Floyd-Warshal Algorithm
And
Johnson's Algorithm
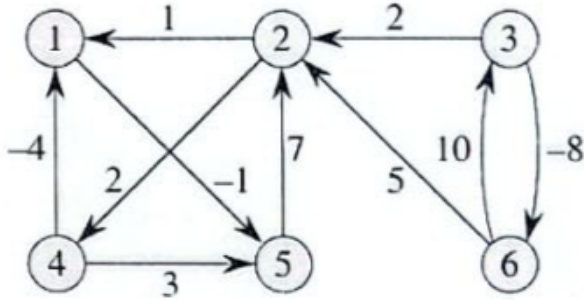
Floyd-Warshal Algorithm

$\alpha \pm 5 = \alpha$
$\alpha \pm \alpha = \alpha$

$n(n-1)(n-1)$
$n \cdot n \cdot n = n^3$

k=0

APSP Table Format:

	1	2	3	4	5	6
1	0	α	α	α	-1	α
2	1	0	α	2	α	α
3	α	2	0	α	α	-8
4	-4	α	α	0	3	α
5	α	7	α	α	0	α
6	α	5	10	α	α	0

k=1

APSP Table Format:

	1	2	3	4	5	6
1	0	α	α	α	-1	α
2	1	0	α	2	0	α
3	α	2	0	α	α	-8
4	-4	α	α	0	-5	α
5	α	7	α	α	0	α
6	α	5	10	α	α	0

k=2

APSP Table Format:

	1	2	3	4	5	6
1	0	α	α	α	-1	α
2	1	0	α	2	0	α
3	3	2	0	4	2	-8
4	-4	α	α	0	-5	α
5	8	7	α	9	0	α
6	6	5	10	7	5	0

k = 3

APSP Table Format:

	1	2	3	4	5	6
1	0	α	α	α	-1	α
2	1	0	α	2	0	α
3	3	2	0	4	2	-8
4	-4	α	α	0	-5	α
5	8	7	α	9	0	α
6	6	5	10	7	5	0

k=4

APSP Table Format:

	1	2	3	4	5	6
1	0	α	α	α	-1	α
2	-2	0	α	2	-3	α
3	0	2	0	4	-1	-8
4	-4	α	α	0	-5	α
5	5	7	α	9	0	α
6	3	5	10	7	2	0

k=5

APSP Table Format:

	1	2	3	4	5	6
1	0	6	α	8	-1	α
2	-2	0	α	2	-3	α
3	0	2	0	4	-1	-8
4	-4	2	α	0	-5	α
5	5	7	α	9	0	α
6	3	5	10	7	2	0

k=6

APSP Table Format:

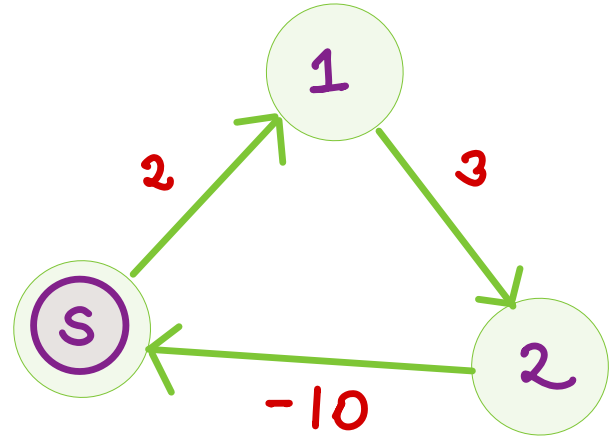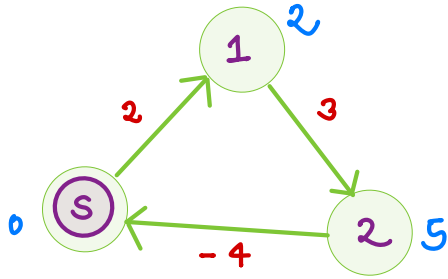|	1	2	3	4	5	6
1	0	6	α	8	-1	α
2	-2	0	α	2	-3	α
3	-5	-3	0	-1	-6	-8
4	-4	2	α	0	-5	α
5	5	7	α	9	0	α
6	3	5	10	7	2	0

Johnson's Algorithm
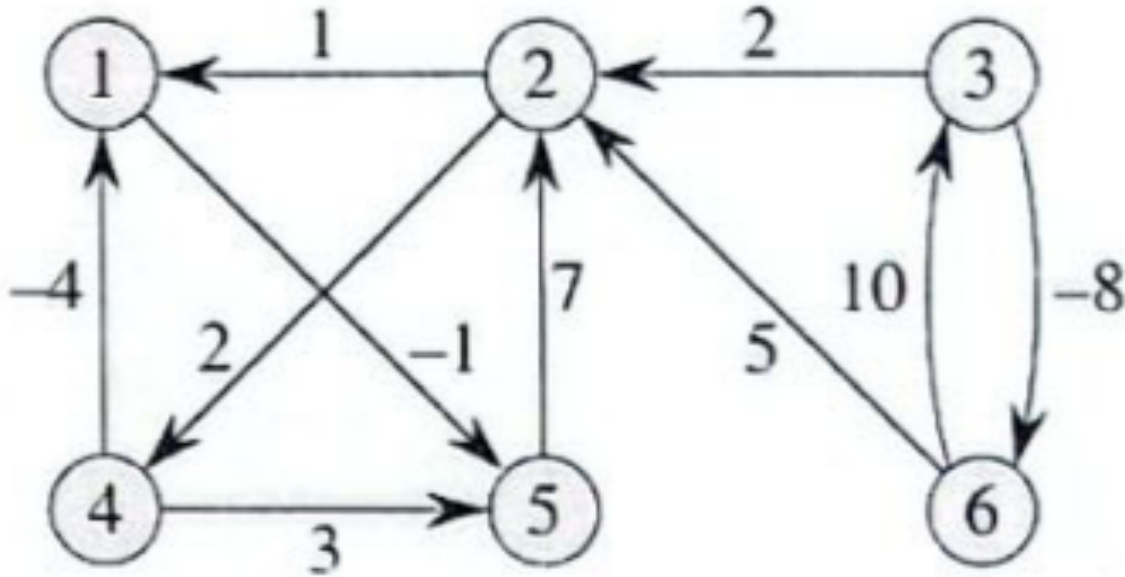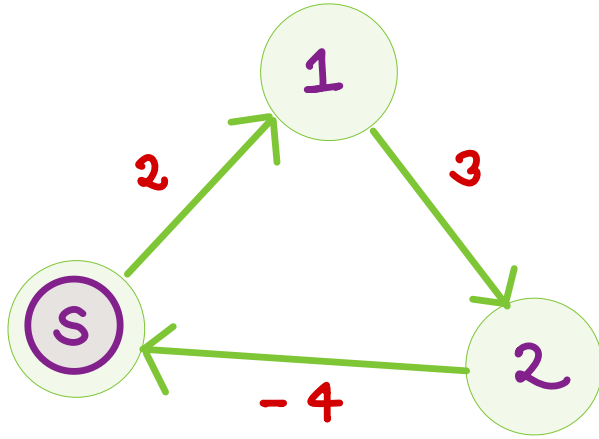
PWC (Positive Weighted Cycle)

$$2 + 3 - 4 = 1 \geq 0$$
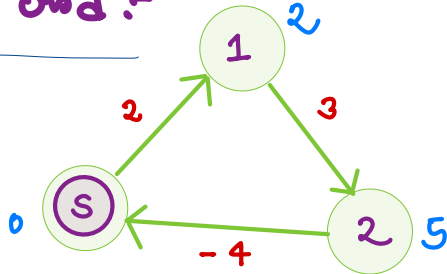
NWC (Negative Weighted Cycle)

$$2 + 3 - 10 = -5 < 0$$

Made with Goodnotes

Real Solution:-

Djkastra:-
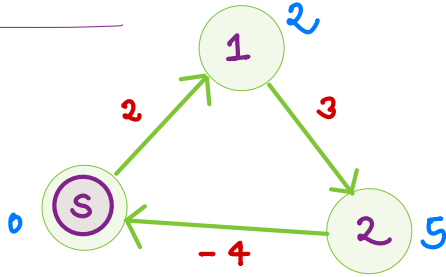
Bellman - Ford:-

S-1	2 5 0
1-2	2 5 0
2-S	2 5 0

Real Solution:-

Djkastra:-
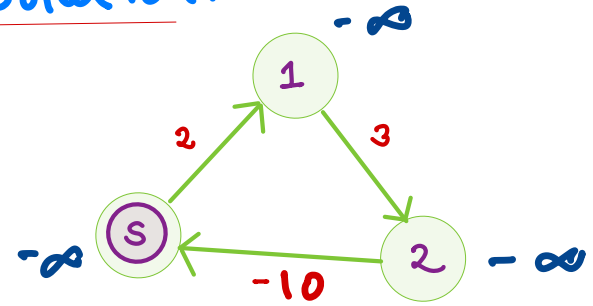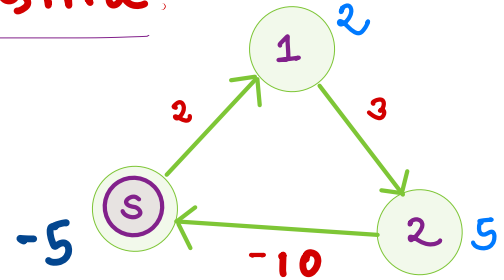
Bellman - Ford:-

S-1	2, 5, -5
1-2	-3, 0, -10
2-S	-8, -5, -15
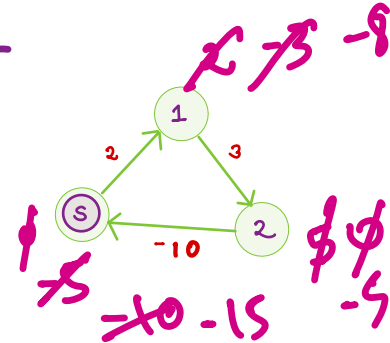
Johnson's Algorithm

Bellman Ford

Source

VE

$h[1] = -5$
$h[2] = -3$
$h[3] = 0$

$h[4] = -1$
$W[5] = -6$
$W[6] = -8$
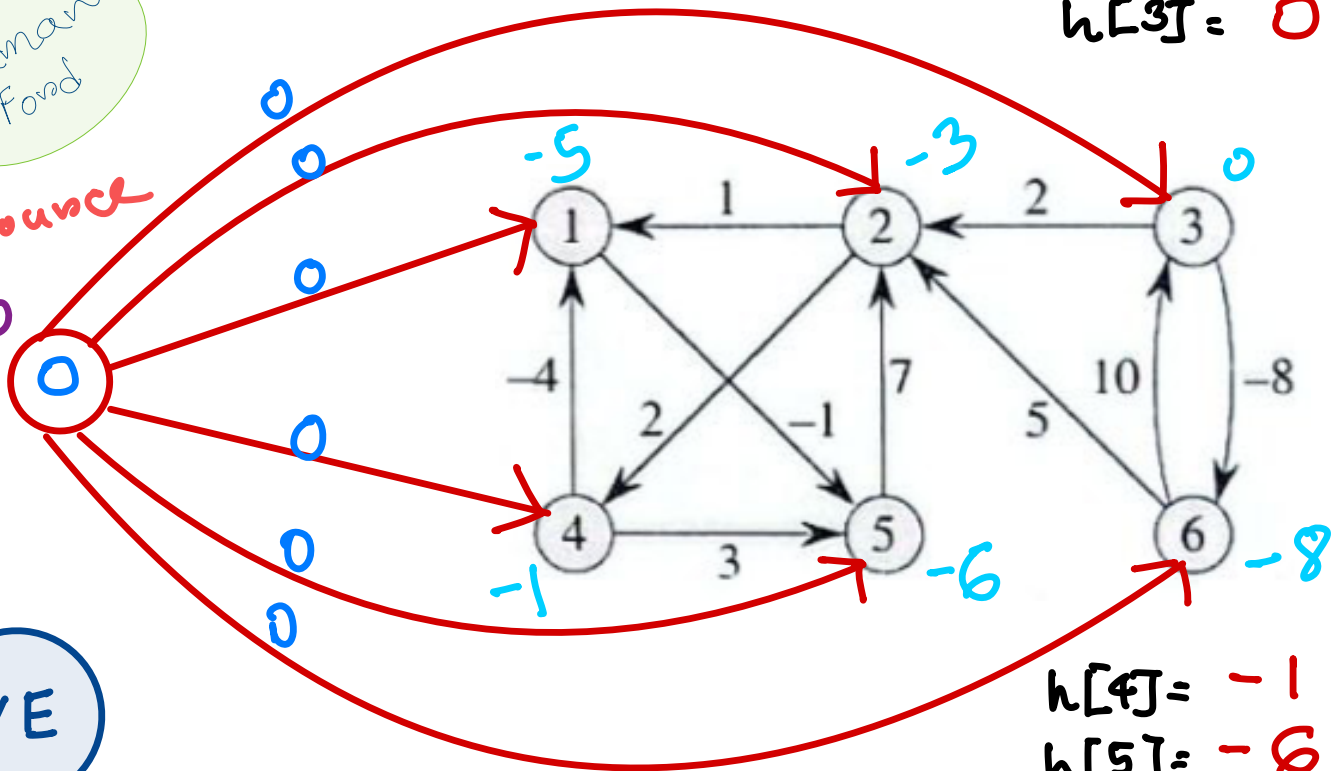
Left column:
0 1
0 2
0 3
0 4
0 5
0 6
1 5
2 1
2 4
3 2
3 6
4 1
4 5
5 2
6 2
6 3

Graph edge weights: 1, 2, −4, 2, −1, 7, 10, −8, 5, 3

Node labels: −5, −3, 0, −1, −6, −8

Source weights: 0, 0, 0, 0, 0, 0

-5   3   -3   5   0

2

0   0   0   0

0   4   0

-1   8   -6   -8

$$w'(u,v) = w(u,v) + h[u] - h[v]$$
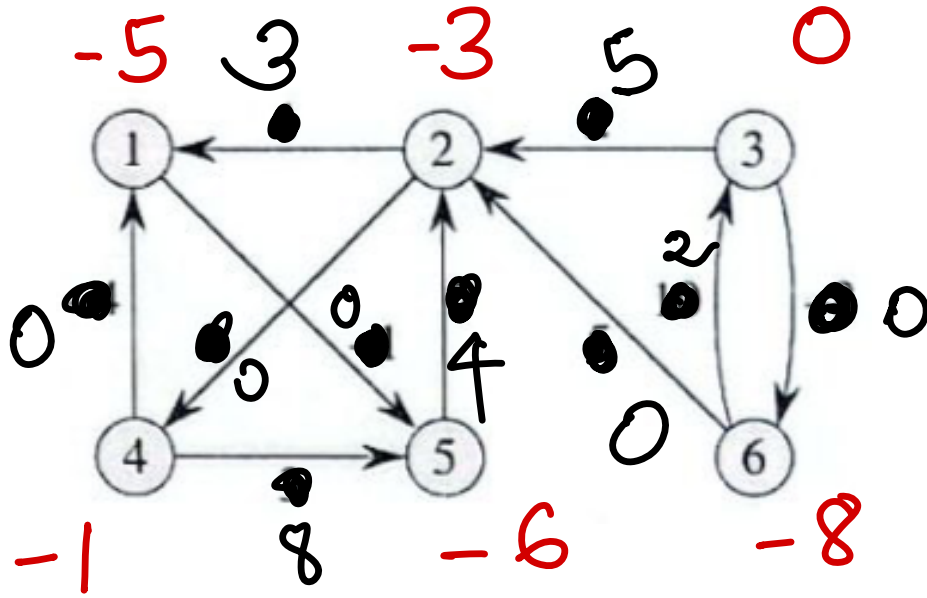
start   end

$$w'(2,1) = w(2,1) + h[2] - h[1]$$
$$= 1 + (-3) - (-5) = 1-3+5 = 3$$

Johnson's Algorithm

APSP
SSSP
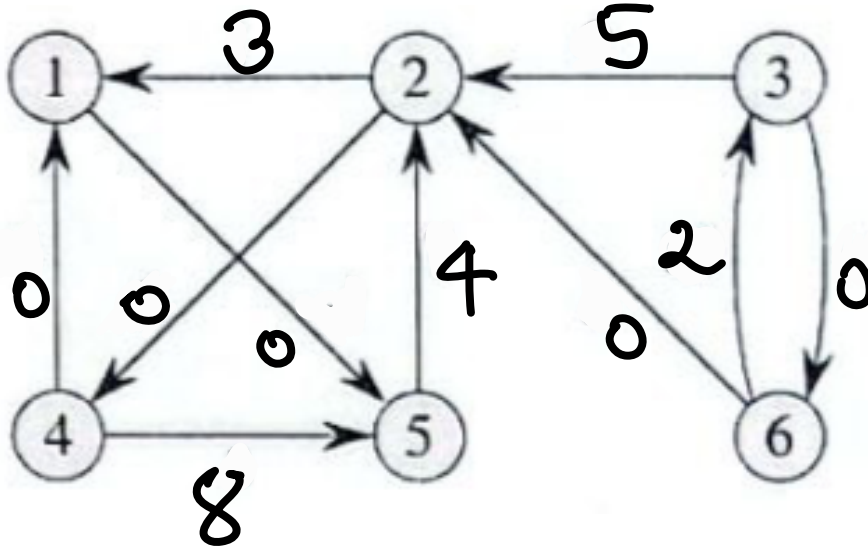
APSP Table Format:

	1	2	3	4	5	6
1	0	4	∝	4	0	∝
2	0	0	∝	0	0	∝
3	0	0	0	0	0	0
4	0	4	∝	0	0	∝
5	4	4	∝	4	0	∝
6	0	0	2	0	0	0

$V \cdot V \log V$

$V^2 \cdot \log V < V^3$

Johnson's Algorithm

APSP Table Format:

	1	2	3	4	5	6
1	0	4	∝	4	0	∝
2	0	0	∝	0	0	∝
3	0	0	0	0	0	0
4	0	4	∝	0	0	∝
5	4	4	∝	4	0	∝
6	0	0	2	0	0	0
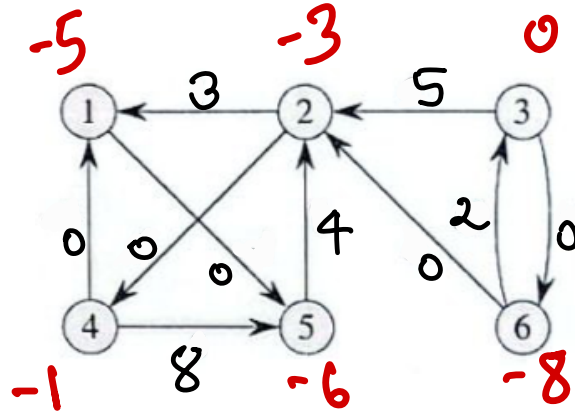
APSP Table Format:

	1	2	3	4	5	6
1	0	6	∝	8	-1	∝
2	-2	0	∝	2	-3	∝
3	-5	-3	0	-1	-6	-8
4	-4	2	∝	0	-5	∝
5	5	7	∝	9	0	∝
6	3	5	10	7	2	0

$$d'(u,v) = d(u,v) + h(\underset{\text{end}}{v}) - h(\underset{\text{start}}{u})$$

$$d'(5,4) = d(5,4) + h(4) - h(5)$$

$$= 4 + (-1) - (-6) = 9$$

$$\underline{\text{Johnson}} \rightarrow VE \quad +V^2 \log V$$

$$\uparrow \qquad\qquad \uparrow$$

$$\text{Bellman} \qquad \text{Dijkstra}$$
$$\text{Fond}$$

$$x^2 \log x$$

$$\underline{\text{Floyd Warshal}} \rightarrow V^3$$

$$x^2 \cdot x = x^3$$