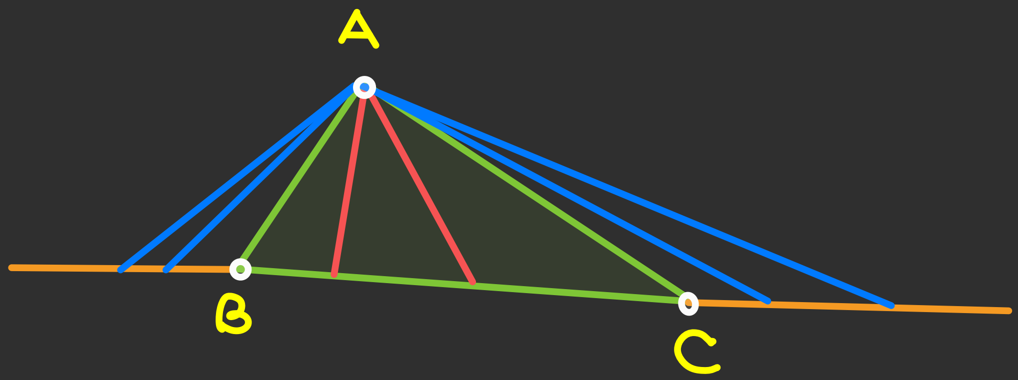
Cheva & Menalaus

-Jubain

Chevian

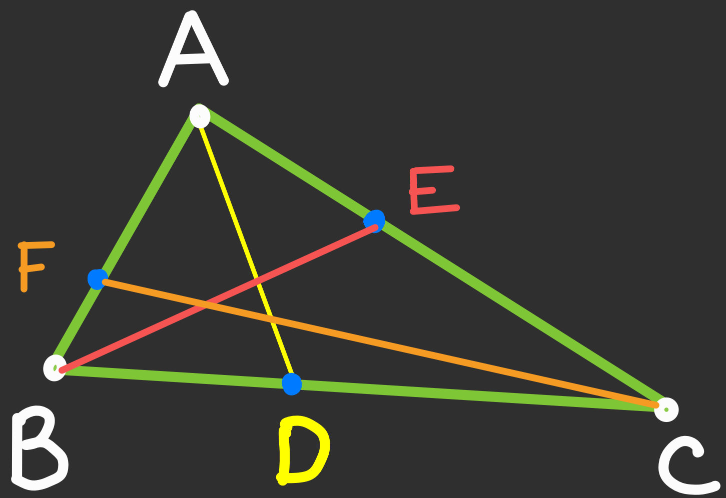


ABC is a triangle.

Red lines are internal A-chevian.

Blue lines are external A-chevian.

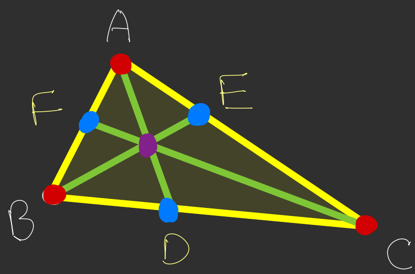
Rules for theorem



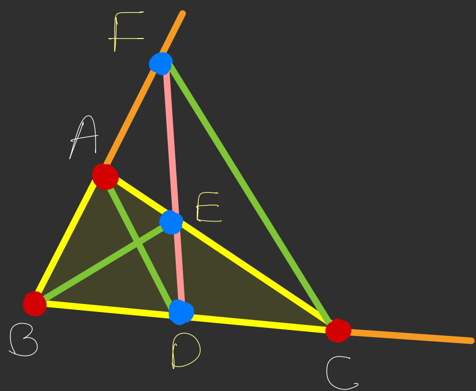
A- chevian meets BC at D

B - chevian meets AC at E

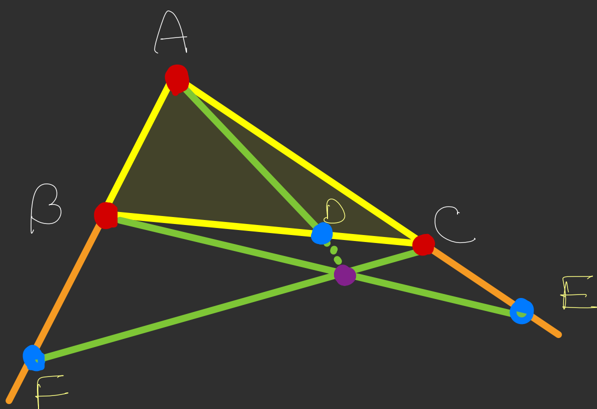
C - chevian meets AB at F



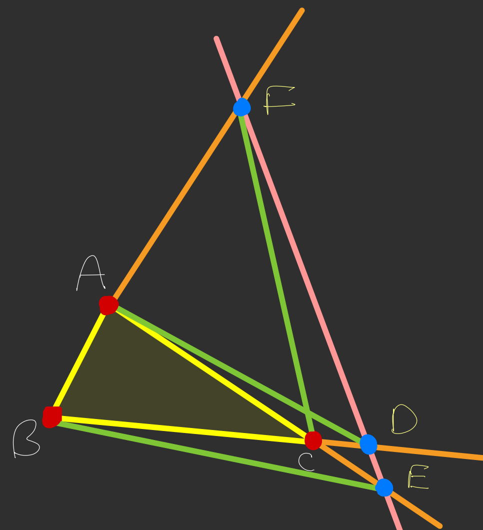
3 internal chevian
0 extarnal chevian
followed chevas theorem



2 internal chevian
1 extarnal chevian
followed menalaus theorem



1 internal chevian
0 extarnal chevian
followed chevas theorem



0 internal chevian
3 extarnal chevian
followed menalaus theorem

Every combination for cheva & menalaus theorem

For every picture,

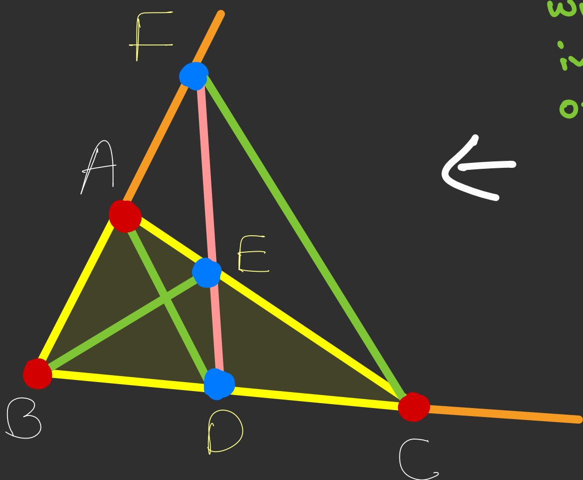
$$\frac{AF}{FB}\times\frac{BD}{DC}\times\frac{CE}{EA}=1$$

Either green lines (chevian) are concurrent or,

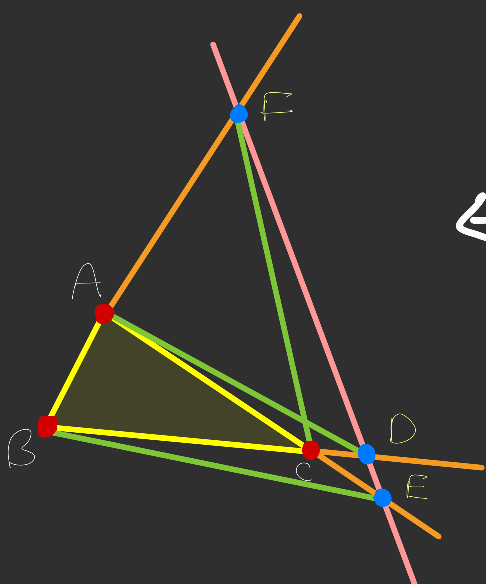
blue points are collinear

Why 2 combination will not work for cheva's T.?

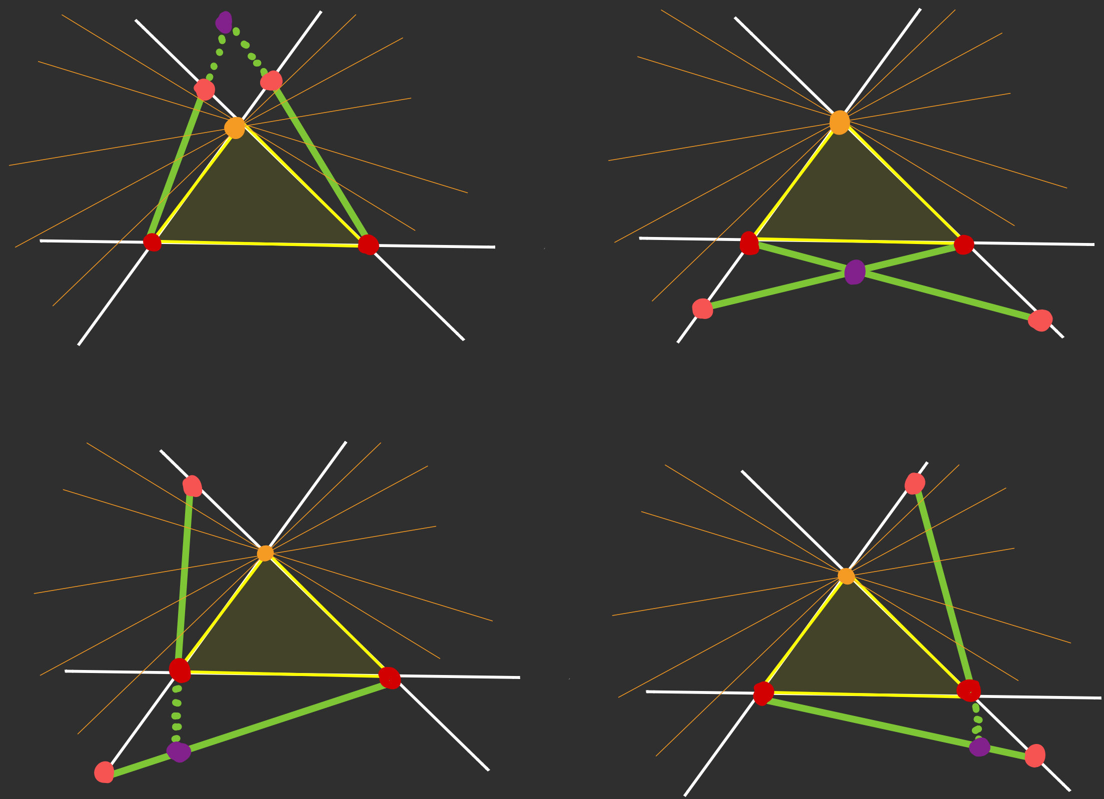
2 internal chevian will meet internally in the inner side of the triangle

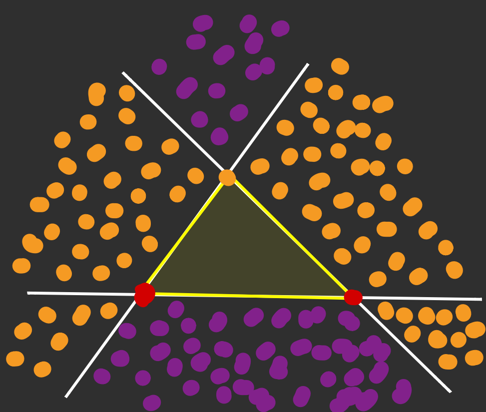


So the 3rd external chevian can't not meet that common point.



Details on next page





orange and violet areas are different. That's why no possibility to meet

Basic Ratio Operations

1) if $\frac{a}{b} = \frac{c}{d}$ then

i) * $\frac{a \pm b}{b} = \frac{c \pm d}{d}$

$$\boxed{\begin{aligned} &\frac{a}{b} \pm 1 = \frac{c}{d} \pm 1 \\ \Rightarrow &\frac{a \pm b}{b} = \frac{c \pm d}{d} \end{aligned}}$$

* $\frac{a}{a \pm b} = \frac{c}{c \pm d}$

* $\frac{a}{b \pm a} = \frac{c}{d \pm c}$

* $\frac{b \pm a}{b} = \frac{d \pm c}{d}$

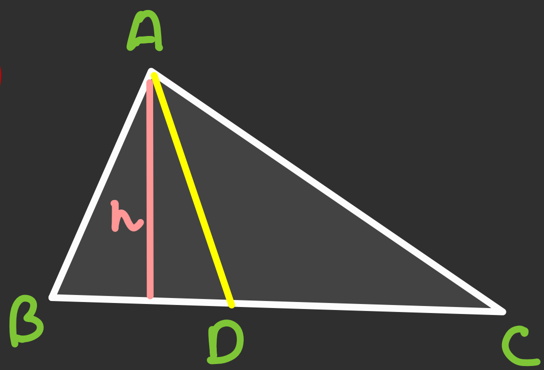
iii) $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

iii) if $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots$

then $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots = \frac{a+c+e}{b+d+f}$

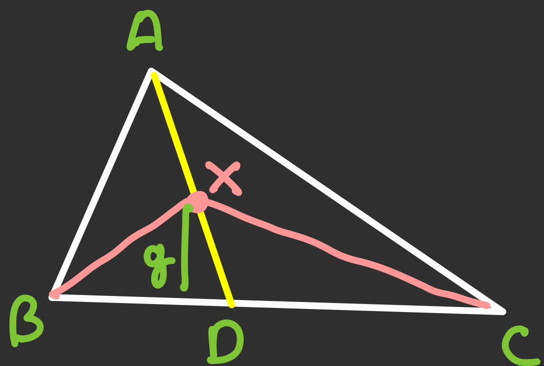
Basic Geometry

1)



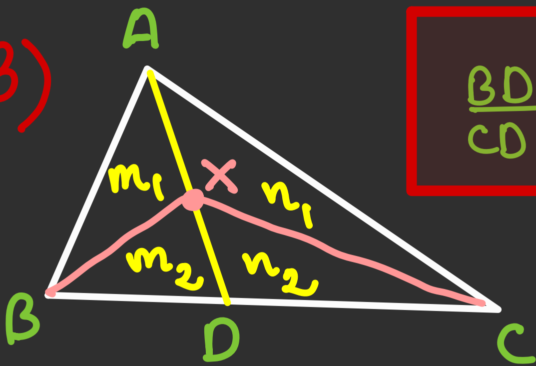
$$\frac{BD}{CD} = \frac{\frac{1}{2} \times BD \times h}{\frac{1}{2} \times CD \times h} = \frac{(\Delta ABD)}{(\Delta ACD)}$$

2)



$$\frac{BD}{CD} = \frac{(\Delta ABD)}{(\Delta ACD)} = \frac{(\Delta xBD)}{(\Delta xCD)} = \frac{\frac{1}{2} BD \times g}{\frac{1}{2} \times CD \times g}$$

3)



$$\boxed{\frac{BD}{CD} = \frac{(\Delta ABX)}{(\Delta ACX)} = \frac{m_1}{m_2}}$$

$$\frac{BD}{CD} = \frac{(\Delta ABD)}{(\Delta ACD)} = \frac{(\Delta XBD)}{(\Delta XCD)}$$

$$\Rightarrow \frac{m_1+m_2}{n_1+n_2} = \frac{m_2}{n_2}$$

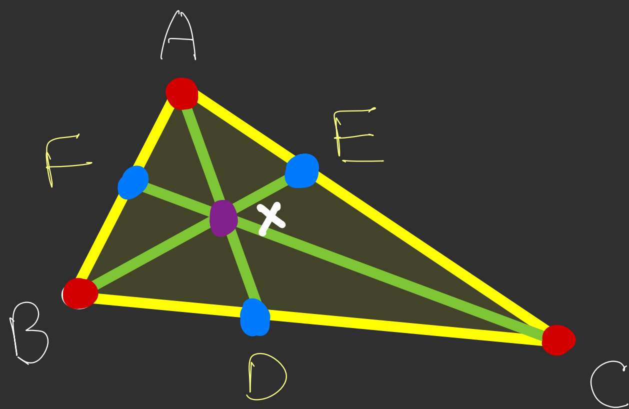
$$\Rightarrow \frac{m_1+m_2}{m_2} = \frac{n_1+n_2}{n_2}$$

$$\Rightarrow \frac{m_1}{m_2} + 1 = \frac{n_1}{n_2} + 1$$

$$\therefore \frac{m_2}{n_2} = \frac{m_1}{n_1}$$

$$\boxed{\frac{BD}{CD} = \frac{m_1+m_2}{n_1+n_2} = \frac{m_2}{n_2} = \frac{m_1}{n_1}}$$

Prove of Cheva's Theorem



If AD, BE and CF are concurrent, then

$$\frac{AF}{FB} \times \frac{BD}{DC} \times \frac{CE}{EA} = 1$$

$$\Rightarrow \frac{\cancel{(\Delta AXC)}}{\cancel{(\Delta BXC)}} \times \frac{\cancel{(\Delta AXB)}}{\cancel{(\Delta AXC)}} \times \frac{\cancel{(\Delta BXC)}}{\cancel{(\Delta AXB)}}$$

$$= 1$$