Geometry

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1 Angles and Area of Triangles

1.1 Acute Triangle

Definition: All angles less than 90°.

1.2 Right-Angled Triangle

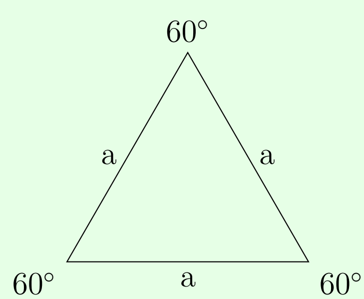
Definition: One angle equals 90°.

1.3 Obtuse Triangle

Definition: One angle greater than 90°.

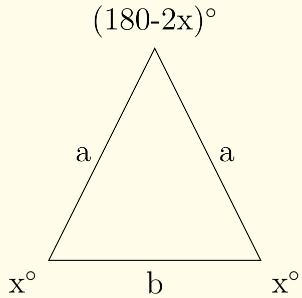
1.4 Equilateral Triangle

Definition: A triangle with all sides equal and all angles equal to 60°.
Note: Equilateral triangle is an acute triangle and also an isosceles triangle.
Area: $\frac{\sqrt{3}}{4}a^2$, **Perimeter:** $3a$



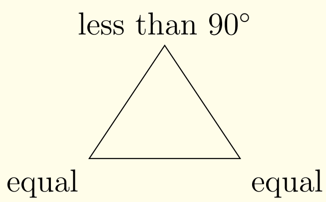
1.5 Isosceles Triangle

Definition: A triangle with two equal sides and two equal angles.
Area: $\frac{b}{4}\sqrt{4a^2 - b^2}$, **Perimeter:** $2a + b$

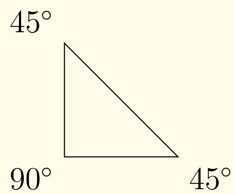


Types of Isosceles Triangles:

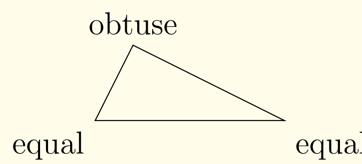
Acute



Right-Angled

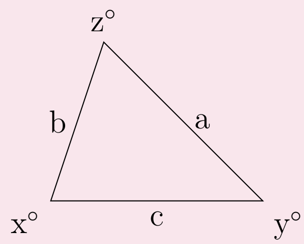


Obtuse



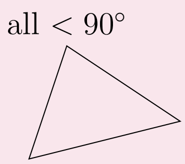
1.6 Scalene Triangle

Definition: A triangle with no equal sides or angles.
Area: $\sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{a+b+c}{2}$, **Perimeter:** $a+b+c$

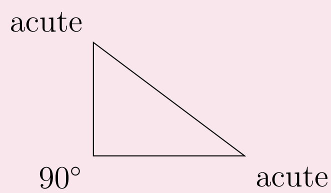


Types of Scalene Triangles:

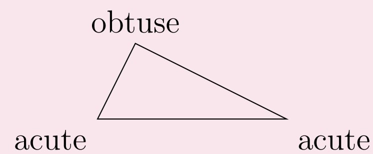
Acute



Right-Angled



Obtuse



1.7 Area Formulas for Triangles

There are several ways to calculate the area of a triangle, depending on the given information:

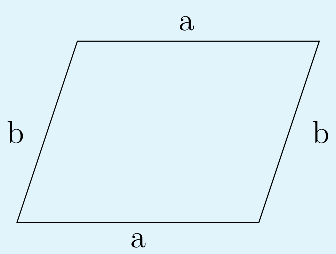
- **Base and Height:** $A = \frac{1}{2} \times b \times h$
- **Using All Three Sides (Heron's Formula):** $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{a+b+c}{2}$ is the semi-perimeter
- **Using Semiperimeter and Inradius:** $A = s \times r$, where r is the inradius
- **Using Two Sides and Angle:** $A = \frac{1}{2} \times a \times b \times \sin(\theta)$
- **Circumradius Formula:** $A = \frac{abc}{4R}$, where R is the circumradius

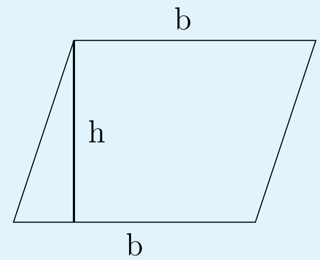
2 Area of Quadrilaterals

2.1 Parallelogram

Definition: A quadrilateral with opposite sides parallel and equal. The opposite angles are equal.

Area: $A = b \times h$, **Perimeter:** $2(a + b)$

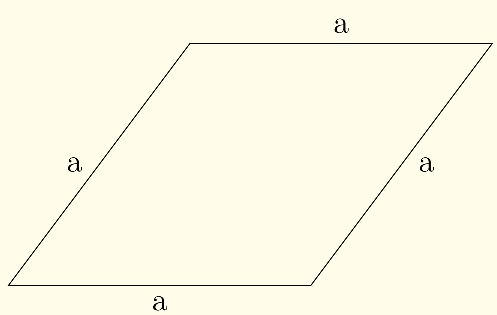




2.2 Rhombus

Definition: A quadrilateral with all sides equal and opposite angles equal.

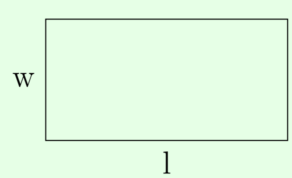
Area: $A = \frac{1}{2} \times d_1 \times d_2$, **Perimeter:** $4a$



2.3 Rectangle

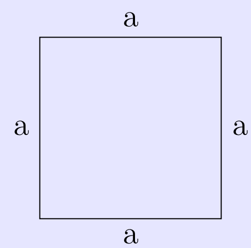
Definition: A quadrilateral with equal opposite sides and all angles 90°.

Area: $A = l \times w$, **Perimeter:** $2(l + w)$



2.4 Square

Definition: A quadrilateral with all sides equal and all angles 90°.
Area: $A = a^2$, **Perimeter:** $4a$

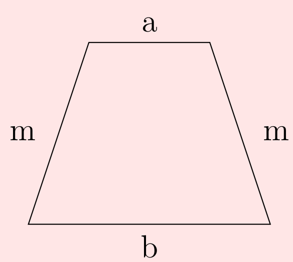


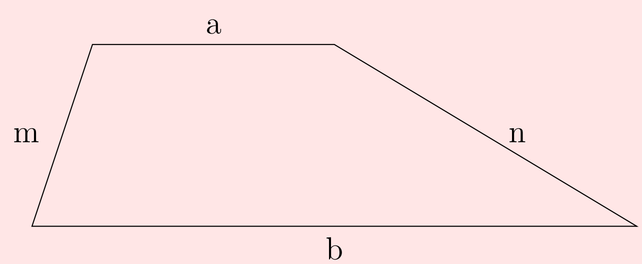
2.5 Trapezium

Definition: A quadrilateral with a pair of opposite sides parallel.

Types of Trapezium:

Isosceles Trapezium:





Non-Isosceles Trapezium:

2.6 Notes

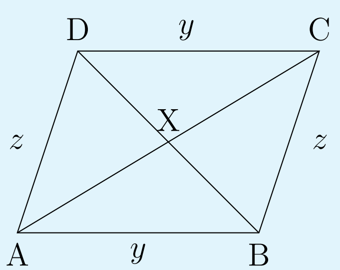
- a) Square, Rectangle, and Rhombus are all types of parallelograms.
- b) All squares are rhombuses.
- c) All squares are rectangles.

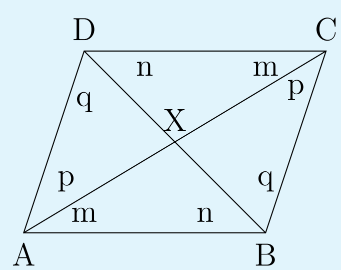
3 Quadrilateral

3.1 Parallelogram

Theorem:

- $AB = CD = y$
- $BC = AD = z$ (Opposite sides of a parallelogram are equal)
- $AX = CX$
- $BX = DX$ (Diagonals of a parallelogram are equally divided into two parts)
- $\angle BAC = \angle ACD = m$
- $\angle ADB = \angle CBD = q$
- $\angle CAD = \angle ACB = p$
- $\angle ABD = \angle BDC = n$ (Alternate Angle Theorem)
- $\angle ABC = \angle CDA = n + q$
- $\angle DAB = \angle BCD = m + p$ (Opposite angles are equal)
- The sum of the angles $m + n + p + q = 180°$ (Angles around the intersection point X sum to 360°)

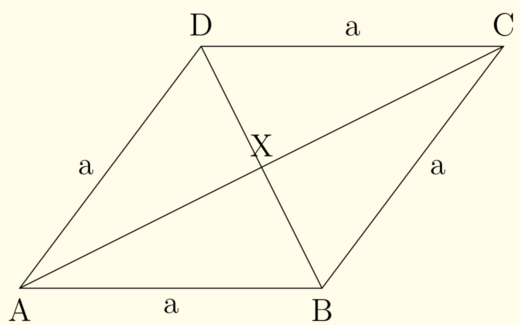


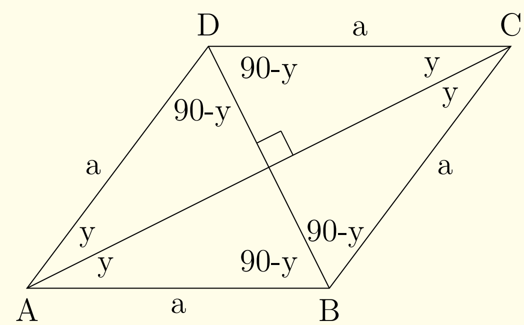


3.2 Rhombus

Theorem:

- $AB = CD = BC = AD = a$ (All sides of a rhombus are equal)
- $AX = CX$
- $BX = DX$ (Diagonals of a rhombus are equally divided into two parts)
- $\angle BAC = \angle ACD = \angle CAD = \angle ACB = y$
- $\angle ADB = \angle CBD = \angle ABD = \angle BDC = 90° - y$ (Alternate Angle Theorem)
- $\angle ABC = \angle CDA = 180° - 2y$
- $\angle DAB = \angle BCD = 2y$ (Opposite angles are equal)
- $\angle AXB = \angle CXD = \angle AXD = \angle BXC = 90°$ (Diagonal of a rhombus perpendicularly divide one another)

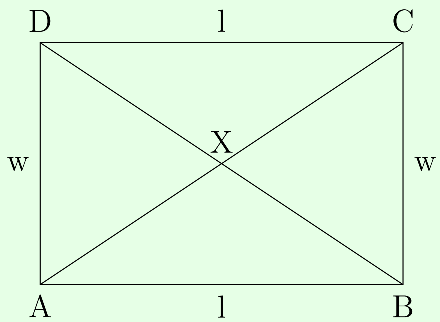


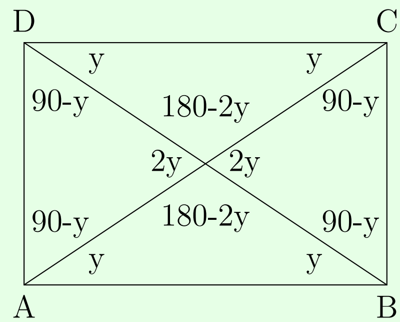


3.3 Rectangle

Theorem:

- $AB = CD = l$ and $BC = AD = w$ (Opposite sides of a rectangle are equal)
- $AC = BD$ (Diagonals of a rectangle are equal)
- $AX = CX = BX = DX$ (Diagonals of a rectangle are equally divided into two parts)
- $\angle BAC = \angle ACD = \angle ABD = \angle BDC = y$
- $\angle ADB = \angle CBD = \angle CAD = \angle ACB = 90° - y$ (Alternate Angle Theorem)
- $\angle ABC = \angle BCD = \angle CDA = \angle DAB = 90°$ (All angles are right angles)

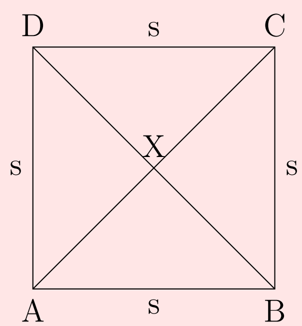


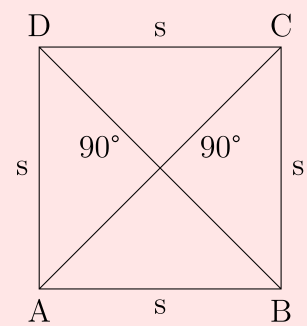


3.4 Square

Theorem:

- $AB = CD = BC = AD = s$ (All sides of a square are equal)
- $AC = BD$ (Diagonals of a square are equal)
- $AX = CX = BX = DX$ (Diagonals of a square are equally divided into two parts)
- $\angle BAC = \angle ACD = \angle ABD = \angle BDC = \angle ADB = \angle CBD = \angle CAD = \angle ACB = 45°$
- $\angle ABC = \angle CDA = \angle DAB = \angle BCD = 90°$ (All angles are right angles)
- $\angle AXB = \angle CXD = \angle DXA = \angle BXC = 90°$ (All angles are right angles)

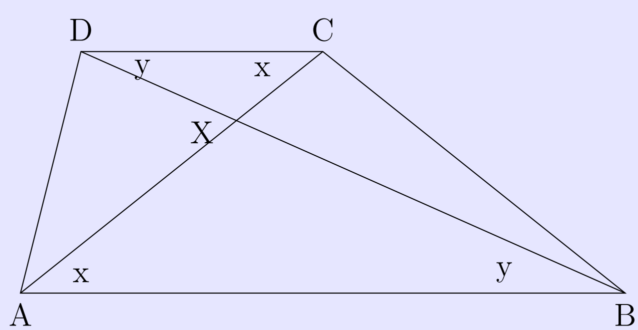




3.5 Trapezium Characteristics

Trapezium Characteristics:

- A trapezium has one pair of parallel sides. (AB —— CD)
- The non-parallel sides are called legs. item



3.6 Area of Trapezium

Proof of Area of Trapezium

Given: A trapezium with parallel sides a and b, and height h.

1st Diagram (Trapezium with Parallel Sides a and b):

We divide the trapezium into three parts:
1. A rectangle with width a and height h.
2. Two right triangles with bases x and $b - x - a$, and height h.

Adding these areas together, we get the total area of the trapezium:

$$\text{Total Area} = \frac{1}{2} \times x \times h + \frac{1}{2}2 \times a \times h + \frac{1}{2} \times (b - x - a) \times h$$

Simplifying:

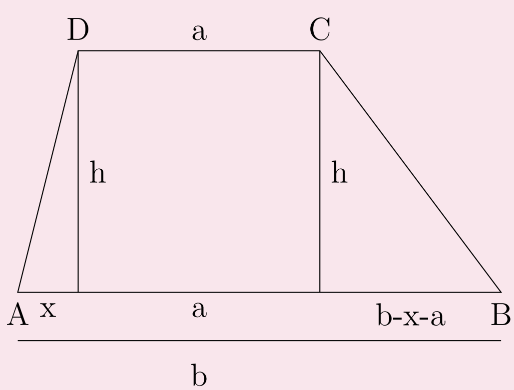
$$\begin{aligned} \text{Total Area} &= \frac{1}{2} \times h \times [x + 2a + (b - x - a)] \\ &= \frac{1}{2} \times h \times (b + b - x) \\ &= \frac{1}{2} \times (a + b) \times h \end{aligned}$$

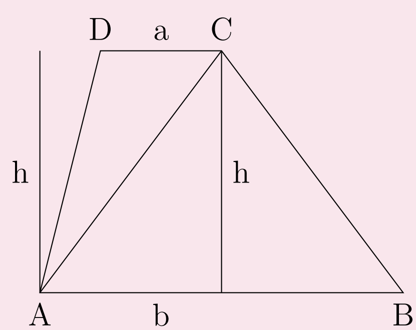
2nd Diagram (Trapezium with Parallel Sides a and b):

We divide the trapezium into two parts: Both are triangle with different bases a and b and height will be h

$$\text{Total Area} = \frac{1}{2} \times a \times h + \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times (a + b) \times h$$

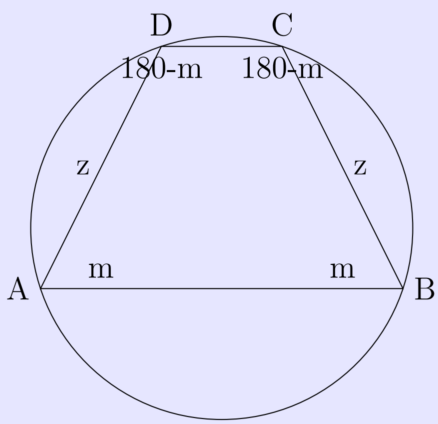


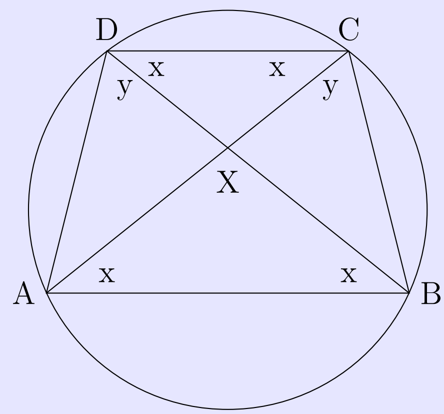


3.7 Isosceles Trapezium

Isosceles Trapezium:

- $AD = BC = z$ (In an isosceles trapezium, the non-parallel sides (legs) are equal.)
- $AC = BD$ (The diagonals are also equal in length.)
- $AX = BX$ and $CX = DX$
- $\angle DAB = \angle ABC = m$
- $\angle CDA = \angle BCD = 180 - m$ (The angles at the base are equal, and the isosceles trapezium is cyclic, meaning it can be inscribed in a circle.)
- $\triangle AXD \cong \triangle BXC$ ($\triangle AXD$ is congruent to $\triangle BXC$)
- $\angle XAB = \angle XBA = \angle XCD = \angle XDC = x$
- $\angle XDA = \angle XCB = y$
- $\angle XAD = \angle XBC = 180 - 2x - y$



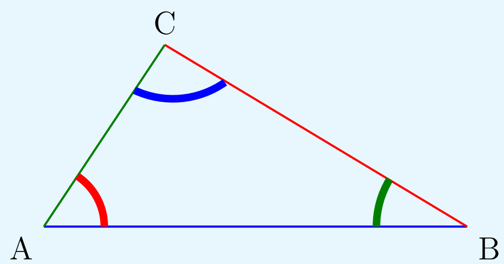


4 Triangle Congruence

SSS (Side-Side-Side) Theorem

Statement:
If $AB = DE$, $BC = EF$, and $CA = FD$,
then $\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$.

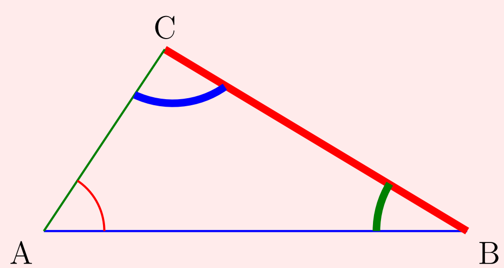
Explanation: From the known equal sides, we can conclude that the corresponding angles are also equal, which were unknown initially.



SAS (Side-Angle-Side) Theorem

Statement:
If $AB = DE$, $\angle A = \angle D$, and $AC = DF$,
then $\angle B = \angle E$, $\angle C = \angle F$, and $BC = EF$,

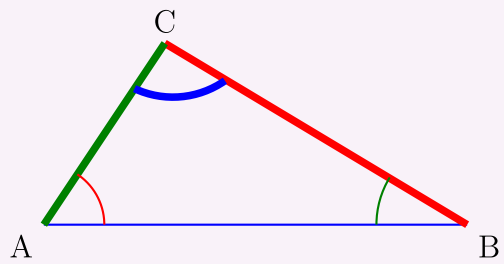
Explanation: If two sides and the included angle in one triangle are congruent to two sides and the included angle in another triangle, then the triangles are congruent.

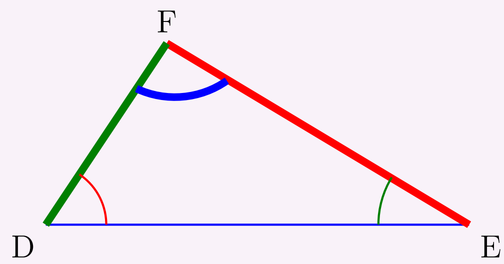


ASA (Angle-Side-Angle) Theorem

Statement:
If $\angle A = \angle D$, $AB = DE$, and $\angle B = \angle E$,
then $\angle C = \angle F$, $AC = DF$ and $BC = EF$.

Explanation: If two angles and the included side in one triangle are congruent to two angles and the included side in another triangle, then the triangles are congruent.





RHS (Right Angle-Hypotenuse-Side) Theorem

Statement:
If the hypotenuse and one leg of a right triangle are congruent to the hypotenuse and one leg of another right triangle, then the two triangles are congruent.
Given, $AB = DE$, $AC = DF$ and $\angle C = \angle F = 90^\circ$.
then, $BC = EF$, $\angle A = \angle D$ and $\angle B = \angle E$.

Explanation: This theorem is used for right-angled triangles, where the hypotenuse and one leg are congruent.