INDUCTION

Prove that every amount of postage of **ab-a-b+1** cents or more can be formed using just **a**-cent and **b**-cent stamps, where **a<b** and **gcd(a,b)=1**

1) Prove that every amount of postage of 8 cents or more can be formed using just 3-cent and 5-cent stamps.

2) Prove that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps.

3) Prove that every amount of postage of 18 cents or more can be formed using just 4-cent and 7-cent stamps.

4) Prove that every amount of postage of 18 cents or more can be formed using just 3-cent and 10-cent stamps.

5) Prove that every amount of postage of 30 cents or more can be formed using just 4-cent and 11-cent stamps.

These problems can be solved by two methods. One is normal induction and the another is strong induction.

 # Prove that every amount of postage of 30 cents or more can be formed using just 4-cent and 11-cent stamps.

NORMAL INDUCTION

Base Case/Basic Step: P (30) is true, because 30=4x+11y has a solution (x,y)=(2,2). 30=4x2+11x2

Inductive Step: If P(k) is true then P(k+1) is also true.

Now we'll find out a single solution of two Diophantine equation 4x-11y=1, 11x-4y=1.

So, 4x3-11x1=1, 11x3-4x8=1, which means if you want to add 1 cent then,

Either add 3 four cents (12) and remove 1 eleven cents (11),

or add 3 eleven cents (33) and remove 8 four cents (32)

30=4x2+11x2

31=4x5+11x1

32=4x8+11x0

33=4x0+11x3

34=4x3+11x2

35=4x6+11x1

36=4x9+11x0

37=4x1+11x3

38=4x4+11x2

39=4x7+11x1

40=4x10+11x0

41=4x2+11x3

STRONG INDUCTION

Base Case/Basic Step: P (30), P (31), P (32), P (33) is true, because

30=4x2+11x2

31=4x5+11x1

32=4x8+11x0

33=4x0+11x3

Inductive Step: If P(k) is true then P(k+4) is also true. Just increase only 1 four cents.

30=4x2+11x2	31=4x5+11x1	32=4x8+11x0	33=4x0+11x3
34=4x3+11x2	35=4x6+11x1	36=4x9+11x0	37=4x1+11x3
38=4x4+11x2	39=4x7+11x1	40=4x10+11x0	41=4x2+11x3