Comparative Performance Study of Lyapunov Based MRAC Technique and MRAC Augmented with PID Controller for Speed Control of a DC motor

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***Abstract*— The speed control of Direct Current (DC) motor is one of the widely used industrial control due its specific characteristics. This paper discusses about the application of MRAC using Lyapunov rule and Model Reference Adaptive Control (MRAC) augmented with Proportional Integral Derivative (PID) method to control the speed of a DC motor system. Different values of adaptation gains are taken for comparative analysis. Simulation is done in MATLAB Simulink environment. A detail comparative performance has been stated with different MRAC strategies applied to the DC motor system.**

Keywords*— *Adaptive Control, Lyapunov rule, MIT rule, MRAC, PID Tuning

I. Introduction

A control system is basically an interconnection between different components forming a system configuration which provides a desired system response. Adaptive Control is a technique which adjusts the controller automatically in real time, for achieving or for maintaining a desired level of control system performance whenever the parameters of the plant dynamic model are unidentified and/or change in time [1].

When the plant parameters are fixed, then by adjusting the PID gains of the controller, desired performance can be achieved. But the performance of the system using PID is not satisfactory due to low bandwidth, dependency of error on plant for its parameter variation and noise sensitivity [2], [3], [4]. So when the plant parameters are not accurately known, adaptive control technique is used. Here an additional loop is situated over and above the ordinary feedback loop, is necessary for adapting/adjusting the controller parameters to minimize the difference of output between plant and reference model [2]. In this paper a comparative study is discussed on MRAC using Lyapunov rule and MRAC augmented with PID method, on speed control of a DC motor.

II. Model Reference Adaptive Control

The system where the parameters are not accurately known the response of a system using ordinary feedback loop turns inaccurate, in that case adaptive control is used. The MRAC adjustment mechanism can be done by gradient method known as Massachusetts Institute of Technology (MIT) rule or by using a stability theory known as Lyapunov method or by augmented error [1], [5].

In MRAC, a reference model as designed by the designer is chosen that describes the desired response. It consists of four blocks i) Reference Model ii) Plant iii) Controller iv) Adaptive mechanism, that adjusts controller to drive the error between output of plant and reference model towards zero (Fig. 1), [2], [3], [6]. The MRAC using Lyapunov method is used to determine overall stability of the system [4], [5]. The MRAC augmented with PID technique uses MIT rule to adjust the gain of PID controller.

To design MRAC, tracking error $e(t)$, is expressed as the difference between plant output and reference model output.

$$e(t) = y(t) - y_m(t) \tag{1}$$

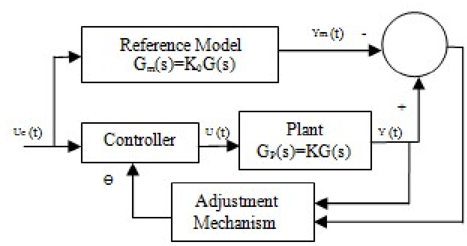

Fig. 1. Model Reference Adaptive Control

From this error a cost function $J(\theta)$ is given as

$$J(\theta) = \frac{e^2}{2} \tag{2}$$

As per the MIT rule, $\dot{\theta}$ is directly proportional to the negative gradient of cost function, is given by

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma e \frac{\partial e}{\partial \theta} \tag{3}$$

Where, $\frac{\partial e}{\partial \theta}$ is sensitivity derivative and it determines how the parameter theta is updated. γ is adaptation gain.

III. DC Motor

The Direct Current motor is used in different industrial control applications such as medical, automobile and aircraft etc. It is smooth, efficient, provides high starting torque and also has a property of quick reversal.

The transfer function of a DC Motor is given by, [7]

$$G(s) = \frac{\dot{\theta}(s)}{E_a(s)} = \frac{K_T}{(R_a + sL_a)(Js + f_0) + K_T K_b} \tag{4}$$

The parameters of the DC motor are given in Table. I, [8].

TABLE I. PARAMETERS OF DC MOTOR

	Parameter	Value
1	Moment of inertia of motor & load as referred to motor shaft, J	0.5 Kg-m^2
2	Viscous friction coefficient in rotating parts of motor & load as referred to motor shaft, f_0	0.5Nm/rad/s
3	Motor torque constant, K_T	1 Nm/A
4	Back emf constant, K_b	1 V-s/rad
5	Inductance of armature winding, L_a	1 Henry
6	Resistance of armature, R_a	1 Ohm

IV. Performance Indices

Performance indices such as Integral of the Absolute magnitude of Error (IAE), Integral Square Error (ISE), Integral Time Absolute Error (ITAE) and Integral Time Square Error (ITSE) are used to inspect accuracy, sensitivity, selectivity of adaptive control system. Extremum value of this index corresponds to the optimum set of parameter values [7]. The mathematical notations of the above indices are as follows:

$$IAE \ = \int_0^{\propto} |e(t)|\, dt \tag{5}$$

$$ISE \ = \int_0^{\propto} [\,e(t)]^2 dt \tag{6}$$

$$ITAE = \int_0^{t} t\,|e(t)| dt \tag{7}$$

$$ITSE = \int_0^{\propto} t[e(t)]^2\, dt \tag{8}$$

V. Performance Analysis

The performance analyses of a DC motor using different adaptive techniques are illustrated as follows:

A. MRAC using Lyapunov Stability Method

To ensure the system stability MRAC using Lyapunov method is used. A positive definite Lyapunov function (V) is chosen, in such a way that derivative of it, along the solution is negative semi definite, then the system turns stable. If derivative is negative definite, then system is asymptotically stable. In this technique adaptation mechanism is such that the error between plant and model output goes to zero [4], [5].

Let the transfer function of the plant is represented as

$$\frac{d^2y}{dt^2} = -a\frac{dy}{dt} + bu \tag{9}$$

The reference model is represented as

$$\frac{d^2y_m}{dt^2} = -a_m\frac{dy_m}{dt} + b_m u_c \tag{10}$$

Where, y & y_m are the output of the plant and the reference model respectively, u_c is the reference input signal.

Let, the control input is $u = \theta_1 u_c - \theta_2\frac{dy}{dt}$ (11)

θ_1 and θ_2 are control parameters with adjustable gain γ.

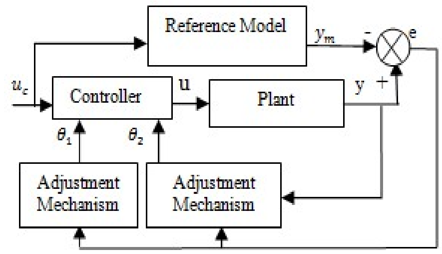

Fig. 2. Block diagram of Lyapunov based direct MRAC

From Eqs. (1), (9), (10) and (11), it is found that

$$\frac{de}{dt} = -a_m e - (b\theta_2 + a - a_m)y + (b\theta_1 - b_m)u_c \tag{12}$$

From the error dynamics it is clear error turns zero if,

$$\theta_1 = \frac{b_m}{b} \tag{13}$$

$$\theta_2 = \frac{a_m - a}{b} \tag{14}$$

The Lyapunov Function is chosen as [9], [10]

$$V(e,\theta_1,\theta_2) = \frac{1}{2}[e^2 + \frac{1}{b\gamma}(b\theta_2 + a - a_m)^2 + \frac{1}{b\gamma}(b\theta_1 - b_m)]^2 \tag{15}$$

The above Lyapunov function turns zero for error is zero. The parameters of the controller are equal to the correct values. The time derivative of this valid Lyapunov function should be negative, so

$$\frac{dV}{dt} = e\frac{de}{dt} + \frac{1}{\gamma}(b\theta_2 + a - a_m)\frac{d\theta_2}{dt} + \frac{1}{\gamma}(b\theta_1 - b_m)\frac{d\theta_1}{dt} \tag{16}$$

From Eq. (12) and Eq. (16)

$$\frac{dV}{dt} = -a_m e^2 + \frac{1}{\gamma}(b\theta_2 + a - a_m)\left(\frac{d\theta_2}{dt} - \gamma y e\right) + \frac{1}{\gamma}(b\theta_1 - b_m)(\frac{d\theta_1}{dt} + \gamma u_c e) \tag{17}$$

The parameters are updated as follows

$$\theta_1 = -\frac{\gamma}{s}u_c e \tag{18}$$

$$\theta_2 = \frac{\gamma}{s}ye \tag{19}$$

Here K_0 for reference model and K for plant are taken as 1.5 and 1 respectively.

Therefore the transfer function of Reference Model is $G_m(s) = \frac{1.5}{0.5s^2+s+1.5}$ and the transfer function of the Plant is $G_p(s) = \frac{1}{0.5s^2+s+1.5}$ [Table. I], [8].

By choosing $\gamma = 0.5, 1, 2$, the responses of plant and reference model are shown in (Fig. 3). It is observed, for high value of γ the response of the system becomes fast with high value of overshoots, whereas for low value of γ the response of the system turns slow with low value of overshoot. For high value of γ accuracy of the system turns poor as indicated by the plot of different performance indices Figs. (4), (5), (6), (7), [Table. II].

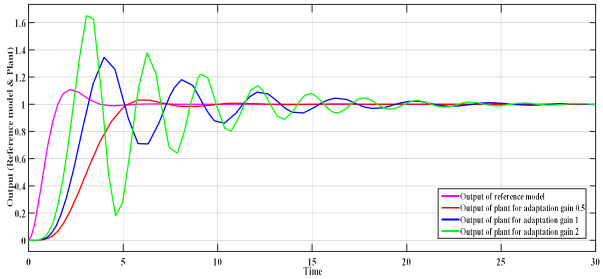

Fig. 3. Tracking performance with Lyapunov method

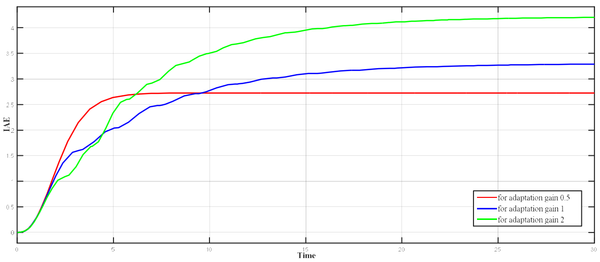

Fig. 4. IAE plot using Lyapunov method

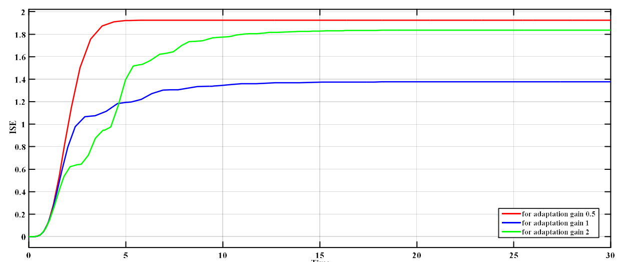

Fig. 5. ISE plot using Lyapunov method

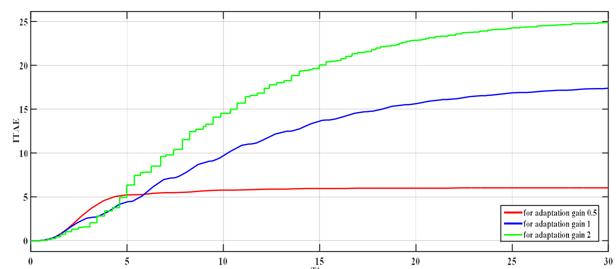

Fig. 6. ITAE plot using Lyapunov method

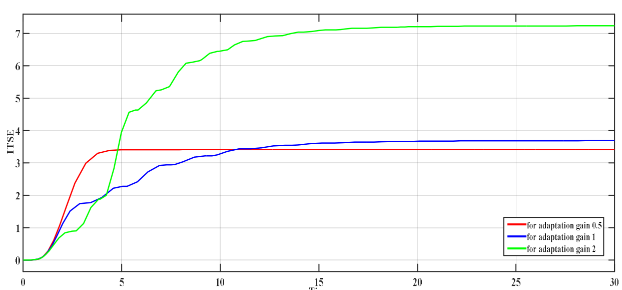

Fig. 7. ITSE plot using Lyapunov method

B. MRAC Augmented with PID Method

The Control signal of PID controller is given by, [11]

$$u(t) = K_P[e(t) + \frac{1}{T_i}\int_0^t e(t)dt + T_d\frac{de(t)}{dt}] \tag{20}$$

$$u(t) = K_p e(t) + K_I\int_0^t e(t)dt + K_D\,\frac{de(t)}{dt} \tag{21}$$

Where, K_P is proportional gain, T_i is integral gain, T_d is derivative time. Also $K_I = \frac{Kp}{T_i}$ gain reset part and $K_D = K_P T_d$ gain derivative part of the PID controller [11], [12], [13], [14].

In the modified PID algorithm technique, the derivative action is operated directly on feedback signal instead of error signal (Fig. 8). Here the portion of the proportional action acts only on the fraction of the reference value. The derivative action is replaced by an approximation that decreases the gain at large frequencies. There is a modification of integral action such that, it will not keep integrating when the control variable saturates [11], [13].

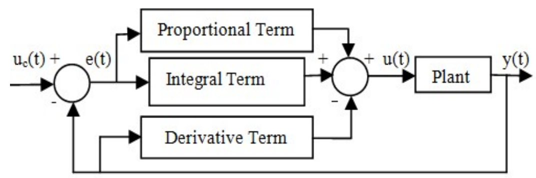

Fig. 8. Block diagram of a Modified PID Control System

From the above block diagram u(t) can be expressed as

$$u(t) = K_P e(t) + K_I\,\int_0^t e(t)\,dt - K_D\,\frac{dy(t)}{dt} \tag{22}$$

The MRAC augmented with PID is an adaptive control law where K_P, K_I, K_D are updated/tuned according to the control mechanism based on MRAC and MIT rule, so that the plant follows the reference model (Fig. 9), [13].

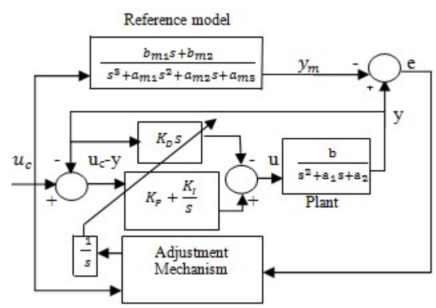

Fig. 9. Block diagram of proposed MRAC PID diagram

Suppose the second order plant has the transfer function

$$G_P(s) = \frac{b}{s^2+a_1 s+a_2} \tag{23}$$

From Eq. (22), the control signal is given as

$$u(s) = K_P\,[u_c(s) - y(s)] + \frac{K_I[u_c(s)-y(s)]}{s} - K_D y(s)s \tag{24}$$

Also from Fig. 1, $G_P(s) = \frac{y(s)}{u(s)}$ (25)

From Eq. (24) and Eq. (25), it is found

$$\frac{y(s)}{u_c(s)} = \frac{b(K_P s+K_I)}{s^3+(a_1+bK_D)s^2+(a_2+bK_P)s+bK_I} \tag{26}$$

The equation of PID controller can be obtained as [13]

$$s^3 + (a_1 + bK_D)s^2 + (a_2 + bK_P)s + bK_I \approx s^3 + a_{m1}s^2 + a_{m2}s + a_{m3} \tag{27}$$

By Eq. (26) & Eq. (27), the reference model is given by

$$\frac{y_m(s)}{u_m(s)} = \frac{b_{m1}s+b_{m2}}{s^3+a_{m1}s^2+a_{m2}s+a_{m3}} \tag{28}$$

From the MIT rule, K_P, K_I, K_D can be calculated as

$$\frac{dK_P}{dt} = -\gamma_P \frac{\partial J}{\partial K_P} = -\gamma_P \times \frac{\partial J}{\partial e} \times \frac{\partial e}{\partial y} \times \frac{\partial y}{\partial K_P} \tag{29}$$

$$\frac{dK_I}{dt} = -\gamma_I \frac{\partial J}{\partial K_I} = -\gamma_I \times \frac{\partial J}{\partial e} \times \frac{\partial e}{\partial y} \times \frac{\partial y}{\partial K_I} \tag{30}$$

$$\frac{dK_D}{dt} = -\gamma_D \frac{\partial J}{\partial K_D} = -\gamma_D \times \frac{\partial J}{\partial e} \times \frac{\partial e}{\partial y} \times \frac{\partial y}{\partial K_D} \tag{31}$$

From Eq. (1) and Eq. (2),

$$\frac{de}{dy} = 1 \tag{32}$$

$$\frac{\partial J}{\partial e} = e \tag{33}$$

By solving Eq. (26), (29), (30) and (31), the updated PID parameters can be determined as

$$\frac{dK_P}{dt} = -\gamma_P . e . \frac{bs.[u_c-y]}{s^3+(a_1+bK_D)s^2+(a_2+bK_P)s+bK_I} \tag{34}$$

$$\frac{dK_I}{dt} = -\gamma_I . e . \frac{b.[u_c-y]}{s^3+(a_1+bK_D)s^2+(a_2+bK_P)s+bK_I} \tag{35}$$

$$\frac{dK_D}{dt} = \gamma_D . e \frac{b.s^3[y]}{s^3+(a_1+bK_D)s^2+(a_2+bK_P)s+bK_I} \tag{36}$$

Eq. (34), (35), (36) represent the change of different PID parameters with time.

By PID Tuning method applied to the DC motor as given in Eq. (4), it is found $K_P = 3.149$, $K_I = 3.393$, $K_D = 0.7253$

Using these values the reference model is given by,

$$\frac{y_{m(s)}}{u_m(s)} = \frac{6.298s+6.786}{s^3+3.4506\ ^2+9.298s+6.786} \tag{37}$$

For $\gamma = 0.5, 1, 2$, and for step input, it is seen that the plant takes some time for tracking the reference model, known as adaptation time (Fig. 10). The plot indicates the proposed controller is tracking the reference model. From the plot of tracking performance and the performance indices, it is clear that the performance as well as accuracy of the system will be better for high value of adaptation gain Figs. (11), (12), (13), (14), [Table. II], [15], [16].

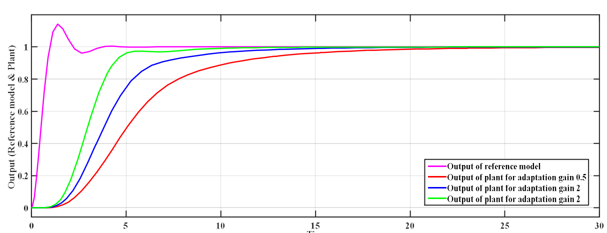

Fig. 10. Tracking performance using MRAC augmented with PID

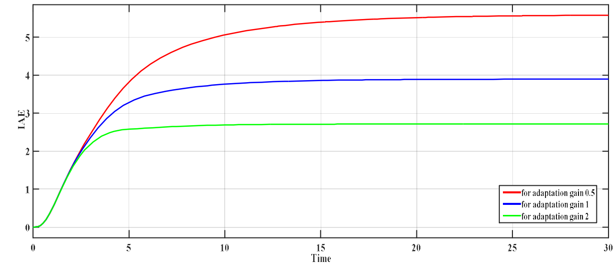

Fig. 11. IAE plot using MRAC augmented with PID

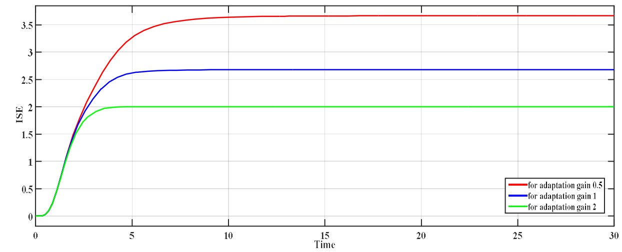

Fig. 12. ISE plot using MRAC augmented with PID

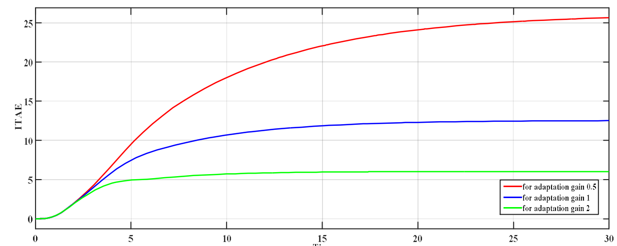

Fig. 13. ITAE plot using MRAC augmented with PID

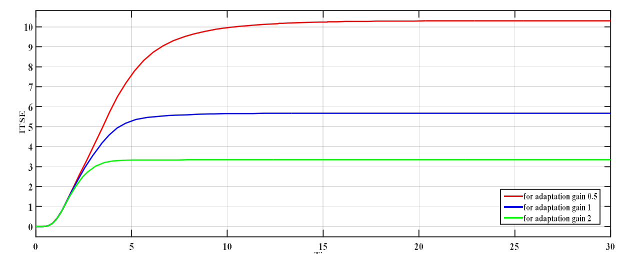

Fig. 14. ITSE plot using MRAC augmented with PID

TABLE II. COMPARISON OF VARIOUS PERFORMANCE INDICES USING DIFFERENT ADAPTIVE CONTROL TECHNIQUES

Technique used	Performance index	Adaptation gain		
		0.5	1	2
MRAC using Lyapunov method	IAE	2.524	3.310	4.22
	ISE	1.706	1.378	1.837
	ITAE	6.006	16.83	24.27
	ITSE	3.413	3.684	7.228
MRAC augmented with PID	IAE	5.581	3.896	2.715
	ISE	3.666	2.680	2.003
	ITAE	25.65	12.51	6.031
	ITSE	10.31	5.676	3.334

During the adaptation time K_P, K_I, K_D are adjusted automatically, which can be determined by graphically. From the values of these parameters thus obtained, it is observed that the values of these PID parameter increases for increasing values of adaptation gain Figs. (15), (16), (17), [Table III].

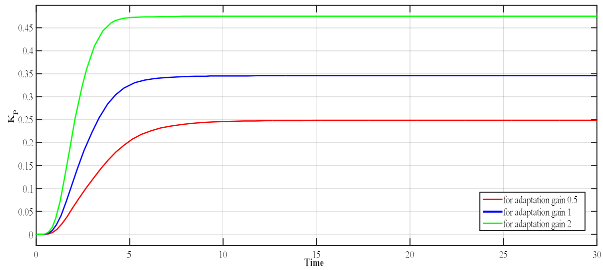

Fig. 15. Update value of K_P

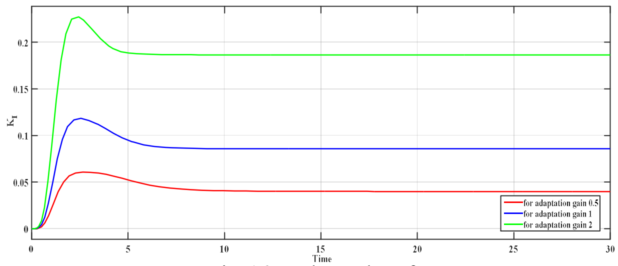

Fig. 16. Update value of K_I

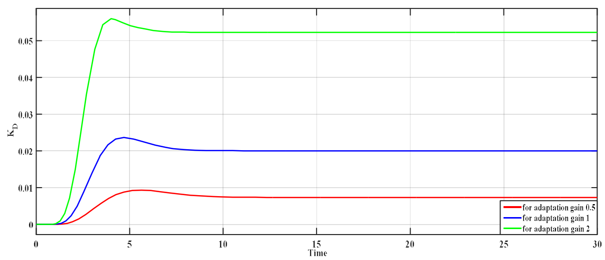

Fig. 17. Update value of K_D

TABLE III. UPDATE VALUE OF PID PARAMETERS

PID Parameters	Adaptation gain		
	0.5	1	2
K_P	0.25	0.35	0.47
K_I	0.061	0.115	0.226
K_D	0.009	0.023	0.055

VI. CONCLUSIONS

This paper describes about speed control of a DC motor employing MRAC using Lyapunov rule and MRAC augmented with PID method. By applying unit step input and taking different adaptation gains, the tracking performances and performance indices are studied in MATLAB SIMULINK environment. It is observed, for the same adaptation gain, the controller using MRAC augmented with PID tracks the reference model better than Lyapunov rule. The analysis of performance indices demonstrates that if adaptation gains increases, accuracy of the system increases for PID method and decreases for Lyapunov rule. In Lyapunov rule the system is stable in the given range of adaptation gain. In MRAC augmented with PID method, values of PID parameters are automatically adjusted and calculated in this paper. It can be concluded that performance using MRAC augmented with PID method is better than the Lyapunov rule.

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