pert_bulk_incomp

November 16, 2020

[1]:
```
# perturbed bulk flow equations
from sympy import *
import numpy as np
init_printing()
import numpy as np
```

continuity

$$\sigma\frac{\partial\left(\rho h\right)}{\partial t}+\frac{\partial\left(\rho h v_x\right)}{\partial x}+\frac{\partial\left(\rho h v_y\right)}{\partial y}=0 \tag{1}$$

axial momentum

$$\sigma\frac{\partial\left(\rho h v_x\right)}{\partial t}+\frac{\partial\left(\rho h v_x^2\right)}{\partial x}+\frac{\partial\left(\rho h v_x v_y\right)}{\partial y}=-\frac{\partial p}{\partial x}h-0.5\frac{R}{C}\rho\left(f_r v_r+f_s v_s\right)v_x \tag{2}$$

The axial momentum equation can be simplifed using the continuity equation to…

$$\rho h\left(\sigma\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}\right)=-\frac{\partial p}{\partial x}h-0.5\frac{R}{C}\rho\left(f_r v_r+f_s v_s\right)v_x \tag{3}$$

but it may advantageous to retain the previous conservative form to facilitate finite volume discretization

circumferential momentum

$$\sigma\frac{\partial\left(\rho h v_y\right)}{\partial t}+\frac{\partial\left(\rho h v_x v_y\right)}{\partial x}+\frac{\partial\left(\rho h v_y^2\right)}{\partial y}=-\frac{\partial p}{\partial y}h-0.5\frac{R}{C}\rho\left(f_r v_r+f_s v_s\right)v_y+0.5\frac{R}{C}\rho f_r v_r\frac{\Omega R}{u_*} \tag{4}$$

Pertubed variables

$h=h_0+\epsilon_\psi h_\psi e^{it}$

$v_x=v_{x0}+\epsilon_\psi v_{x,\psi}e^{it}$

$v_y=v_{y0}+\epsilon_\psi v_{y,\psi}e^{it}$

$p=p_0+\epsilon_\psi p_\psi e^{it}$

$v_r=v_{r0}+\epsilon_\psi v_{r,\psi}e^{it}$

$v_s = v_{s0} + \epsilon_\psi v_{s,\psi} e^{it}$

$f_r = f_{r0} + \epsilon_\psi f_{r,\psi} e^{it}$

$f_s = f_{s0} + \epsilon_\psi f_{s,\psi} e^{it}$

Note that t in the harmonic component is really dimensionless time ωt and $\psi = X, Y$

[2]:
```
e,x,y,t,sigma = symbols('epsilon x y t sigma')
R, C, n, m, rot = symbols('R C n m Omega')
#rho,mu = symbols('rho mu')
mu = symbols('mu')
u_scale = symbols('u_*')
#us0, ur0, fs0 = symbols('u_{s0} u_{r0} f_{s0}')
```

[3]:
```
h0 = Function('h_0')(x,y)
h1 = Function('h_{\psi}')(x,y)
#h = h0 + dx*diff(h0,x)*exp(I*omega*t)+dy*diff(h0,y)*exp(I*omega*t)
h = h0 + e*h1*exp(I*t)
p0 = Function('p_0')(x,y)
p1 = Function('p_{\psi}')(x,y)
#p = p0 + dx*diff(p0,x)*exp(I*omega*t)+dy*diff(p0,y)*exp(I*omega*t)
p = p0 + e*p1*exp(I*t)
vx0 = Function('v_{x0}')(x,y)
vx1 = Function('v_{x,\psi}')(x,y)
#u = u0 + dx*diff(u0,x)*exp(I*omega*t)+dy*diff(u0,y)*exp(I*omega*t)
vx = vx0 + e*vx1*exp(I*t)
vy0 = Function('v_{y0}')(x,y)
vy1 = Function('v_{y,\psi}')(x,y)
#v = v0 + dx*diff(v0,x)*exp(I*omega*t)+dy*diff(v0,y)*exp(I*omega*t)
vy = vy0 + e*vy1*exp(I*t)

vs0 = Function('v_{s0}')(x,y)
vs1 = Function('v_{s,\psi}')(x,y)
vs = vs0 + e*vs1*exp(I*t)

vr0 = Function('v_{r0}')(x,y)
vr1 = Function('v_{r,\psi}')(x,y)
vr = vr0 + e*vr1*exp(I*t)

fs0 = Function('f_{s0}')(x,y)
fs1 = Function('f_{s,\psi}')(x,y)
fs = fs0 + e*fs1*exp(I*t)

fr0 = Function('f_{r0}')(x,y)
fr1 = Function('f_{r,\psi}')(x,y)
fr = fr0 + e*fr1*exp(I*t)

# density varies
```

```
#rho0 = Function('rho_0')(x,y)
#rho1 = Function('rho_1')(x,y)
#rho = rho0 + e*rho1*exp(I*omega*t)

# density is constant
rho0, rho = symbols('rho_0 rho')

#us0 = sqrt(u0**2 + v0**2)
#ur0 = sqrt(u0**2 + (v0 - R * rot)**2)

# Taylor series expanded rotor and stator velocities

#us = us0 + diff(us0, u0) * (e*u1)*exp(I*omega*t) + diff(us0, v0) *␣
↪(e*v1)*exp(I*omega*t)
#ur = ur0 + diff(ur0, u0) * (e*u1)*exp(I*omega*t) + diff(ur0, v0) *␣
↪(e*v1)*exp(I*omega*t)
```

0.1 Conservation of mass / continuity

[4]:
```
c1 = diff( expand( sigma * rho * h ) , t )
```

[5]:
```
c2 = diff( expand( rho * h * vx  ) , x )
```

[6]:
```
c3 = diff( expand( rho * h * vy  ) , y )
```

0.1.1 zeroth order

[7]:
```
zeroth_c1 = c1.coeff(e,0)
#zeroth_cxy_c1 = zeroth_cx_c1.coeff(dy,0)
#
zeroth_c2 = c2.coeff(e,0)
#zeroth_cxy_c2 = zeroth_cx_c2.coeff(dy,0)
#
zeroth_c3 = c3.coeff(e,0)
#zeroth_cxy_c3 = zeroth_cx_c3.coeff(dy,0)
```

[8]:
```
zeroth_c1
```

[8]:

0

[9]:
```
zeroth_c2
```

[9]:

$$\rho \,\mathrm{h}_0\left(x, y\right) \frac{\partial}{\partial x} \mathrm{v}_{x0}\left(x, y\right) + \rho \,\mathrm{v}_{x0}\left(x, y\right) \frac{\partial}{\partial x} \mathrm{h}_0\left(x, y\right)$$

[10]:
```
zeroth_c3
```

[10]:

$$\rho\,\mathrm{h}_{0}\left(x,y\right)\frac{\partial}{\partial y}\,\mathrm{v}_{\mathrm{y}0}\left(x,y\right)+\rho\,\mathrm{v}_{\mathrm{y}0}\left(x,y\right)\frac{\partial}{\partial y}\,\mathrm{h}_{0}\left(x,y\right)$$

0.1.2 first-order, ϵ

[11]:
```
first_c1 = c1.coeff(e,1)
#
first_c2 = c2.coeff(e,1)
#
first_c3 = c3.coeff(e,1)
```

[12]:
```
first_c1
```

[12]:

$$i\rho\sigma\,\mathrm{h}_{\psi}\left(x,y\right)e^{it}$$

[13]:
```
first_c2
```

[13]:

$$\rho\,\mathrm{h}_{0}\left(x,y\right)e^{it}\frac{\partial}{\partial x}\,\mathrm{v}_{\mathrm{x},\psi}\left(x,y\right)+\rho\,\mathrm{h}_{\psi}\left(x,y\right)e^{it}\frac{\partial}{\partial x}\,\mathrm{v}_{\mathrm{x}0}\left(x,y\right)+\rho\,\mathrm{v}_{\mathrm{x},\psi}\left(x,y\right)e^{it}\frac{\partial}{\partial x}\,\mathrm{h}_{0}\left(x,y\right)+\rho\,\mathrm{v}_{\mathrm{x}0}\left(x,y\right)e^{it}\frac{\partial}{\partial x}\,\mathrm{h}_{\psi}\left(x,y\right)$$

[14]:
```
first_c3
```

[14]:

$$\rho\,\mathrm{h}_{0}\left(x,y\right)e^{it}\frac{\partial}{\partial y}\,\mathrm{v}_{\mathrm{y},\psi}\left(x,y\right)+\rho\,\mathrm{h}_{\psi}\left(x,y\right)e^{it}\frac{\partial}{\partial y}\,\mathrm{v}_{\mathrm{y}0}\left(x,y\right)+\rho\,\mathrm{v}_{\mathrm{y},\psi}\left(x,y\right)e^{it}\frac{\partial}{\partial y}\,\mathrm{h}_{0}\left(x,y\right)+\rho\,\mathrm{v}_{\mathrm{y}0}\left(x,y\right)e^{it}\frac{\partial}{\partial y}\,\mathrm{h}_{\psi}\left(x,y\right)$$

0.2 Axial momentum

[15]:
```
ax1 = diff( expand( sigma * rho * h * vx ) , t )
```

[16]:
```
ax2 = diff( expand( rho * h * vx**2  ) , x )
```

[17]:
```
ax3 = diff( expand( rho * h * vx * vy ) , y )
```

[18]:
```
ax4 = expand( - h * diff( p,  x ) )
```

[19]:
```
ax5 = expand( - 0.5 * R / C * rho * (fr * vr + fs * vs) * vx )
#ax5 = expand( - (fr * ur - fs * us) * u )
```

0.2.1 Zeroth order

[20]:
```
zeroth_ax1 = ax1.coeff(e,0)
#
zeroth_ax2 = ax2.coeff(e,0)
#
zeroth_ax3 = ax3.coeff(e,0)
```

```
#
zeroth_ax4 = ax4.coeff(e,0)
#
zeroth_ax5 = ax5.coeff(e,0)
```

[21]:
```
zeroth_ax1
```

[21]:

$$0$$

[22]:
```
zeroth_ax2
```

[22]:

$$2\rho\,\mathrm{h_0}(x,y)\,\mathrm{v_{x0}}(x,y)\frac{\partial}{\partial x}\mathrm{v_{x0}}(x,y)+\rho\,\mathrm{v_{x0}}^2(x,y)\frac{\partial}{\partial x}\mathrm{h_0}(x,y)$$

[23]:
```
zeroth_ax3
```

[23]:

$$\rho\,\mathrm{h_0}(x,y)\,\mathrm{v_{x0}}(x,y)\frac{\partial}{\partial y}\mathrm{v_{y0}}(x,y)+\rho\,\mathrm{h_0}(x,y)\,\mathrm{v_{y0}}(x,y)\frac{\partial}{\partial y}\mathrm{v_{x0}}(x,y)+\rho\,\mathrm{v_{x0}}(x,y)\,\mathrm{v_{y0}}(x,y)\frac{\partial}{\partial y}\mathrm{h_0}(x,y)$$

[24]:
```
zeroth_ax4
```

[24]:

$$-\mathrm{h_0}(x,y)\frac{\partial}{\partial x}\mathrm{p_0}(x,y)$$

[25]:
```
zeroth_ax5
```

[25]:

$$-\frac{0.5R\rho\,\mathrm{f_{r0}}(x,y)\,\mathrm{v_{r0}}(x,y)\,\mathrm{v_{x0}}(x,y)}{C}-\frac{0.5R\rho\,\mathrm{f_{s0}}(x,y)\,\mathrm{v_{s0}}(x,y)\,\mathrm{v_{x0}}(x,y)}{C}$$

[26]:
```
#ax5
```

0.2.2 first-order, ϵ

[27]:
```
first_ax1 = ax1.coeff(e,1)
#
first_ax2 = ax2.coeff(e,1)
#
first_ax3 = ax3.coeff(e,1)
#
first_ax4 = ax4.coeff(e,1)
#
first_ax5 = ax5.coeff(e,1)
```

[28]:
```
first_ax1
```

[28]:

$$i\rho\sigma\,\mathrm{h_0}(x,y)\,\mathrm{v_{x,\psi}}(x,y)e^{it}+i\rho\sigma\,\mathrm{h_\psi}(x,y)\,\mathrm{v_{x0}}(x,y)e^{it}$$

[29]:
```
first_ax2
```

[29]:

$$2\rho\,\mathrm{h}_0(x,y)\,\mathrm{v}_{x,\psi}(x,y)e^{it}\frac{\partial}{\partial x}\mathrm{v}_{x0}(x,y) + 2\rho\,\mathrm{h}_0(x,y)\,\mathrm{v}_{x0}(x,y)e^{it}\frac{\partial}{\partial x}\mathrm{v}_{x,\psi}(x,y) + 2\rho\,\mathrm{h}_\psi(x,y)\,\mathrm{v}_{x0}(x,y)e^{it}\frac{\partial}{\partial x}\mathrm{v}_{x0}(x,y) + 2\rho\,\mathrm{v}_{x,\psi}(x,y)\,\mathrm{v}_{x0}(x,y)e^{it}\frac{\partial}{\partial x}\mathrm{h}_0(x,y) + \rho\,\mathrm{v}_{x0}^2(x,y)e^{it}\frac{\partial}{\partial x}\mathrm{h}_\psi(x,y)$$

[30]:
```
first_ax3
```

[30]:

$$\rho\,\mathrm{h}_0(x,y)\,\mathrm{v}_{x,\psi}(x,y)e^{it}\frac{\partial}{\partial y}\mathrm{v}_{y0}(x,y) + \rho\,\mathrm{h}_0(x,y)\,\mathrm{v}_{x0}(x,y)e^{it}\frac{\partial}{\partial y}\mathrm{v}_{y,\psi}(x,y) + \rho\,\mathrm{h}_0(x,y)\,\mathrm{v}_{y,\psi}(x,y)e^{it}\frac{\partial}{\partial y}\mathrm{v}_{x0}(x,y) + \rho\,\mathrm{h}_0(x,y)\,\mathrm{v}_{y0}(x,y)e^{it}\frac{\partial}{\partial y}\mathrm{v}_{x,\psi}(x,y) + \rho\,\mathrm{h}_\psi(x,y)\,\mathrm{v}_{x0}(x,y)e^{it}\frac{\partial}{\partial y}\mathrm{v}_{y0}(x,y) + \rho\,\mathrm{h}_\psi(x,y)\,\mathrm{v}_{y0}(x,y)e^{it}\frac{\partial}{\partial y}\mathrm{v}_{x0}(x,y) + \rho\,\mathrm{v}_{x,\psi}(x,y)\,\mathrm{v}_{y0}(x,y)e^{it}\frac{\partial}{\partial y}\mathrm{h}_0(x,y) + \rho\,\mathrm{v}_{x0}(x,y)\,\mathrm{v}_{y,\psi}(x,y)e^{it}\frac{\partial}{\partial y}\mathrm{h}_0(x,y) + \rho\,\mathrm{v}_{x0}(x,y)\,\mathrm{v}_{y0}(x,y)e^{it}\frac{\partial}{\partial y}\mathrm{h}_\psi(x,y)$$

[31]:
```
first_ax4
```

[31]:

$$-\mathrm{h}_0(x,y)e^{it}\frac{\partial}{\partial x}\mathrm{p}_\psi(x,y) - \mathrm{h}_\psi(x,y)e^{it}\frac{\partial}{\partial x}\mathrm{p}_0(x,y)$$

[32]:
```
# us00, ur00 = symbols('u_s0 u_r0')
# fr00, fs00 = symbols('f_r0 f_s0')
# first_ax5 = first_ax5.subs(us0, us00)
# first_ax5 = first_ax5.subs(ur0, ur00)
# first_ax5 = first_ax5.subs(expand(ur0), ur00)
# first_ax5 = first_ax5.subs(fr0, fr00)
# first_ax5 = first_ax5.subs(fs0, fs00)

display(first_ax5)
```

$$-\frac{0.5R\rho\,\mathrm{f}_{r,\psi}(x,y)\,\mathrm{v}_{r0}(x,y)\,\mathrm{v}_{x0}(x,y)e^{it}}{C} - \frac{0.5R\rho\,\mathrm{f}_{r0}(x,y)\,\mathrm{v}_{r,\psi}(x,y)\,\mathrm{v}_{x0}(x,y)e^{it}}{C} - \frac{0.5R\rho\,\mathrm{f}_{r0}(x,y)\,\mathrm{v}_{r0}(x,y)\,\mathrm{v}_{x,\psi}(x,y)e^{it}}{C} - \frac{0.5R\rho\,\mathrm{f}_{s,\psi}(x,y)\,\mathrm{v}_{s0}(x,y)\,\mathrm{v}_{x0}(x,y)e^{it}}{C} - \frac{0.5R\rho\,\mathrm{f}_{s0}(x,y)\,\mathrm{v}_{s,\psi}(x,y)\,\mathrm{v}_{x0}(x,y)e^{it}}{C} - \frac{0.5R\rho\,\mathrm{f}_{s0}(x,y)\,\mathrm{v}_{s0}(x,y)\,\mathrm{v}_{x,\psi}(x,y)e^{it}}{C}$$

0.3 Circumferential momentum

[33]:
```
circ1 = diff( expand( sigma * rho * h * vy ) , t )
```

[34]:
```
circ2 = diff( expand( rho * h * vx * vy  ) , x )
```

[35]:
```
circ3 = diff( expand( rho * h * vy **2 ) , y )
```

[36]:
```
circ4 = expand( - h * diff( p,  y ) )
```

[37]:
```
circ5 = expand( - 0.5 * R / C * rho * (fr * vr + fs * vs) * vy ) + \
    expand( 0.5 * R / C * rho * fr * vr * R * rot / u_scale)
```

0.3.1 zeroth order

[38]:
```
zeroth_circ1 = circ1.coeff(e,0)
#
zeroth_circ2 = circ2.coeff(e,0)
#
zeroth_circ3 = circ3.coeff(e,0)
#
zeroth_circ4 = circ4.coeff(e,0)
#
zeroth_circ5 = circ5.coeff(e,0)
```

[39]:
```
zeroth_circ1
```

[39]:

$$0$$

[40]:
```
zeroth_circ2
```

[40]:

$$\rho\,\mathrm{h}_0(x,y)\,\mathrm{v}_{\mathrm{x}0}(x,y)\frac{\partial}{\partial x}\mathrm{v}_{\mathrm{y}0}(x,y)+\rho\,\mathrm{h}_0(x,y)\,\mathrm{v}_{\mathrm{y}0}(x,y)\frac{\partial}{\partial x}\mathrm{v}_{\mathrm{x}0}(x,y)+\rho\,\mathrm{v}_{\mathrm{x}0}(x,y)\,\mathrm{v}_{\mathrm{y}0}(x,y)\frac{\partial}{\partial x}\mathrm{h}_0(x,y)$$

[41]:
```
zeroth_circ3
```

[41]:

$$2\rho\,\mathrm{h}_0(x,y)\,\mathrm{v}_{\mathrm{y}0}(x,y)\frac{\partial}{\partial y}\mathrm{v}_{\mathrm{y}0}(x,y)+\rho\,{\mathrm{v}_{\mathrm{y}0}}^2(x,y)\frac{\partial}{\partial y}\mathrm{h}_0(x,y)$$

[42]:
```
zeroth_circ4
```

[42]:

$$-\mathrm{h}_0(x,y)\frac{\partial}{\partial y}\mathrm{p}_0(x,y)$$

[43]:
```
zeroth_circ5
```

[43]:

$$\frac{0.5\Omega R^2\rho\,\mathrm{f}_{\mathrm{r}0}(x,y)\,\mathrm{v}_{\mathrm{r}0}(x,y)}{Cu_*}-\frac{0.5R\rho\,\mathrm{f}_{\mathrm{r}0}(x,y)\,\mathrm{v}_{\mathrm{r}0}(x,y)\,\mathrm{v}_{\mathrm{y}0}(x,y)}{C}-\frac{0.5R\rho\,\mathrm{f}_{\mathrm{s}0}(x,y)\,\mathrm{v}_{\mathrm{s}0}(x,y)\,\mathrm{v}_{\mathrm{y}0}(x,y)}{C}$$

0.3.2 first-order, ϵ

[44]:
```
first_circ1 = circ1.coeff(e,1)
#
first_circ2 = circ2.coeff(e,1)
#
first_circ3 = circ3.coeff(e,1)
#
first_circ4 = circ4.coeff(e,1)
#
first_circ5 = circ5.coeff(e,1)
```

[45]:
```
first_circ1
```

[45]:

$$i\rho\sigma\,\mathrm{h}_0(x,y)\,\mathrm{v}_{\mathrm{y},\psi}(x,y)e^{it} + i\rho\sigma\,\mathrm{h}_{\psi}(x,y)\,\mathrm{v}_{\mathrm{y}0}(x,y)e^{it}$$

[46]:
```
first_circ2
```

[46]:

$$\rho\,\mathrm{h}_0(x,y)\,\mathrm{v}_{\mathrm{x},\psi}(x,y)e^{it}\frac{\partial}{\partial x}\mathrm{v}_{\mathrm{y}0}(x,y) + \rho\,\mathrm{h}_0(x,y)\,\mathrm{v}_{\mathrm{x}0}(x,y)e^{it}\frac{\partial}{\partial x}\mathrm{v}_{\mathrm{y},\psi}(x,y) + \rho\,\mathrm{h}_0(x,y)\,\mathrm{v}_{\mathrm{y},\psi}(x,y)e^{it}\frac{\partial}{\partial x}\mathrm{v}_{\mathrm{x}0}(x,y) + \rho\,\mathrm{h}_0(x,y)\,\mathrm{v}_{\mathrm{y}0}(x,y)e^{it}\frac{\partial}{\partial x}\mathrm{v}_{\mathrm{x},\psi}(x,y) + \rho\,\mathrm{h}_{\psi}(x,y)\,\mathrm{v}_{\mathrm{x}0}(x,y)e^{it}\frac{\partial}{\partial x}\mathrm{v}_{\mathrm{y}0}(x,y) + \rho\,\mathrm{h}_{\psi}(x,y)\,\mathrm{v}_{\mathrm{y}0}(x,y)e^{it}\frac{\partial}{\partial x}\mathrm{v}_{\mathrm{x}0}(x,y) + \rho\,\mathrm{v}_{\mathrm{x},\psi}(x,y)\,\mathrm{v}_{\mathrm{y}0}(x,y)e^{it}\frac{\partial}{\partial x}\mathrm{h}_0(x,y) + \rho\,\mathrm{v}_{\mathrm{x}0}(x,y)\,\mathrm{v}_{\mathrm{y},\psi}(x,y)e^{it}\frac{\partial}{\partial x}\mathrm{h}_0(x,y) + \rho\,\mathrm{v}_{\mathrm{x}0}(x,y)\,\mathrm{v}_{\mathrm{y}0}(x,y)e^{it}\frac{\partial}{\partial x}\mathrm{h}_{\psi}(x,y)$$

[47]:
```
first_circ3
```

[47]:

$$2\rho\,\mathrm{h}_0(x,y)\,\mathrm{v}_{\mathrm{y},\psi}(x,y)e^{it}\frac{\partial}{\partial y}\mathrm{v}_{\mathrm{y}0}(x,y) + 2\rho\,\mathrm{h}_0(x,y)\,\mathrm{v}_{\mathrm{y}0}(x,y)e^{it}\frac{\partial}{\partial y}\mathrm{v}_{\mathrm{y},\psi}(x,y) + 2\rho\,\mathrm{h}_{\psi}(x,y)\,\mathrm{v}_{\mathrm{y}0}(x,y)e^{it}\frac{\partial}{\partial y}\mathrm{v}_{\mathrm{y}0}(x,y) + 2\rho\,\mathrm{v}_{\mathrm{y},\psi}(x,y)\,\mathrm{v}_{\mathrm{y}0}(x,y)e^{it}\frac{\partial}{\partial y}\mathrm{h}_0(x,y) + \rho\,\mathrm{v}_{\mathrm{y}0}{}^2(x,y)e^{it}\frac{\partial}{\partial y}\mathrm{h}_{\psi}(x,y)$$

[48]:
```
first_circ4
```

[48]:

$$-\mathrm{h}_0(x,y)e^{it}\frac{\partial}{\partial y}\mathrm{p}_{\psi}(x,y) - \mathrm{h}_{\psi}(x,y)e^{it}\frac{\partial}{\partial y}\mathrm{p}_0(x,y)$$

[49]:
```
# first_circ5 = first_circ5.subs(us0, us00)
# first_circ5 = first_circ5.subs(ur0, ur00)
# first_circ5 = first_circ5.subs(expand(ur0), ur00)
# first_circ5 = first_circ5.subs(fr0, fr00)
# first_circ5 = first_circ5.subs(fs0, fs00)

display(first_circ5)
```

$$\frac{0.5\Omega R^2\rho\,\mathrm{f}_{\mathrm{r},\psi}(x,y)\,\mathrm{v}_{\mathrm{r}0}(x,y)e^{it}}{Cu_*} + \frac{0.5\Omega R^2\rho\,\mathrm{f}_{\mathrm{r}0}(x,y)\,\mathrm{v}_{\mathrm{r},\psi}(x,y)e^{it}}{Cu_*} - \frac{0.5R\rho\,\mathrm{f}_{\mathrm{r},\psi}(x,y)\,\mathrm{v}_{\mathrm{r}0}(x,y)\,\mathrm{v}_{\mathrm{y}0}(x,y)e^{it}}{C} -$$

$$\frac{0.5R\rho\,\mathrm{f}_{\mathrm{r0}}\left(x,y\right)\mathrm{v}_{\mathrm{r},\psi}\left(x,y\right)\mathrm{v}_{\mathrm{y0}}\left(x,y\right)e^{it}}{C} - \frac{0.5R\rho\,\mathrm{f}_{\mathrm{r0}}\left(x,y\right)\mathrm{v}_{\mathrm{r0}}\left(x,y\right)\mathrm{v}_{\mathrm{y},\psi}\left(x,y\right)e^{it}}{C} - \frac{0.5R\rho\,\mathrm{f}_{\mathrm{s},\psi}\left(x,y\right)\mathrm{v}_{\mathrm{s0}}\left(x,y\right)\mathrm{v}_{\mathrm{y0}}\left(x,y\right)e^{it}}{C} - \frac{0.5R\rho\,\mathrm{f}_{\mathrm{s0}}\left(x,y\right)\mathrm{v}_{\mathrm{s},\psi}\left(x,y\right)\mathrm{v}_{\mathrm{y0}}\left(x,y\right)e^{it}}{C} - \frac{0.5R\rho\,\mathrm{f}_{\mathrm{s0}}\left(x,y\right)\mathrm{v}_{\mathrm{s0}}\left(x,y\right)\mathrm{v}_{\mathrm{y},\psi}\left(x,y\right)e^{it}}{C}$$

```
[ ]:
```