What is a Logic?

When most people say 'logic', they mean either propositional logic or first-order predicate logic.

• However, the precise definition is quite broad, and literally hundreds of logics have been studied by philosophers, computer scientists and mathematicians.

• Any 'formal system' can be considered a logic if it has:
– a well-defined syntax;
– a well-defined semantics; and
– a well-defined proof-theory.

What is a Logic?

- The *syntax* of a logic defines the syntactically acceptable objects of the language, which are properly called well-formed formulae (wff). (We shall just call them formulae.)

- The *semantics* of a logic associate each formula with a meaning.

- The *proof theory* is concerned with manipulating formulae according to certain rules.

Propositional Logic

- A **proposition** is a declarative sentence (a sentence that declares a fact) that is either true or false, but not both.
- Propositional Logic – the area of logic that deals with propositions
- Are the following sentences propositions?
  - Toronto is the capital of Canada.
  - Read this carefully.
  - 1+2=3
  - x+1=2
  - What time is it?

Propositional Logic

- A **proposition** is a declarative sentence (a sentence that declares a fact) that is either true or false, but not both.

- Propositional Logic – the area of logic that deals with propositions

- Are the following sentences propositions?
  - Toronto is the capital of Canada. (Yes)
  - Read this carefully. (No)
  - 1+2=3 (Yes)
  - x+1=2 (No)
  - What time is it? (No)

Propositional Variables

- Propositional Variables – variables that represent propositions: p, q, r, s
  - E.g. Proposition p – "Today is Friday."
- Truth values – T, F

Negation

DEFINITION 1

Let p be a proposition. The negation of p, denoted by $\neg p$, is the statement "It is not the case that p."

The proposition $\neg p$ is read "not p." The truth value of the negation of p, $\neg p$ is the opposite of the truth value of p.

- Examples
  - Find the negation of the proposition "Today is Friday." and express this in simple English.

Solution: The negation is "It is not the case that *today is Friday*.
In simple English, "Today is not Friday." or "It is not Friday today."

Negation

DEFINITION 1

Let *p* be a proposition. The negation of *p*, denoted by ¬*p*, is the statement "It is not the case that *p*."

The proposition ¬*p* is read "not *p*." The truth value of the negation of *p*, ¬*p* is the opposite of the truth value of *p*.

- Examples
  - Find the negation of the proposition "At least 10 inches of rain fell today in Miami." and express this in simple English.

    Solution: The negation is "It is not the case that *at least 10 inches of rain fell today in Miami.*" In simple English, "Less than 10 inches of rain fell today in Miami."

Truth Table

- Truth table:

The Truth Table for the Negation of a Proposition.	
p	$\neg p$
T	F
F	T

- Logical operators are used to form new propositions from two or more existing propositions. The logical operators are also called connectives.

Conjunction

DEFINITION 2

Let p and q be propositions. The *conjunction* of p and q, denoted by $p \wedge q$, is the proposition "p and q". The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

- Examples
  - Find the conjunction of the propositions p and q where p is the proposition "Today is Friday." and q is the proposition "It is raining today.", and the truth value of the conjunction.

    Solution: The conjunction is the proposition "Today is Friday and it is raining today." The proposition is true on rainy Fridays.

Disjunction

DEFINITION 3

Let p and q be propositions. The *disjunction* of p and q, denoted by $p \vee q$, is the proposition "p or q". The conjunction $p \vee q$ is false when both p and q are false and is true otherwise.

- Note:

  inclusive or : The disjunction is true when at least one of the two propositions is true.
  - E.g. "Students who have taken calculus or computer science can take this class." – those who take one or both classes.

  exclusive or : The disjunction is true only when one of the proposition is true.
  - E.g. "Students who have taken calculus or computer science, but not both, can take this class." – only those who take one of them.
- Definition 3 uses *inclusive or*.

Exclusive

DEFINITION 4

Let p and q be propositions. The *exclusive or* of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

The Truth Table for the Exclusive Or (XOR) of Two Propositions.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Conditional Statements

DEFINITION 5

Let p and q be propositions. The *conditional statement* $p \rightarrow q$, is the proposition "if p, then q." The conditional statement is false when p is true and q is false, and true otherwise. In the conditional statement $p \rightarrow q$, p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

- A conditional statement is also called an implication.
- Example: "If I am elected, then I will lower taxes." $p \rightarrow q$

implication:

elected, lower taxes.	T	T	T
not elected, lower taxes.	F	T	T
not elected, not lower taxes.	F	F	T
elected, not lower taxes.	T	F	F

Conditional Statement (Cont')

- Example:
  - Let p be the statement "Maria learns discrete mathematics." and q the statement "Maria will find a good job." Express the statement $p \longrightarrow q$ as a statement in English.

Solution: Any of the following -

"If Maria learns discrete mathematics, then she will find a good job.

"Maria will find a good job when she learns discrete mathematics."

"For Maria to get a good job, it is sufficient for her to learn discrete mathematics."

Conditional Statement (Cont')

- Other conditional statements:
  - Converse of $p \rightarrow q : q \rightarrow p$
  - Contrapositive of $p \rightarrow q : \neg q \rightarrow \neg p$
  - Inverse of $p \rightarrow q : \neg p \rightarrow \neg q$

Biconditional Statement

DEFINITION 6

Let p and q be propositions. The *biconditional statement* $p \leftrightarrow q$ is the proposition "p if and only if q." The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called *bi-implications*.

- $p \leftrightarrow q$ has the same truth value as $(p \rightarrow q) \land (q \rightarrow p)$
- "*if and only if*" can be expressed by "*iff*"
- Example:
  - Let p be the statement "You can take the flight" and let q be the statement "You buy a ticket." Then $p \leftrightarrow q$ is the statement
    "You can take the flight if and only if you buy a ticket."
    Implication:
    If you buy a ticket you can take the flight.
    If you don't buy a ticket you cannot take the flight.

Biconditional Statement (Cont')

The Truth Table for the Biconditional $p \leftrightarrow q$.		
p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Truth Tables of Compound Propositions

- We can use connectives to build up complicated compound propositions involving any number of propositional variables, then use truth tables to determine the truth value of these compound propositions.
- Example: Construct the truth table of the compound proposition

$$(p \vee \neg q) \longrightarrow (p \wedge q).$$

The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Precedence of Logical Operators

- We can use parentheses to specify the order in which logical operators in a compound proposition are to be applied.
- To reduce the number of parentheses, the precedence order is defined for logical operators.

Precedence of Logical Operators.	
Operator	Precedence
$\neg$	1
$\wedge$ $\vee$	2 3
$\rightarrow$ $\leftrightarrow$	4 5

E.g. $\neg p \wedge q = (\neg p) \wedge q$

$p \wedge q \vee r = (p \wedge q) \vee r$

$p \vee q \wedge r = p \vee (q \wedge r)$

Translating English Sentences

- English (and every other human language) is often ambiguous. Translating sentences into compound statements removes the ambiguity.
- Example: How can this English sentence be translated into a logical expression?

  "You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old."

Solution: Let q, r, and s represent "You can ride the roller coaster,"

"You are under 4 feet tall," and "You are older than

16 years old." The sentence can be translated into:

$$(r \wedge \neg s) \rightarrow \neg q.$$

Translating English Sentences

- Example: How can this English sentence be translated into a logical expression?

  "You can access the Internet from campus only if you are a computer science major or you are not a freshman."

Translating English Sentences

- Example: How can this English sentence be translated into a logical expression?

  "You can access the Internet from campus only if you are a computer science major or you are not a freshman."

Solution: Let *a*, *c*, and *f* represent "You can access the Internet from campus," "You are a computer science major," and "You are a freshman." The sentence can be translated into:

$$a \rightarrow (c \vee \neg f).$$

Translating English Sentences

Let's use three borrowed from an elementary school reader:
T: Tom hit the ball.
J: Jane caught the ball.
S: Spot chased the ball.

English Sentence	PL Translation
Tom did not hit the ball	
Either Tom hit the ball or Jane caught the ball	
Spot chased the ball, but Jane caught it.	
If Jane caught the ball, then Spot did not chase it.	
Spot chased the ball if and only if Tom hit the ball.	

Translating English Sentences

Let's use three borrowed from an elementary school reader:
T: Tom hit the ball.
J: Jane caught the ball.
S: Spot chased the ball.

English Sentence	**PL Translation**
Tom did not hit the ball	$\neg T$
Either Tom hit the ball or Jane caught the ball	$T \vee J$
Spot chased the ball, but Jane caught it.	$S \wedge J$
If Jane caught the ball, then Spot did not chase it.	$J \rightarrow \neg S$
Spot chased the ball if and only if Tom hit the ball.	$S \leftrightarrow T$

Translating English Sentences

Let's use three borrowed from an elementary school reader:
T: Tom hit the ball.
J: Jane caught the ball.
S: Spot chased the ball.

More Complex Example

➔Tom hit the ball, and if Jane caught the ball, then Spot chased it.

➔If Tom hit the ball and Jane caught it, then Spot chased it.

Translating English Sentences

Let's use three borrowed from an elementary school reader:
T: Tom hit the ball.
J: Jane caught the ball.
S: Spot chased the ball.

More Complex Example

➔Tom hit the ball, and if Jane caught the ball, then Spot chased it.

$$T \land (J \rightarrow S)$$

➔If Tom hit the ball and Jane caught it, then Spot chased it.

$$(T \land J) \rightarrow S$$

Exercise

Let p and q be the propositions “Swimming at the New Jersey shore is allowed” and “Sharks have been spotted near the shore,” respectively.

Express each of these compound propositions as an English sentence.

a) $\neg q$

b) $p \wedge q$

c) $\neg p \vee q$

d) $p \rightarrow \neg q$

e) $\neg q \rightarrow p$

Exercise

Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively.

Express each of these compound propositions as an English sentence.

f) $\neg p \to \neg q$

g) $p \leftrightarrow \neg q$

h) $\neg p \wedge (p \vee \neg q)$

Exercise

Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively.

Express each of these compound propositions as an English sentence.

a) $\neg q$ — Sharks have not been spotted near the shore.

b) $p \wedge q$ — Swimming at the New Jersey shore is allowed, and sharks have been spotted near the shore.

c) $\neg p \vee q$ — Swimming at the New Jersey shore is not allowed, or sharks have been spotted near the shore.

d) $p \rightarrow \neg q$ — If swimming at the New Jersey shore is allowed, then sharks have not been spotted near the shore.

e) $\neg q \rightarrow p$ — If sharks have not been spotted near the shore, then swimming at the New Jersey shore is allowed.

Exercise

Let p and q be the propositions “Swimming at the New Jersey shore is allowed” and “Sharks have been spotted near the shore,” respectively.

Express each of these compound propositions as an English sentence.

f) $\neg p \rightarrow \neg q$ If swimming at the New Jersey shore is not allowed, then sharks have not been spotted near the shore.

g) $p \leftrightarrow \neg q$ Swimming at the New Jersey shore is allowed if and only if sharks have not been spotted near the shore.

h) $\neg p \wedge (p \vee \neg q)$ Swimming at the New Jersey shore is not allowed, and either swimming at the New Jersey shore is allowed or sharks have not been spotted near the shore. Note that we were able to incorporate the parentheses by using the word "either" in the second half of the sentence.

Exercise

Let p and q be the propositions
p :You drive over 65 miles per hour.
q :You get a speeding ticket.
Write these propositions using p and q and logical connectives (including negations).

a) You do not drive over 65 miles per hour.

b) You drive over 65 miles per hour, but you do not get a speeding ticket.

c) You will get a speeding ticket if you drive over 65 miles per hour.

d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.

Exercise

Let p and q be the propositions
p :You drive over 65 miles per hour.
q :You get a speeding ticket.
Write these propositions using p and q and logical connectives (including negations).

e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.

f) You get a speeding ticket, but you do not drive over 65 miles per hour.

g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

Exercise

Let p and q be the propositions
p :You drive over 65 miles per hour.
q :You get a speeding ticket.
Write these propositions using p and q and logical connectives (including negations).

a) You do not drive over 65 miles per hour.

This is just the negation of p, so we write $\neg p$.

b) You drive over 65 miles per hour, but you do not get a speeding ticket.

This is a conjunction ("but" means "and"): $p \land \neg q$.

c) You will get a speeding ticket if you drive over 65 miles per hour.

The position of the word "if" tells us which is the antecedent and which is the consequence: $p \to q$.

d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.

$$\neg p \to \neg q$$

Exercise

Let p and q be the propositions
p :You drive over 65 miles per hour.
q :You get a speeding ticket.
Write these propositions using p and q and logical connectives (including negations).

e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.

The sufficient condition is the antecedent: $p \to q$

f) You get a speeding ticket, but you do not drive over 65 miles per hour.

$q \wedge \neg q$

g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

''Whenever" means "if": $q \to p$.

Exercise

Let p, q, and r be the propositions

p : Grizzly bears have been seen in the area.
q : Hiking is safe on the trail.
r : Berries are ripe along the trail.

Write these propositions using p, q, and r and logical connectives (including negations).

a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.
b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.
c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.
d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.

Exercise

Let *p*, *q*, and *r* be the propositions

p : Grizzly bears have been seen in the area.
q : Hiking is safe on the trail.
r : Berries are ripe along the trail.

Write these propositions using *p*, *q*, and *r* and logical connectives (including negations).

a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.

"But" is a logical synonym for "and" (although it often suggests that the second part of the sentence is likely to be unexpected).
So this is $r \land \neg p$.

b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.

Because of the agreement about precedence, we do not need parentheses in this expression: $\neg p \land q \land r$.

Exercise

Let p, q, and r be the propositions

p : Grizzly bears have been seen in the area.
q : Hiking is safe on the trail.
r : Berries are ripe along the trail.

Write these propositions using p, q, and r and logical connectives (including negations)

c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

The outermost structure here is the conditional statement, and the conclusion part of the conditional statement is itself a biconditional: $r \rightarrow (q \leftrightarrow \neg p)$

d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.

This is similar to part **(b):** $\neg q \land \neg p \land r$

Logic and Bit Operations

- Computers represent information using bits.
- A **bit** is a symbol with two possible values, 0 and 1.
- By convention, 1 represents T (true) and 0 represents F (false).
- A variable is called a Boolean variable if its value is either true or false.
- Bit operation – replace true by 1 and false by 0 in logical operations.

Table for the Bit Operators *OR*, *AND*, and *XOR*.

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

Logic and Bit Operations

> DEFINITION 7
>
> A *bit string* is a sequence of zero or more bits. The *length* of this string is the number of bits in the string.

- Example: Find the bitwise *OR*, bitwise *AND*, and bitwise *XOR* of the bit string 01 1011 0110 and 11 0001 1101.

Solution:

01 1011 0110	
11 0001 1101	
11 1011 1111	bitwise *OR*
01 0001 0100	bitwise *AND*
10 1010 1011	bitwise *XOR*

Propositional Equivalences

DEFINITION 1

A compound proposition that is always true, no matter what the truth values of the propositions that occurs in it, is called a *tautology*. A compound proposition that is always false is called a *contradiction*. A compound proposition that is neither a tautology or a contradiction is called a *contingency*.

Examples of a Tautology and a Contradiction.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

Logical Equivalences

DEFINITION 2

The compound propositions p and q are called *logically equivalent* if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ denotes that p and q are logically equivalent.

- Compound propositions that have the same truth values in all possible cases are called logically equivalent.
- Example: Show that $\neg p \vee q$ and $p \rightarrow q$ are logically equivalent.

Truth Tables for $\neg p \vee q$ and $p \rightarrow q$.

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Logical Equivalence

Two statements have the same truth table

De Morgan's Law

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

De Morgan's Law

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Constructing New Logical Equivalences

- Example: Show that $\neg(p \rightarrow q)$ and $p \wedge \neg q$ are logically equivalent.

  Solution:

$\neg(p \rightarrow q)$	$\equiv \neg(\neg p \vee q)$	*by example on earlier slide*
	$\equiv \neg(\neg p) \wedge \neg q$	*by the second De Morgan law*
	$\equiv p \wedge \neg q$	*by the double negation law*

Constructing New Logical Equivalences

- *Example: Show that* $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

  Solution: To show that this statement is a tautology, we will use logical equivalences to demonstrate that it is logically equivalent to T.

  $(p \wedge q) \rightarrow (p \vee q) \equiv \neg(p \wedge q) \vee (p \vee q)$ *by example on earlier slides*

  $\equiv (\neg p \vee \neg q) \vee (p \vee q)$ *by the first De Morgan law*

  $\equiv (\neg p \vee p) \vee (\neg q \vee q)$ *by the associative and communicative law for disjunction*

  $\equiv T \vee T$

  $\equiv T$

- *Note: The above examples can also be done using truth tables.*

Important Logic Equivalence

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \rightarrow q \equiv \neg p \vee q$

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

$p \vee q \equiv \neg p \rightarrow q$

$p \wedge q \equiv \neg(p \rightarrow \neg q)$

$\neg(p \rightarrow q) \equiv p \wedge \neg q$

$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$

$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

Predicates

A predicate is an expression of one or more variables defined on some specific domain. A predicate with variables can be made a proposition by either assigning a value to the variable or by quantifying the variable.

The following are some examples of predicates –

- Let E(x, y) denote "x = y"
- Let X(a, b, c) denote "a + b + c = 0"
- Let M(x, y) denote "x is married to y"

Predicates

- Statements involving variables are neither true nor false.
- E.g. "$x > 3$", "$x = y + 3$", "$x + y = z$"
- "x is greater than 3"
  - "x": subject of the statement
  - "is greater than 3": the ***predicate***
- We can denote the statement "x is greater than 3" by $P(x)$, where P denotes the predicate and x is the variable.
- Once a value is assigned to the variable x, the statement $P(x)$ becomes a proposition and has a truth value.

Predicates

- Example: Let $P(x)$ denote the statement "$x > 3$." What are the truth values of $P(4)$ and $P(2)$?

  Solution: $P(4)$ – "$4 > 3$", *true*
  $P(2)$ – "$2 > 3$", *false*

- Example: Let $Q(x,y)$ denote the statement "$x = y + 3$." What are the truth values of the propositions $Q(1,2)$ and $Q(3,0)$?

  Solution: $Q(1,2)$ – "$1 = 2 + 3$" , *false*
  $Q(3,0)$ – "$3 = 0 + 3$", *true*

Predicates

- Example: Let *A(c,n)* denote the statement "Computer *c* is connected to network *n*", where *c* is a variable representing a computer and *n* is a variable representing a network. Suppose that the computer MATH1 is connected to network CAMPUS2, but not to network CAMPUS1. What are the values of A(MATH1, CAMPUS1) and A(MATH1, CAMPUS2)?

Solution: *A(MATH1, CAMPUS1) – "MATH1 is connect to CAMPUS1", false*

A(MATH1, CAMPUS2) – "MATH1 is connect to CAMPUS2", true

Propositional Function (Predicate)

- A statement involving n variables $x_1, x_2, \ldots, x_n$ can be denoted by $P(x_1, x_2, \ldots, x_n)$.
- A statement of the form $P(x_1, x_2, \ldots, x_n)$ is the value of the propositional function P at the n-tuple $(x_1, x_2, \ldots, x_n)$, and P is also called a ***n*-place predicate** or a ***n*-ary predicate**.

Quantifiers

- Quantification: express the extent to which a predicate is true over a range of elements.
- **Universal quantification**: a predicate is true for every element under consideration
- **Existential quantification**: a predicate is true for one or more element under consideration
- A domain must be specified.

Universal Quantifier

The *universal quantification* of $P(x)$ is the statement
"$P(x)$ for all values of x in the domain."
The notation $xP(x)$ denotes the universal quantification of $P(x)$. Here $\forall$ is called the **Universal Quantifier**. We read $xP(x)$ as "for all $xP(x)$" or "for every $xP(x)$." A$\forall$n element for which $P(x)$ is false is called a **counterex$\forall$ample** of $xP(x)$.

Example: Let $P(x)$ be the statement "$x + 1 > x$." What is the truth value of the quantification $\forall xP(x)$, where the domain consists of all real numbers?

Solution: *Because $P(x)$ is true for all real numbers, the quantification is true.*

Universal Quantification

- A statement $\forall xP(x)$ is false, if and only if $P(x)$ is not always true where x is in the domain. One way to show that is to find a counterexample to the statem$\forall$ent $xP(x)$.
- Example: Let $Q(x)$ be the statement "x < 2". What is the truth value of the quantification $\forall xQ(x)$, where the domain consists of all real numbers?

Solution: *Q(x) is not true for every real numbers, e.g. Q(3) is false. x = 3 is a counterexample for the statement $xQ(x)$.*

Thus the quantification is false.

- $\forall xP(x)$ is the same as the conjunction

$$P(x_1) \wedge P(x_2) \wedge .... \wedge P(x_n)$$

Universal Quantifier

- Example: What does the statement $\forall x N(x)$ mean if $N(x)$ is "Computer x is connected to the network" and the domain consists of all computers on campus?

  Solution: *"Every computer on campus is connected to the network."*

Existential Quantification

DEFINITION 2

The *existential quantification* of $P(x)$ is the statement

"There exists an element x in the domain such that $P(x)$."

We use the notation $\exists x P(x)$ for the existential quantification of $P(x)$. Here

$\exists$ is called the **Existential Quantifier**.

- The existential quantificatio∃n $xP(x)$ is read as
  "There is an x such that $P(x)$," or
  "There is at least one x such that $P(x)$," or
  "For some x, $P(x)$."

Existential Quantification

- Example: Let $P(x)$ denote the statement "$x > 3$". What is the truth value of the quantific∃ation $xP(x)$, where the domain consists of all real numbers?

Solution: *"$x > 3$" is sometimes true – for instance when $x = 4$. The existential quantification is true.*

- $\exists xP(x)$ is false if and only if $P(x)$ is false for every element of the domain.
- Example: Let $Q(x)$ denote the statement "$x = x + 1$". What is the true value of the quant∃ification $xQ(x)$, where the domain consists for all real numbers?

Solution: *$Q(x)$ is false for every real number. The existential quantification is false.*

Existential Quantification

- If the domain is empty, $\exists xQ(x)$ is false because there can be no element in the domain for which $Q(x)$ is true.
- The existential quantification $\exists xP(x)$ is the same as the disjunction $P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n)$

Quantifiers		
Statement	**When True?**	**When False?**
$\forall xP(x)$	*xP(x) is true for every x.*	*There is an x for which xP(x) is false.*
$\exists xP(x)$	*There is an x for which P(x) is true.*	*P(x) is false for every x.*

∃

Uniqueness Quantifier

- Uniqueness quantifier $\exists!$ or $\exists_1$
  - $\exists! x P(x)$ or $\exists_1 P(x)$ states "There exists a unique x such that $P(x)$ is true."
- Quantifiers with restricted domains
  - Example: What do the following statements mean? The domain in each case consists of real numbers.
    - $\forall x < 0\ (x^2 > 0)$: For every real number x with $x < 0$, $x^2 > 0$. "The square of a negative real number is positive." It's the same as $\forall x(x < 0 \rightarrow x^2 > 0)$
    - $\forall y \neq 0\ (y^3 \neq 0)$: For every real number y with $y \neq 0$, $y^3 \neq 0$. "The cube of every non-zero real number is non-zero." It's the same as $\forall y(y \neq 0 \rightarrow y^3 \neq 0)$.
    - $\exists z > 0\ (z^2 = 2)$: There exists a real number z with $z > 0$, such that $z^2 = 2$. "There is a positive square root of 2." It's the same as $\exists z(z > 0 \wedge z^2 = 2)$:

Precedence of Quantifiers

- Precedence of Quantifiers
  - $\forall$ and $\exists$ have higher precedence than all logical operators.
  - E.g. $\forall x P(x) \vee Q(x)$ is the same as $(\forall x P(x)) \vee Q(x)$

Translating from English into Logical Expressions

- Example: Express the statement "Every student in this class has studied calculus" using predicates and quantifiers.

Solution:

If the domain consists of students in the class –

$$\forall x C(x)$$

where $C(x)$ is the statement "x has studied calculus.

If the domain consists of all people –

$$\forall x(S(x) \rightarrow C(x))$$

where $S(x)$ represents that person x is in this class.

If we are interested in the backgrounds of people in subjects besides calculus, we can use the two-variable quantifier $Q(x,y)$ for the statement "student x has studies subject y." Then we would replace $C(x)$ by $Q(x, \text{calculus})$ to obtain $\forall x Q(x, \text{calculus})$ or $\forall x(S(x) \rightarrow Q(x, \text{calculus}))$

Translating from English into Logical Expressions

- Example: Consider these statements. The first two are called *premises* and the third is called the *conclusion*. The entire set is called an *argument*.

  "All lions are fierce."

  "Some lions do not drink coffee."

  "Some fierce creatures do not drink coffee."

Solution: Let $P(x)$ be "x is a lion."

$Q(x)$ be "x is fierce."

$R(x)$ be "x drinks coffee."

$\forall x(P(x) \rightarrow Q(x))$

$\exists x(P(x) \land \neg R(x))$

$\exists x(Q(x) \land \neg R(x))$

Exercises Part1

Suppose that during the most recent fiscal year, the annual revenue of Acme Computer was 138 billion dollars and its net profit was 8 billion dollars, the annual revenue of Nadir Software was 87 billion dollars and its net profit was 5 billion dollars, and the annual revenue of Quixote Media was 111 billion dollars and its net profit was 13 billion dollars. Determine the truth value of each of these propositions for the most recent fiscal year.

a) Quixote Media had the largest annual revenue.
b) Nadir Software had the lowest net profit and Acme Computer had the largest annual revenue.
c) Acme Computer had the largest net profit or Quixote Media had the largest net profit.
d) If Quixote Media had the smallest net profit, then Acme Computer had the largest annual revenue.
e) Nadir Software had the smallest net profit if and only if Acme Computer had the largest annual revenue.

Suppose that during the most recent fiscal year, the annual revenue of Acme Computer was 138 billion dollars and its net profit was 8 billion dollars, the annual revenue of Nadir Software was 87 billion dollars and its net profit was 5 billion dollars, and the annual revenue of Quixote Media was 111 billion dollars and its net profit was 13 billion dollars. Determine the truth value of each of these propositions for the most recent fiscal year.

a) Quixote Media had the largest annual revenue.
b) Nadir Software had the lowest net profit and Acme Computer had the largest annual revenue.
c) Acme Computer had the largest net profit or Quixote Media had the largest net profit.
d) If Quixote Media had the smallest net profit, then Acme Computer had the largest annual revenue.
e) Nadir Software had the smallest net profit if and only if Acme Computer had the largest annual revenue.

a) This is false, because Acme's revenue was larger.
b) Both parts of this conjunction are true, so the statement is true.
c) The second part of this disjunction is true, so the statement is true.
d) The hypothesis of this conditional statement is false and the conclusion is true, so by the truth-table definition this is a true statement. (Either of those conditions would have been enough to make the statement true.)
e) Both parts of this biconditional statement are true, so by the truth-table definition this is a true statement.

Let p and q be the propositions “Swimming at the New Jersey shore is allowed” and “Sharks have been spotted near the shore,” respectively. Express each of these compound propositions as an English sentence.

(a) $\neg q$

(b) b) $p \wedge q$

*(***c)** $\neg p \vee q$

d) $p \rightarrow \neg q$

e) $\neg q \rightarrow p$

f) $\neg p \rightarrow \neg q$

g) $p \leftrightarrow \neg q$

h) $\neg p \wedge (p \vee \neg q)$

Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express each of these compound propositions as an English sentence.
a) $\neg q$ **b)** $p \wedge q$ **c)** $\neg p \vee q$
d) $p \rightarrow \neg q$ **e)** $\neg q \rightarrow p$ **f)** $\neg p \rightarrow \neg q$
g) $p \leftrightarrow \neg q$ **h)** $\neg p \wedge (p \vee \neg q)$

a) Sharks have not been spotted near the shore.
b) Swimming at the New Jersey shore is allowed, and sharks have been spotted near the shore.
c) Swimming at the New Jersey shore is not allowed, or sharks have been spotted near the shore.
d) If swimming at the New Jersey shore is allowed, then sharks have not been spotted near the shore.
e) If sharks have not been spotted near the shore, then swimming at the New Jersey shore is allowed.
f) If swimming at the New Jersey shore is not allowed, then sharks have not been spotted near the shore.
g) Swimming at the New Jersey shore is allowed if and only if sharks have not been spotted near the shore.
h) Swimming at the New Jersey shore is not allowed, and either swimming at the New Jersey shore is allowed or sharks have not been spotted near the shore. Note that we were able to incorporate the parentheses by using the word "either" in the second half of the sentence.

Let p and q be the propositions
p : It is below freezing.
q : It is snowing.
Write these propositions using p and q and logical connectives (including negations).
a) It is below freezing and snowing.
b) It is below freezing but not snowing.
c) It is not below freezing and it is not snowing.
d) It is either snowing or below freezing (or both).
e) If it is below freezing, it is also snowing.
f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
g) That it is below freezing is necessary and sufficient for it to be snowing.

Let p and q be the propositions
p : It is below freezing.
q : It is snowing.
Write these propositions using p and q and logical connectives (including negations).
a) It is below freezing and snowing.
b) It is below freezing but not snowing.
c) It is not below freezing and it is not snowing.
d) It is either snowing or below freezing (or both).
e) If it is below freezing, it is also snowing.
f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
g) That it is below freezing is necessary and sufficient for it to be snowing.

a) Here we have the conjunction $p \wedge q$.
b) Here we have a conjunction of p with the negation of q, namely $p \wedge \neg q$. Note that "but" logically means the same thing as "and."
c) Again this is a conjunction: $\neg p \wedge \neg q$.
d) Here we have a disjunction, $p \vee q$. Note that $\vee$ is the inclusive *or*, so the "(or both)" part of the English sentence is automatically included.
e) This sentence is a conditional statement, $p \rightarrow q$.
f) This is a conjunction of propositions, both of which are compound: $(p \vee q)\ A\ (p \rightarrow \neg q)$.
g) This is the biconditional $p \leftrightarrow q$.

Construct a truth table for each of these compound propositions.

a) $(p \vee q) \rightarrow (p \oplus q)$
b) $(p \oplus q) \rightarrow (p \wedge q)$
c) $(p \vee q) \oplus (p \wedge q)$
d) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
e) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg q)$
f) $(p \oplus q) \rightarrow (p \oplus \neg q)$

e) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg q)$

P	q	$\neg p$	$\neg q$	$(p \leftrightarrow q)$	$\neg p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg q)$
T	T	F	F	T	T	F
T	F	F	T	F	F	F
F	T	T	F	F	F	F
F	F	T	T	T	T	F

Explain, without using a truth table, why $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is true when at least one of p, q, and r is true and at least one is false, but is false when all three variables have the same truth value.

Explain, without using a truth table, why $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is true when at least one of p, q, and r is true and at least one is false, but is false when all three variables have the same truth value.

Answer:

The first clause $(p \vee q \vee r)$ is true if and only if at least one of p, q, and r is true. The second clause $(\neg p \vee \neg q \vee \neg r)$ is true if and only if at least one of the three variables is false. Therefore both clauses are true, and therefore the entire statement is true, if and only if there is at least one T and one F among the truth values of the variables, in other words, that they don't all have the same truth value.

Express these system specifications using the propositions p "The message is scanned for viruses" and q "The message was sent from an unknown system" together with logical connectives (including negations).

a) "The message is scanned for viruses whenever the message was sent from an unknown system."

b) "The message was sent from an unknown system but it was not scanned for viruses."

c) "It is necessary to scan the message for viruses whenever it was sent from an unknown system."

d) "When a message is not sent from an unknown system it is not scanned for viruses."

Express these system specifications using the propositions p “The message is scanned for viruses” and q “The message was sent from an unknown system” together with logical connectives (including negations).

a) “The message is scanned for viruses whenever the message was sent from an unknown system.”

b) “The message was sent from an unknown system but it was not scanned for viruses.”

c) “It is necessary to scan the message for viruses whenever it was sent from an unknown system.”

d) “When a message is not sent from an unknown system it is not scanned for viruses.”

Answer:

a) Since "whenever" means "if," we have $q \rightarrow p$.

b) Since "but" means "and," we have $q \wedge \neg P$.

c) This sentence is saying the same thing as the sentence in part (a), so the answer is the same: $q \rightarrow p$.

d) Again, we recall that "when" means "if" in logic: $\neg q \rightarrow \neg p$.

Exercises Part 2

For each of these sentences, state what the sentence means if the logical connective or is an inclusive or (that is, a disjunction) versus an exclusive or. Which of these meanings of or do you think is intended?

a) To take discrete mathematics, you must have taken calculus or a course in computer science.

b) When you buy a newcar fromAcme Motor Company, you get \$2000 back in cash or a 2% car loan.

c) Dinner for two includes two items from column A or three items from column B.

d) School is closed if more than 2 feet of snow falls or if the wind chill is below −100.

For each of these sentences, state what the sentence means if the logical connective or is an inclusive or (that is, a disjunction) versus an exclusive or. Which of these meanings of or do you think is intended?

a) To take discrete mathematics, you must have taken calculus or a course in computer science.

b) When you buy a newcar fromAcme Motor Company, you get \$2000 back in cash or a 2% car loan.

c) Dinner for two includes two items from column A or three items from column B.

d) School is closed if more than 2 feet of snow falls or if the wind chill is below −100.

a) If this is an inclusive *or*, then it is allowable to take discrete mathematics if you have had calculus or computer science or both. If this is an exclusive *or*, then a person who had both courses would not be allowed to take discrete mathematics-only someone who had taken exactly one of the prerequisites would be allowed in. Clearly the former interpretation is intended; if anything, the person who has had both calculus and computer science is even better prepared for discrete mathematics.

b) If this is an inclusive *or*, then you can take the rebate, or you can sign up for the low-interest loan, or you can demand both of these incentives. If this is an exclusive *or*, then you will receive one of the incentives but not both. Since both of these deals are expensive for the dealer or manufacturer, surely the exclusive *or* was intended.

c) If this is an inclusive *or*, you can order two items from column A (and none from B), or three items from column B (and none from A), or five items (two from A and three from B). If this is an exclusive *or*, which it surely is here, then you get your choice of the two A items or the three B items, but not both.

d) If this is an inclusive *or*, then lots of snow, or extreme cold, or a combination of the two will close school. If this is an exclusive *or*, then one form of bad weather would close school but if both of them happened then school would meet. This latter interpretation is clearly absurd, so the inclusive *or* is intended.

Let $Q(x, y)$ denote the statement "x is the capital of y."
What are these truth values?
a) Q(Denver, Colorado)
b) Q(Detroit, Michigan)
c) Q(Massachusetts, Boston)
d) Q(NewYork, NewYork)

Let *Q(x, y)* denote the statement "*x* is the capital of *y*."
What are these truth values?

a) *Q*(Denver, Colorado)
b) *Q*(Detroit, Michigan)
c) *Q*(Massachusetts, Boston)
d) *Q*(NewYork, NewYork)

a) This is true.
b) This is false, since Lansing, not Detroit, is the capital.
c) This is false (but Q(Boston, Massachusetts) is true).
d) This is false, since Albany, not New York, is the capital.

Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

a) $\exists x P(x)$
b) $\forall x P(x)$
c) $\exists x \neg P(x)$
d) $\forall x \neg P(x)$

Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

a) $\exists x P(x)$
b) **b)** $\forall x P(x)$
c) $\exists x \neg P(x)$
d) $\forall x \neg P(x)$

a) There is a student who spends more than five hours every weekday in class.
b) Every student spends more than five hours every weekday in class.
c) There is a student who does not spend more than five hours every weekday in class.
d) No student spends more than five hours every weekday in class. (Or, equivalently, every student spends
less than or equal to five hours every weekday in class.)

Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

a) $\forall x(C(x) \rightarrow F(x))$

b) $\forall x(C(x) \land F(x))$

c) $\exists x(C(x) \rightarrow F(x))$

d) $\exists x(C(x) \land F(x))$

Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

a) $\forall x(C(x) \rightarrow F(x))$

b) **b)** $\forall x(C(x) \wedge F(x))$

c) $\exists x(C(x) \rightarrow F(x))$

d) $\exists x(C(x) \wedge F(x))$

a) This statement is that for every *x*, if *x* is a comedian, then *x* is funny. In English, this is most simply stated, "Every comedian is funny."
b) This statement is that for every *x* in the domain (universe of discourse), *x* is a comedian *and* *x* is funny. In English, this is most simply stated, "Every person is a funny comedian." Note that this is not the sort of thing one wants to say. It really makes no sense and doesn't say anything about the existence of boring comedians; it's surely false, because there exist lots of *x* for which *C(x)* is false. This illustrates the fact that you rarely want to use conjunctions with universal quantifiers.
c) This statement is that there exists an *x* in the domain such that if *x* is a comedian then *x* is funny. In English, this might be rendered, "There exists a person such that ifs/he is a comedian, then s/he is funny." Note that this is not the sort of thing one wants to say. It really makes no sense and doesn't say anything about the existence of funny comedians; it's surely true, because there exist lots of *x* for which *C(x)* is false (recall the definition of the truth value of $p \rightarrow q$). This illustrates the fact that you rarely want to use conditional statements with existential quantifiers.
d) This statement is that there exists an *x* in the domain such that *x* is a comedian and *x* is funny. In English, this might be rendered, "There exists a funny comedian" or "Some comedians are funny" or "Some funny people are comedians."

Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language C++." Express each of these sentences in terms of $P(x)$, $Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

a) There is a student at your school who can speak Russian and who knows C++.

b) There is a student at your school who can speak Russian but who doesn't know C++.

c) Every student at your school either can speak Russian or knows C++.

d) No student at your school can speak Russian or knows C++.

Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language C++." Express each of these sentences in terms of $P(x)$, $Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

a) There is a student at your school who can speak Russian and who knows C++.
b) There is a student at your school who can speak Russian but who doesn't know C++.
c) Every student at your school either can speak Russian or knows C++.
d) No student at your school can speak Russian or knows C++.

a) We assume that this sentence is asserting that the same person has both talents. Therefore we can write $\exists x(P(x) \land Q(x))$.
b) Since "but" really means the same thing as "and" logically, this is $\exists x\ (P(x) \land \neg Q(x))$
c) This time we are making a universal statement: $\forall x\ (P(x) \lor Q(x))$
d) This sentence is asserting the nonexistence of anyone with either talent, so we could write it as $\neg\exists x(P(x) \lor Q(x))$. Alternatively, we can think of this as asserting that everyone fails to have either of these talents, and we obtain the logically equivalent answer $\forall x\ \neg(P(x) \lor Q(x))$. Failing to have either talent is equivalent to having neither talent (by De Morgan's law), so we can also write this as $\forall x\ ((\neg P(x)) \land (\neg Q(x))$. Note that it would *not* be correct to write $\forall x((\neg P(x)) \lor (\neg Q(x))$ nor to write $\forall x\ \neg(P(x) \land Q(x))$.

Suppose that the domain of the propositional function $P(x)$ consists of the integers 0, 1, 2, 3, and 4. Write out each of these propositions using disjunctions, conjunctions, and negations.

a) $\exists x P(x)$ **b)** $\forall x P(x)$ **c)** $\exists x \neg P(x)$
d) $\forall x \neg P(x)$ **e)** $\neg \exists x P(x)$ **f)** $\neg \forall x P(x)$

Suppose that the domain of the propositional function $P(x)$ consists of the integers 0, 1, 2, 3, and 4. Write out each of these propositions using disjunctions, conjunctions, and negations.

a) $\exists x P(x)$ **b)** $\forall x P(x)$ **c)** $\exists x \neg P(x)$

d) $\forall x \neg P(x)$ **e)** $\neg \exists x P(x)$ **f)** $\neg \forall x P(x)$

Existential quantifiers are like disjunctions, and universal quantifiers are like conjunctions.

a) We want to assert that $P(x)$ is true for some x in the universe, so either P(O) is true or $P(l)$ is true or P(2) is true or P(3) is true or P(4) is true. Thus the answer is $P(O) \vee P(l) \vee P(2) \vee P(3) \vee P(4)$. The other parts of this exercise are similar. Note that by De Morgan's laws, the expression in part (c) is logically equivalent to the expression in part **(f),** and the expression in part **(d)** is logically equivalent to the expression in part (e).

b) $P(O) \wedge P(l) \wedge P(2) \wedge P(3) \wedge P(4)$

c) $\neg P(0) \vee \neg P(l) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4)$

d) $\neg P(O) \wedge \neg P(l) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4)$

e) This is just the negation of part (a): $\neg(P(O) \vee P(l) \vee P(2) \vee P(3) \vee P(4))$

f) This is just the negation of part **(b):** $\neg(P(O) \wedge P(l) \wedge P(2) \wedge P(3) \wedge P(4))$

Are these system specifications consistent?
“The system is in multiuser state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. The kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode.”

Are these system specifications consistent?
"The system is in multiuser state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. The kernel is not functioning, or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode."

Let *m, n, k,* and i represent the propositions "The system is in multiuser state," "The system is operating normally," "The kernel is functioning," and "The system is in interrupt mode," respectively. Then we want to make the following expressions simultaneously true by our choice of truth values for *m*, n, *k,* and i:

$$m \leftrightarrow n,\ n \rightarrow k,\ \neg k \vee i,\ \neg m \rightarrow i,\ \neg i$$

In order for this to happen, clearly i must be false. In order for $\neg m \rightarrow i$ to be true when i is false, the hypothesis $\neg m$ must be false, so *m* must be true. Since we want $m \leftrightarrow n$ to be true, this implies that *n* must
also be true. Since we want $n \rightarrow k$ to be true, we must therefore have *k* true. But now if *k* is true and i is false, then the third specification, $\neg k \vee i$ is false. Therefore we conclude that this system is not consistent.

Use truth tables to verify the commutative laws
a) $p \vee q \equiv q \vee p$. **b)** $p \wedge q \equiv q \wedge p$.

Determine the truth value of each of these statements if the domain consists of all integers.

a) $\forall n(n + 1 > n)$ **b)** $\exists n(2n = 3n)$

c) $\exists n(n = -n)$ **d)** $\forall n(3n \leq 4n)$

Determine the truth value of each of these statements if the domain consists of all integers.

a) $\forall n(n + 1 > n)$ **b)** $\exists n(2n = 3n)$

c) $\exists n(n = -n)$ **d)** $\forall n(3n \leq 4n)$

a) Since adding 1 to a number makes it larger, this is true.

b) Since $2 \cdot 0 = 3 \cdot 0$, this is true.

c) This statement is true, since $0 = -0$.

d) This is true for the nonnegative integers but not for the negative integers. For example, $3(-2) \not\leq 4(-2)$.

Therefore the universally quantified statement is false.

For each of these statements find a domain for which the statement is true and a domain for which the statement is false.

a) Everyone is studying discrete mathematics.
b) Everyone is older than 21 years.
c) Every two people have the same mother.
d) No two different people have the same grandmother.

Domain: Rahim and Karim two friend -T
Domain: Rahim and Karim are cousins - F

For each of these statements find a domain for which the statement is true and a domain for which the statement is false.
a) Everyone is studying discrete mathematics.
b) Everyone is older than 21 years.
c) Every two people have the same mother.
d) No two different people have the same grandmother.

a) One would hope that if we take the domain to be the students in your class, then the statement is true. If we take the domain to be all students in the world, then the statement is clearly false, because some of them are studying only other subjects.
b) If we take the domain to be United States Senators, then the statement is true. If we take the domain to be college football players, then the statement is false, because some of them are younger than 21.
c) If the domain consists of just Princes William and Harry of Great Britain (sons of the late Princess Diana), then the statement is true. It is also true if the domain consists of just one person (everyone has the same mother as him- or herself). If the domain consists of all the grandchildren of Queen Elizabeth II of Great Britain (of whom William and Harry are just two), then the statement is false.
d) If the domain consists of Bill Clinton and George W. Bush, then this statement is true because they do not have the same grandmother. If the domain consists of all residents of the United States, then the statement is false, because there are many instances of siblings and first cousins, who have at least one grandmother in common.

For each of these statements find a domain for which the statement is true and a domain for which the statement is false.

a) Everyone is studying discrete mathematics.

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d) If the domain consists of Bill Clinton and George W. Bush, then this statement is true because they do not have the same grandmother. If the domain consists of all residents of the United States, then the statement is false, because there are many instances of siblings and first cousins, who have at least one grandmother in common.

Exercise Part 3

Translate these specifications into English where $F(p)$ is "Printer p is out of service," $B(p)$ is "Printer p is busy," $L(j)$ is "Print job j is lost," and $Q(j)$ is "Print job j is queued."

a) $\exists p(F(p) \wedge B(p)) \rightarrow \exists j L(j)$

b) $\forall p B(p) \rightarrow \exists j Q(j)$

c) $\exists j\, (Q(j) \wedge L(j)) \rightarrow \exists p F(p)$

d) $(\forall p B(p) \wedge \forall j Q(j)) \rightarrow \exists j L(j)$

a) If there is a printer that is both out of service and busy, then some job has been lost.

b) For every printer is busy, then there is a job in the queue.

c) If there is a job that is both queued and lost, then some printer is out of service.

d) If every printer is busy and every job is queued, then some job is lost.

Translate these specifications into English where $F(p)$ is "Printer p is out of service," $B(p)$ is "Printer p is busy," $L(j)$ is "Print job j is lost," and $Q(j)$ is "Print job j is queued."

a) $\exists p(F(p) \wedge B(p)) \rightarrow \exists j L(j)$

b) $\forall p B(p) \rightarrow \exists j Q(j)$

c) $\exists j\, (Q(j) \wedge L(j)) \rightarrow \exists p F(p)$

d) $(\forall p B(p) \wedge \forall j Q(j)) \rightarrow \exists j L(j)$

In each case we pretty much just write what we see.

a) If there is a printer that is both out of service and busy, then some job has been lost.

b) If every printer is busy, then there is a job in the queue.

c) If there is a job that is both queued and lost, then some printer is out of service.

d) If every printer is busy and every job is queued, then some job is lost.

Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. let the domain consist of the students in your class.

a) Someone in your class can speak Hindi.
b) Everyone in your class is friendly.
c) There is a person in your class who was not born in California.
d) A student in your class has been in a movie.
e) No student in your class has taken a course in logic programming.

a) Let H(x) be "x can speak hindi" Then $\exists x H(x)$

b) Let F(x) be "x is friendly". The we have $\forall x F(x)$.

c) Let B(x) be "x was born in California". Then we have $\exists x \neg B(x)$

d) Let M(x) be "x has been in a Movie". Then we have $\exists x\, M(x)$

e) This is saying that everyone has failed to take the course. The answer is $\forall x \neg L(x)$

Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

a) Someone in your class can speak Hindi.

b) Everyone in your class is friendly.

c) There is a person in your class who was not born in California.

d) A student in your class has been in a movie.

e) No student in your class has taken a course in logic programming.

let *C(x)* be the propositional function *"x* is in your class." Note that for the second way, we always want to use conditional statements with universal quantifiers and conjunctions with existential quantifiers.

a) Let H (x) be "x can speak Hindi." Then we have $\exists x$ H (x) the first way, or $\exists x$ (C (x) $\wedge$ H (x)) the second way.

b) Let F(x) be "x is friendly." Then we have $\forall x$ F(x) the first way, or $\forall$ x(C(x)$\rightarrow$ F(x)) the second way.

c) Let B(x) be "x was born in California." Then we have $\exists x \neg$B(x) the first way, or $\exists x$(C(x) $\wedge$ $\neg$ B(x)) theconjunctions with existential quantifiers.

d) Let M(x) be "x has been in a movie." Then we have $\exists$ xM(x) the first way, or $\exists$ x(C(x) $\wedge$ M(x)) the second way.

e) This is saying that everyone has failed to take the course. So the answer here is $\forall x \neg$L(x) the first way, or $\forall$x (C(x) $\rightarrow$ $\neg$ L(x)) the second way, where L(x) is "x has taken a course in logic programming."

Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

a) No one is perfect.

b) Not everyone is perfect.

c) All your friends are perfect.

d) At least one of your friends is perfect.

e) Everyone is your friend and is perfect.

f) Not everybody is your friend or someone is not perfect.

Let P(x) "x is perfect" F(x) "x is friend"

a) $\forall x \neg P(x)$ alternatively, $\neg \exists x P(x)$

b) $\neg \forall x P(x)$

c) $\forall x (F(x) \rightarrow P(x))$

d) $\exists x (F(x) \wedge P(x))$

e) $\forall x (F(x) \wedge P(x))$

f) $(\neg \forall x F(x)) \vee (\exists x \neg P(x))$

Translate each of these statements into logical expressions in three different ways by varying the domain and by using predicates with one and with two variables.

a) A student in your school has lived in Vietnam.

b) There is a student in your school who cannot speak Hindi.

c) A student in your school knows Java, Prolog, and C++.

d) Everyone in your class enjoys Thai food.

e) Someone in your class does not play hockey.

a) $\exists\, x Y(x)$ or $\exists\, x\, (Y(x) \wedge V(x, vietnam))$

b) $\exists x \neg H(x)$ or $\exists x\, (Y(x) \wedge \neg H(x, Hindi))$

c) x is a student J(x), P(x) and C(x). $\exists x\, (J(x) \wedge P(x) \wedge C(x))$, considering domain is your class mates. Domain all students in a university, x is student and y subject D(x,y)
$\exists x\, (Y(x) \wedge J(x, Java) \wedge P(x, Prolong) \wedge C(x, C{+}{+}))$

Translate each of these statements into logical expressions in three different ways by varying the domain and by using predicates with one and with two variables.

a) A student in your school has lived in Vietnam.

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d) Everyone in your class enjoys Thai food.

e) Someone in your class does not play hockey.

d) $\forall x\, P(x)$

e) $\exists x \neg H(x)$

Questions: Consider these statements, of which the first three are premises and the fourth is a valid conclusion.
"All hummingbirds are richly colored."
"No large birds live on honey."
"Birds that do not live on honey are dull in color."
"Hummingbirds are small."
Answer:
x is a bird P(x) = "*x* is a hummingbird" Q(x) = "x is large" R(x)="x live on honey"
S(x) = "x is richly colored"
Let *P(x)*, *Q(x)*, *R(x)*, and *S(x)* be the statements "*x* is a hummingbird," "*x* is large," "*x* lives on honey," and "*x* is richly colored," respectively. Assuming that the domain consists of all birds, express the statements in the argument using quantifiers and *P(x)*, *Q(x)*, *R(x)*, and *S(x)*.

$\forall x\, (P(x) \rightarrow S(x))$
$\neg \exists x (Q(x) \wedge R(x))$
$\forall x\, (\neg R(x) \rightarrow \neg S(x))$
$\forall x (P(x) \rightarrow \neg Q(x))$

Questions: Consider these statements. The first two are called *premises* and the third is called the *conclusion*. The entire set is called an *argument*.

"All lions are fierce."

"Some lions do not drink coffee."

"Some fierce creatures do not drink coffee."

Answer:

Let $P(x)$, $Q(x)$, and $R(x)$ be the statements "x is a lion," "x is fierce," and "x drinks coffee," respectively. Assuming that the domain consists of all creatures, express the statements in the argument using quantifiers and $P(x)$, $Q(x)$, and $R(x)$.

$\forall x\ (P(x) \rightarrow Q(x))$

$\exists x\ (P(x) \rightarrow \neg R(x))$

$\exists x\ (Q(x) \rightarrow \neg R(x))$