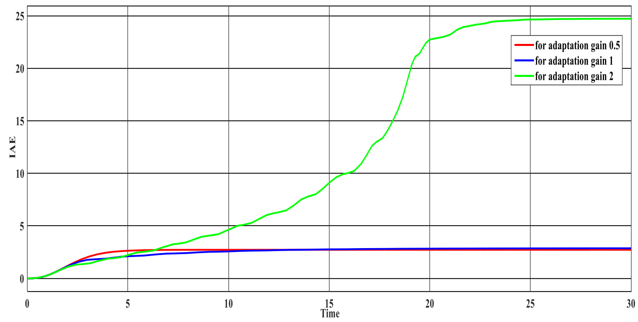

Fig. 3. Plot of IAE using MIT rule

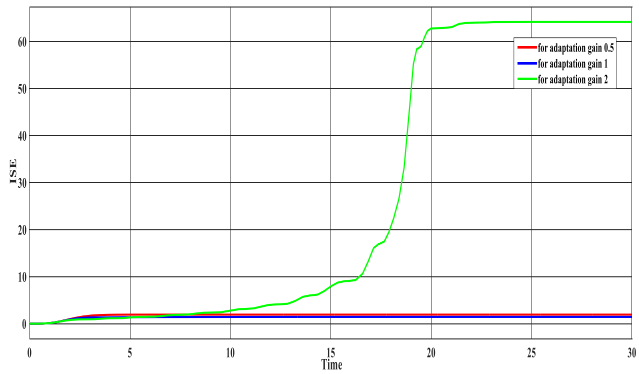

Fig. 4. Plot of ISE using MIT rule

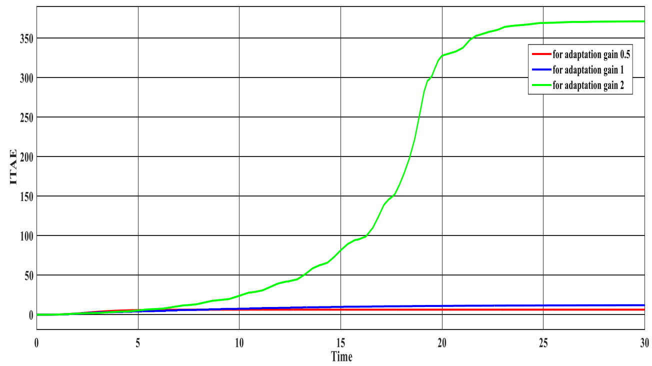

Fig. 5. Plot of ITAE using MIT rule

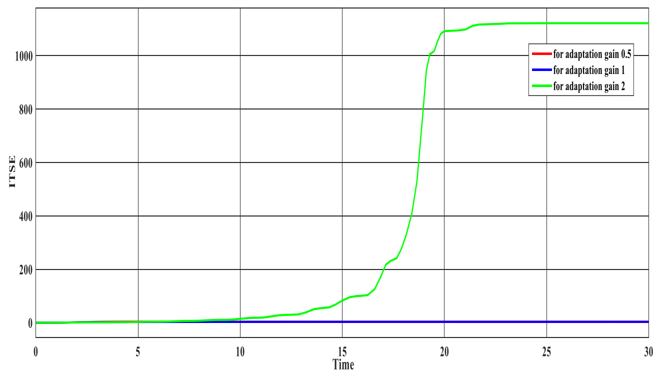

Fig. 6. Plot of ITSE using MIT rule

B. *MRAC using Lyapunov Stability Method*

By applying MRAC with MIT rule stability can't be determined, so to ascertain the system stability MRAC with Lyapunov Stability Theory is employed. Generally it is used for first and second order systems. In this technique different adaptation law is not necessary if there is a change in reference model or plant, unless the performance seems to be insufficient. Here it is chosen a Lyapunov function (V), which is positive definite, so that its derivative along the solution is negative semi definite, then the system becomes stable. If derivative is negative definite, then system is asymptotically stable. In this method adaptation mechanism is in such a way that the error between plant and model output goes towards zero [4], [5],[9],[13].

Suppose the transfer function of the plant is expressed as

$$\frac{d^2y}{dt^2} = -a\frac{dy}{dt} + bu \tag{14}$$

The reference model is expressed as

$$\frac{d^2y_m}{dt^2} = -a_m\frac{dy_m}{dt} + b_m u_c \tag{15}$$

Here, y and y_m denotes plant output and reference model output respectively, u_c represents reference input signal.

Suppose, the control input is given by

$$u = \theta_1 u_c - \theta_2\frac{dy}{dt} \tag{16}$$

Here, θ_1 along with θ_2 represents the control parameters having adjustable gain γ.

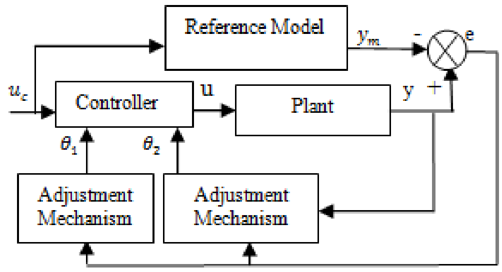

Fig. 7. Block diagram of Lyapunov based MRAC

From Eqs. (1), (14), (15) and (16), it is found that

$$\frac{de}{dt} = -a_m e - (b\theta_2 + a - a_m)y + (b\theta_1 - b_m)u_c \tag{17}$$

It is recognized using error dynamics that, the tracking error turns zero provided,

$$\theta_1 = \frac{b_m}{b} \tag{18}$$

$$\theta_2 = \frac{a_m - a}{b} \tag{19}$$

The Lyapunov Function is chosen as [9], [13]

$$V(e,\theta_1,\theta_2) = \frac{1}{2}[e^2 + \frac{1}{b\gamma}(b\theta_2 + a - a_m)^2 + \frac{1}{b\gamma}(b\theta_1 - b_m)]^2 \tag{20}$$

This Lyapunov function turns zero for the case when error is zero. The controller parameters are equal to the correct values. The time derivative of the said valid Lyapunov function must be negative [9],[13],[14]

$$\frac{dV}{dt} = e\frac{de}{dt} + \frac{1}{\gamma}(b\theta_2 + a - a_m)\frac{d\theta_2}{dt} + \frac{1}{\gamma}(b\theta_1 - b_m)\frac{d\theta_1}{dt} \tag{21}$$

From Eq. (17) and Eq. (21)

$$\frac{dV}{dt} = -a_m e^2 + \frac{1}{\gamma}(b\theta_2 + a - a_m)\left(\frac{d\theta_2}{dt} - \gamma y e\right) + \frac{1}{\gamma}(b\theta_1 - b_m)\left(\frac{d\theta_1}{dt} + u_c e\right) \tag{22}$$

Hence the parameters are updated as given by

$$\frac{d\theta_1}{dt} = -\gamma u_c e \tag{23}$$

$$\frac{d\theta_2}{dt} = \gamma y e \tag{24}$$

Hence, $\theta_1 = -\frac{\gamma}{s} u_c e$ (25)

$$\theta_2 = \frac{\gamma}{s} y e \tag{26}$$

In the present case, by taking adaptation gain having the values 0.5, 1, 2, the responses of the plant as well as the reference model are plotted in (Fig. 8).

From the plot it is recognized that, for high value of adaptation gain the response of the given system turns fast having large value of overshoots, while for low value of adaptation gain the response of the said system becomes slow having low value of overshoot.

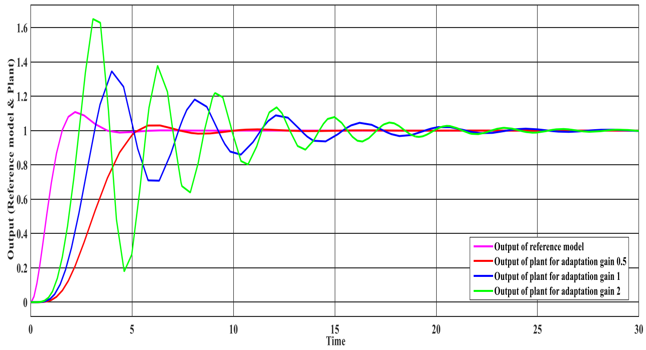

Fig. 8. Tracking performance with Lyapunov method

Here if the adaptations gain of the system increases then the system turns less accurate as indicated by the plot of different performance indices Figs. (9), (10), (11), (12), [Table. II], [10], [11], [12].

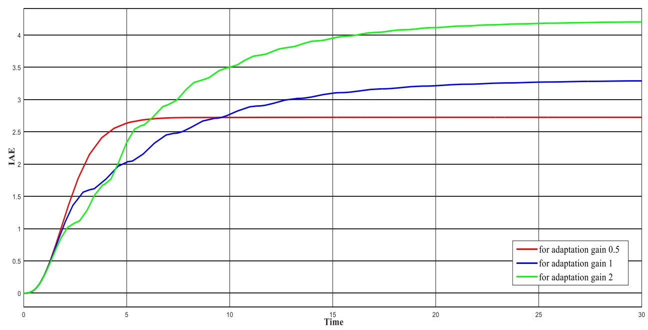

Fig. 9. Plot of IAE using Lyapunov method

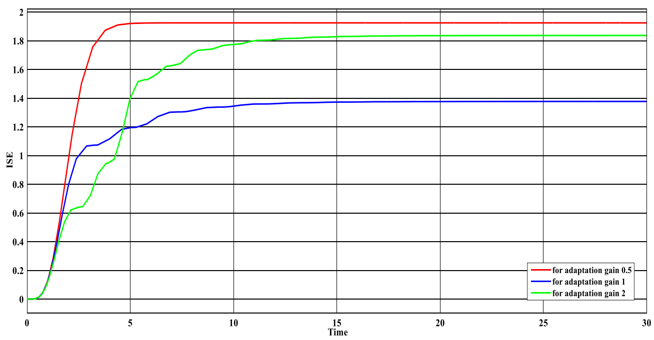

Fig. 10. Plot of ISE using Lyapunov method

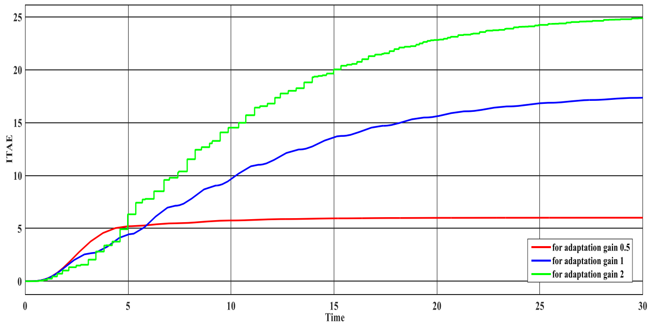

Fig. 11. Plot of ITAE using Lyapunov method

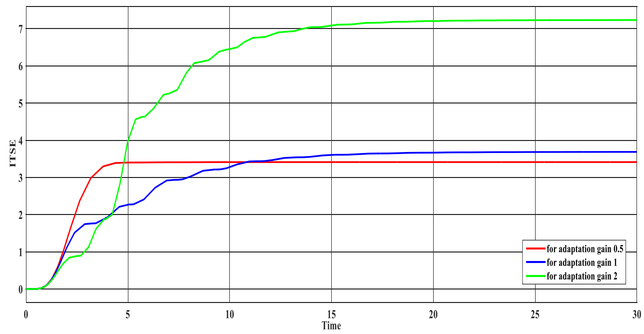

Fig. 12. Plot of ITSE using Lyapunov method

TABLE II. COMPARISON OF DIFFERENT PERFORMANCE INDICES BY APPLYING DIFFERENT ADAPTIVE CONTROL STRATEGIES

Technique used	Performance index	Adaptation gain		
		0.5	1	2
MRAC with MIT Rule	IAE	2.725	2.876	24.76
	ISE	1.925	1.449	64.17
	ITAE	6.408	11.79	369.2
	ITSE	3.978	3.065	1121
MRAC using Lyapunov method	IAE	2.524	3.310	4.22
	ISE	1.706	1.378	1.837
	ITAE	6.006	16.83	24.27
	ITSE	3.413	3.684	7.228

VI. CONCLUSIONS

This paper illustrates about speed control of a DC motor applying MRAC with MIT rule and MRAC using Lyapunov method. In the above methods, by using unit step input and choosing different adaptation gains, the tracking performance as well as the performance indices are studied in MATLAB/SIMULINK environment. It is seen that, for the same adaptation gain, the controller using MRAC using Lyapunov rule tracks the reference model better than MIT rule. In case of MRAC using Lyapunov method, the system is stable in the given range of adaptation gain. With suitable value of adaptation gain, both MRAC using MIT rule and MRAC using Lyapunov method force the tracking error towards zero. From the analysis of performance indices, it is observed that, if the adaptation gain increases, accuracy of the system increases for Lyapunov method and decreases for MIT rule. It can be concluded, the performance of MRAC using Lyapunov method is more acceptable than MRAC using MIT rule.

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