Design and Implementation of MRAC and Modified MRAC technique for Inverted Pendulum

R. J. Pawar, B.J. Parvat
Department of Instrumentation and control,
P.R.E.C. Loni, India
Email: rupalijpawar@gmail.com, pbhagsen@yahoo.com

***Abstract*—This paper is concerned with the combination of model reference adaptive control (MRAC) and PID. Incorporating application for inverted pendulum system, subjected to it finds the analogy with various control system applications like robotic arm, satellite launching system etc. The main aim is to find further improvement of traditional MRAC method and to provide more accurate control to the inverted pendulum and to minimize drawbacks of the traditional MRAC method. This is examined when combining the MRAC method with the PID control. The performance of the application system is examined from the simulation results in MATLAB/SIMULINK. For showing its effectiveness its simulated results are compared with the traditional control strategies like PID and MRAC.**

***Keywords*—Lyapunov Theory, MIT rule, MRAC, Modified MRAC PID etc.**

I. INTRODUCTION

MRAC finds wide applications in linear as well as non linear system. The comparative study of MRAC, modified MRAC and RGA based modified MRAC has been carried out on hybrid tank process [1]. The adaptive controller techniques are used for balancing the inverted pendulum system namely MRAC with lyapunov theory approach and fuzzy learning control technique [2]. The adaptive control is dynamic field of research and industrial applications. It can modify its behavior in response to changes in the process parameters and various disturbances. Hence it plays an important role in control system. Amongst the various types of adaptive controller the model reference adaptive controller is an important adaptive controller which uses the reference model for the adaptation of controller parameters [3]. Though PID algorithm is most popular approach for industrial process control, the major issue is to tune the parameters of PID. The MRAC approach can be used for autotuning of PID controller [6].The MRAC also proved effective on multimodal piecewise affine and piecewise linear system [7]. MRAC also found effective in regulation of air mass coming from internal combustion engine. To cope up with the non linear torques acting on the plant new technique is introduced called novel discrete time MRAC [8]. Direct model reference adaptive internal model controller is proposed which provides variable gain adjustment mechanism [9]. To reduce the problems like large gain requirements, fast actuating requirements as well as improve the noise performance the implementation of MRAC using simultaneous probing, estimation and control is carried out [10]. The application of MRAC is done on sheet metal stamping process, which shows better performance over fixed PI process controller [11]. The single reference model design is replaced with tube model reference adaptive control with the help of two alternative adaptive control schemes, while maintaining the fundamental idea of MRAC [12]. With the help of maximum power tracking algorithm, MRAC is implemented for active and reactive power regulation of grid connected wind turbine based on doubly fed induction generator [13].

II. MODEL REFERENCE ADAPTIVE CONTROL

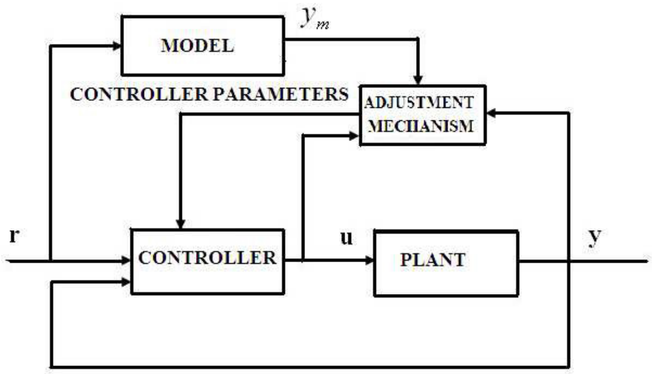

Fig. 1. Model Reference Adaptive Control

The model reference adaptive controller is a control system in which the desired specifications are given in the form of reference model. The schematic diagram of such system is shown in Fig. (1). Basically it consist of two loops, first is for normal feedback control and second loop for controller parameter adjustment. The reference model tells how the process output should give response to the command signal [3][4]. The output of reference model and plant is compared and error between them is given as a feedback through parameter adjustment loop. The parameters of the controller are updated such as to minimize the error till it becomes zero. There are mainly two approaches to implement the MRAC, namely, MIT rule and Lyapunov theory.

A. MIT Rule

We will consider a closed loop system in which controller has one adjustable parameter θ. The desired closed loop response is specified by a model whose output is y_m. Let e be the error between output y of closed loop system and output y_m of

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reference model. The variable control parameter θ is adapted such a way that the cost function,

$$J(\theta)=\frac{1}{2}e^2 \tag{1}$$

is minimized, the given cost function can be minimized if we change the parameter in the direction of negative gradient of J, generally known as gradient descent approach, in the following manner,

$$\frac{d\theta}{dt}=-\gamma\frac{\partial J}{\partial\theta}=-\gamma e\frac{\partial e}{\partial\theta} \tag{2}$$

Equation (2) is known as MIT rule [3].

B. Lyapunov Theory

The fundamental contribution to stability theory for non linear system were made by the Russian mathematician Lyapunov in the end of nineteenth century. Lyapunov investigated the nonlinear differential equation,

$$\frac{dx}{dt}=f(x) \quad \text{where } f(0)=0 \tag{3}$$

As $f(0)=0$ the equation has the solution $x(t)=0$. To guarantee that the solution exist and unique, it is necessary to make some assumptions is that $f(x)$ is locally Lipschitz, that is,

$$\|f(x)-f(y)\|\le L\|x-y\| \text{ where L>0} \tag{4}$$

in the neighborhood of the origin. If there exist a function $V:R^n\rightarrow R$ is positive definite such that its derivative along the solution of equation (3),

$$\frac{dV}{dt}=\frac{\partial V^T}{\partial x}\frac{dx}{dt}=\frac{\partial V^T}{\partial x}f(x)=-W(x) \tag{5}$$

is negative semidefinite, then the solution $x(t)=0$ to equation (3) is stable. If $\frac{dV}{dt}$ is negative definite, then the solution is also asymptotically stable. The function V is called a Lyapunov function for the system (3).

Moreover if $\frac{dV}{dt}<0$ and $V(x)\rightarrow\infty$ when $\|x\|\rightarrow\infty$ then the solution is globally asymptotically stable [3].

Assume that the linear system

$$\frac{dx}{dt}=Ax \tag{6}$$

is asymptotically stable. Then for each symmetric positive definite matrix Q there exists a unique symmetric positive definite matrix P such that

$$A^TP+PA=-Q \tag{7}$$

Furthermore, the function

$$V(x)=x^TPx \tag{8}$$

is a Lyapunov function for equation(6).

III. Modified Model Reference Adaptive Control Scheme

In recent development in MRAC have introduced the another control structure of MRAC called as modified MRAC which consist of combination of MRAC and PID control [1]. This control structure was proposed to improve the transient response of the plant. It can be used with MIT approach as well as Lyapunov approach. The structure of Modified MRAC is shown in Fig. (2).

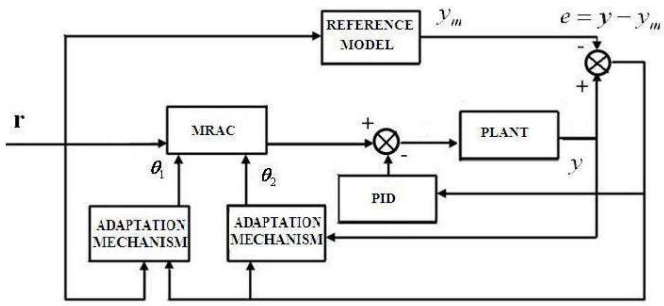

Fig. 2. Stucture of Modified MRAC

e.g. A simple first or second order system having two control parameters θ_1 and θ_2. The MRAC for the system using control law as,

$$u=\theta_1 r-\theta_2 y \tag{9}$$

If Modified MRAC approach is implemented on the same system then the control law u is modified as [1],

$$u=\theta_1 r-\theta_2 y-(k_p e+k_i\int e dt+k_d\frac{de}{dt}) \tag{10}$$

IV. Inverted Pendulum System

The Inverted pendulum system on motor driven cart is shown in Fig. (3).

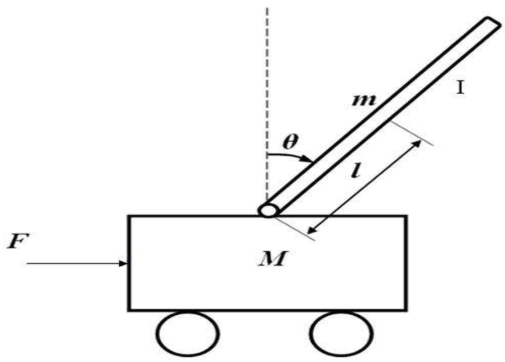

Fig. 3. Inverted Pendulum System

The aim is to keep the shaft in vertical position. As the pendulum is in inverted position is so unstable that it may fall over at any time. control force must be applied to maintain the vertical position of shaft. Here we have considered only two dimensional problem. The control force F is applied to the cart. The center of gravity of the pendulum rod is assumed at its geometric center. Pendulum is assumed as a uniform rod, its moment of Inertia is $I=ml^2/3$. The motion of inverted pendulum is described by two non linear equations given below [2][5]:

$$(M+m)\ddot{X}+ml\ddot{\theta}\cos\theta-ml\dot{\theta}^2\sin\theta=F \quad (11)$$

$$mgl\sin\theta-ml^2\ddot{\theta}-m\ddot{X}l\cos\theta=I\ddot{\theta} \quad (12)$$

Where,
θ= angle of pendulum
F=force applied to the cart
X=position of the pivoting point
m=mass of the pendulum
M=mass of cart
l= distance between the pivot point and center of gravity of pendulum
g=gravitational constant
I=Inertia of pendulum

The model is linearized about equilibrium point θ=0 by using Taylor's series approximation as follows: From Taylor's series expansion the approximation of any function of θ is given by,

$$f(\theta)\approx f(\theta_0)+\varepsilon\frac{df}{d\theta}|\theta_0 \quad (13)$$

$$\cos\theta\approx\cos(0)+\theta[-\sin(0)]=1 \quad (14)$$

$$\sin\theta\approx\sin(0)+\theta[-\cos(0)]=\theta \quad (15)$$

Hence for small angle the following approximations are assumed, $\sin\theta\approx\theta,\cos\theta\approx 1$ and $\theta^2\dot{\theta}\approx 0$

The transfer function model is obtained as follows:

$$\frac{\theta}{F}=\frac{y}{u}=\frac{\frac{-1}{4/3(M+m)l-ml}}{s^2-\frac{(M+m)}{4/3(M+m)l-ml}g} \quad (16)$$

where y=θ is angle of the pendulum and u=F is force applied on cart.

The linear state space model is obtained as follows [2]:

$$\dot{x}_1=x_2 \quad (17)$$

$$\dot{x}_2=\frac{g}{\frac{4}{3}l-\frac{ml}{M+m}}x_1-\frac{1}{(M+m)(\frac{4}{3}l-\frac{ml}{M+m})}u \quad (18)$$

$$y=x_1 \quad (19)$$

Where x_1=θ is the angle of pendulum, x_2 is the rotational speed of the rod, u=F is the input to the system and y=x_1 is the systems output. The plant parameters values selected as shown in Table (1) [2].

Table 1. Plant Parameter Values

Parameters	Values
M	1 kg
m	0.5 kg
g	9.81m/s^2
l	0.5 m
I	0.0833 kg.m^2

A. MRAC for Inverted Pendulum system

The reference model is selected as follows: The standard second order differential equation is given by,

$$\frac{y_m(s)}{r(s)}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \quad (20)$$

To obtain critically damped response we select $\omega_n=3$ rad/sec and $\zeta=1$ [2].

The same reference model is used for both approaches of MRAC namely MIT rule and Lyapunov theory.

1) MRAC using MIT rule:

The control law selected as,

$$u=\theta_1 r-\theta_2 y \quad (21)$$

where θ_1 and θ_2 are controller parameters. The transfer function obtained of Inverted Pendulum system is,

$$\frac{y}{u}=\frac{\frac{-1}{4/3(M+m)l-ml}}{s^2-\frac{(M+m)}{4/3(M+m)l-ml}g} \quad (22)$$

The above equation is simplified as,

$$\frac{y}{u}=\frac{-b}{s^2-a} \quad (23)$$

By substituting the control law in above equation we obtain,

$$y=\frac{-b\theta_1 r}{s^2-(b\theta_2+a)} \quad (24)$$

by comparing above equation with equation(20), we get,

Error! Bookmark not defined. $\theta_1=-\omega_n^2/b$ (25)

$$\theta_2=-\left(\frac{\omega_n^2+a}{b}\right) \quad (26)$$

this means that controller parameters should converge to these values would result the perfect model following.

By using MIT rule the controller parameters equations obtained as,

$$\frac{d\theta_1}{dt}=\gamma_1 e\frac{br}{s^2-(b\theta_2+a)} \quad (27)$$

$$\frac{d\theta_2}{dt}=-\gamma_2 e\frac{by}{s^2-(b\theta_2+a)} \quad (28)$$

where γ_1 and γ_2 are adaptation gains for controller parameters θ_1 and θ_2 respectively.

2) MRAC using Lyapunov Theory:

This section is concerned with implementation of MRAC on Inverted Pendulum System. The general strategy while designing MRAC for general linear system using state space includes selection of controller structure, derivation of error equation and selection of Lyapunov function which is used to derive a parameter updating laws such that error will go to zero [2][3].

The reference model is given by following state equation [2],

$$\dot{x}_m = A_m x_m + B_m r \quad (29)$$

Where, $x_m = \begin{bmatrix} y_m \\ \dot{y}_m \end{bmatrix}$

$$A_m = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \text{ and } B_m = \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix}$$

The selection of control law is done by considering nature of the plant and the reference model. Therefore three parameters θ_1, θ_2 and θ_3 are selected for the controller of Inverted Pendulum system. The control law takes the following form [2],

$$u = \theta_1 r - \theta_2 y - \theta_3 \dot{y} \quad (30)$$

For $\theta_1 = \theta_2$ the controller becomes proportional derivative controller with derivative component on feedback loop [2],

$$i.e. \quad u = \theta_1 e - \theta_3 \dot{y} \quad (31)$$

applying control law to plant and substituting values of plant parameters it takes the following form [2],

$$A = \begin{bmatrix} 0 & 1 \\ \dfrac{0.66\theta_2 + 9.8}{0.5} & \dfrac{0.66\theta_3}{0.5} \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ -\dfrac{0.66\theta_1}{0.5} \end{bmatrix}$$

The tracking error represents the deviation of plant output from desired output which is given by,

$$e = y - y_m \quad (32)$$

The following differential equation describes tracking error,

$$\dot{x}_e = A_m x_e + (A - A_m)x + (B - B_m)r \quad (33)$$

Where, $x_e = \begin{bmatrix} e \\ \dot{e} \end{bmatrix}$

The most important part is selection of Lyapunov equation. For this system let us select following function which is positive definite [3],

$$V = \frac{1}{2}\{\gamma x_e^T P x_e + tr\{(A - A_m)^T (A - A_m)\} + tr\{(B - B_m)^T (B - B_m)\}\} \quad (34)$$

According to Lyapunov theory, the equilibrium point $x_e = 0$ is asymptotically stable if, $V(0) = 0$, V is positive definite and $\dot{V}$ is negative definite.

The derivative of the function V is obtained as,

$$\dot{V} = -\frac{1}{2}\gamma x_e^T Q x_e + tr\{(A - A_m)^T (\dot{A} + \gamma P x_e x^T)\} + tr\{(B - B_m)^T (\dot{B} + \gamma P x_e r^T)\} \quad (35)$$

Where, $A_m^T P + P A_m = -Q \quad (36)$

To satisfy the Lyapunov equation following adaptation laws are chosen,

$$\dot{A} = -\gamma P x_e x^T \quad (37)$$

$$\dot{B} = -\gamma P x_e r^T \quad (38)$$

Solving equation(36) we get symmetric matrix,

$$p = \begin{bmatrix} \dfrac{1}{\omega_n}\left(\zeta + \dfrac{1 + \omega_n^2}{4\zeta}\right) & \dfrac{1}{2\omega_n^2} \\ \dfrac{1}{2\omega_n^2} & \dfrac{1}{4\zeta\omega_n}\left(1 + \dfrac{1}{\omega_n^2}\right) \end{bmatrix}$$

solving adaptation laws controller's parameters are obtained as follows:

$$\dot{\theta}_1 = \gamma_1 [P_{12} e + P_{22} \dot{e}] r \quad (39)$$

$$\dot{\theta}_2 = -\gamma_2 [P_{12} e + P_{22} \dot{e}] y \quad (40)$$

$$\dot{\theta}3 = -\gamma_3 [P_{12} e + P_{22} \dot{e}] \dot{y} \quad (41)$$

Where P_{12}, P_{22} are elements of matrix P and γ_1, γ_2, γ_3 are adaptation gains.

3) Modified MRAC:

In this section we discuss about implementation of Modified MRAC scheme on inverted pendulum system. As per discussion in previous section Modified MRAC is nothing but combination of control strategy of MRAC and PID controller, here we combine the control law of MRAC using MIT rule as well as Lyapunov theory discussed in earlier section with PID control law. Therefore the controller law takes the following form [1]:

In case of MIT rule,

$$u = \theta_1 r - \theta_2 y - (k_p e + k_i \int e dt + k_d \frac{de}{dt}) \quad (42)$$

In case of Lyapunov theory,

$$u = \theta_1 r - \theta_2 y - \theta_3 \dot{y} - (k_p e + k_i \int e dt + k_d \frac{de}{dt}) \quad (43)$$

V. SIMULATION AND RESULT

In this section the simulation results of PID, MRAC as well as modified MRAC are compared.

As the plant model is open loop unstable, hence Ziegler Nichols PID tuning method can not be used. Therefore, the tuning of the PID controller is carried out by SISO design toolbox *i.e.* sisotool of MATLAB using robust response time tuning algorithm. The values of PID gains are obtained as follows: k_p=-343.82, k_i=-175.33 and k_d=-20.32.

Fig. (4) shows response of various controller for unit step input. Table (2) shows the comparative analysis for PID, MRAC and modified MRAC for unit step input.

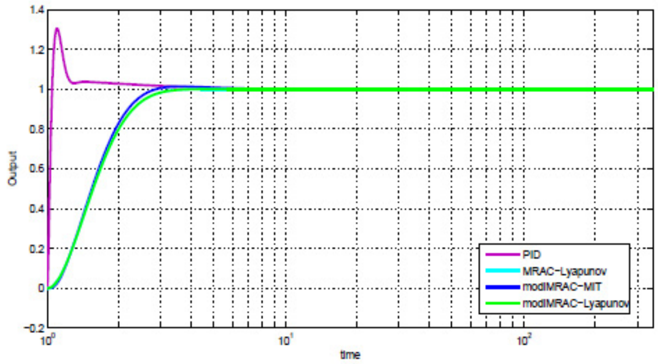

Fig. 4. Simulation result for unit step input

Table. 2. Comparative analysis of PID, MRAC and modified MRAC for unit step input

Controller	Rise time (sec)	Settling time (sec)	Peak Value	Peak time (sec)
PID	30.86	212.02	1.304	60
MRAC-MIT	-	-	-	-
MRAC-Lyapunov	154.48	348.41	1.0001	856
Modified MRAC –MIT	37.58	68.03	1.0132	107
Modified MRAC - Lyapunov	101.25	271.99	1.0001	682

The attempt is made to simulate inverted pendulum system with MRAC using MIT approach but MIT rule approach is unable to handle this system alone due to limited range of adaptation gains. Therefore attempt have been made with modified MRAC. In case of modified MRAC using MIT rule adaptation gain values are γ_1=1 and γ_2=-1. In case of MRAC using lyapunov theory the best parameter values obtained are, γ_1=90, γ_2=-700000 and γ_3=-8000.

Fig.(5), Fig.(6) and Fig.(7)shows response of MRAC and modified MRAC using MIT and Lyapunov approach respectively. For the application of square wave with amplitude of 0.25 and frequency 0.5 rad/sec.

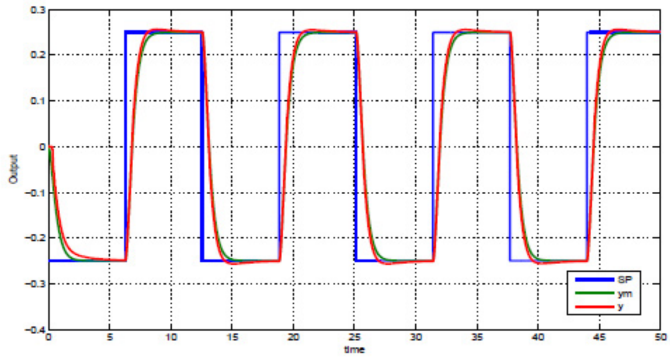

Fig. 5. Simulation Result with MRAC alone with square wave input (Lyapunov Theory)

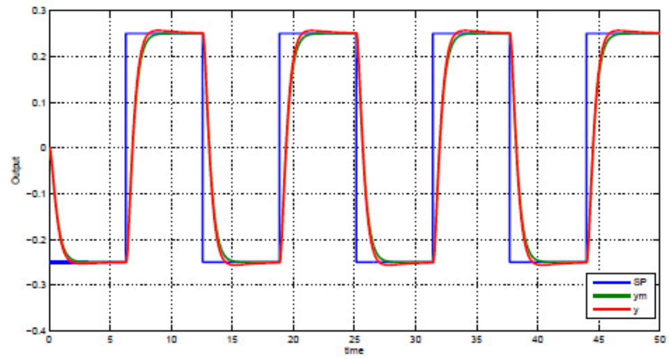

Fig. 6. Simulation Result with modified MRAC with square wave input using MIT rule

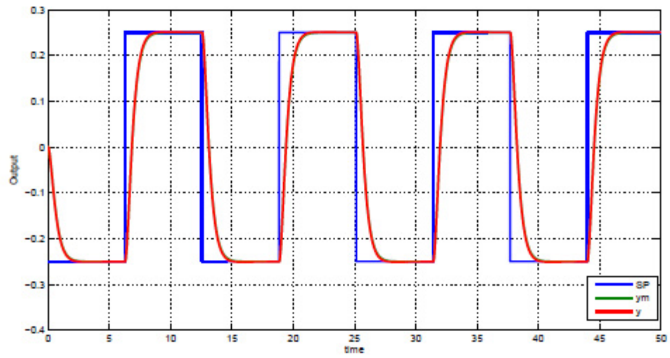

Fig. 7. Simulation Result with modified MRAC with square wave input using Lyapunov theory

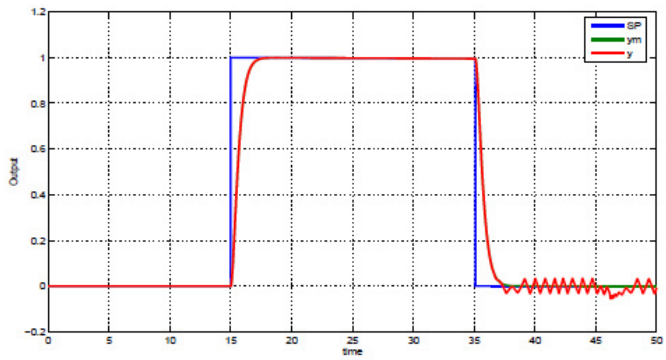

Fig. 8. Simulation Result with MRAC alone (Lyapunov theory) for pulse input

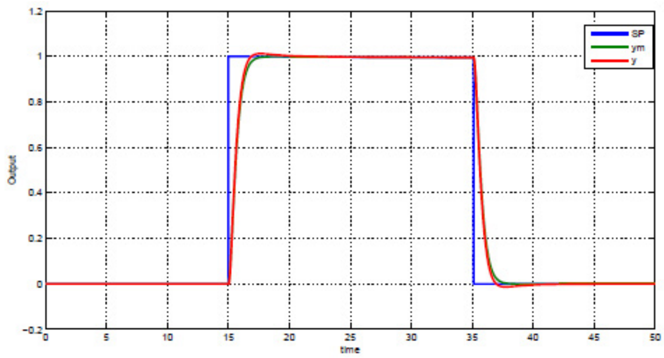

Fig. 9. Simulation Result with modified MRAC for pulse input using MIT rule

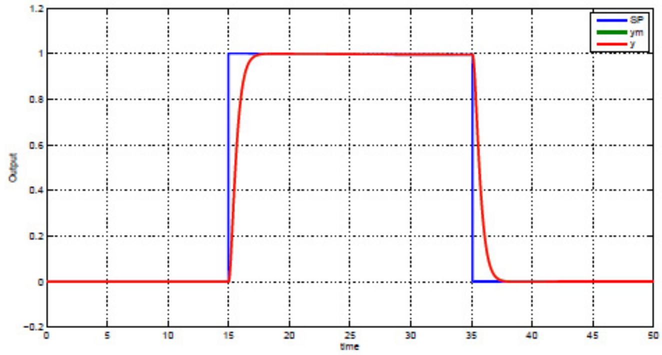

Fig. 10. Simulation Result with modified MRAC for pulse input using Lyapunov theory

Fig.(8), Fig.(9) and Fig.(10) shows response of MRAC and modified MRAC respectively for the application of pulse with amplitude 1 and duration of 20 seconds.

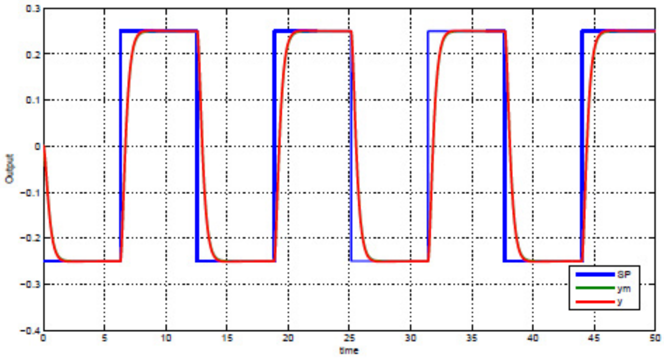

Fig. 11. Simulation Result with modified MRAC for change in plant parameters using Lyapunov approach

The change in the value of plant parameters is made and also desired natural frequency of oscillation changed to ω_n =4 rad/sec. The response of modified MRAC with lyapunov approach is shown in Fig. (11).

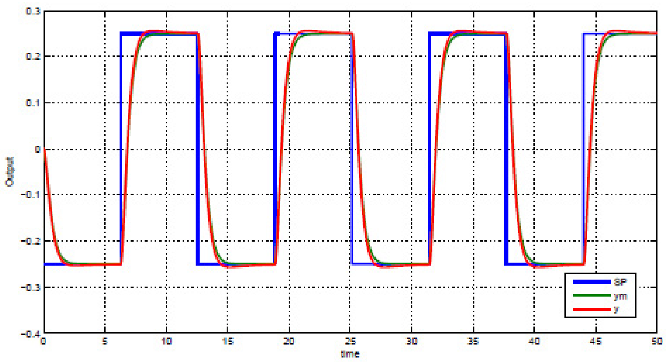

Fig. 12. Simulation Result with modified MRAC for smaller adaptation gains using Lyapunov approach

The values of controller adaptation gains are reduced as γ_1=90, γ_2=-70 and γ_3=-0.8. The response of modified MRAC with reduced gains is shown in Fig. (12).

VI. CONCLUSION

The concept of modified MRAC implemented on Inverted Pendulum System. The comparison between PID, MRAC and modified MRAC has been carried out, which shows that, due to minimum range of adaptation gains, MRAC using MIT rule alone is unable to control this system and MRAC with Lyapunov's approach requires higher values of adaptation gains. Though PID and MRAC alone not suitable for this kind of nonlinear system, but the combination of both working satisfactory. The modified MRAC shows marked improvement in transient response specifications, perform well even at reduced adaptation gain values as well as in presence of disturbances like change in plant parameter values, change in desired specification etc. which proves its robustness.

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