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• Function approximation

Taylor series

input f^n $\longrightarrow$ polynomials

• easy to deal

$f(x)$ is a differentiable at the point $x = a$

then,

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots\cdots$$

0^{th} order $\longrightarrow$ 1 term

1^{st} order $\longrightarrow$ 2 terms

2^{nd} order $\longrightarrow$ 3 terms

3^{rd} order $\longrightarrow$ 4 terms

$h = x_{i+1} - x_i$

$$f(x_{i+1}) = f(x_i) + \frac{f'(x_i)}{1!}h + \frac{f'(x_i)}{2!}h^2 + \cdots$$

Q.

$f(x) = \ln(x)$ predict $f(1.5)$

base point $x = 1$ with 4^{th} order TS

sol^n: $f(1) = \ln(1) = 0$

$f'(x) = \frac{1}{x}$	$f'(1) = 1$	$x_{i+1} = 1.5$
$f''(x) = -\frac{1}{x^2}$	$f''(1) = -1$	$x_i = 1$
$f'''(x) = \frac{2}{x^3}$	$f'''(1) = 2$	$h = 0.5$
$f^{iv}(x) = -\frac{6}{x^4}$	$f^{iv}(1) = -6$	

$$f(x_{i+1}) = f(1) + \frac{f'(1)}{1!}(0.5) + \frac{f''(1)}{2!}(0.5)^2$$

$$+ \frac{f'''(1)}{3!}(0.5)^3 + \frac{f^{iv}(1)}{4!}(0.5)^4$$

$$f(1.5)_{approx} = 0.40104$$

$$f(1.5)_{actual} = 0.4054$$

Absolute error

$$\varepsilon_a = |\text{Current app.} - \text{Previous app.}|$$

Relative %

$$\% \varepsilon = \frac{|\text{Current app.} - \text{Previous app.}|}{\text{Actual value}}$$

Example-1: Approximate the function, $f(x) = 1.2 - 0.25x - 0.5x^2 - 0.15x^3 - 0.1x^4$

from $x_i = 0$ *and predict* $f(x)$ *at* $x_{i+1} = 1$

Approximation

```
clear
close all
clc

x1=0;
x2 = 10;
y1 = 1.2 - 0.25*x1-0.5*x1^2-0.15*x1^3-0.1*x1^4;
actual_y2 = 1.2 - 0.25*x2-0.5*x2^2-0.15*x2^3-0.1*x2^4;

%0th order approximation
pred_y2_0 = y1;

%1st order approximation
y1d = -0.25-x1-0.45*x1^2-0.4*x1^3;
pred_y2_1 = y1 + y1d*(x2-x1);

%2nd order approximation
y2d = -1-0.9*x1-1.2*x1^2;
pred_y2_2 = y1 + y1d*(x2-x1) + (y2d*(x2-x1)^2)/2;

%3rd order approximation
y3d = -0.9-2.4*x1;
pred_y2_3 = y1 + y1d*(x2-x1) + (y2d*(x2-x1)^2)/2 + (y3d*(x2-x1)^3)/6;

%4th order approximation
y4d = -2.4;
pred_y2_4 = y1 + y1d*(x2-x1) + (y2d*(x2-x1)^2)/2 + (y3d*(x2-x1)^3)/6
+ (y4d*(x2-x1)^4)/24;

%%
%take only 2nd order approximation but improve the result through iteration


rel_error1= abs(actual_y2-pred_y2_2)/abs(actual_y2);

x=0:0.5:10;
n=size(x,2);
pred_y=zeros(1,n);
pred_y(1)=y1;

for i=1:n-1
    y1d = -0.25-x(i)-0.45*x(i)^2-0.4*x(i)^3;
    y2d = -1-0.9*x(i)-1.2*x(i)^2;
    pred_y(i+1) = pred_y(i) + y1d*(x(i+1)-x(i)) + (y2d*(x(i+1)-x(i))^2)/2;

end

rel_error2= abs(actual_y2-pred_y(n))/abs(actual_y2);
fprintf('%f',rel_error2);
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