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Indirect Model Reference Adaptive Control with Online Parameter Estimation

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INDIRECT MODEL REFERENCE ADAPTIVE CONTROL WITH ONLINE AIRCRAFT PARAMETER ESTIMATION

By

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ABSTRACT

Over the years, parameter estimation has focused on approaches in both the time and frequency domains. The parameter estimation process is particularly important for aerospace vehicles that have considerable uncertainty in the model parameters, as might be the case with unmanned aerial vehicles (UAVs). This thesis investigates the use of an Indirect Model Reference Adaptive Controller (MRAC) to provide online, adaptive estimates of uncertain aerodynamic coefficients, which are in turn used in the MRAC to enable an aircraft to track reference trajectories. The performance of the adaptive parameter estimator is compared to that of the Extended Kalman Filter (EKF), a classical time-domain approach. The algorithms will be implemented on simulation models of a general aviation aircraft, which would be representative of the dynamics of a medium-scale fixed-wing UAV. The relative performance of the parameter estimation algorithms within an adaptive control framework is assessed in terms of parameter estimation error and tracking error under various conditions. It was found that limitations exist with the adaptive update laws in terms of number of parameters estimated within the Indirect MRAC system. The Indirect MRAC-EKF was determined to be a viable option to estimate multiple parameters simultaneously.

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NOMENCLATURE

A	System Matrix
A_{RM}	System Reference Model Matrix
B	Input Matrix
B_{RM}	Reference Input Matrix
x	System States
x_{RM}	Reference System States
x_a	Augmented States
h_a	Observation Matrix
Γ	Control Distribution Matrix
K	Kalman Gain
λ	Forgetting Factor
P	Covariance Matrix
Q	Process Noise Covariance
v	Process Noise
R	Measurement Noise Covariance
w	Measurement Noise
K_x	Control Gain
K_u	Control Gain
u_s	Control Input
u_r	Reference Input

1. Introduction

Medium scale UAV development has been plagued with the challenge of implementing an efficient and cost-effective growth structure. The challenge arises from the general cost intensive nature of the majority of the aircraft development processes for manned aircraft, including extensive wind tunnel and flight test programs. To address these difficulties, many strategies have been administered within the UAV development phase. One of the notable strategies that has been implemented is Model Reference Adaptive Control (MRAC).

1.1. Indirect Model Reference Adaptive Control (MRAC)

Model Reference Adaptive Control (MRAC) is designed to automatically tune the controller parameters to control the response of the system (Zhang, Pan, & Zeng, 2018). The control response is then used to track the desired characteristics of the reference model. An MRAC can either be distinguished as indirect control or direct control. Indirect control occurs when parameters or state variables of the unknown plant are estimated and in turn used to adjust the controller; control systems defined in such a manner are sometimes referred to as self-tuning regulators (Narendra & Valavani, 1979; Astrom & Wittenmark, 1973). There is no explicit identification in direct control; however, the error between the plant and the reference model consistently updates to zero, an approach typically called model reference adaptive control (Narendra & Valavani, 1979; Landau, 1974). MRAC provides a means for developers to design an aircraft that meets the general requirements within the natural system, while at the same time improving the system dynamics through the tracking of a reference model that represents the ideal dynamics.

Typically, in aircraft development, significant hours in a wind tunnel or in flight test are necessary in order to determine an aircraft's aerodynamic stability derivatives. This process is notably expensive. The costly nature of wind tunnel tests has resulted in many aircraft designs not coming to fruition. An indirect MRAC strategy has the ability to reduce the numbers of hours needed in a wind tunnel. With the MRAC approach, designers can potentially manufacture aircraft with parameters that are not known with a high level of accuracy. Unknown or uncertain parameters can then be estimated through an adaptive estimation process. The indirect MRAC, through the reference model, can be used to steer an aircraft towards specified dynamic characteristics. For instance, it can be applied in scenarios where an aircraft would need to maneuver more aggressively. The reference aerodynamic parameters in such an instance would need to reflect the aggressive nature of the dynamics required from the system. The same process would be applied if an aircraft required more subtle characteristics within a specified flight envelope. As it relates to UAV modeling, the Indirect MRAC would also provide clarity to these systems which often have more uncertainty. Therefore, the Indirect MRAC has the potential to make aircraft design more efficient and also provide the ability to develop flight controllers that can operate with uncertain parameters over a range of flight conditions.

1.2. Objectives and Methodology

This thesis will investigate the parameter estimation performance of the EKF and an adaptive estimator within the Indirect MRAC framework. The Indirect MRAC in its original form adaptively estimates specified parameters within the dynamic system; however, the use of traditional estimation approaches such as the EKF may be found to provide more robust and accurate parameter estimation and, as a result, better controller performance. Therefore, the main objective of the thesis to determine whether the EKF algorithm is superior to the adaptive estimator in terms of its parameter estimation performance and tracking error under various conditions.

1.2.1. Parameter Estimation Algorithms

Aircraft parameter estimation can be defined as the process of estimating aerodynamic coefficients with respect to motion variables as well as flight control variables (Chaunhan & Singh, 2017). In other words, the output of an aircraft's dynamic system can be used to better determine uncertain aerodynamic parameters within the system. While parameter estimation algorithms are commonly implemented in the time and frequency domains, this thesis will focus on time domain approaches. There have primarily been two time domain parameter estimation approaches used over the years. First, a traditional approach such as the Recursive Least Squares (RLS) has long been an accepted means of estimating parameters in the time domain. The simple nature of its approach, along with its ability to adjust to varying requirements, allows for many types of implementations. In its simplest form, the RLS is quite capable in its estimation abilities, but a robust form of RLS is often implemented for online applications. Robust forms of the RLS is comprised of the use of a forgetting factor to address the use of sampling data. Also, configurations can include the use of a weighting matrix to ensure that each parameter is on the same order of magnitude during each update. The simple and malleable nature of the RLS makes it quite suitable to implement in many aircraft control system architectures.

The second time domain traditional approach to parameter estimation is the Extended Kalman Filter (EKF). The EKF is a well-known algorithm that stems from the general structure of a simple Kalman Filter, which was originally derived as an optimal estimator for linear systems subject to Gaussian white process noise and measurement noise. The Kalman Filter, in its various forms, is well known for its estimation abilities and is widely used in industry. Given the nonlinear formulation that is often used in parameter estimation, the EKF has been a primary candidate for application to nonlinear systems. A more recent approach to parameter estimation has been adaptive estimation which is within the MRAC. Early indications suggest that adaptive estimation, which is derived from stability analysis, can potentially be competitive when compared to traditional time domain approaches. With that said, traditional time domain estimation algorithms will be the primary focus of the thesis.

1.2.2. Indirect Model Reference Adaptive Control (MRAC)

A lateral-directional aircraft model of the Cessna 182 aircraft is used for the simulation studies presented in the thesis. The C182 model represents the lateral-directional dynamics of a medium scale fixed-wing unmanned aerial vehicle (UAV). Within the Indirect MRAC formulation, the baseline system will consist of the aircraft's intrinsic dynamics, which will include uncertain aerodynamic stability derivatives that are not well known. The reference model dynamics within the Indirect MRAC formulation will serve as a basis for the desired dynamics for the natural system. As the control system seeks to attain the reference model's dynamics, various parameter estimation techniques will be used to continuously improve the estimation of selected parameters.

The MATLAB/Simulink environment is used to design and implement the Indirect MRAC. Both the EKF and the adaptive estimation methods are implemented during the indirect MRAC simulations. The thesis seeks to investigate the use of the EKF and adaptive estimation algorithms within the Indirect MRAC, and compare the performance in terms of metrics based on parameter estimation and tracking. To replicate real-word sensor measurements, Gaussian noise is added to the sensor measurements before they are used for parameter estimation.

1.3. Organization of Thesis

A review of the literature is first provided along with how each estimation technique has been implemented into various systems. Conclusions can then be made about how to move forward given the current literature. In Chapter Three, a detailed description of the RLS, EKF and Adaptive Estimation algorithms is presented. A brief discussion about the necessity of persistence of excitation is also presented. Chapter Four introduces a thorough formulation of the lateral-dynamics model to be used for the EKF and adaptive estimation algorithms; as well as their implementation in the Indirect MRAC algorithm. The formulation includes the lateral-directional equations of motion, the augmented model and the linearization process required for the EKF implementation, and the Indirect MRAC implementation. Chapter Five presents simulation results of the EKF and adaptive estimation techniques and their relative performance within an indirect model reference adaptive control framework. Conclusions about the results are then discussed in Chapter Six along with further points for investigation in future work.

2. Literature Review

This chapter examines the current parameter estimation literature. Various approaches are assessed and conclusions are made about the best approaches to implement various estimation techniques.

2.1. State of Parameter Estimation

The parameter estimation literature extends as far back as the 1970s, with some of the earlier research originating from NASA. One of the many focuses at the time was to implement parameter estimation that is capable of being used in conjunction with an adaptive control system (Calise, Lee, & Sharma, 1998). Notable benefits of such a model included the ability for reconfigurable control and efficient flight testing. At the same time, the overarching goal presented sizeable challenges given the real-time nature needed for its application.

The techniques involved in parameter identification vary across the literature, with a few similarities. The use of a least squares method has been a common factor which offers an insight into the most efficient and effective parameter identification algorithms. While a least squares method and an extended Kalman filter may be beneficial, the time domain nature of these approaches faces notable challenges. For instance, the RLS does not allow for a noise structure to be modelled in the system (Ljung, 1976), and the EKF can potentially be computationally intensive (Seo, Kim, & Saderla, 2019). When compared to the Discrete Fourier Transform (DFT) used in frequency domain methods, it is noted that the hardware requirements are less invasive for the DFT (Manry & Huddleston, 1987).

2.1.1. Earlier Studies

Earlier reports, such as research from (Nikolaev, Teryaev, & Shamrikov, 1979), implemented algorithms based the least squares method. (Nikolaev, Teryaev, & Shamrikov, 1979) discusses both the properties of the estimator and also the computational procedures involved. The key with (Nikolaev, Teryaev, & Shamrikov, 1979) parameter identification is that the method depends on input and output data used in multiple aspects. One of these is the uniqueness of the solution. As it relates to the computational procedures involved, it has been determined that the algorithm is an effective means to limit data accumulation time while also increasing the confidence of the results. Limiting the data accumulation has been a pivotal subject in ensuring the progression of parameter estimation.

2.2. The Limited Least Squares Method

A more recent investigation into parameter identification is explored by (Wei, Yang, Zhang, & Shen, 2013). (Wei, Yang, Zhang, & Shen, 2013) also focuses on the limited recursive least squares method, and in addition the online application in the case of a damaged wing is investigated. The identification of mutational parameters was also studied, as well as exploring the nature in which lateral and longitudinal components become highly coupled. The limited recursive least squares method focuses on three main sections. The first is the Forgetting Factor, which addresses the issue of data accumulation. Old identification results may lead estimates far away from their true dynamic characteristics. The forgetting factor is used to address the accumulation of sampling data by only using data that is relevant to the current estimation. The second is the Limit Item, which makes use of transcendental experience that is stored offline. The last section of focus is the Weighting Matrix, which ensures that all limit parameters are of the same order of magnitude. (Wei, Yang, Zhang, & Shen, 2013) addressed the issue of data accumulation; in addition, in an earlier study of the recursive least squares parameter estimation method (Basappa & Jategaonkar, 2004), it is noted that a judicious choice of the Forgetting Factor is necessary to ensure good performance.

2.3. The Extended Kalman Filter

Variations of the Kalman Filter have continuously progressed across academia. Each variation is focused on addressing a specific issue within a system, consequently making the algorithms more robust. In the case of parameter estimation, the Extended Kalman Filter (EKF) has consistently been used. The EKF was primarily designed for nonlinear system estimation and filtering. Therefore, the EKF is well suited for the nonlinear nature of the augmented system required for implementation. (Grillo & Montano, 2015) focused on the EKF's implementation on a coupled longitudinal and lateral dynamics model, an approach that is interesting because most parameter estimation strategies have focused on first decoupling the system. The issue of tuning, which is often an argument brought against time-domain techniques such as the RLS and EKF, is also addressed. (Grillo & Montano, 2015) argues that tuning can be implemented through the effects of dynamic derivatives, derivatives that are also estimated in the model. In the case of UAV development, (Grillo & Montano, 2015) also notes that the use of the EKF can present considerable cost savings and efficiency in development.

2.4. Major Challenges and Resolutions

(Basappa & Jategaonkar, 2004) presented a comparison study of the RLS, EKF and frequency domain methods applied to aircraft parameter identification. The comparisons focused on the accuracy of the parameter identification, robustness in the presence of noise and lastly the computational effort involved. Noise presents a special issue in the real-time application of these systems. The problem occurs because of the fairly long data sets that are often needed for parameter estimation. A balance must be found to ensure that data accumulation is not excessive but instead sufficient to distinguish between required data and noise. (Morelli, 2000) suggests two methodologies to address the issue. The first is the use of a recursive least squares method alongside a forgetting factor, a fact which once again proves the robustness of the least squares method. Secondly, it was suggested that an extended Kalman filter be used. However, it was noted that the downside of the extended Kalman filter was that it required significant tuning during its usage. Nonetheless, (Morelli, 2000) cautions against the use of a time-domain approach, concluding that such an approach can lead to inaccurate results because of the low signal to noise ratio.

Given the difficulties of the time-domain approach, (Morelli, 2000) focused his research on a different approach centered around the frequency domain. In real-time, a Recursive Fourier Transform alongside an equation error was used for analysis. The flight test data from the research showed that the frequency domain approach produced accurate parameter estimation within reasonable margins. It was also concluded that the approach has lower computational requirements, which effectively made it suitable for real-time aircraft application. In later studies, such as (Morelli & Grauer, 2018), greater focus was placed on understanding the advantages of the frequency domain methods. Foremost, the basis of the method depended on the Finite Fourier Transform, which is important since the data must be accurately transformed from the time domain to the frequency domain. As it relates to advantages, the research first showed that the frequency domain was able to readily provide a physical insight into particular types of dynamics. Secondly, it was also found that its applicability was more direct to common control design techniques, while at the same time being able to maintain a lower dimensionality for model parameter estimation.

2.5. More Recent Studies

Over the years, significant study has been placed on understanding the least squares method and its applicability in adaptive control and online parameter estimation. It can be surmised that it is indeed a resilient approach, especially in the presence of a forgetting factor, limit item and weighting matrix, as concluded by (Wei, Yang, Zhang, & Shen, 2013). However, the benefits of a frequency domain approach are unprecedented in terms of not only the physical insight obtained, but also the benefits of reduced computational requirements. (NASA, 1973) at the time understood that most parameter estimation techniques required extensive calculations, which meant that existing hardware could often not handle such calculations.

2.5.1. The Introduction of Adaptive Control

Later studies published by (Hageman, Smith, & Stachowiak, 2003) from NASA notes the advances in computational power, which have now made real-time online parameter estimation possible. Along with real-time parameter estimation came the development of adaptive control, which in turn resulted in the possibility of significant improvement of in-flight aircraft dynamics. (Nguyen N. , 2011), however, discussed many difficulties associated with the implementation of adaptive control techniques. The research makes a notable point about the robustness that is needed in the presence of unmodeled dynamics and disturbances. It also states that the system must be able to adequately adapt in the presence of actuator rate and position limits. With adaptive control implementations, it should be noted that there still does not exist a formal certification process for aircraft flight controllers (Balas, Noriega, & Anderson, 2017). Therefore, the fidelity across various adaptive control systems may not be standardized. In such a case, the onus falls upon individual adaptive control systems to ensure that the necessary requirements are met with sufficient margins.

Even without the formal certification process, research into adaptive control implementations on aerospace systems has continued. (Prabhakar, Painter, Prazenica, & Balas, 2018) focused on a direct adaptive control system that enables UAVs to track reference trajectories within adverse operating conditions, such as urban environments where the vehicle is consistently exposed to external disturbances. Research has also extended to space vehicles. (Tiwari, Prazenica, & Henderson, 2020) developed a direct adaptive controller that is implemented on a space vehicle dynamic model to track hovering and orbital trajectories in the vicinity of an asteroid. Therefore, numerous examples of adaptive control applied to aerospace systems can be found in the literature. However, as it relates specifically to indirect model reference adaptive control, (Kersting & Buss, 2017) notes that the pseudo inverse within the algorithm remains a major obstacle as it can introduce singularities into the system.

2.6. Summary

In summary, most of the literature has focused on the progression and benefits of time-domain parameter estimation approaches. Specifically, variations in the least squares methods and Kalman Filters have allowed for online estimation of system parameters. At the same time, the computationally intense nature of the RLS and EKF algorithms remains a challenge especially when compared to frequency domain approaches such as the DFT. With that said, the literature has not focused much on other approaches such as adaptive estimation through the use of an MRAC. Also, estimation of parameters through the use of the RLS and EKF inside an adaptive control framework has not been thoroughly explored. It is the goal of this thesis to further explore adaptive estimation, along with traditional estimation approaches such as the EKF, within an indirect model reference adaptive control framework.

3. Parameter Estimation Methods

Many parameter estimation methods exist across the literature, each with varying advantages and disadvantages. Chapter Three presents the RLS and EKF as two traditional approaches to parameter estimation in the time domain. The descriptions of these methods and their subsequent implementation is based on (Basappa & Jategaonkar, 2004). The necessity of persistence of excitation is also discussed along with its importance to parameter estimation. Lastly, the Indirect MRAC to be utilized in simulation is outlined and examined for a simplified aircraft model.

3.1. Recursive Least Squares (RLS)

The RLS algorithm is simply an extension of the more commonly known least squares (LS) method. Whereas the LS method uses all available data in its estimation process, the RLS instead uses small sets of data to recursively update its estimate. The general form of the RLS algorithm is commonly defined as follows:

$$\theta_{k+1} = \theta_k + K(y_{k+1} - \hat{y}_{k+1}) \tag{3.1}$$

The residual error $(y_{k+1} - \hat{y}_{k+1})$, which is defined as the difference between the output (sensor) measurement y_{k+1} and the current computed output $\hat{y}_{k+1}$, is multiplied by a gain K which gives the direction of the update. That update is then added to the previous estimate θ_k to finally determine the current estimate θ_{k+1}. Defining the parameter update in such a manner allows for not only an online application but also an offline application where k can be defined as the step within a set number of iterations.

The research focuses on implementing the RLS algorithm on a dynamic system where the state equations are arranged into augmented states. The augmented states are comprised of not only the state variables but also the parameters to be estimated. A dynamic system consisting of this formulation is defined below:

$$x_k = \phi_k x_{k-1} + v_k, \quad k = 0,1, \ldots, \tag{3.2}$$

$$y_k = H_k x_k + w_k, \quad k = 0,1, \ldots, \tag{3.3}$$

where ϕ_k represents the state transition matrix, and x_k represents a vector of the states and parameters to be estimated, which are both varying with time. H_k is defined as the observation matrix, and lastly v_k and w_k are defined as process and measurement noise in the system.

An outline from both (Basappa & Jategaonkar, 2004) and (Zhu, 1999) summarizes the RLS algorithm. First, the algorithm requires that both the linearized augmented dynamics and covariance matrix be initialized. Initialization of the augmented states can take place at the trim values or the values that were previously obtained from wind tunnel or flight tests. It should also be noted that the covariance matrix is typically initialized to fairly high values, on the order of $10^3 I$, where I represents the identity matrix. The next step in the algorithm is to choose a constant value for the forgetting factor λ. The choice of λ is heavily dependent on the dynamic system being tested, but as a general rule values ranging between 0.98 and 0.995 are accepted for most implementations. It should be noted that, for cases in which the dynamics of the system are not rapidly changing, a smaller value for λ can be used. The algorithm then requires that the input and output of the system be collected, after which the state transition matrix ϕ_k is updated. The RLS algorithm is recursively implemented as follows:

$$K_k = P_{k-1}\phi_{k-1}^T H_k^T (\lambda I + H_k \phi_{k-1} P_{k-1} \phi_{k-1}^T H_k^T)^{-1} \tag{3.4}$$

$$\epsilon_k = y_k - H_k \phi_{k-1} x_{k-1} \tag{3.5}$$

$$x_k = \phi_{k-1}[x_{k-1} + K_k(\epsilon_k)] \tag{3.6}$$

$$P_k = \lambda^{-1} \phi_{k-1}(I - K_k H_k \phi_{k-1}) P_{k-1} \phi_{k-1}^T \tag{3.7}$$

The gain matrix K_k is updated at each time step and used along with the residual error ϵ_k to update the augmented state vector x_k. Then the covariance matrix P_k is updated. The algorithm is then iterated over k until parameters converge to their respective estimates, or in the case of an online application, continues to iterate as data are received.

3.2. Extended Kalman Filter (EKF)

Many similarities exist with the EKF when compared to the RLS. For instance, both algorithms make use of a gain update structure in which the gain provides the direction in which the parameters are updated. Prior to implementation, which follows the development in (Basappa & Jategaonkar, 2004), the EKF requires that the augmented state vector be defined. The augmented state vector x_a can be defined as:

$$x_a^T(t) = [x^T(t), \theta^T] \qquad (3.8)$$

where x is a vector of the state variables of the system and θ is vector of parameters to be estimated. With the augmented state vector x_a defined, the augmented system model can then be defined in terms of the following state and observation equations:

$$\dot{x}_a(t) = f_a[x_a(t), u(t)] + w(t) \qquad (3.9)$$

$$y(t) = h_a[x_a(t)] + v(t) \qquad (3.10)$$

The functions f_a and h_a define the possibly nonlinear state and measurement equations, while w and v are defined as the process and measurement noise within the system, respectively. Augmenting the state vector with parameters often results in the system becoming nonlinear. As a result, the nonlinear system must be linearized using the Jacobian matrices given by:

$$A(k-1) = \left(\frac{\partial f_a}{\partial x_a}\right)_{\hat{x}_a(k-1),u(k-1)} \qquad (3.11)$$

$$B(k-1) = \left(\frac{\partial f_a}{\partial u}\right)_{\hat{x}_a(k-1),u(k-1)} \qquad (3.12)$$

$$C(k-1) = \left(\frac{\partial h_a}{\partial x_a}\right)_{\hat{x}_a(k-1)} \tag{3.13}$$

where $\hat{x}_a(k-1)$ is defined as the previous state estimate and $u(k-1)$ as the control input. The model is, therefore, linearized at each time step about the previous state estimate. At each time step, the linearized augmented system is discretized using the state transition matrix ϕ and control distribution matrix Γ shown below:

$$\phi(k, k-1) = I + A(k-1)\Delta t + A^2(k-1)\frac{\Delta t^2}{2!} + \cdots \tag{3.14}$$

$$\Gamma(k, k-1) = \left[I\Delta t + A(k-1)\frac{\Delta t^2}{2!} + A^3(k-1)\frac{\Delta t^3}{3!} + \cdots\right]B(k-1) \tag{3.15}$$

The EKF algorithm consists of two major sections, which are iterated consistently over many iterations. The first section is the prediction step in which the EKF propagates the augmented state vector using the nonlinear dynamics equations. Following the prediction step, the EKF then moves to the correction section of the algorithm, which involves using current measurements from the system to update the prediction. The prediction and correction steps are described below.

EKF Prediction

State prediction and state covariance prediction:

$$\tilde{x}_a(k/k-1) = \hat{x}_a(k-1/k-1) + \int_{t_{k-1}}^{t_k} f_a[x_a(\tau), u(k-1)]d\tau \tag{3.16}$$

$$\tilde{P}(k/k-1) = \phi(k, k-1)\hat{P}(k-1/k-1)\phi^T(k, k-1) + Q(k-1) \tag{3.17}$$

EKF Correction

The corrected optimal estimate (measurement update):

$$\hat{x}_a(k/k) = \tilde{x}_a(k/k-1) + K(k)\{y(k) - h_a[\tilde{x}_a(k/k-1)]\} \tag{3.18}$$

where the Kalman gain is defined as:

$$K(k) = \tilde{P}(k/k-1)C^T(k)[C(k)\tilde{P}(k/k-1)C^T(k) + R(k)]^{-1} \tag{3.19}$$

The covariance matrix is then updated as follows:

$$\hat{P}(k/k) = [I - K(k)C(k)]\tilde{P}(k/k-1) \tag{3.20}$$

It should be noted that, in order to acquire a desirable performance from the EFK, the process noise covariance $Q = E\{\underline{v}\ \underline{v}^T\}$ and the measurement noise covariance $R = E\{\underline{w}\ \underline{w}^T\}$ should be suitably tuned. The process and measurement noise are modeled as uncorrelated (white) and normally distributed (Gaussian) processes. In the case of an offline application, the EKF would be iterated over the length of data previously accumulated. However, in the case of an online application, the EKF would continue to estimate parameters as data are received. It should be noted that, in either case, the estimated parameters should converge to their true values as time progresses.

3.3. Indirect Model Reference Adaptive Control (MRAC)

MRAC is an adaptive control implementation that drives the natural system to track reference trajectories. The MRAC uses a reference model, which contains the desired dynamics, and an adaptive controller to generate the necessary control inputs for the natural system. In state-space representation, $\underline{x}$ defines the state vector of the natural system, while $\underline{x}_{RM}$ defines the state vector of the reference model. The system and reference models are defined as:

$$\underline{\dot{x}} = A\,\underline{x} + B\underline{u}_s \tag{3.21}$$

$$\underline{\dot{x}}_{RM} = A_{RM}\underline{x}_{RM} + B_{RM}\underline{u}_r \tag{3.22}$$

It should be noted that the reference model uses a reference input u_r, which is derived by the user to generate reference tracking trajectories. On the other hand, in order to drive the system to track the desired dynamics, the natural system uses a different input u_s that is derived from the adaptive control law. The matrices A and B are system and input matrices respectively, while matrices A_{RM} and B_{RM} are the reference model's system and input matrices.

3.3.1. Adaptive Control Law

An indirect adaptive control law is developed in a manner that estimates parameters of both the A and B matrices, which are in turn used to update the control gains. Following the formulation provided in (Nguyen N. T., 2018), the adaptive controller is formulated as follows:

$$u_s(t) = -K_x(t)\underline{x}(t) - K_u(t)u_r(t) \tag{3.23}$$

where K_x and K_u are time varying gains to the system. The gains are determined by first defining the close-loop dynamics of the natural system.

$$\underline{\dot{x}}(t) = A\underline{x}(t) - BK_x(t)\underline{x}(t) - BK_u(t)u_r(t) \tag{3.24}$$

Given the closed-loop structure, the reference model can then be matched to the natural system. Consequently, the matching of the reference system to the closed-loop system results in the time-varying control gains $K_x(t)$ and $K_u(t)$ being defined as:

$$K_x(t) = \left(\hat{B}^T(t)\hat{B}(t)\right)^{-1}\hat{B}^T(t)(A_{RM} - \hat{A}(t)) \tag{3.25}$$

$$K_u(t) = \left(\hat{B}^T(t)\hat{B}(t)\right)^{-1}\hat{B}^T(t)B_{RM} \tag{3.26}$$

where $\hat{A}$ and $\hat{B}$ represents the estimation of the respective matrixes, with $\hat{B}$ being full column rank.

3.3.2. Adaptive Parameter Estimation

The indirect adaptive control law includes an adaptive estimation process. Stability analysis through a candidate Lyapunov function allows for an adaptive update of $\hat{A}$ and $\hat{B}$ to be chosen as follows:

$$\dot{\hat{A}}(t) = -P\underline{e}\,\underline{x}^T\sigma_A \tag{3.27}$$

$$\dot{\hat{B}}(t) = -P\underline{e}\,\underline{u}^T\sigma_B \tag{3.28}$$

It should be noted that σ_A and σ_B are positive definite matrices used to tune the adaptive positive definite control law, and $\underline{e}$ refers to the error between the system states and the reference states. $P = P^T > 0$ is an $n \times n$ matrix which is determined from the solution of the algebraic Lyapunov equation:

$$A_{RM}^T P + PA_{RM} = -Q \tag{3.29}$$

Since A_{RM} is Hurwitz (i.e., A_{RM} has stable eigenvalues), a solution $P = P^T > 0$ is guaranteed to exist for any $Q = Q^T > 0$. In order to estimate specified parameters, $\dot{\hat{A}}(t)$ and $\dot{\hat{B}}(t)$ are expanded in terms of their individual elements.

The adaptive estimator is derived in such a manner to guarantee asymptotic tracking of the reference trajectories as well as bounded (stable) adaptive parameters. This can be proven using the candidate Lyapunov function $V = \frac{1}{2}\underline{e}^T P\underline{e} + tr(\Delta A\sigma_A^{-1}\Delta A^T) + tr(\Delta B\sigma_B^{-1}\Delta B^T)$. When the adaptation is selected as shown above, $\dot{V} = \frac{1}{2}\underline{e}^T P\underline{e} \leq 0$. Therefore, the closed-loop system is guaranteed to be stable (bounded) by the Lyapunov direct method. Barbalat's Lemma can then be used to prove asymptotic stability of the tracking error ($\underline{e} \to \underline{0}$ as $t \to \infty$). Refer to (Nguyen N. T., 2018) for more details. Moreover, the proposed method can allow for parameter convergence when the closed-loop system is persistently excited.

3.4. Persistence of Excitation

Persistence of excitation refers to the control inputs providing sufficient excitation of the system dynamics to enable successful parameter estimation. For example, it is not generally possible to generate reliable aircraft parameter estimates using data collected while the aircraft is flying straight and level. Persistence of excitation can be a critical issue in the convergence of parameter estimates to their true values. Given a general parameter estimation algorithm, persistence of excitation conditions can be expressed as follows (Nguyen N. T., 2018):

There exists some $\alpha > 0$ and $T > 0$ such that, for all $t > 0$,

$$\int_{t}^{t+T} \underline{\phi}(\tau)\, \underline{\phi}^{T}(\tau)\, d\tau > \alpha I_N \tag{3.30}$$

where N represents the number of parameters to be estimated and $\underline{\phi}$ is a vector of variables or functions used to estimate the parameters. For linear systems, the persistence of excitation implies that, in order to estimate N parameters, the reference input should be composed of at least $N/2$ sinusoidal frequencies. In general, persistence of excitation conditions become much more complex for nonlinear systems.

In this work, persistence of excitation conditions are addressed in the simulation studies by applying sinusoidal reference inputs that contain multiple frequencies, whereas in flight test they are often achieved using flight maneuvers such as doublets. It should be noted that applying these reference inputs can be somewhat contradictory to typical flight control objectives such as tracking flight trajectories for waypoint navigation.

3.5. Case Model

A relatively simple test case is presented as a means to establish the fundamentals of the parameter estimation algorithms. A linear, time-invariant (LTI) system that represents the simplified dutch roll dynamics of an aircraft is used as a case model. The equations of motion for the dutch roll dynamics are shown below:

$$\dot{\beta} = -r \tag{3.31}$$

$$\dot{r} = N_{\beta}\beta + N_r r + N_{\delta r}\delta r \tag{3.32}$$

where β and r represent the sideslip and yaw rate. The model has a single input corresponding to the rudder deflection δr. The equations of motion were first placed into state-space form. The parameters N_β and N_r are stability derivatives to be estimated. The true values for these parameters are noted to be $N_\beta = 2$ and $N_r = 0.5$ respectively. The parameter $N_{\delta r} = -2$ is assumed to be known in this case. In the simulation, both parameters were initialized to zero.

The EKF algorithm is formulated as described in Section 3.2 by first defining the augmented state vector as:

$$x_a = \left[\beta, r, \widehat{N}_\beta, \widehat{N}_r\right]^T \tag{3.33}$$

The augmented state-space model can then be developed using the equations of motion and the augmented state vector:

$$\dot{x}_1 = -x_2 \tag{3.34}$$

$$\dot{x}_2 = x_1 x_3 + x_2 x_4 + u N_{\delta r} \tag{3.35}$$

$$\dot{x}_3 = 0 \tag{3.36}$$

$$\dot{x}_4 = 0 \tag{3.37}$$

$$y = [x_1 \quad x_2] \tag{3.38}$$

Following the augmented state-space model, the nonlinear system is linearized and takes the form:

$$A = \begin{bmatrix} 0 & -1 & 0 & 0 \\ x_3 & x_4 & x_1 & x_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.39}$$

$$B = \begin{bmatrix} 0 \\ -2 \\ 0 \\ 0 \end{bmatrix} \tag{3.40}$$

The linearized augmented state-space model is then implemented into the EKF algorithm, and iterated over a one hundred second time period.

The Indirect MRAC is formulated as described in Section 3.3 by first defining the linear, time-invariant dutch roll dynamics system in state-space form:

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -1 \\ N_\beta & N_r \end{bmatrix}}_{A} \begin{bmatrix} \beta \\ r \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ -2 \end{bmatrix}}_{B} u_s \tag{3.41}$$

$$y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} \tag{3.42}$$

The update laws to estimate the parameters N_β and N_r are defined as:

$$\dot{\hat{N}}_\beta(t) = -x_1 \sigma_{A_1} \underline{e}^T \overline{P} \tag{3.43}$$

$$\dot{\hat{N}}_r(t) = -x_2 \sigma_{A_2} \underline{e}^T \overline{P} \tag{3.44}$$

With the adaptive parameter estimates of N_β and N_r, the control gains can then be formed as:

$$K_x(t) = \left(\hat{B}^T(t)\hat{B}(t)\right)^{-1} \hat{B}^T(t)(A_{RM} - \hat{A}(t)) \tag{3.45}$$

$$K_x(t) = \frac{-2}{5}\left[(\overline{N}_\beta - \hat{N}_\beta(t)) \quad (\overline{N}_r - \hat{N}_r(t))\right] \tag{3.46}$$

$$K_u(t) = \left(\hat{B}^T(t)\hat{B}(t)\right)^{-1} \hat{B}^T(t) B_{RM} \tag{3.47}$$

$$K_u(t) = -1 \tag{3.48}$$

Lastly, with the control gains $K_x(t)$ and $K_u(t)$ in terms of the parameters to be estimated, the control law can then be defined as:

$$u_s(t) = -K_x(t)\underline{x}(t) - K_u(t)u_r(t) \tag{3.49}$$

The results of the estimation using the EKF and adaptive estimator from the Indirect MRAC are shown in Figure 3.1.

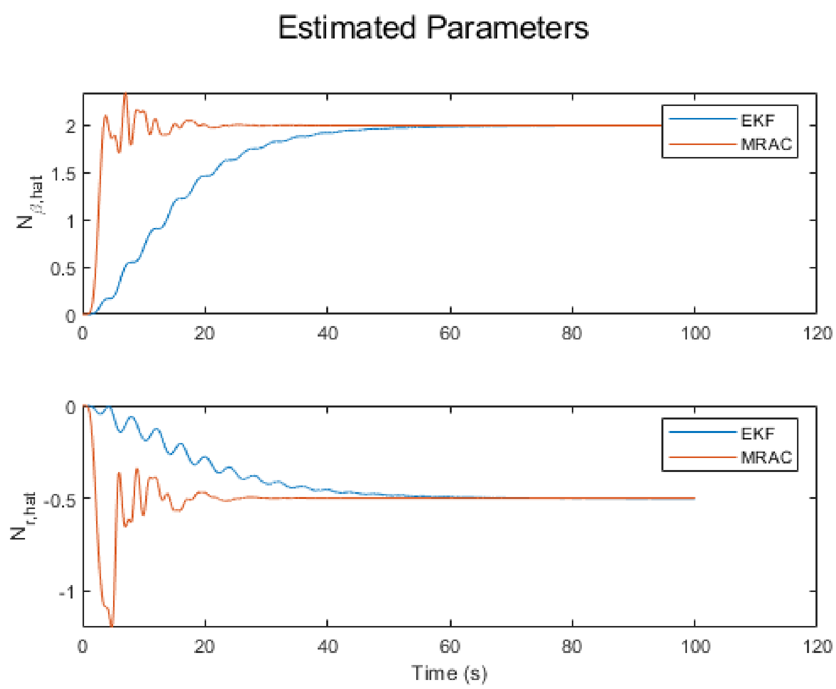

Figure 3.1 Case Model Estimation using the EKF and Indirect MRAC

It can be seen from Figure 3.1 that the Indirect MRAC converges to the true values fairly quickly, while on the other hand, the EKF is fairly slow to converge. It should be noted that the Indirect MRAC required little tuning to estimate the two parameters. In contrast, the EKF required significant tuning to ensure that it converges to the true values. It may be possible to improve the aggressiveness of the EKF convergence with further tuning.

4. Lateral-Directional Aircraft Model

To represent the lateral dynamics of a medium scale UAV, a Cessna 182 model is used for simulation studies. Specifically, the lateral-directional dynamics model is isolated and used as the base system for simulation. The equations of motion for the lateral dynamics model, which consists of the states v, p, r and ϕ, are shown below.

$$v = Y_v v + \left(Y_p + w_\star\right)p + (Y_r - u_\star)r + gcos(\theta_\star)\phi + Y_{\delta r}\delta r$$

$$p = (I_2 L_v + I_3 N_v)v + \left(I_2 L_p + I_3 N_p\right)p + (I_2 L_r + I_3 N_r)r + (I_2 L_{\delta a} + I_3 N_{\delta a})\delta a + (I_2 L_{\delta r} + I_3 N_{\delta r})\delta r$$

$$r = (I_2 N_v + I_4 L_v)v + \left(I_2 N_p + I_4 L_p\right)p + (I_2 N_r + I_4 L_r)r + (I_2 N_{\delta a} + I_4 L_{\delta a})\delta a + (I_2 N_{\delta r} + I_4 L_{\delta r})\delta r$$

$$\phi = p + \tan{(\theta_\star)} r$$

The state v is defined as the lateral velocity, p as the roll rate, r as the yaw rate and ϕ as the roll angle. The control input δa is defined as the aileron deflection and δr as the rudder deflection. Also, the aircraft was trimmed at $220.5\ fts^{-1}$ at $5{,}000\ ft$. Therefore, the corresponding trim values are: vertical velocity $w_\star = 0.80 fts^{-1}$, longitudinal velocity $u_\star = 220.45 fts^{-1}$, and pitch angle $\theta_\star = 2.16°$. The stability derivatives and their references values are shown in Table 4.1.

Table 4.1

Stability Derivatives and their Reference Values

Stability Derivatives	Reference Value
L_v	-0.1377
L_p	-12.9949
L_r	2.1426
N_v	0.0422
N_p	-0.3597
N_r	-1.2125
$L_{\delta a}$	75.3007
$L_{\delta r}$	4.8337
$N_{\delta a}$	-3.4231
$N_{\delta r}$	-10.2218

I_2, I_3 and I_4 are defined in terms of the inertias as follows:

$$I_2 = \frac{I_x I_z}{I_1} \tag{4.1}$$

$$I_3 = \frac{I_{xz} I_z}{I_1} \tag{4.2}$$

$$I_4 = \frac{I_{xz} I_x}{I_1} \tag{4.3}$$

$$I_1 = I_x I_z - I_{xz}^2 \tag{4.4}$$

where the inertias are listed in Table 4.2.

Table 4.2

Inertias and their Reference Value

Inertia	Reference Value
I_x	948 $lbft^2$
I_z	1967 $lbft^2$
I_{xz}	0

Given the basic configuration of the C182, the aircraft's lateral dynamics model can be reduced further because the product of inertia I_{xz} is approximately zero. Since I_3 and I_4 are representative of that inertia term, the equations of motion then take the form:

$$v = Y_v v + \left(Y_p + w_\star\right)p + (Y_r - u_\star)r + gcos(\theta_\star)\phi + Y_{\delta r}\delta r \tag{4.5}$$

$$p = I_2 L_v v + I_2 L_p p + I_2 L_r r + I_2 L_{\delta a}\delta a + I_2 L_{\delta r}\delta r \tag{4.6}$$

$$r = I_2 N_v v + I_2 N_p p + I_2 N_r r + I_2 N_{\delta a}\delta a + I_2 N_{\delta r}\delta r \tag{4.7}$$

$$\phi = p + tan\,(\theta_\star)r \tag{4.8}$$

With the current reduced form of the equations of motion, the EKF and adaptive estimator techniques are used to estimate parameters corresponding to the roll and yaw rate stability derivatives.

4.1. EKF Formulation

In order to estimate parameters using the EKF approach, the equations of motion must be expressed in the form of a linearized augmented model. In the EKF algorithm, the augmented state vector of the system includes the aircraft states along with the parameters to be estimated. It should be noted that the linearized augmented model will change depending on the parameters to be estimated. The thesis focuses on estimating the parameters L_v, L_p, L_r, $L_{\delta a}$ and $L_{\delta r}$ in the roll rate state, the parameters N_v, N_p, N_r, $N_{\delta a}$ and $N_{\delta r}$ in the yaw rate state. Therefore, the formulation of the linearized augmented model will focus on these parameters.

The augmented model includes the parameters to be estimated as states of the system. With the inclusion of the original four aircraft states, the augmented system has a total of fourteen states. It is important to note that, in augmenting the model which has linear dynamics, the result is a system that is nonlinear with respect to the augmented states.

$$
\begin{array}{r|c}
v = x_1 & \dot{x}_1 = Y_v x_1 + \left(Y_p + w_\star\right) x_2 + (Y_r r - u_\star) x_3 + g(\cos(\theta_\star)) x_4 + Y_{\delta r} u_2 \\
p = x_2 & \dot{x}_2 = I_2 x_5 x_1 + I_2 x_6 x_2 + I_2 x_7 x_3 + I_2 x_{11} u_1 + I_2 x_{12} u_2 \\
r = x_3 & \dot{x}_3 = I_2 x_8 x_1 + I_2 x_9 x_2 + I_2 x_{10} x_3 + I_2 x_{13} u_1 + I_2 x_{14} u_2 \\
\phi = x_4 & \dot{x}_4 = x_2 + \tan\,(\theta_\star) x_3 \\
L_v = x_5 & \dot{x}_5 = 0 \\
L_p = x_6 & \dot{x}_6 = 0 \\
L_r = x_7 & \dot{x}_7 = 0 \\
N_v = x_8 & \dot{x}_8 = 0 \\
N_p = x_9 & \dot{x}_9 = 0 \\
N_r = x_{10} & \dot{x}_{10} = 0 \\
L_{\delta a} = x_{11} & \dot{x}_{11} = 0 \\
L_{\delta r} = x_{12} & \dot{x}_{12} = 0 \\
N_{\delta a} = x_{13} & \dot{x}_{13} = 0 \\
N_{\delta r} = x_{14} & \dot{x}_{14} = 0
\end{array}
$$

The EKF implementation requires linearizing the nonlinear augmented model about the most recent state estimate. The linearized augmented matrix A is shown below, which is a 14×14 matrix that includes the four states and the ten parameters to be estimated.

$$\begin{bmatrix} Y_v & \left(Y_p + w_0\right) & (Y_r r - u_0) & gcos(\theta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I_2 x_5 & I_2 x_6 & I_2 x_7 & 0 & I_2 x_1 & I_2 x_2 & I_2 x_3 & 0 & 0 & 0 & I_2 u_1 & I_2 u_2 & 0 & 0 \\ I_2 x_8 & I_2 x_9 & I_2 x_{10} & 0 & 0 & 0 & 0 & I_2 x_1 & I_2 x_2 & I_2 x_3 & 0 & 0 & I_2 u_1 & I_2 u_2 \\ 0 & 1 & \tan(\theta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The linearized augmented matrix B is shown below, which is 14×2 in size.

$$\begin{bmatrix} 0 & Y_{\delta r} \\ I_2 x_{11} & I_2 x_{12} \\ I_2 x_{13} & I_2 x_{14} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

These matrices are fairly large in structure but relatively simple in form. The A and B matrices are sparse primarily due to the current formulation assuming that parameters remain constant over time. It should be noted that the size of the matrices will increase as the system gets larger or the number of parameters to be estimated increases.

4.2. Indirect Model Reference Adaptive Control (MRAC)

The Indirect MRAC is implemented in Simulink as shown in Figure 4.1. Both the system and reference model are enacted as state-space models of the C182 aircraft dynamics. As introduced in Chapter 3, the adaptive controller shown includes the formulation of the update gains $K_x(t)$ and $K_u(t)$ as well as the update laws for both $\hat{A}$ and $\hat{B}$, which consist of the individual parameter estimates as well as some parameters that are assumed to be known (i.e., side velocity derivatives).

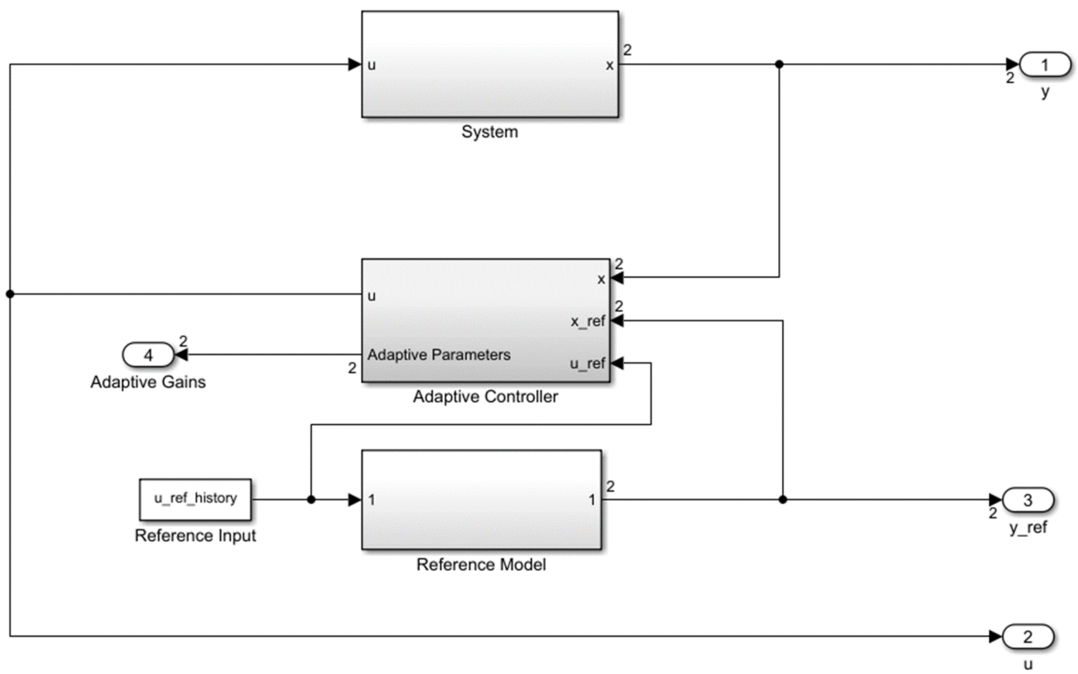

Figure 4.1 Indirect MRAC Implementation in Simulink

The update laws for both $\hat{A}$ and $\hat{B}$ are defined as:

$$\dot{\hat{A}}(t) = \begin{bmatrix} Y_v & Y_p + w_\star & Y_r - u_\star & g\cos(\theta_\star) \\ \hat{L}_v(t) & \hat{L}_p(t) & \hat{L}_r(t) & 0 \\ \hat{N}_v(t) & \hat{N}_p(t) & \hat{N}_r(t) & 0 \\ 0 & 1 & \tan(\theta_\star) & 0 \end{bmatrix} \tag{4.9}$$

$$\dot{\hat{B}}(t) = \begin{bmatrix} 0 & Y_{\delta r} \\ \hat{L}_{\delta a}(t) & \hat{L}_{\delta r}(t) \\ \hat{N}_{\delta a}(t) & \hat{N}_{\delta r}(t) \\ 0 & 0 \end{bmatrix} \quad (4.10)$$

where the parameter estimates are given by:

$$\dot{\hat{L}}_v(t) = \sigma_{a1}x_1(P_{21}e_1 + P_{22}e_2 + P_{23}e_3 + P_{24}e_4) \quad (4.11)$$

$$\dot{\hat{L}}_p(t) = \sigma_{a2}x_2(P_{21}e_1 + P_{22}e_2 + P_{23}e_3 + P_{24}e_4) \quad (4.12)$$

$$\dot{\hat{L}}_r(t) = \sigma_{a3}x_3(P_{21}e_1 + P_{22}e_2 + P_{23}e_3 + P_{24}e_4) \quad (4.13)$$

$$\dot{\hat{N}}_v(t) = \sigma_{a1}x_1(P_{31}e_1 + P_{32}e_2 + P_{33}e_3 + P_{34}e_4) \quad (4.14)$$

$$\dot{\hat{N}}_p(t) = \sigma_{a2}x_2(P_{31}e_1 + P_{32}e_2 + P_{33}e_3 + P_{34}e_4) \quad (4.15)$$

$$\dot{\hat{N}}_r(t) = \sigma_{a3}x_3(P_{31}e_1 + P_{32}e_2 + P_{33}e_3 + P_{34}e_4) \quad (4.16)$$

$$\dot{\hat{L}}_{\delta a}(t) = \sigma_{b1}u_1(P_{21}e_1 + P_{22}e_2 + P_{23}e_3 + P_{24}e_4) \quad (4.17)$$

$$\dot{\hat{L}}_{\delta r}(t) = \sigma_{b2}u_2(P_{21}e_1 + P_{22}e_2 + P_{23}e_3 + P_{24}e_4) \quad (4.18)$$

$$\dot{\hat{N}}_{\delta a}(t) = \sigma_{b1}u_1(P_{31}e_1 + P_{32}e_2 + P_{33}e_3 + P_{34}e_4) \quad (4.19)$$

$$\dot{\hat{N}}_{\delta r}(t) = \sigma_{b2}u_2(P_{31}e_1 + P_{32}e_2 + P_{33}e_3 + P_{34}e_4) \quad (4.20)$$

To use other estimation techniques within the Indirect MRAC, the adaptive update laws for each element are replaced with the estimates provided from the EKF algorithm. These changes are implemented inside the Adaptive Controller block. The adaptive control law takes the form:

$$u_s(t) = -K_x(t)\underline{x}(t) - K_u(t)u_r(t) \quad (4.21)$$

where $K_x(t)$ and $K_u(t)$ are defined as:

$$K_x(t) = \left(\hat{B}^T(t)\hat{B}(t)\right)^{-1}\hat{B}^T(t)(A_{RM} - \hat{A}(t)) \quad (4.22)$$

$$K_u(t) = \left(\hat{B}^T(t)\hat{B}(t)\right)^{-1}\hat{B}^T(t)B_{RM} \quad (4.23)$$

5. Simulation Results

Chapter 5 presents the results of parameter estimation and tracking control using the EKF and adaptive estimator within the Indirect MRAC framework. The convergence results of all methods are depicted in the figures presented. Parameter estimation results using both methods are first presented followed by parameter estimation and tracking performance in the indirect adaptive control law.

5.1. EKF Estimation

The estimation results of the lateral aircraft parameters formulated in Chapter 4 are presented. First, the parameters are estimated without the presence of noise, and then measurement noise is introduced to the system. State and parameter estimates were initialized as shown in Table 5.1. These initial parameter estimates were chosen to have the correct sign of the true parameters but with approximately 50% error in magnitude.

Table 5.1

Parameters and their Initialized Values

Parameter	Initial Value	Reference Values
L_v	-0.1377	-0.1377
L_p	-12.9949	-12.9949
L_r	2.1426	2.1426
N_v	0.0422	0.0422
N_p	-0.3597	-0.3597
N_r	-1.2125	-1.2125
$L_{\delta a}$	75.3007	75.3007
$L_{\delta r}$	4.8337	4.8337
$N_{\delta a}$	-3.4231	-3.4231
$N_{\delta r}$	-10.2218	-10.2218

In all cases, the reference inputs were specified as:

$$\delta a = u_{1,ref} = 0.1 sin\ (0.25\pi t) \tag{5.1}$$

$$\delta r = u_{2,ref} = 0.1 sin\ (0.50\pi t) \tag{5.2}$$

5.1.1. Results Without Noise

The results of the EKF without noise are presented in Figures 5.1-5.5. Each plot shows the estimated parameter as well as the 3-sigma bounds, where sigma is the standard deviation derived from the estimation error covariance. These bounds represent confidence intervals on the parameter estimate.

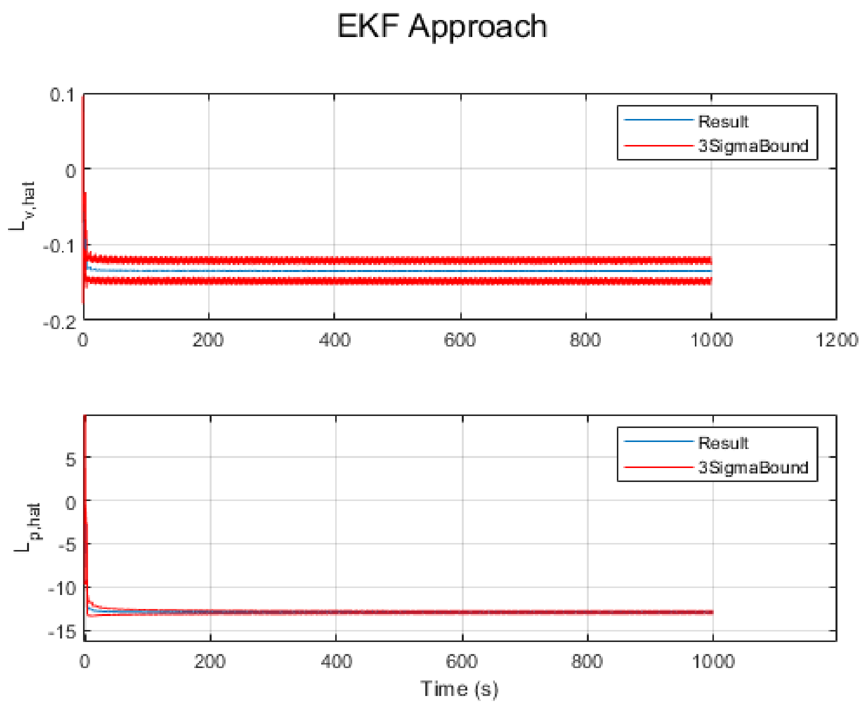

Figure 5.1 EKF Estimation of Parameters L_v and L_p without Noise

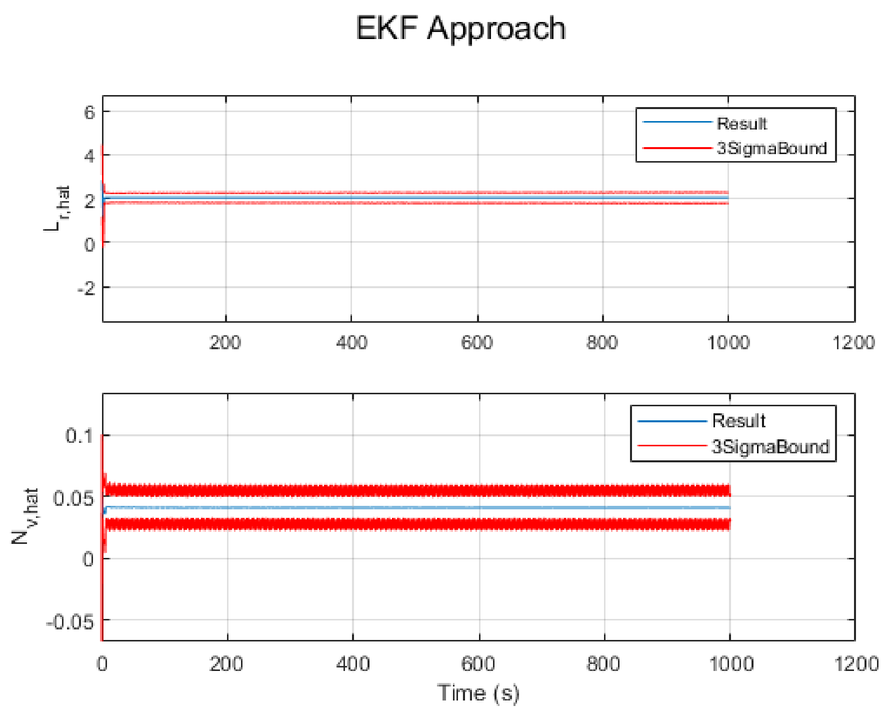

Figure 5.2 EKF Estimation of Parameters L_r and N_v without Noise

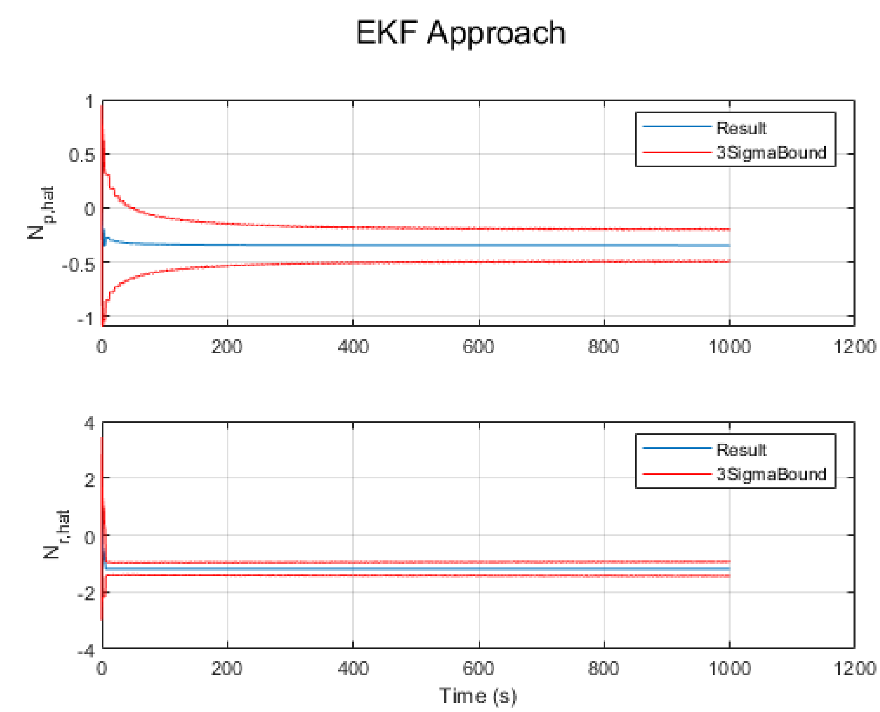

Figure 5.3 EKF Estimation of Parameters N_p and N_r without Noise

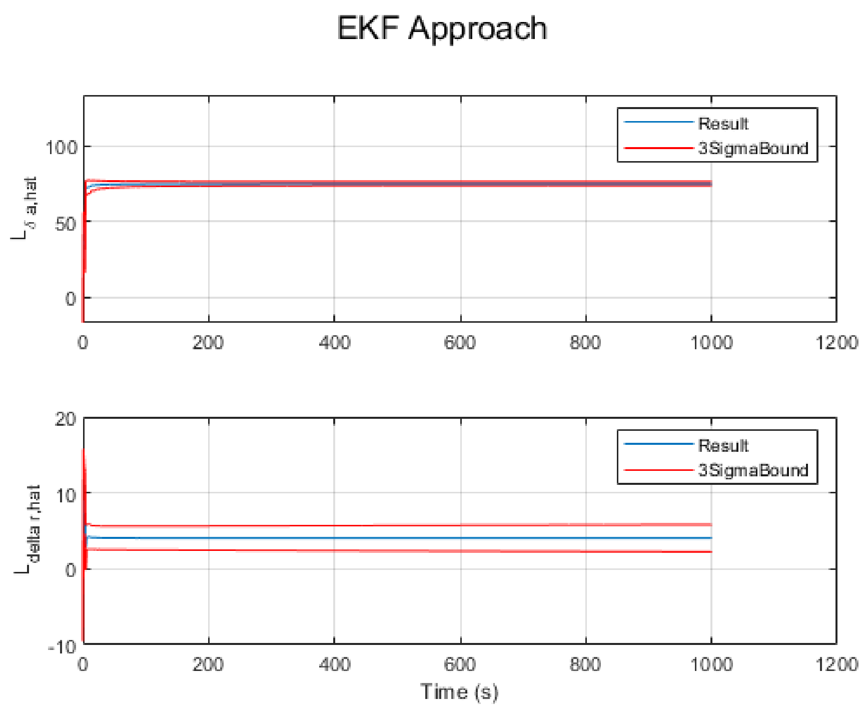

Figure 5.4 EKF Estimation of Parameters $L_{\delta a}$ and $L_{\delta r}$ without Noise

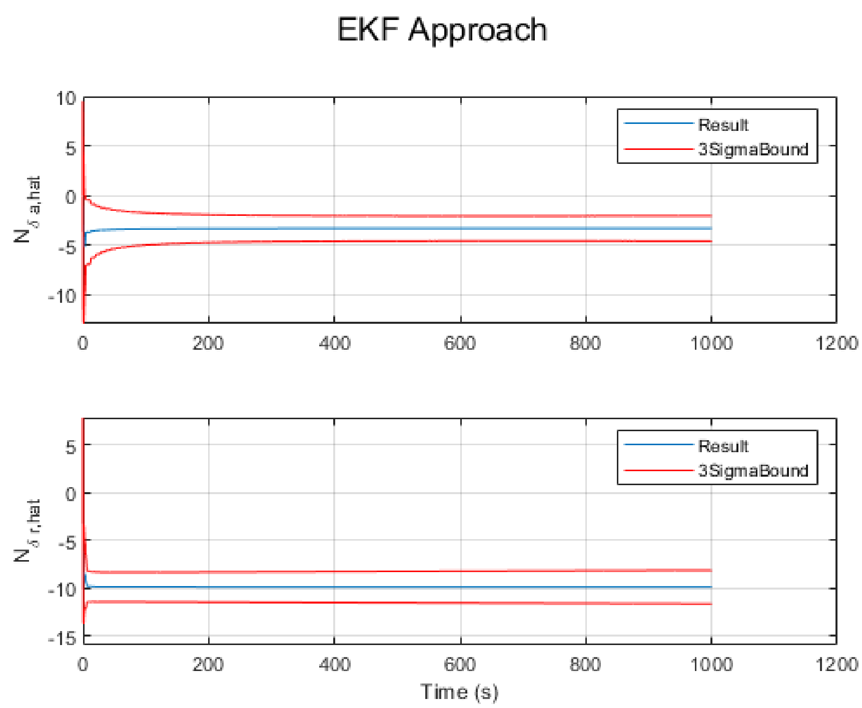

Figure 5.5 EKF Estimation of Parameters $N_{\delta a}$ and $N_{\delta r}$ without Noise

When examining the estimation results, it can be seen that each parameter has converged to its true value relatively fast. In fact, the estimation performance of the EKF is quite acceptable in terms of convergence to true (reference) values. It is noted that only one parameter differed from its reference value by more than 5%. A summary of the EKF results can be seen in Table 5.2. The results show that, in the noisefree case, the EKF can provide accurate estimation of a large number of parameters. Chapter 3 discussed the challenges of tuning time domain approaches such as the EKF. It should therefore be noted that incorrectly tuning the EKF algorithm results in inferior results. With that said, even though noise is not present in the current results, the algorithm still requires that close attention is paid to tuning.

Table 5.2

EKF Estimated Parameters compared to their Reference Values

Estimated Parameter	Reference Value	EKF Estimate	Percent Difference
L_v	-0.1377	-0.1347	2%
L_p	-12.9949	-12.9804	0%
L_r	2.1426	2.0429	5%
N_v	0.0422	0.0412	2%
N_p	-0.3597	-0.3459	4%
N_r	-1.2125	-1.1820	3%
$L_{\delta a}$	75.3007	74.7938	1%
$L_{\delta r}$	4.8337	4.0427	16%
$N_{\delta a}$	-3.4231	-3.3381	2%
$N_{\delta r}$	-10.2218	-9.8934	3%

5.1.2. Results With Measurement Noise

The results of the EKF with additive measurement (sensor) noise are presented in Figures 5.6-5.10. In this case the sensor outputs were corrupted with additive, normally distributed noise with the following variances:

$$\sigma(v) = (2)^2 \ \ (ft/s)^2 \tag{5.3}$$

$$\sigma(p) = (0.1)^2 \ \ (rad/s)^2 \tag{5.4}$$

$$\sigma(r) = (0.1)^2 \ \ (rad/s)^2 \tag{5.5}$$

$$\sigma(\phi) = (0.01)^2 \ \ (rad)^2 \tag{5.6}$$

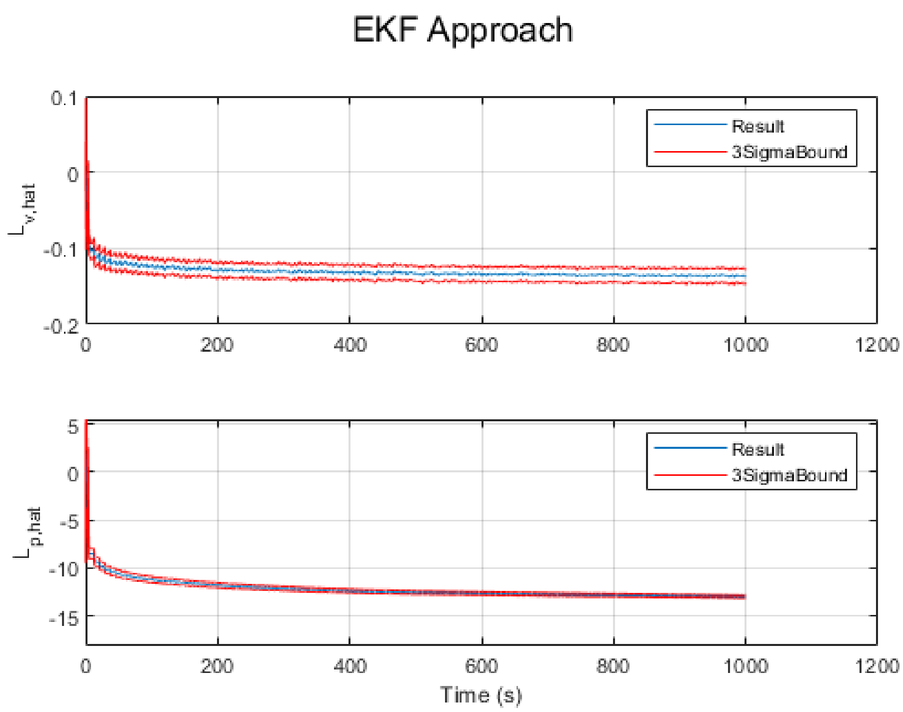

Figure 5.6 EKF Estimation of Parameters L_v and L_p with Noise

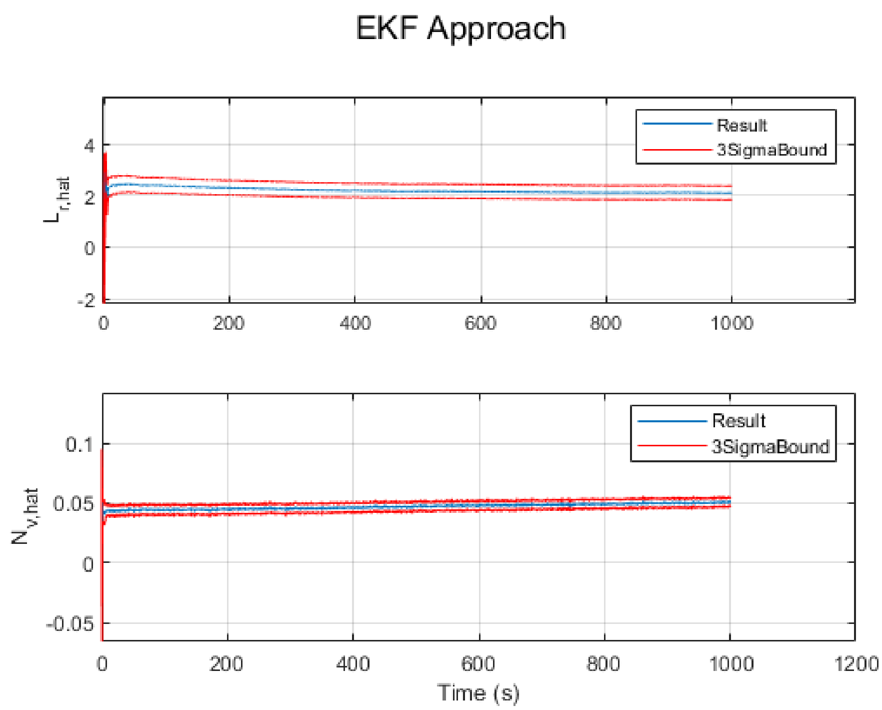

Figure 5.7 EKF Estimation of Parameters L_r and N_v with Noise

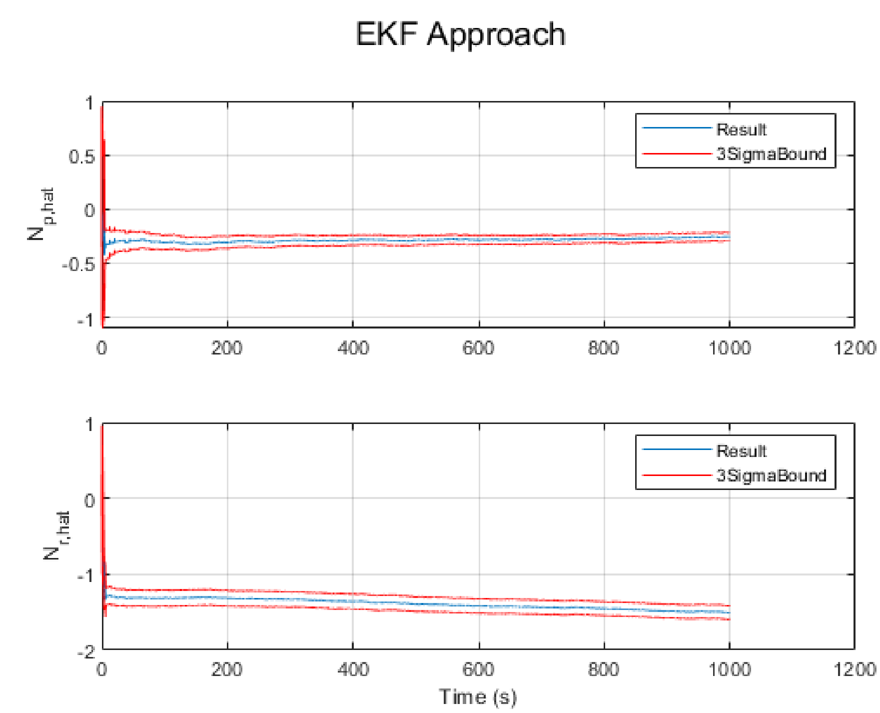

Figure 5.8 EKF Estimation of Parameters N_p and N_r with Noise

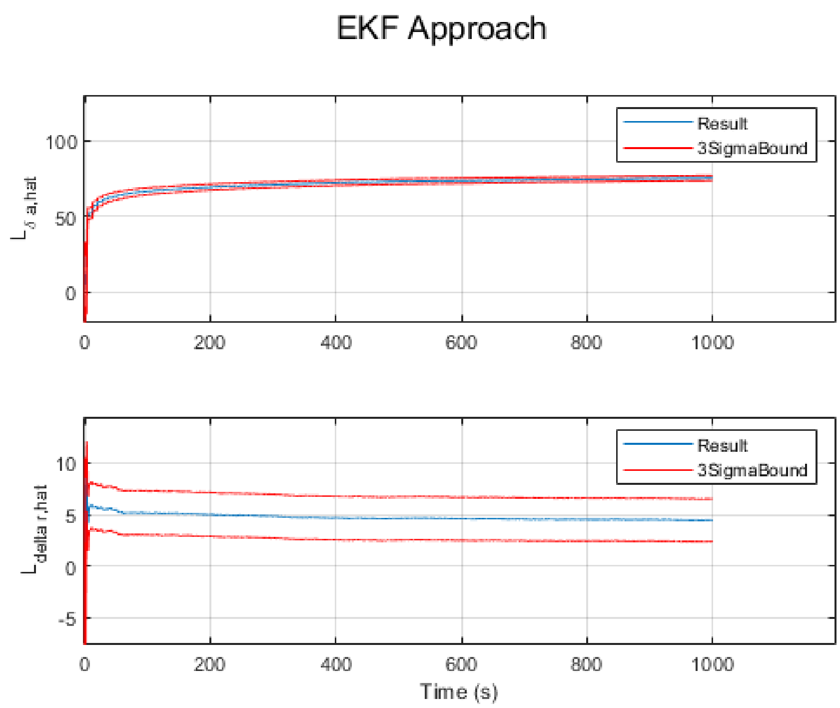

Figure 5.9 EKF Estimation of Parameters $L_{\delta a}$ and $L_{\delta r}$ with Noise

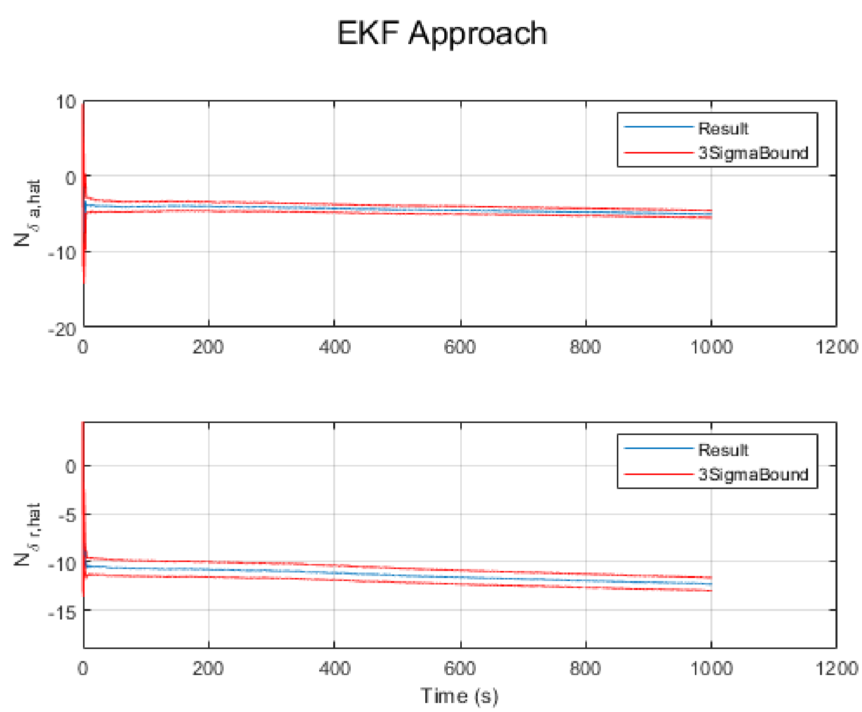

Figure 5.10 EKF Estimation of Parameters $N_{\delta a}$ and $N_{\delta r}$ with Noise

Comparing the estimation results, it can be seen that all parameters converge fairly quickly, just as in the case of the EKF without noise. In this case, however, some parameters either converge a noticeable distance away from their true values or diverge as time progresses. Exactly half of the parameters estimated have a percent difference of over 15% , with one parameter approaching almost 50%. The summary of the EKF results with noise is shown in Table 5.3. The limitations of the EKF become noticeable when such a vast number of parameters are being estimated. Significant time was taken to tune the system, but it still should be noted that further tuning of the system may potentially improve the performance of the EKF. This fact concurs with the literature that the tuning of time domain approaches presents a distinct challenge to parameter estimation.

Table 5.3

EKF Parameters Estimated with noise compared to their Reference Values

Estimated Parameter	Reference Value	EKF Estimate	Percent Difference
L_v	-0.1377	-0.1368	1%
L_p	-12.9949	-13.0018	0%
L_r	2.1426	2.1016	2%
N_v	0.0422	0.0502	19%
N_p	-0.3597	-0.2546	29%
N_r	-1.2125	-1.5069	24%
$L_{\delta a}$	75.3007	75.0981	0%
$L_{\delta r}$	4.8337	4.4465	8%
$N_{\delta a}$	-3.4231	-5.0709	48%
$N_{\delta r}$	-10.2218	-12.3057	20%

5.2. Adaptive Estimator

The adaptive estimator is notably different when compared to traditional approaches such as the EKF. Those differences are further highlighted in simulation, which in turn can lead to further improvements in the system. Initially, the adaptive estimator was formulated to estimate a total of ten parameters, a case that the EKF handled within fair margins. In fact, there are no instances where the EKF failed to estimate a parameter. In the case of the adaptive estimator, the system simply is not able to estimate such a vast number of parameters simultaneously. This can be demonstrated by initializing each parameter at its true value, in which case the system recognizes that no update is needed. Issues occur when multiple parameters need to be updated to their true values. It was determined via simulation studies that a maximum of six parameters could be successfully estimated using the adaptive estimator. However, it will be seen that a drastic difference in performance occurs when noise is added to the system.

5.2.1. Results Without Noise

The results of the adaptive update without noise are presented in Figures 5.11-5.13.

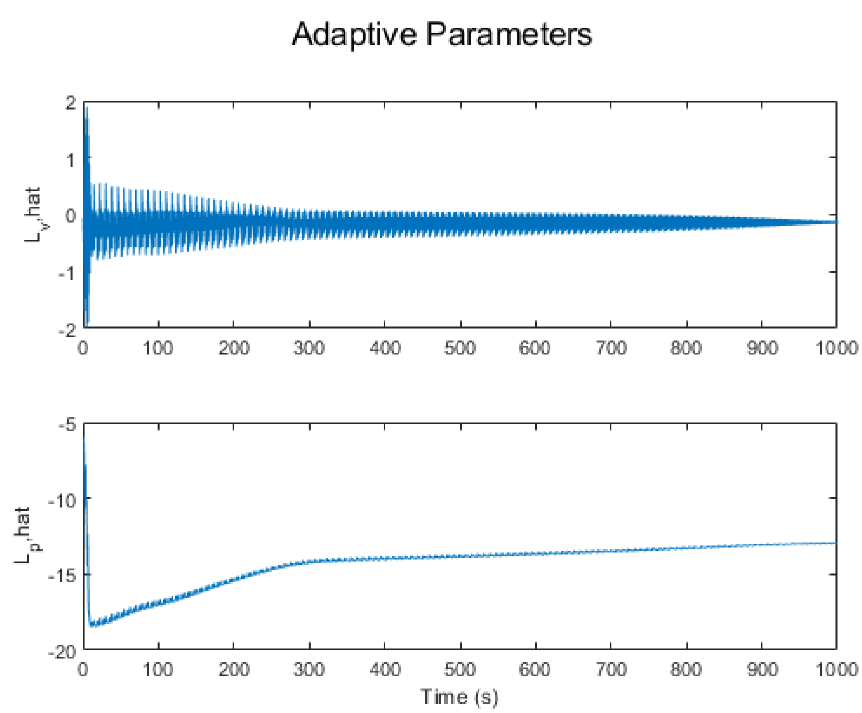

Figure 5.11 Adaptive Estimation of Parameters L_v and L_p without Noise

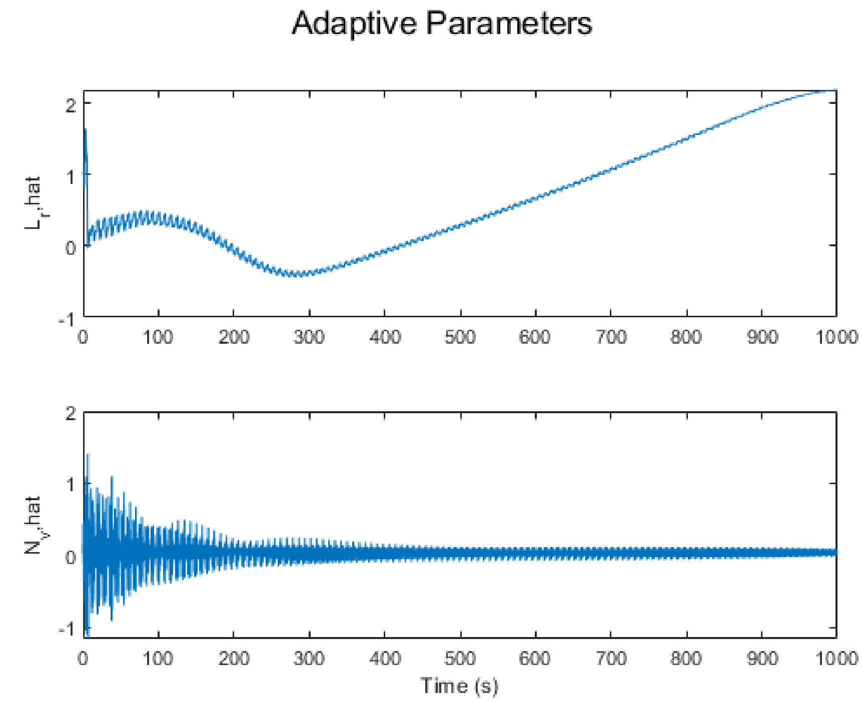

Figure 5.12 Adaptive Estimation of Parameters L_r and N_v without Noise

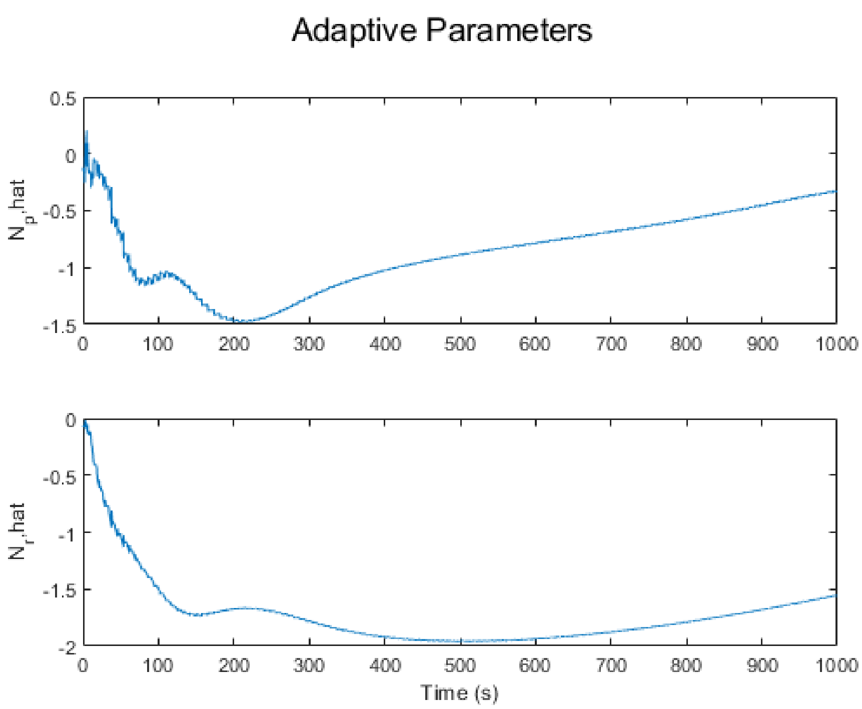

Figure 5.13 Adaptive Estimation of Parameters N_p and N_r without Noise

Aside from the inability of the adaptive estimator to successfully estimate the same number of parameters as the EKF, the accuracy of the six estimated parameters is satisfactory in the noise free case. There are still drawbacks with the system, however, such as slow convergence. It can be seen in the results that it takes a considerable time before the system begins to converge around its true value, a significantly slower convergence rate than the EKF. When compared to the case model presented in Chapter 3, the current performance shows almost the inverse trend, therefore further demonstrating the decrease in performance as the number of parameters is increased. On the other hand, it can be seen from the summarized results in Table 5.3 that the converged results of the adaptive estimator results in a percent difference of no greater than 28% for all parameters. However, it is noted that the distinct challenge of tuning is once again present just as in the case of the EKF.

Table 5.4

Adaptive Estimator Parameters estimated without noise compared to their Reference Values

Estimated Parameter	Reference Value	Adaptive Estimator	Percent Difference
L_v	-0.1377	-0.1493	8%
L_p	-12.9949	-12.9777	0%
L_r	2.1426	2.1922	2%
N_v	0.0422	0.0434	3%
N_p	-0.3597	-0.3322	8%
N_r	-1.2125	-1.5563	28%

5.2.2. Results With Measurement Noise

The results of the adaptive estimator with additive measurement noise are presented in Figure 5.14-5.16. This measurement noise was of the same form applied in the EKF simulations.

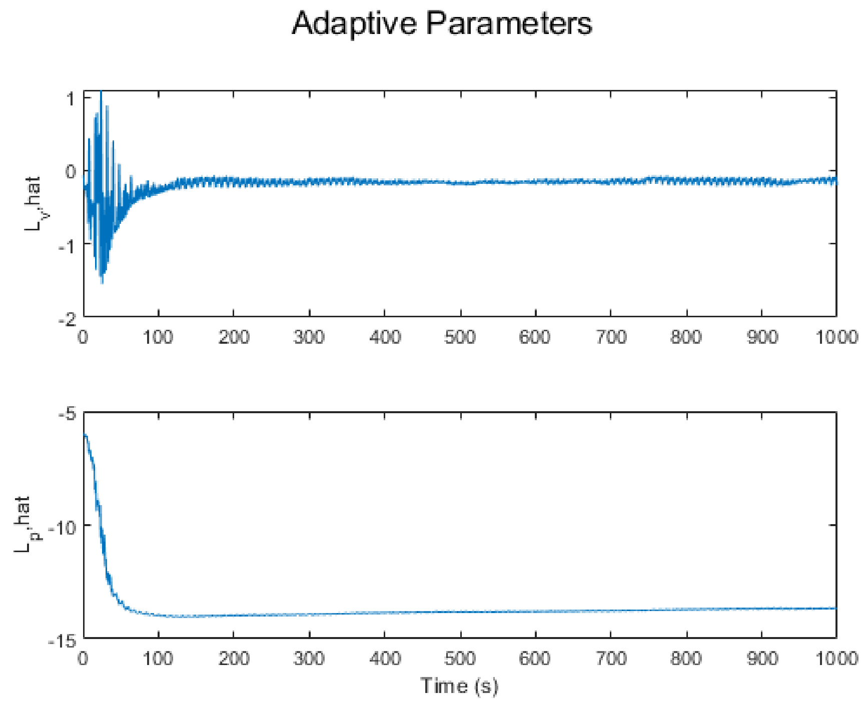

Figure 5.14 Adaptive Estimation of Parameters L_v and L_p with Noise

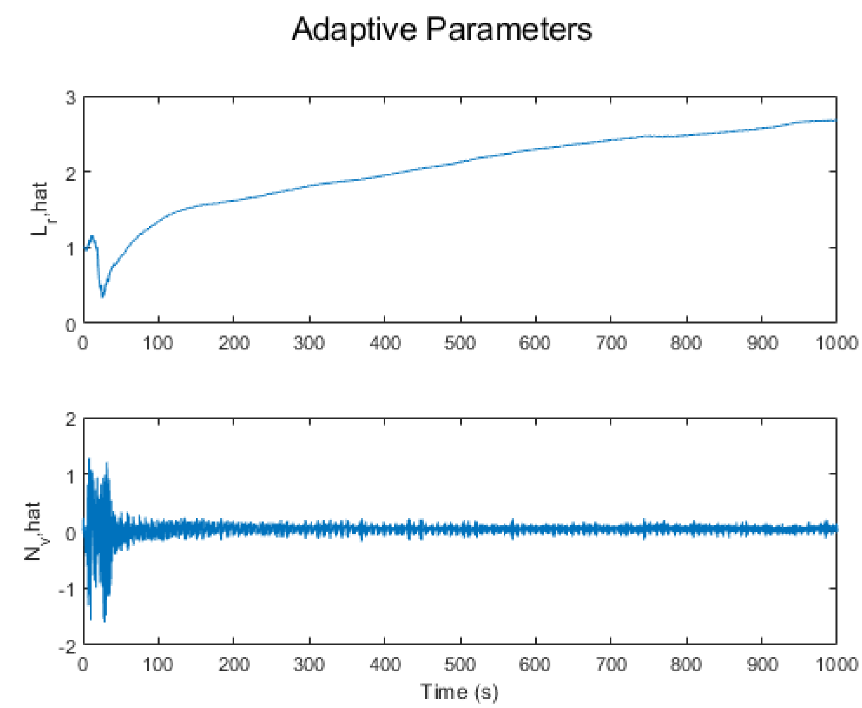

Figure 5.15 Adaptive Estimation of Parameters L_r and N_v with Noise

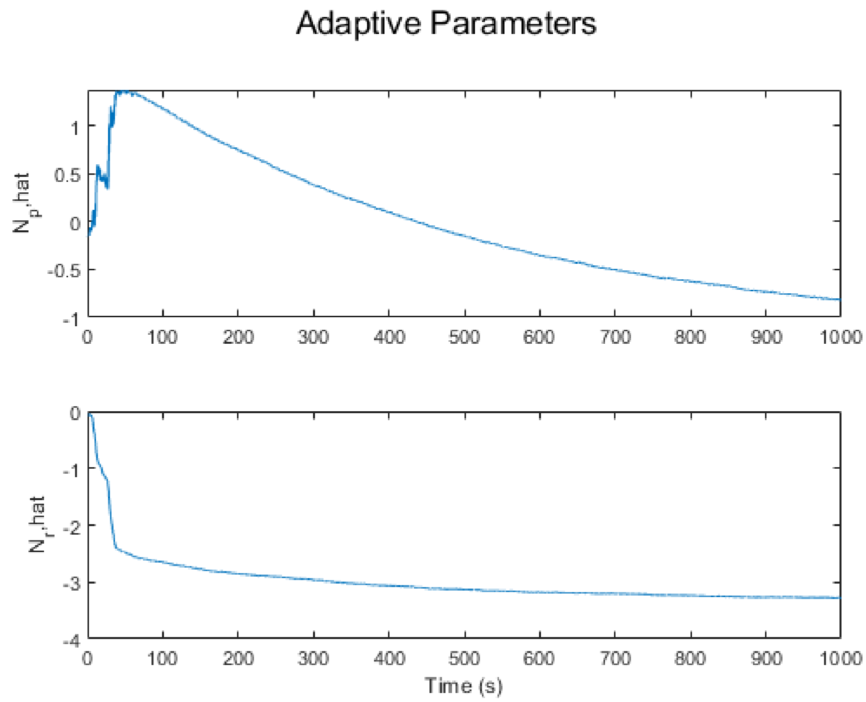

Figure 5.16 Adaptive Estimation of Parameters N_p and N_r with Noise

In the case of measurement noise being added to the system, the adaptive estimator does not yield good results. Not only do some parameters fail to converge anywhere near to their true values, but some parameters seem to diverge away. The EKF had minuscule divergence of some parameters, but in the case of the adaptive estimator, it can be seen that the divergence of some parameters is significantly larger with no indication of converging to their true values. For some parameters, convergence only begins after a significant time has progressed, a fact which is extrapolated from the adaptive update case without noise. In this specific case L_r, convergence to its true value only begins around eight hundred seconds. In the presence of noise, where multiple parameters exist, it is simply not advantageous to use the adaptive estimator for estimation. A summary of the adaptive estimator results can be seen in Table 5.5. Further tuning may potentially improve some parameters; however, it was noted in testing that the presence of noise only further amplifies the challenge of tuning.

Table 5.5

Adaptive Estimator Parameters estimated with noise compared to their Reference Values

Estimated Parameter	Reference Value	Adaptive Estimator	Percent Difference
L_v	-0.1377	-0.2055	49%
L_p	-12.9949	-13.6573	5%
L_r	2.1426	2.6781	25%
N_v	0.0422	0.0518	23%
N_p	-0.3597	-0.0817	77%
N_r	-1.2125	-3.2769	170%

5.3. Indirect MRAC with EKF Estimation

Both the Indirect MRAC and the EKF have notable inherent advantages. The current section presents simulation results where the EKF is used to estimate parameters that are in turn used by the indirect MRAC to control the system. Considering the limitations found with the adaptive estimator, it is expected that the EKF will improve the overall performance of the Indirect MRAC.

5.3.1. Results Without Noise

The results of the indirect MRAC-EKF without noise are presented in Figures 5.17-5.21.

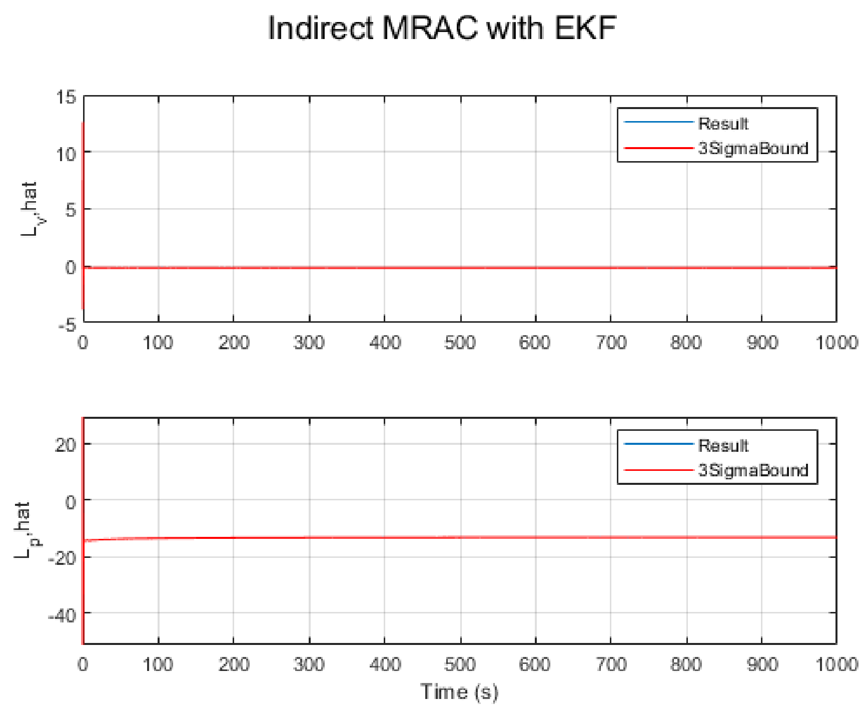

Figure 5.17 Indirect MRAC-EKF Estimation of Parameters L_v and L_p without Noise

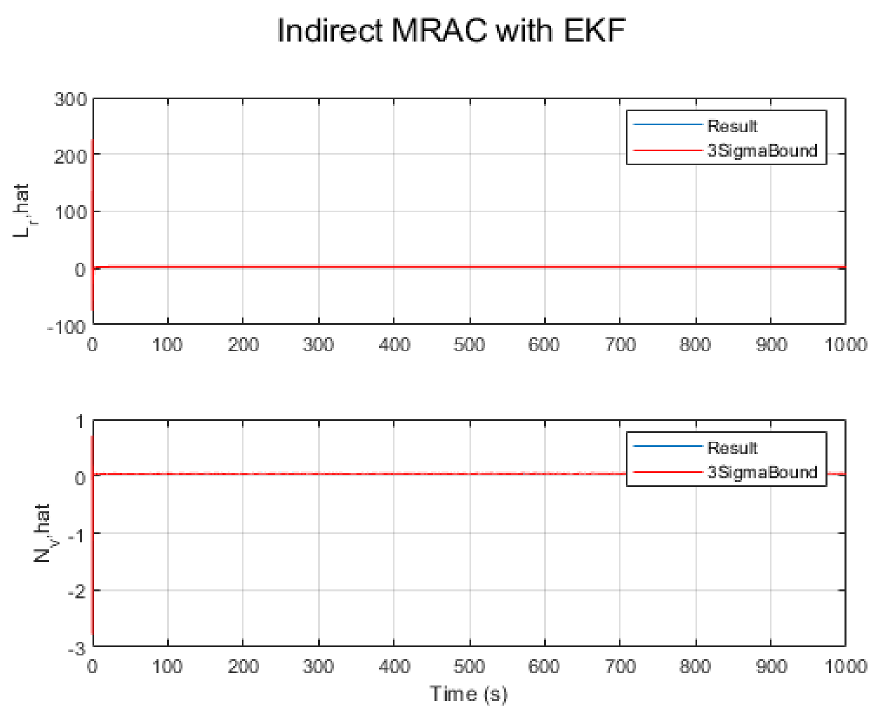

Figure 5.18 Indirect MRAC-EKF Estimation of Parameters L_r and N_v without Noise

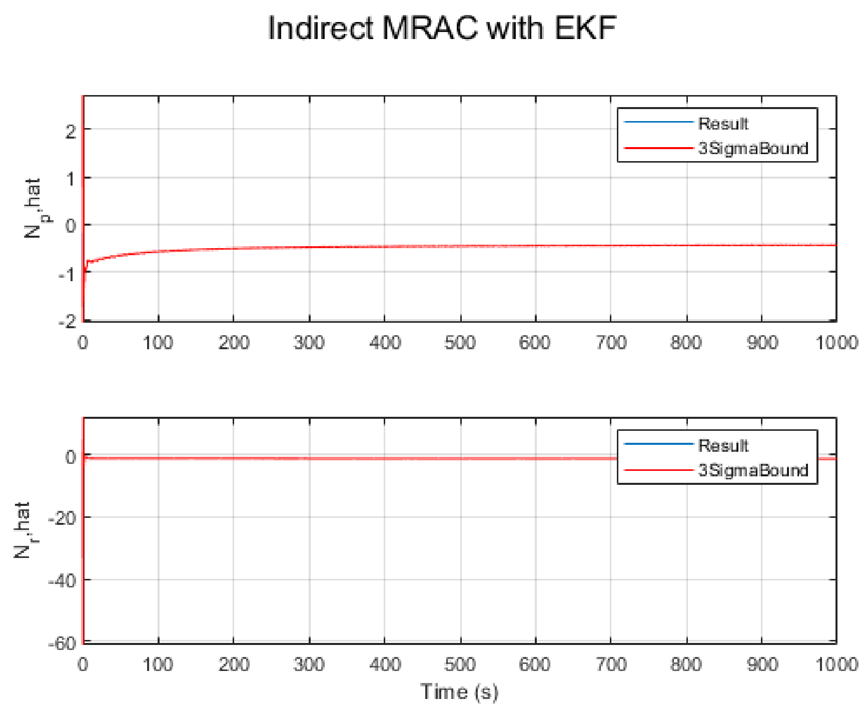

Figure 5.19 Indirect MRAC-EKF Estimation of Parameters N_p and N_r without Noise

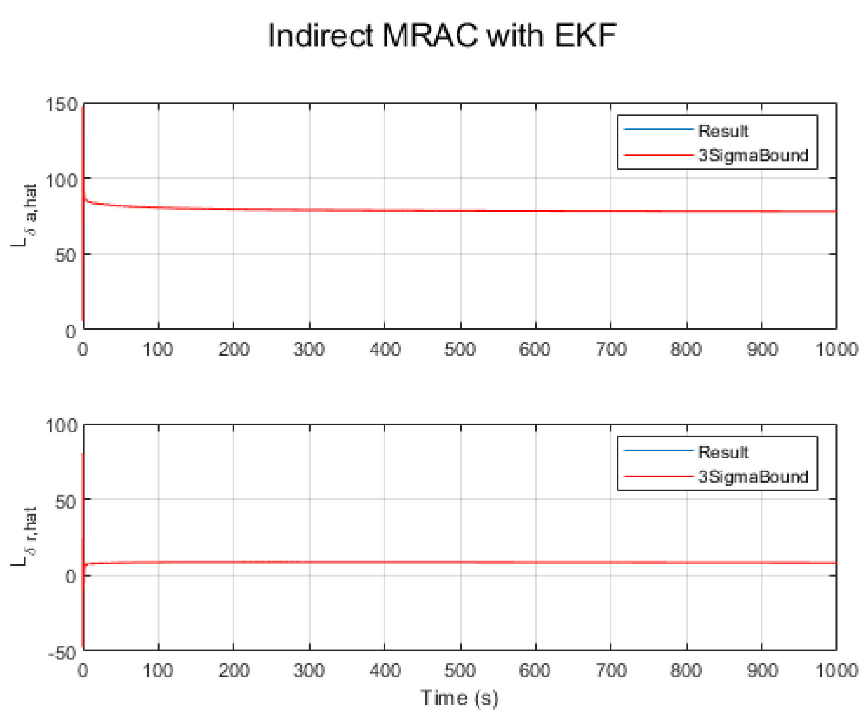

Figure 5.20 Indirect MRAC-EKF Estimation of Parameters $L_{\delta a}$ and $L_{\delta r}$ without Noise

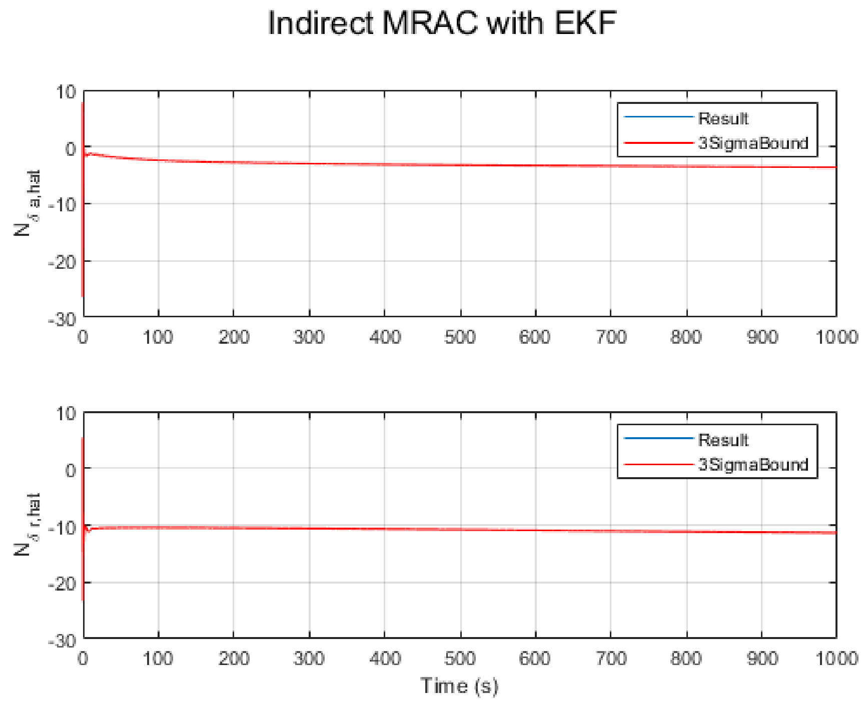

Figure 5.21 Indirect MRAC-EKF Estimation of Parameters $N_{\delta a}$ and $N_{\delta r}$ without Noise

Compared to the adaptive estimator, the EKF is able to estimate all ten parameters successfully. In fact, with the use of the EKF it can be seen that each parameter within seconds begins to converge towards its true value. In the current form, it should be noted that the Indirect MRAC-EKF system is not only estimating parameters but is also tracking the reference model. Regardless, as summarized in Table 5.6, all estimated parameters converge within satisfactory margins, except for the parameter $L_{\delta r}$ which had a percent difference of 68%. Comparing the sole estimation results of the EKF in Table 5.2 it can be seen that $L_{\delta r}$ was also the parameter that had a notably higher percent difference. Further tuning may potentially improve $L_{\delta r}$. However it was noted in testing that the system still remains very sensitive when small changes are made in tuning the EKF.

Table 5.6

Indirect MRAC-EKF Parameters estimated without noise compared to their Reference Values

Estimated Parameter	Reference Value	Indirect MRAC-EKF Estimate	Percent Difference
L_v	-0.1377	-0.1516	10%
L_p	-12.9949	-13.1859	1%
L_r	2.1426	2.5067	17%
N_v	0.0422	0.0469	11%
N_p	-0.3597	-0.4312	20%
N_r	-1.2125	-1.2587	4%
$L_{\delta a}$	75.3007	78.0047	4%
$L_{\delta r}$	4.8337	8.1347	68%
$N_{\delta a}$	-3.4231	-3.6196	6%
$N_{\delta r}$	-10.2218	-11.3603	11%

5.3.2. Results With Measurement Noise

The results of the indirect MRAC-EKF with additive measurement noise are presented Figures 5.22-5.26. The same noise characteristics were applied as in the previous cases.

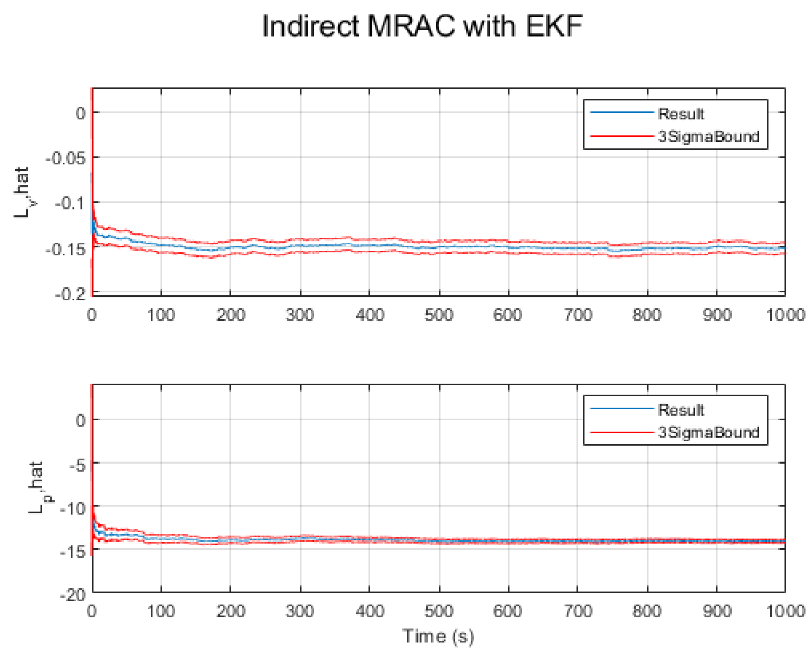

Figure 5.22 Indirect MRAC-EKF Estimation of Parameters L_v and L_p with Noise

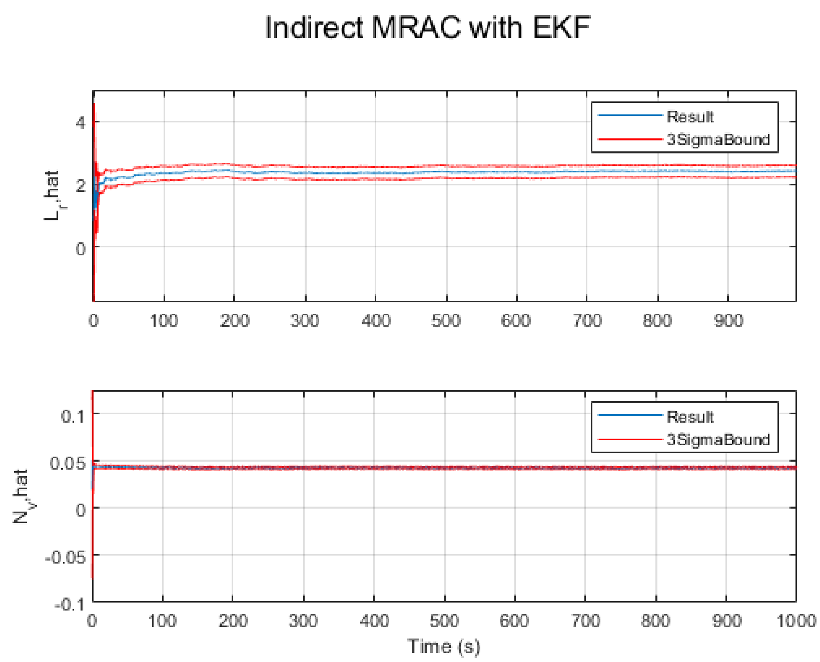

Figure 5.23 Indirect MRAC-EKF Estimation of Parameters L_r and N_v with Noise

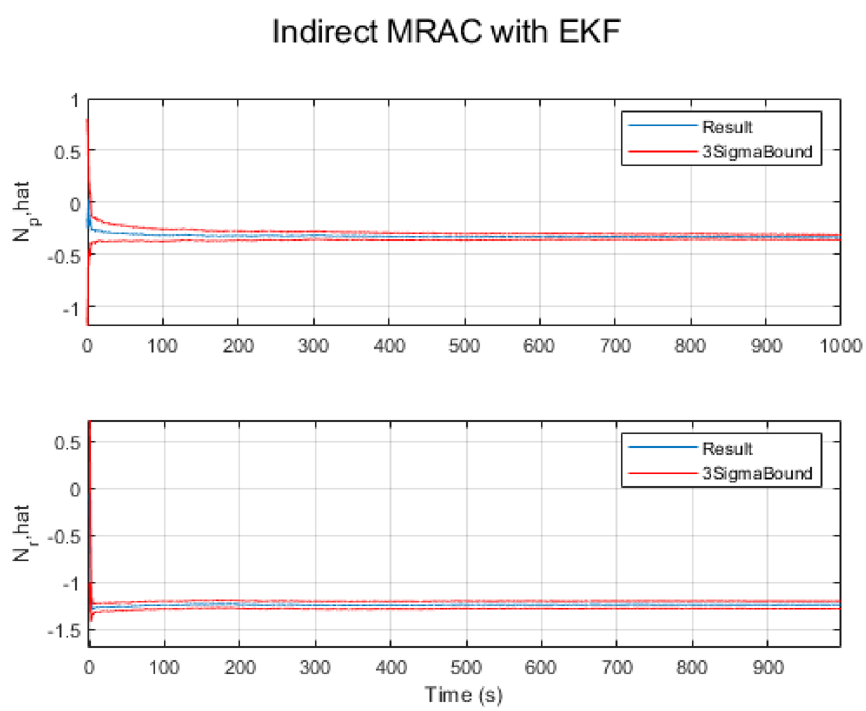

Figure 5.24 Indirect MRAC-EKF Estimation of Parameters N_p and N_r with Noise

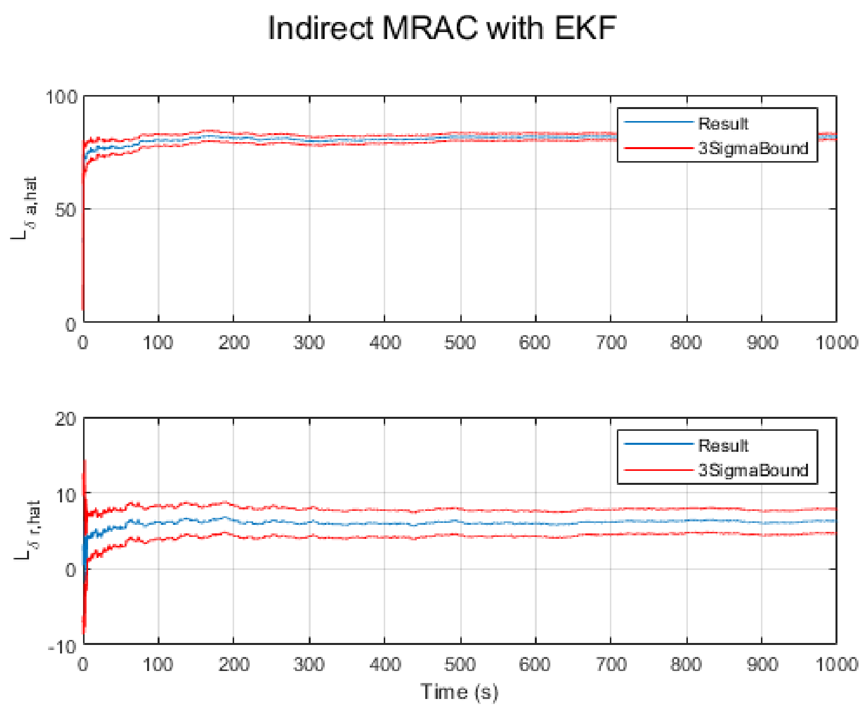

Figure 5.25 Indirect MRAC-EKF Estimation of Parameters $L_{\delta a}$ and $L_{\delta r}$ with Noise

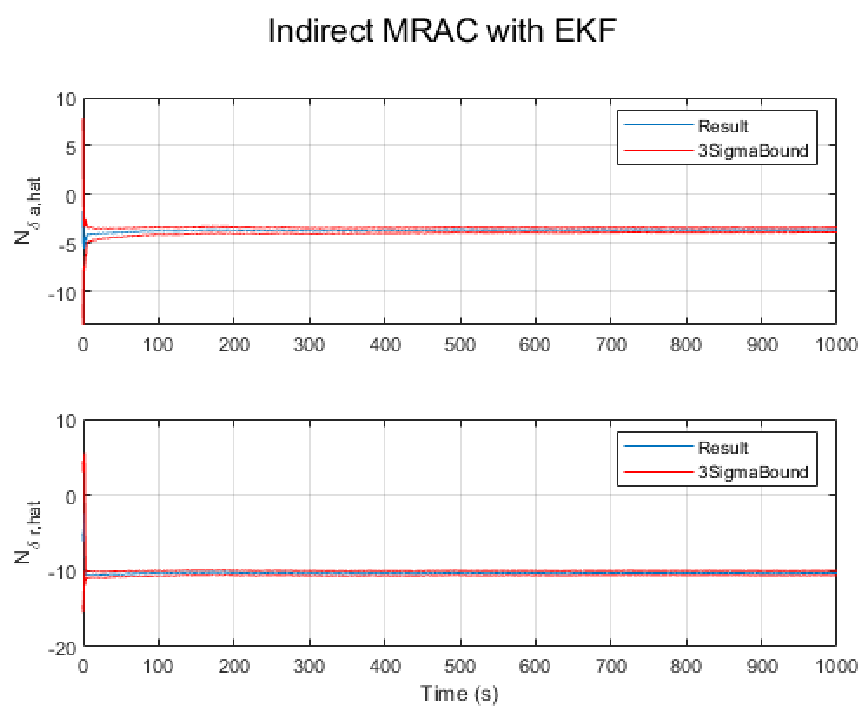

Figure 5.26 Indirect MRAC-EKF Estimation of Parameters $N_{\delta a}$ and $N_{\delta r}$ with Noise

The addition of noise to the system shows very promising results. In fact the estimates of all parameters have improved. Table 5.7 summarizes the results and it can be seen that only three parameters have a percent difference of over 10%. The results also show that, after the first couple of seconds, each parameter estimate remains fairly consistent over the remainder of the simulation. The use of the EKF as the estimator shows that accurate estimates of all parameters can be obtained when using the Indirect MRAC system. However, considerable attention must once again be taken when tuning the EKF in the presence of noise.

Table 5.7

Indirect MRAC-EKF Parameters estimated with noise compared to their Reference Values

Estimated Parameter	Reference Value	Indirect MRAC-EKF Estimate	Percent Difference
L_v	-0.1377	-0.1512	10%
L_p	-12.9949	-14.0399	8%
L_r	2.1426	2.4026	12%
N_v	0.0422	0.0428	1%
N_p	-0.3597	-0.3315	8%
N_r	-1.2125	-1.2408	2%
$L_{\delta a}$	75.3007	81.7589	9%
$L_{\delta r}$	4.8337	6.2560	29%
$N_{\delta a}$	-3.4231	-3.6326	6%
$N_{\delta r}$	-10.2218	-10.2805	1%

5.4. Tracking Control Performance

The tracking of the states v, p, r and ϕ using the indirect MRAC are presented based on the limitations of the adaptive estimator discussed. All parameters of the systems are assumed known by the system, excluding the six parameters being estimated by the adaptive update laws. Given persistently exciting input, it is expected that the closed-loop system should be able to track the reference model within appreciable margins.

5.4.1. Indirect MRAC with Adaptive Estimator

The tracking performance with and without noise is presented in Figures 5.26 to 5.37. The Indirect MRAC makes use of an adaptive update law which can be limited based on the results previously presented. However there still remains cases where its usage is applicable in tracking. It should be noted that further tuning of the adaptive update would not only result in an improvement in estimation but also would result in an improvement in tracking performance.

5.4.1.1. Results Without Noise

Outside the presence of noise, it can be seen that all states tracked the reference states very well within seconds of the simulation beginning, with the exclusion of p, which tracked later. The adaptive gains K_x and K_u provide an insight into what the control gains are doing at various points during simulation. It can be seen that the gains are very active at the early stages, but for some begin to converge approximately two hundred seconds into the simulation.

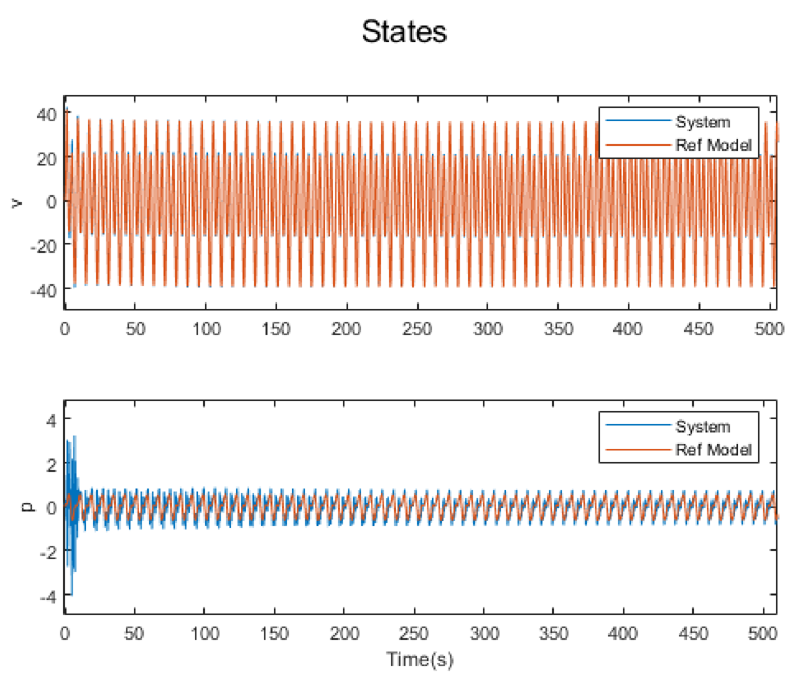

Figure 5.27 Indirect MRAC Tracking Performance of States v and p without Noise

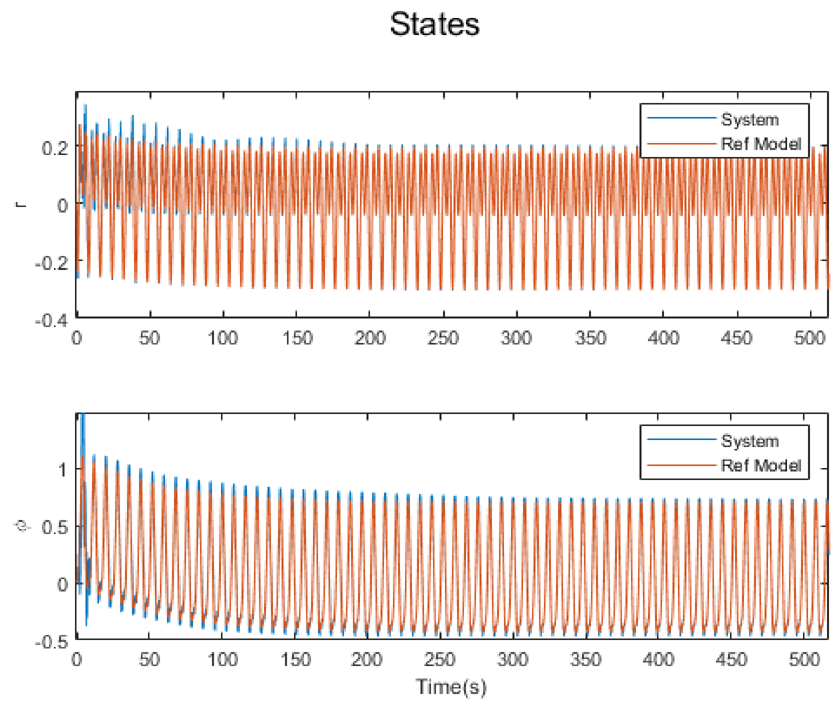

Figure 5.28 Indirect MRAC Tracking Performance of States r and ϕ without Noise

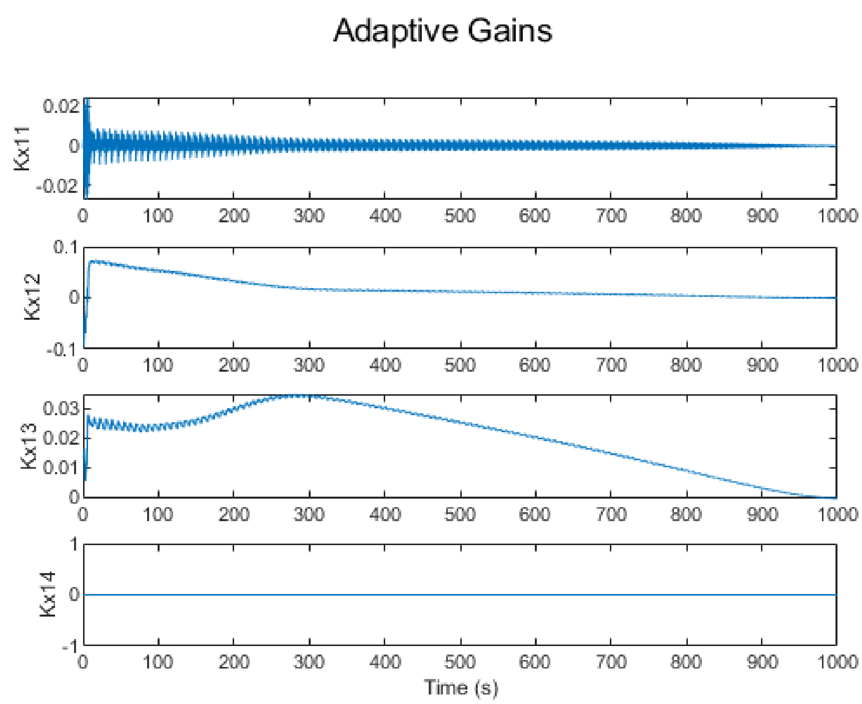

Figure 5.29 Indirect MRAC Adaptive Gains of $K_{x_{11}} - K_{x_{14}}$ without Noise

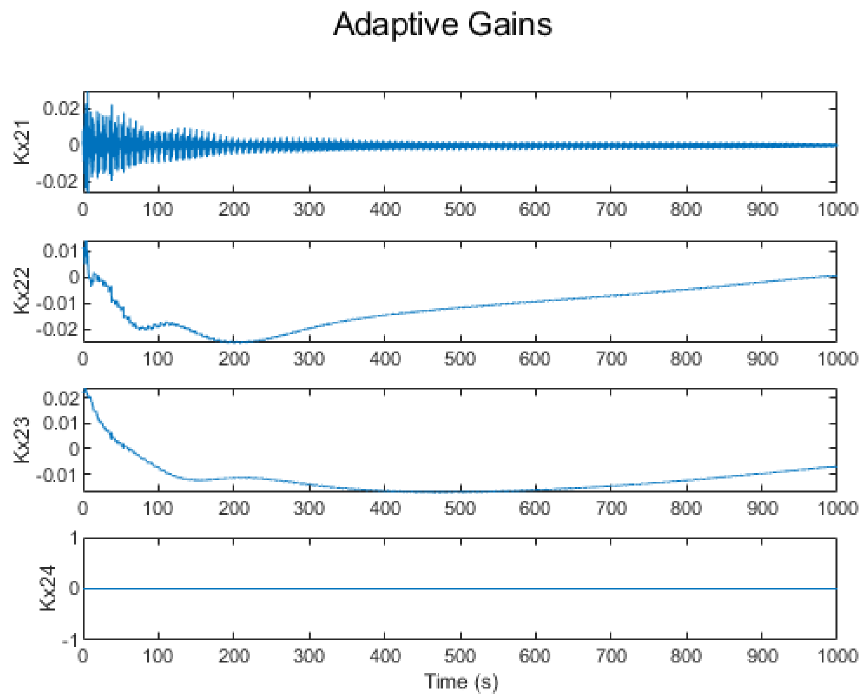

Figure 5.30 Indirect MRAC Adaptive Gains of $K_{x_{21}} - K_{x_{24}}$ without Noise

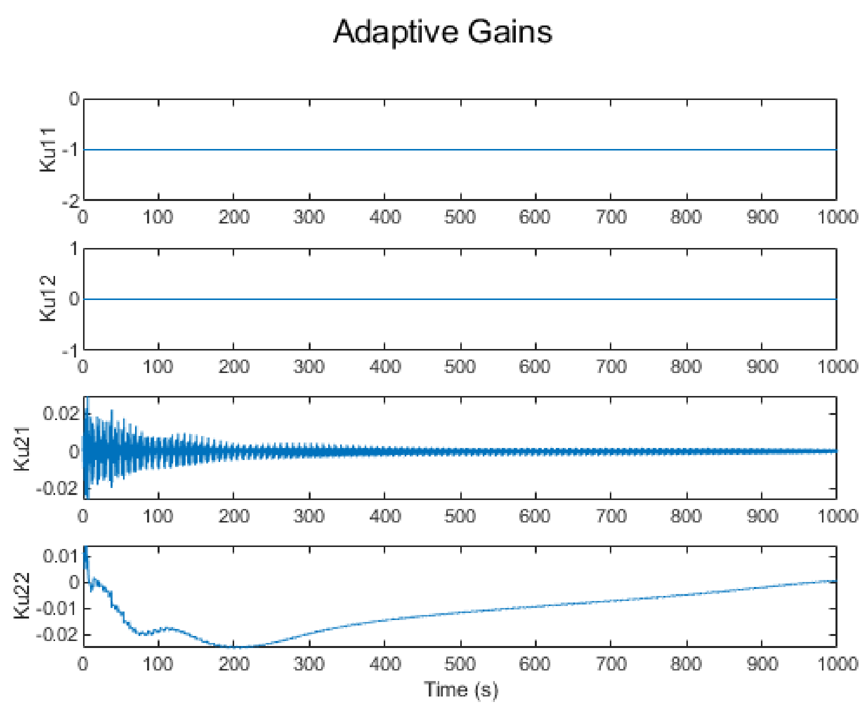

Figure 5.31 Indirect MRAC Adaptive Gains of K_u without Noise

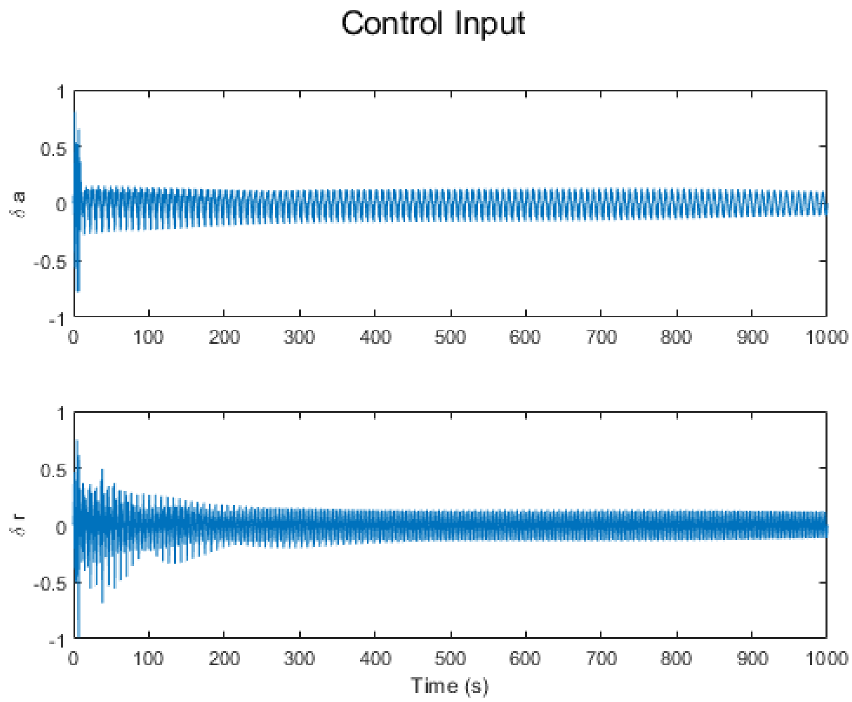

Figure 5.32 Indirect MRAC Control Input without Noise

5.4.1.2. Results With Measurement Noise

When the parameters are estimated within adequate margins, it is expected that the system should also track the reference model well. In the case of noise added to the system, the premise is proven because inferior estimates lead to inferior tracking. The tracking performance of the states is fair but was expected. The states do not perform as well as the case without noise. This is reasonable given the known performance of the adaptive estimator in the presence of noise. It can also be seen that many of the adaptive gains do not settle as in the noise free case. In fact, many of the gains seem to diverge as time progresses. The results further prove that it is first important that the parameters are estimated within reasonable margins.

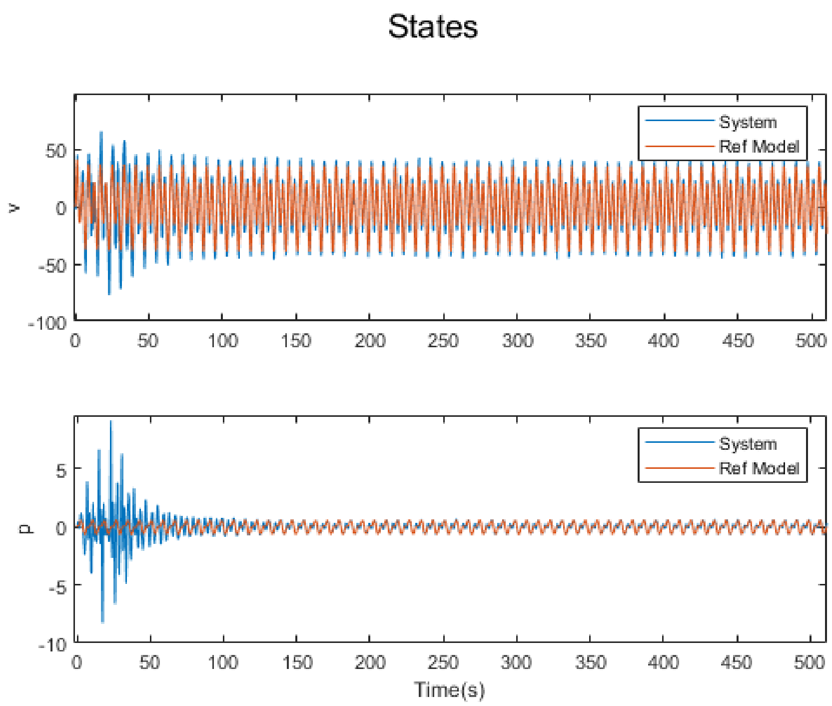

Figure 5.33 Indirect MRAC Tracking Performance of States v and p with Noise

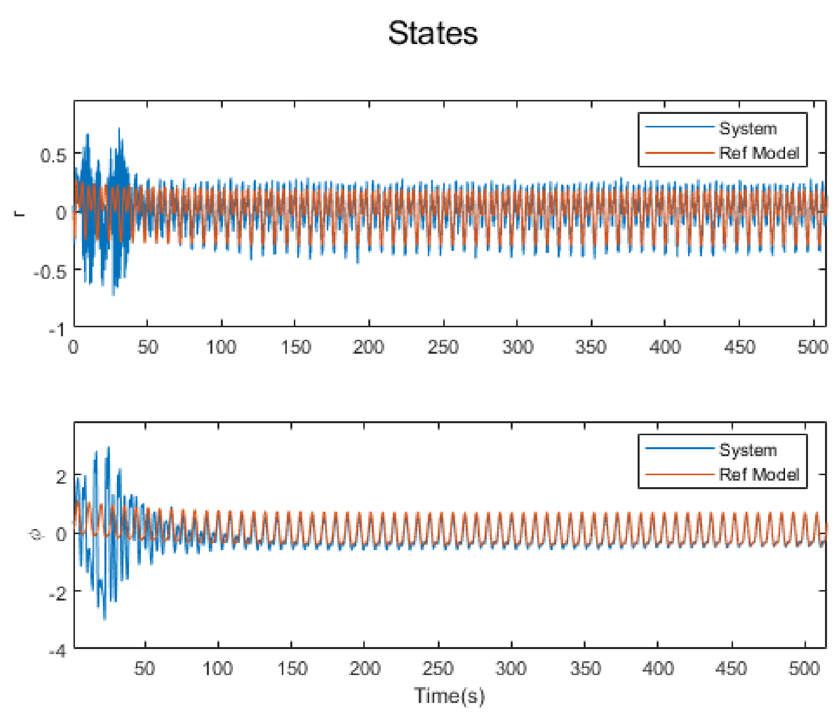

Figure 5.34 Indirect MRAC Tracking Performance of States r and ϕ with Noise

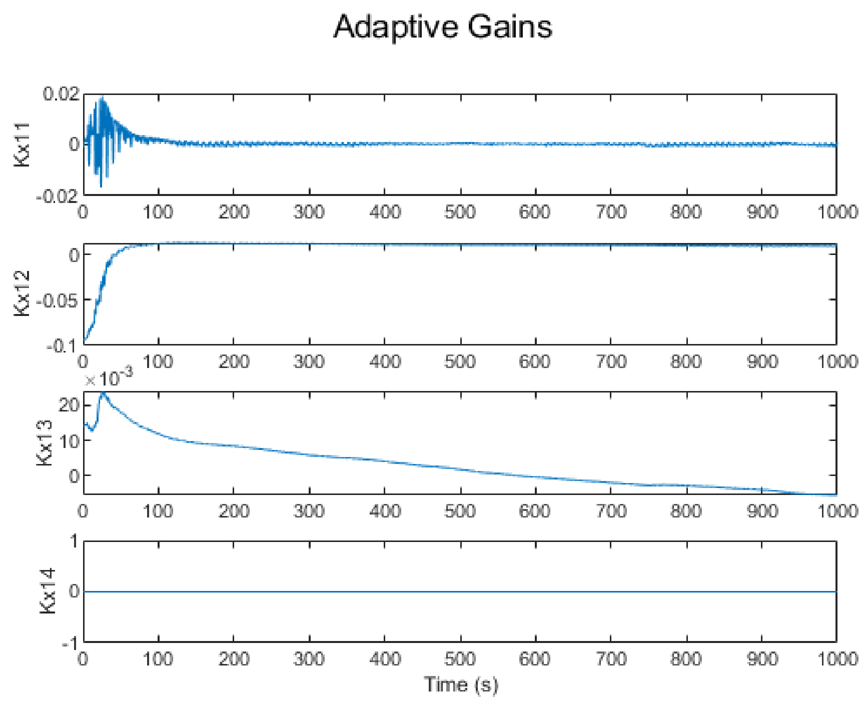

Figure 5.35 Indirect MRAC Adaptive Gains of $K_{x_{11}} - K_{x_{14}}$ with Noise

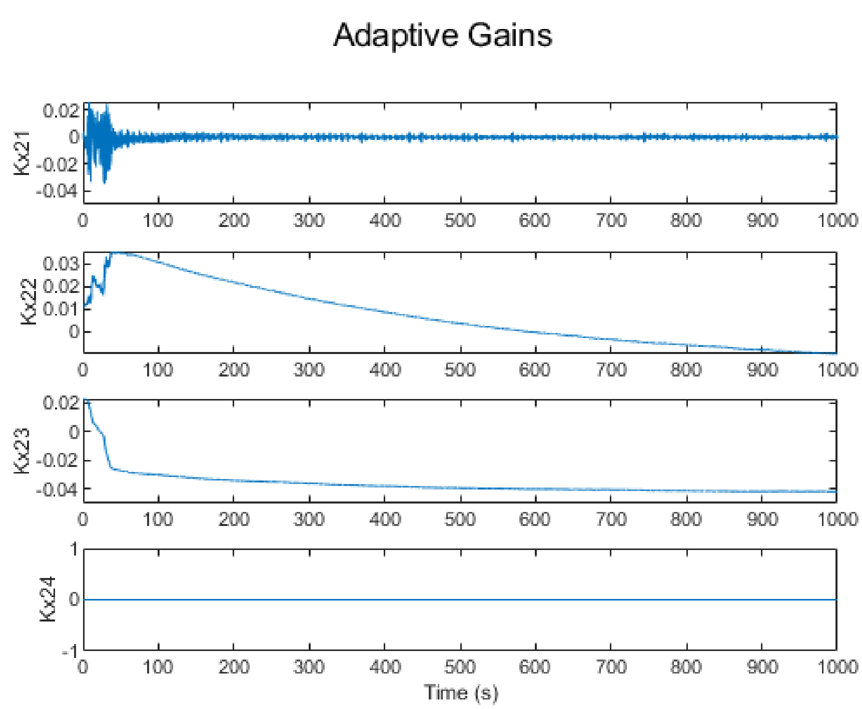

Figure 5.36 Indirect MRAC Adaptive Gains of $K_{x_{21}} - K_{x_{24}}$ with Noise

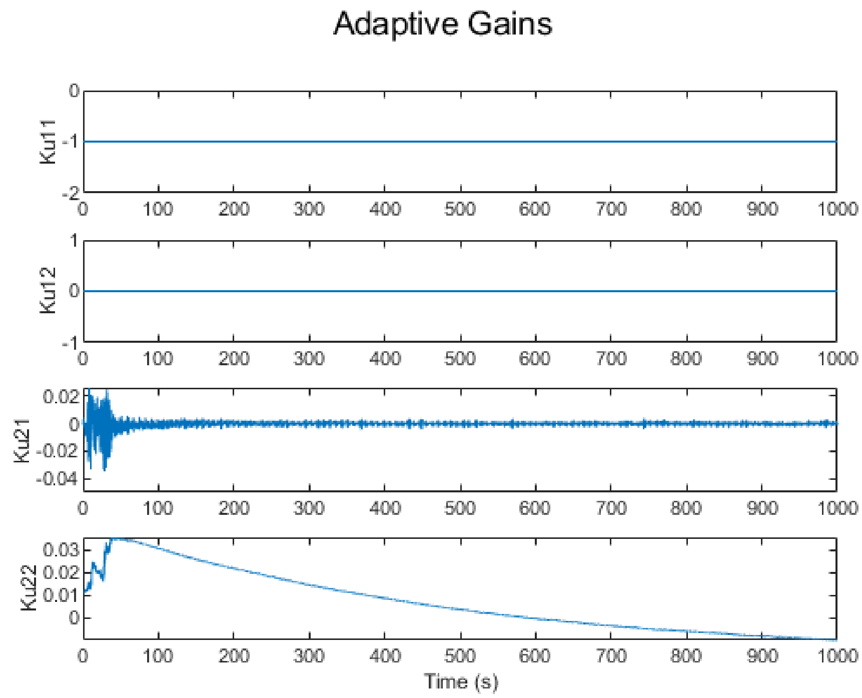

Figure 5.37 Indirect MRAC Adaptive Gains of K_u with Noise

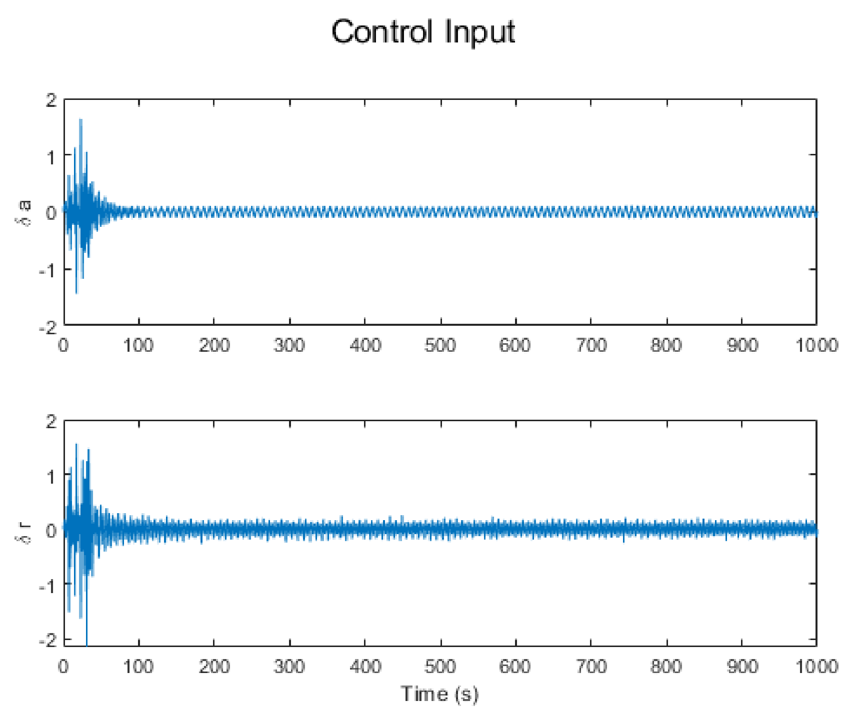

Figure 5.38 Indirect MRAC Control Input with Noise

5.4.2. Indirect MRAC with EKF Estimation

An EKF estimator within an Indirect MRAC addresses the major issue of the number of parameters to be estimated. The use of the EKF as the primary estimator allows for all ten parameters to be estimated simultaneously. The tuning of the system itself remains a significant challenge. The implementation of an EKF within an indirect MRAC results in the system becoming hypersensitive to tuning. Thus, in order to tune the system effectively, considerable a priori information may need to be known about the system. This goes against the premise of a user knowing little about the medium-scale UAV in development. Nevertheless, the EKF within an Indirect MRAC is still promising as the results will show because, not only is it well understood, but it is considerably robust in application.

5.4.2.1. Results Without Noise

The Indirect MRAC-EKF is able to track all four states outside the presence of noise. The system begins to track considerably well within seconds of the simulation beginning. This is conceivable because the EKF estimation results showed that each parameter begins to converge within seconds. The adaptive gains match the same premise, as very little activity occurs after the first few seconds of the simulation. This further solidifies the relationship between the quality of parameters estimated and the tracking performance.

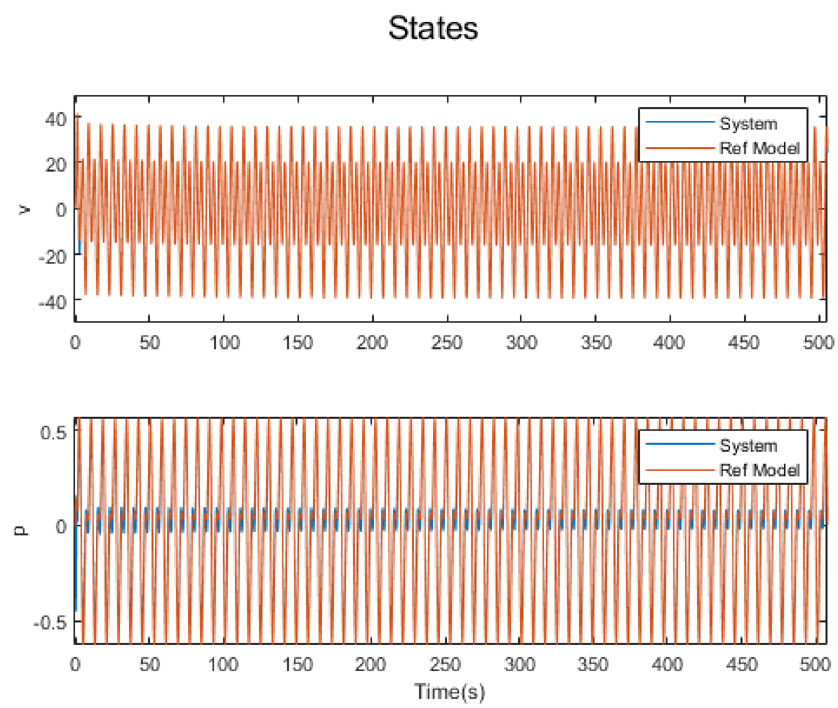

Figure 5.39 Indirect MRAC-EKF Tracking Performance of States v and p without Noise

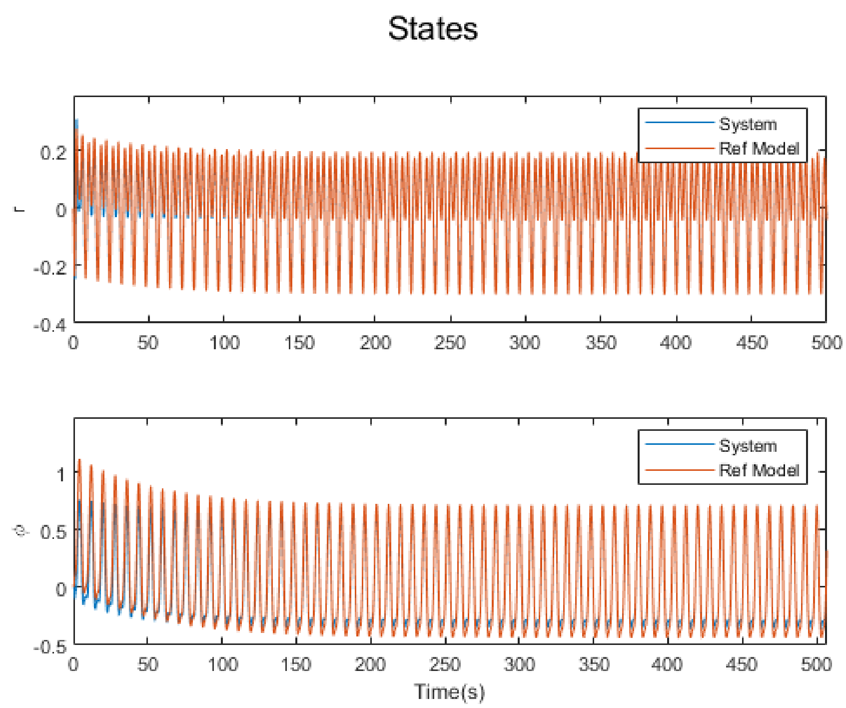

Figure 5.40 Indirect MRAC-EKF Tracking Performance of States r and ϕ without Noise

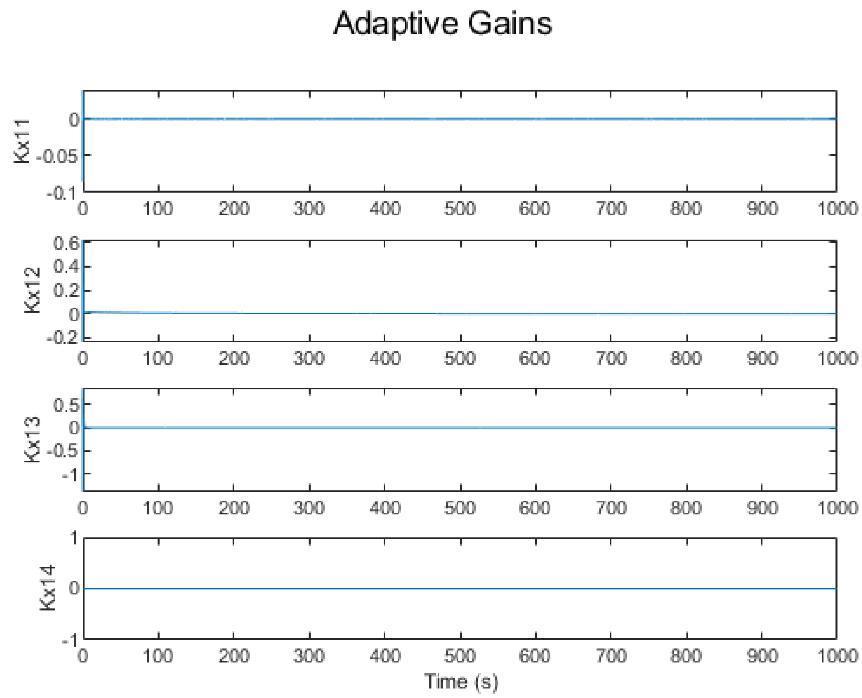

Figure 5.41 Indirect MRAC-EKF Adaptive Gains of $K_{x_{11}} - K_{x_{14}}$ without Noise

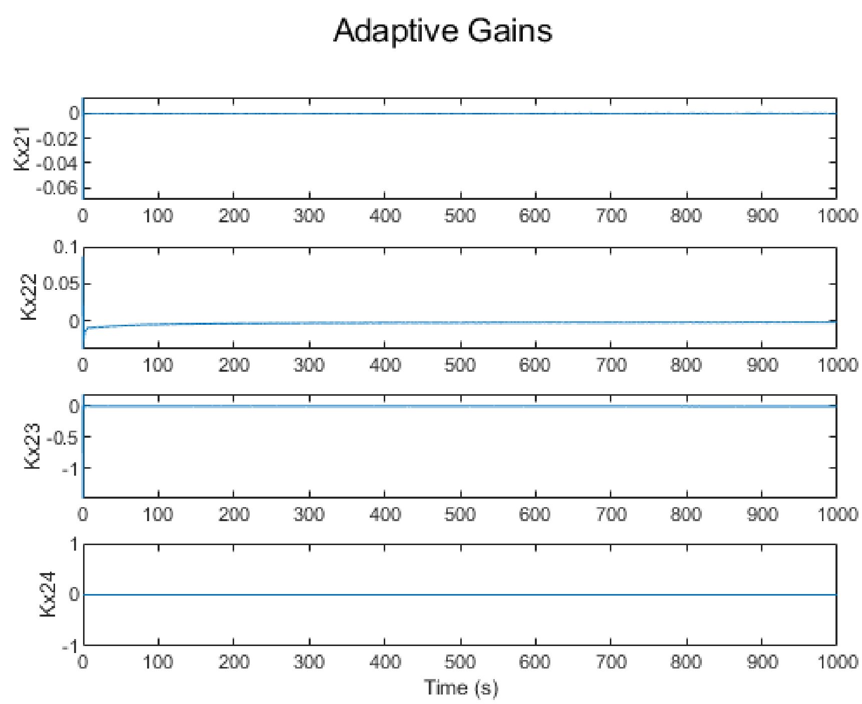

Figure 5.42 Indirect MRAC-EKF Adaptive Gains of $K_{x_{21}} - K_{x_{24}}$ without Noise

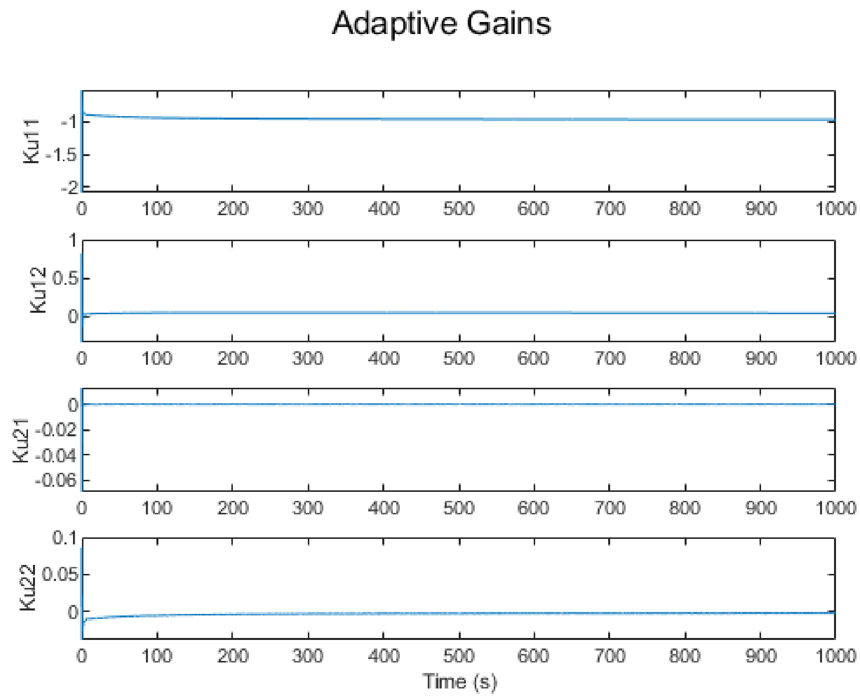

Figure 5.43 Indirect MRAC-EKF Adaptive Gains of K_u without Noise

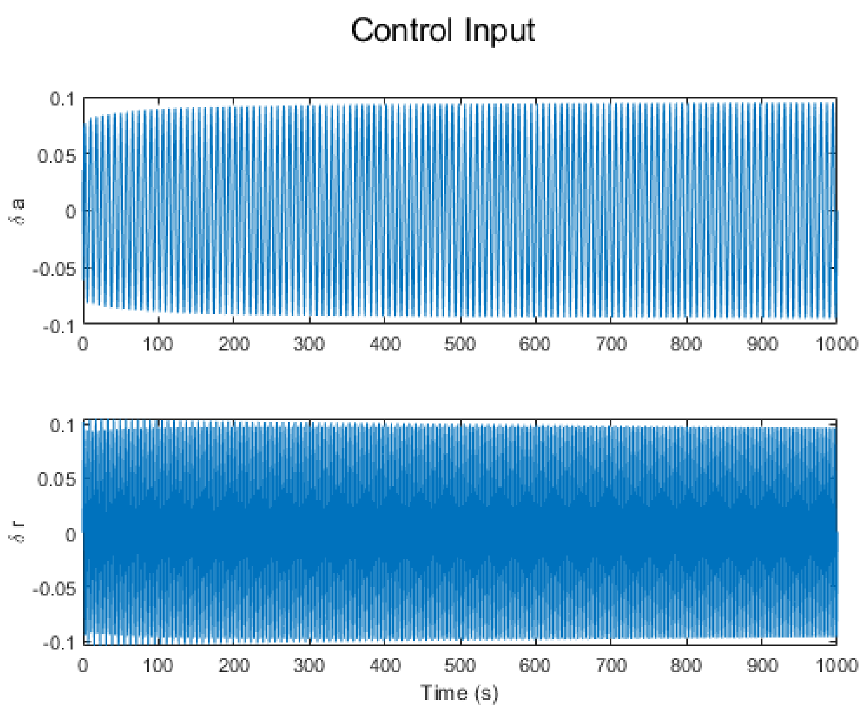

Figure 5.44 Indirect MRAC-EKF Control Input without Noise

5.4.2.2. Results With Measurement Noise

In the case of noise added to the Indirect MRAC-EKF, the results are promising. All four states are reasonably tracked with the exclusion of state r. Further tuning of the system should improve the tracking performance; however, that still remains a challenge given the sensitive nature of the Indirect MRAC-EKF system. It is interesting to note that even with the relatively poor tracking performance of state r, the adaptive gains still remain active only within the first few seconds of the simulation. This indicates that the system believed its parameter estimates were sufficient. Further tuning will then not only drive the estimates closer to their true values but will also improve the tracking performance of all states.

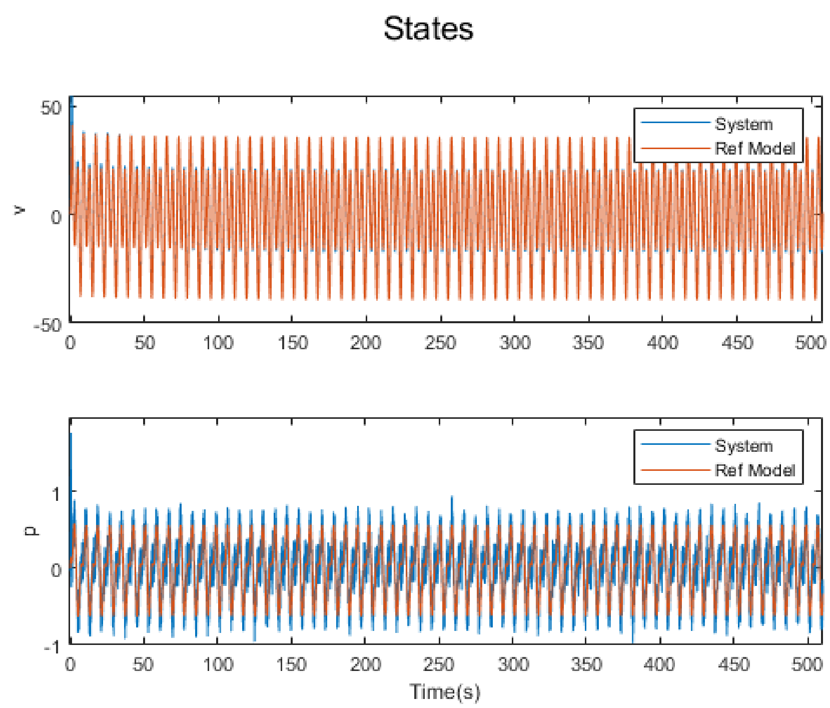

Figure 5.45 Indirect MRAC-EKF Tracking Performance of States v and p with Noise

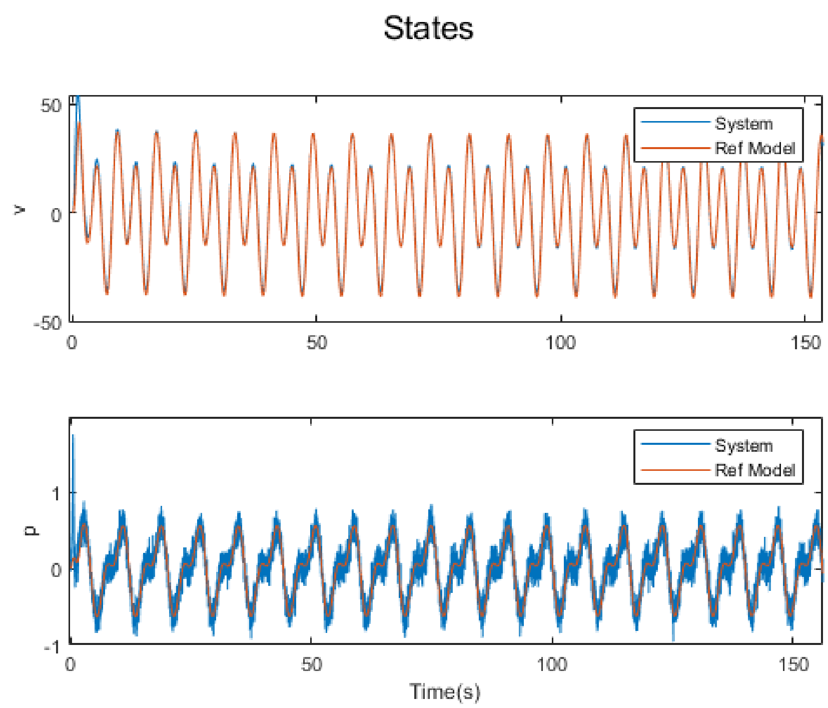

Figure 5.46 Indirect MRAC-EKF Tracking Performance of States v and p with Noise (Zoomed-In)

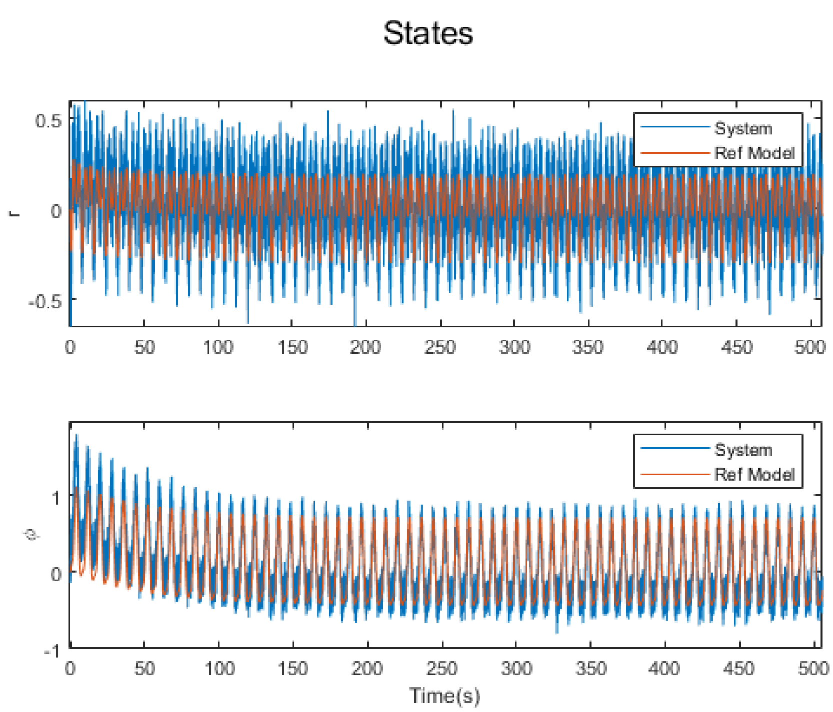

Figure 5.47 Indirect MRAC-EKF Tracking Performance of States r and ϕ with Noise

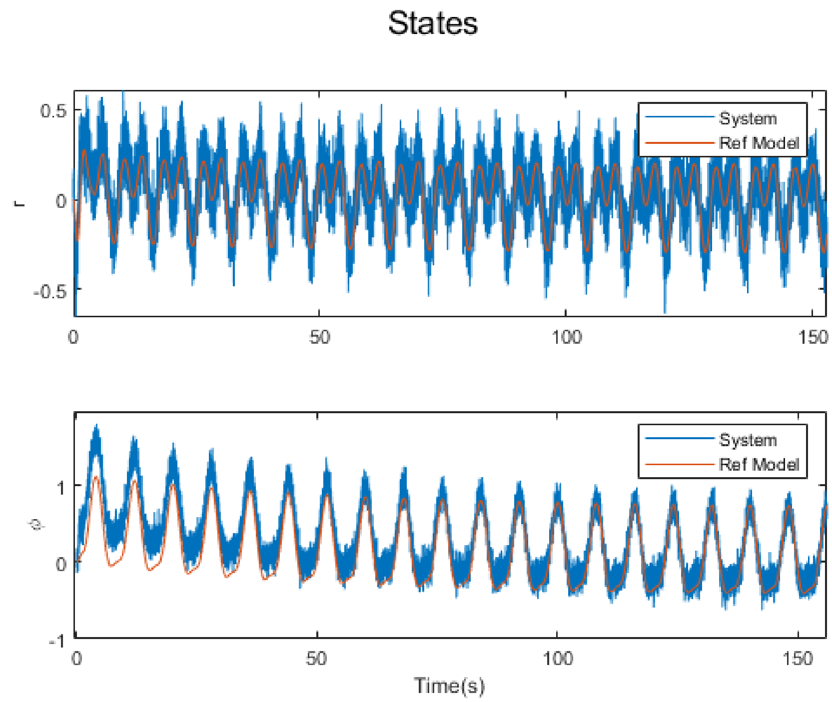

Figure 5.48 Indirect MRAC-EKF Tracking Performance of States r and ϕ with Noise (Zoomed-In)

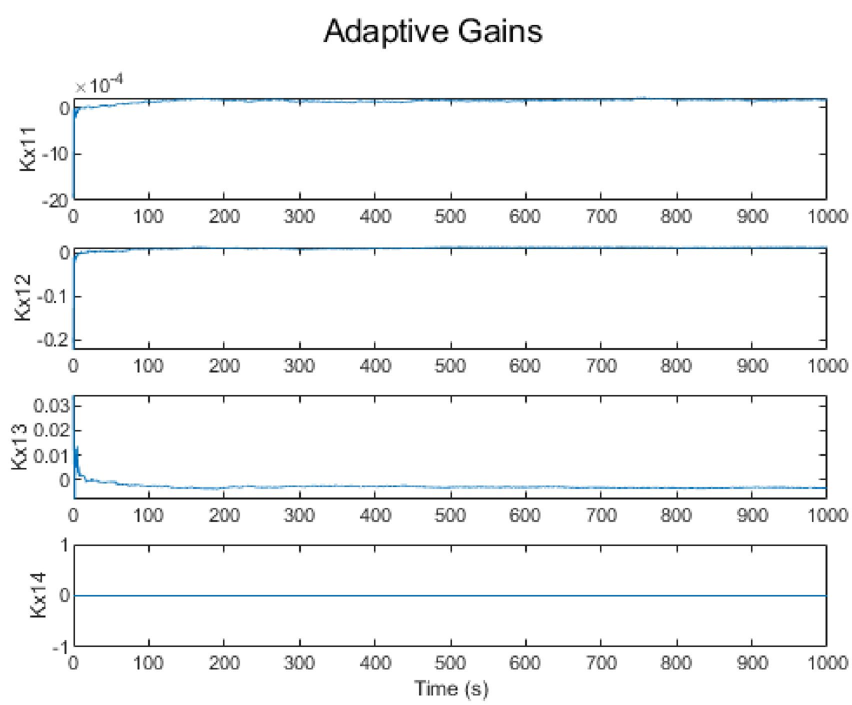

Figure 5.49 Indirect MRAC-EKF Adaptive Gains of $K_{x_{11}} - K_{x_{14}}$ with Noise

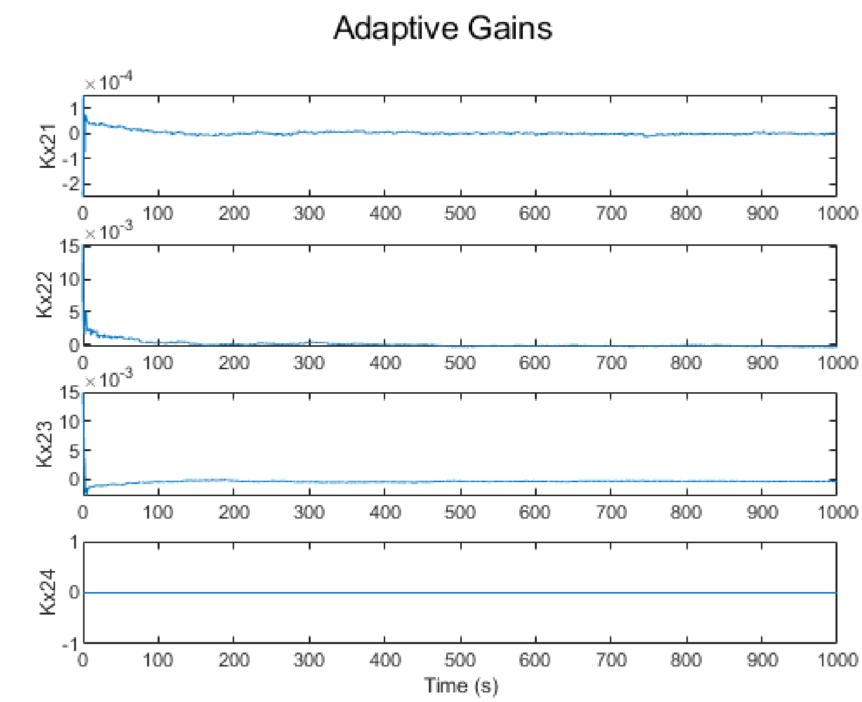

Figure 5.50 Indirect MRAC-EKF Adaptive Gains of $K_{x_{21}} - K_{x_{24}}$ with Noise

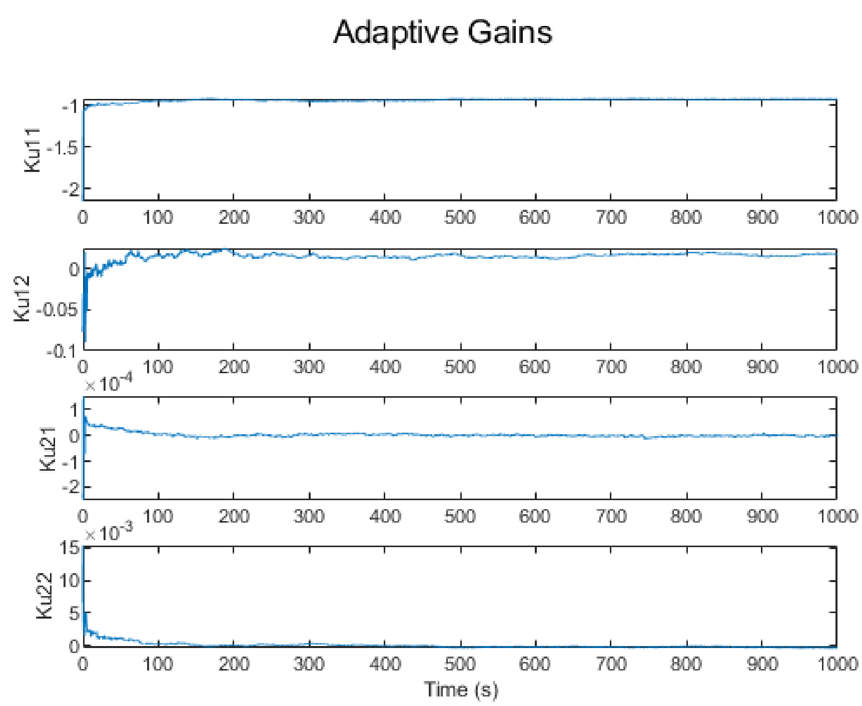

Figure 5.51 Indirect MRAC-EKF Adaptive Gains of K_u with Noise

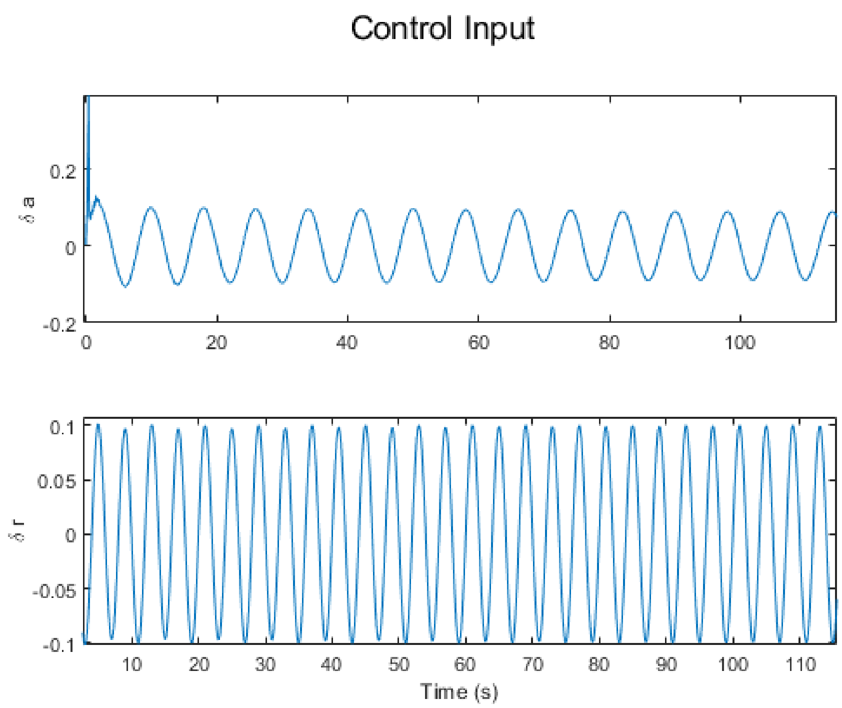

Figure 5.52 Indirect MRAC-EKF Control Input with Noise

5.5. Comparison of Estimation Methods

To directly compare the estimation results, the percent difference for each estimated parameter based on the two approaches is summarized in Table 5.8. It should be noted that the adaptive estimator was not able to estimate all ten parameters and therefore the percent difference for only the first six parameters is shown. The adaptive estimator approach also resulted in the highest percent errors of any case in the presence of noise, while at the same time having minimal percent error for the noise free case. The low percent error can, however, be misleading as the adaptive estimator had significant challenges functioning in a scenario with all parameters estimated. Overall, the Indirect MRAC-EKF shows promising estimation results and only the challenge of tuning remains.

Table 5.8

Percent difference estimation comparison of the EKF, Adaptive Update and the Indirect MRAC-EKF

		Without Noise			With Noise		
Parameter	Reference Value	EKF	Adaptive Estimator	Indirect MRAC-EKF	EKF	Adaptive Estimator	Indirect MRAC-EKF
L_v	-0.1377	2%	8%	10%	1%	49%	10%
L_p	-12.9949	0%	0%	1%	0%	5%	8%
L_r	2.1426	5%	2%	17%	2%	25%	12%
N_v	0.0422	2%	3%	11%	19%	23%	1%
N_p	-0.3597	4%	8%	20%	29%	77%	8%
N_r	-1.2125	3%	28%	4%	24%	170%	2%
$L_{\delta a}$	75.3007	1%	-	4%	0%	-	9%
$L_{\delta r}$	4.8337	16%	-	68%	8%	-	29%
$N_{\delta a}$	-3.4231	2%	-	6%	48%	-	6%
$N_{\delta r}$	-10.2218	3%	-	11%	20%	-	1%

To gain a quantitative understanding of the tracking performance of the Indirect MRAC and the Indirect MRAC-EKF, the root mean square error (RMSE) was calculated for each state and is shown in Table 5.9. It should be noted that because of the limitations in the adaptive estimator, the Indirect MRAC only estimated six of the ten parameters. Therefore, it may seem that the Indirect MRAC may perform better in some aspects, but that is only under certain conditions. Overall, all aspects considered, the Indirect MRAC-EKF outperforms the Indirect MRAC.

Table 5.9

Root Mean Square Error (RMSE) state tracking performance of the Indirect MRAC and Indirect MRAC-EKF

	Without Noise		With Noise	
State	Indirect MRAC	Indirect MRAC-EKF	Indirect MRAC	Indirect MRAC-EKF
v	0.4066	1.6969	3.8213	1.5044
p	0.2071	0.0431	0.5082	0.1103
r	0.0090	0.0169	0.0767	0.1018
ϕ	0.0425	0.0661	0.3854	0.1385

6. Conclusions and Future Work

Estimation has been crucial for the progression of modern technology. It is often in instances where much is not known about a system that estimation is able to offer valuable insight. The presumption is the same in the case of parameter estimation. Smaller companies involved in medium-scale UAV development do not have the means to conduct extensive research compared to manufacturers of larger, manned aircraft. Parameter estimation is able to provide valuable insight where notable uncertainties exist within aircraft parameters. That insight can be used to lead an aircraft towards specific flight characteristics with the use of an Indirect MRAC. As it relates to parameter estimation of manned aircraft, a pilot would need to make certain that they inject consistent and adequate control inputs to excite the system sufficiently, therefore ensuring that parameters are estimated within suitable margins.

6.1. Conclusions

The thesis found that the EKF is a solid approach to parameter estimation especially in the case where a significant number of parameters needs to be estimated. Both in and outside the presence of measurement noise, the EKF is consistent but must be tuned correctly to acquire suitable results. The adaptive estimator within the Indirect MRAC is predominantly different. When a small number of parameters needs estimation, the adaptive estimator is sufficient and, in some instances, may be superior. This reality was seen in the case model presented in Chapter 3. On the other hand, a point exists where the number of parameters estimated may directly affect the estimation abilities of the adaptive estimator. The adaptive estimator implemented in this thesis was only able to estimate a total of six parameters out of the ten that were required. Even with only six parameters, the adaptive estimator failed to estimate all parameters accurately in the presence of noise. It should also be noted that the tuning of the adaptive update became increasingly difficult as the number of parameters increased. To combat the parameter limitations, the EKF was used to replace the adaptive update laws inside the Indirect MRAC system. Not only were all parameters able to be estimated within the MRAC system but all parameters were estimated within suitable margins. The EKF, when placed in the Indirect MRAC implementation, becomes notably sensitive to tuning. Close attention must be made during the tuning process as unfavorable results are possible if incorrectly tuned.

The state tracking performance was found to have an interrelation with the quality of the parameters estimated. Therefore, parameters estimated with very large margins resulted in the system being unable to adequately track the reference model. In the same instance, an observation of the adaptive gains K_x and K_u shows that over the course of the simulation, the gains do not converge. Specifically, as it relates to the Indirect MRAC-EKF, the results of the RMSE shows a promising future in terms of online applications. The ability to estimate a greater number of parameters while still maintaining feasible tracking performance makes the Indirect MRAC-EKF a more attractive method when compared to the adaptive estimator.

6.2. Future Work

The thesis has investigated various estimation techniques and their applicability within an adaptive control framework. Nevertheless, the thesis only provides a basis and there still remain many avenues to continue the research. Some of the possible avenues for future work are presented below:

- Investigate other time-domain and frequency-domain estimation techniques within the adaptive control framework.
- Evaluate possible improvements to the adaptive update laws which could results in multiple parameter estimation. These includes adding robustness terms such as sigma- and e-modification (Nguyen N. T., 2018).
- Address the limitations of the pseudo inverse within the Indirect MRAC and evaluate the new estimation and tracking performance.
- Explore methodologies to improve the tuning properties of the Indirect MRAC and Indirect MRAC-EKF.
- Investigate implementations where parameters do not remain constant over time, but are dependent upon the states of the system.
- Implement the Indirect MRAC and parameter estimation algorithms on a larger set of flight simulation models and possibly in flight test.

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