NETWORK THEOREM

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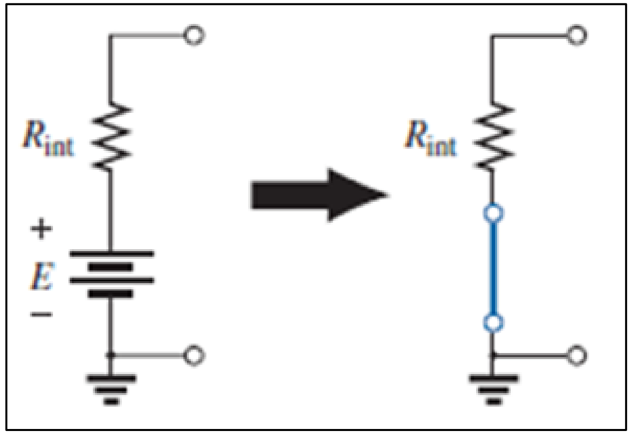
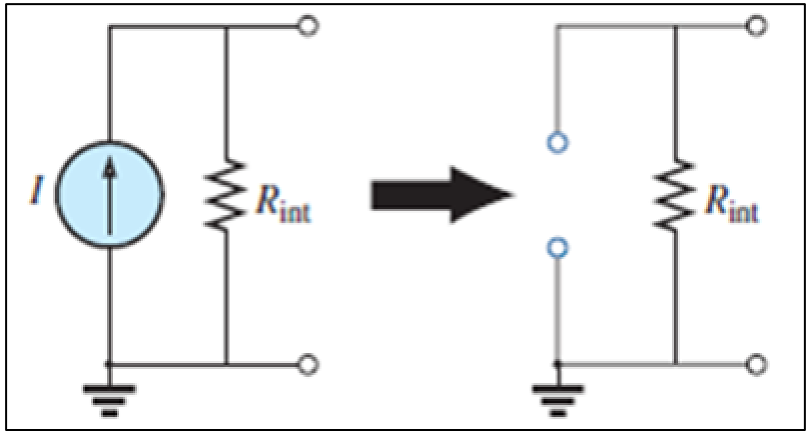
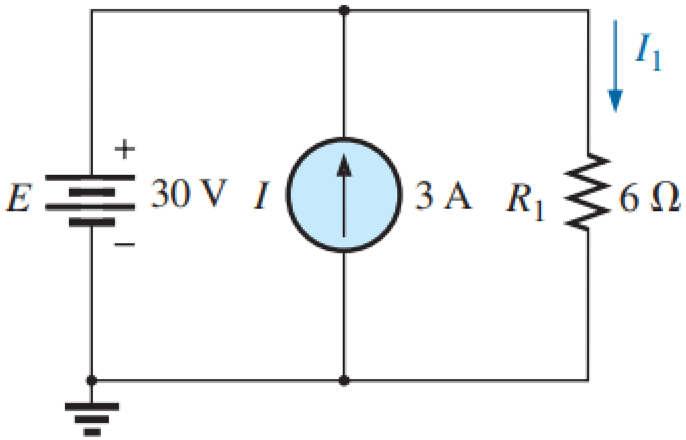
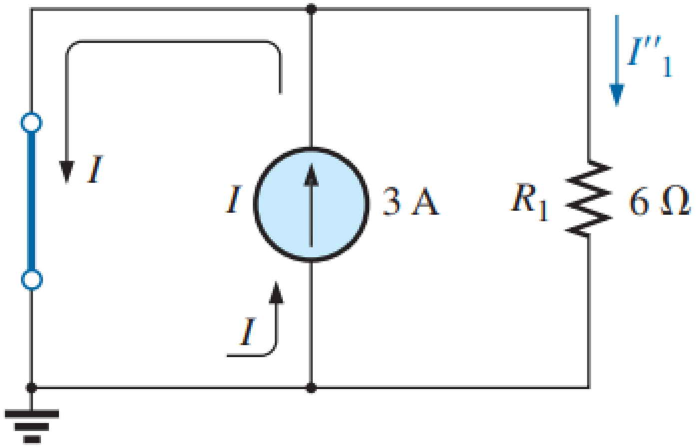
1. Superposition Theorems

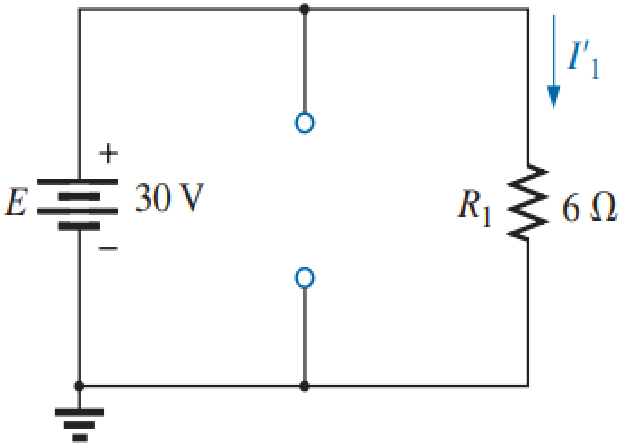
$$\boxed{I_1 = I'_1 + I''_1 = 5\text{ A} + 0\text{ A} = \mathbf{5\ A}}$$

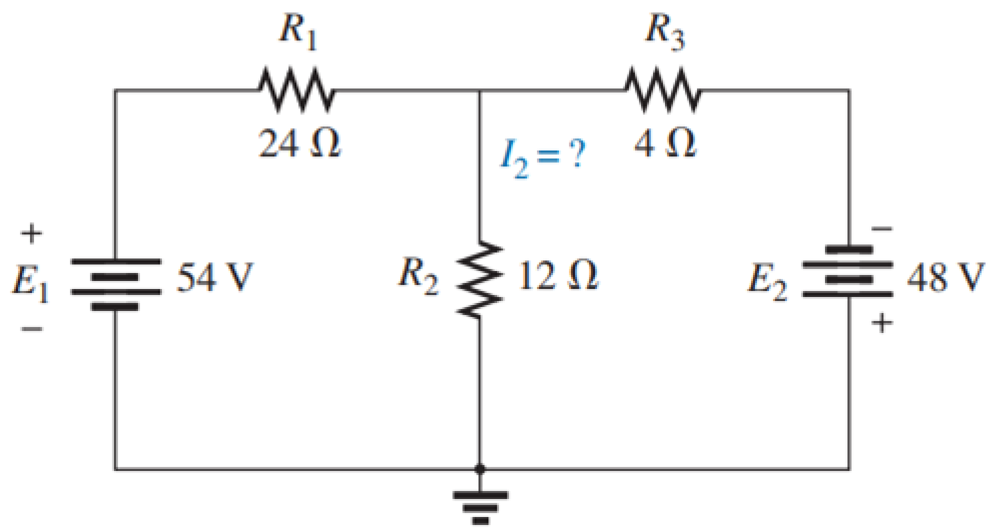


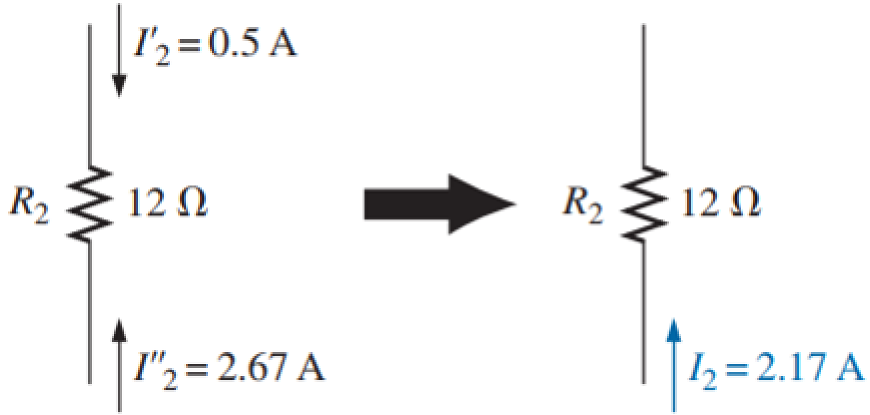


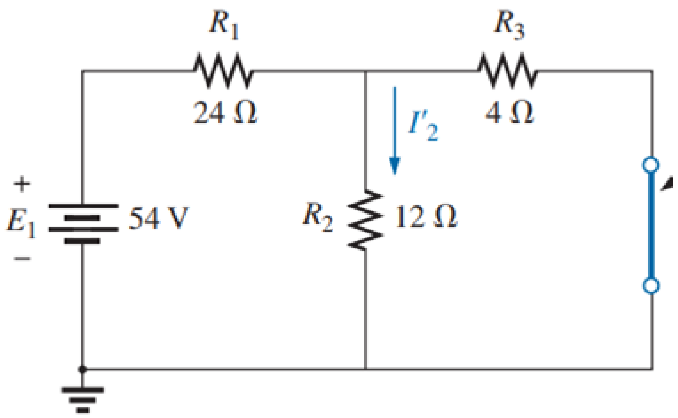


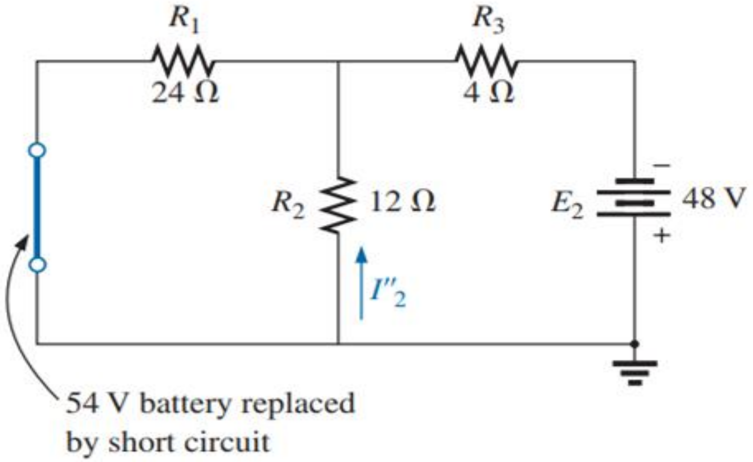












It is now important to realize that current I_2 due to each source has a different direction, as shown in Fig. 9.8. The net current therefore is the difference of the two and the direction of the larger as follows:

$$I_2 = I''_2 - I'_2 = 2.67\text{ A} - 0.5\text{ A} = \mathbf{2.17\text{ A}}$$

2,3. Thevenin's Theorems & Norton's Theorem

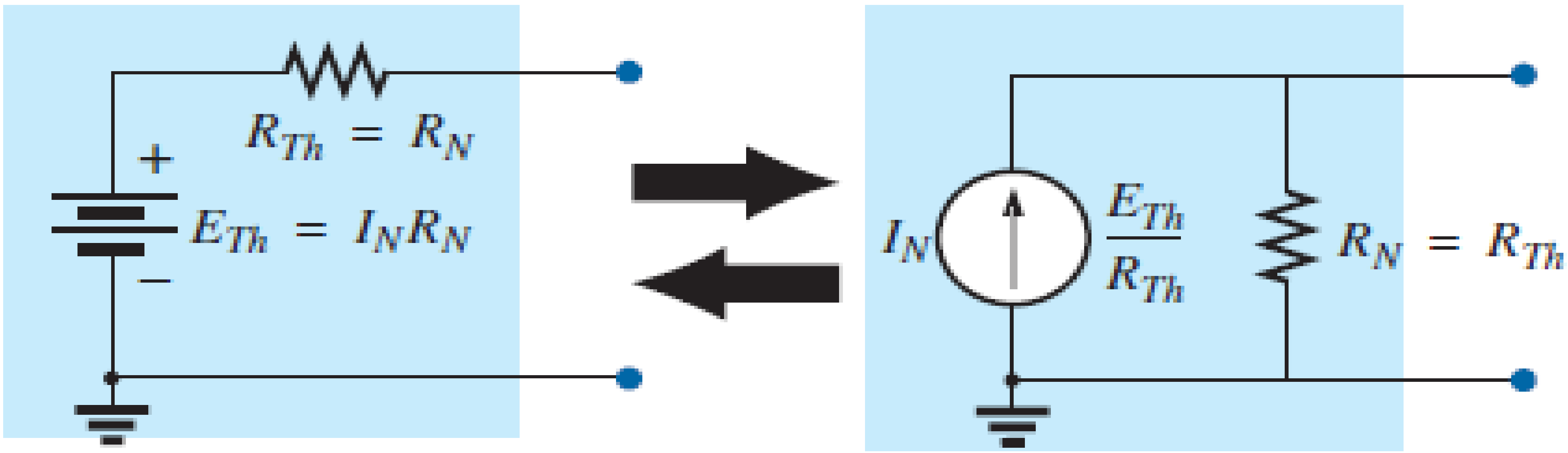


FIG. 9.60

Converting between Thévenin and Norton equivalent circuits.

4. Maximum Power Transfer Theorem

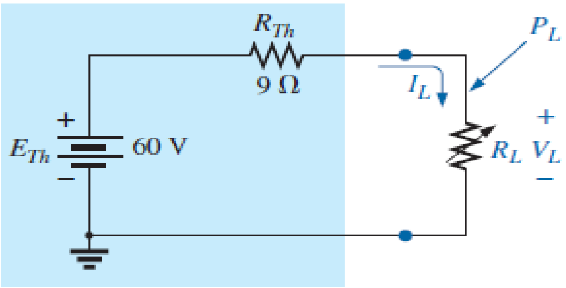


FIG. 9.79
Thévenin equivalent network to be used to validate the maximum power transfer theorem.

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} \qquad \therefore P_L = {I_L}^2 . R_L = \frac{{E_{Th}}^2}{(R_{Th}+R_L)^2} . R_L$$

- ***A load will receive maximum power from a network when its load resistance is exactly equal to the Thévenin resistance of the network applied to the load. That is, $R_L = R_{Th}$***

$$\therefore P_{L(Max)} = \frac{{E_{Th}}^2}{(R_L+R_L)^2} . R_L = \frac{{E_{Th}}^2}{4R_L}$$

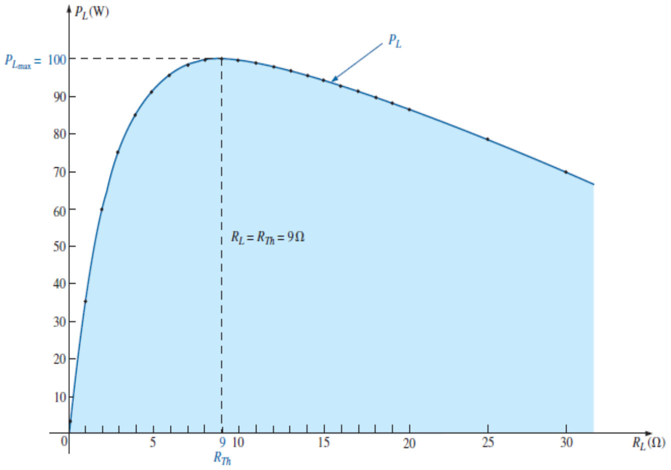


FIG. 9.80
P_L versus R_L for the network in Fig. 9.79.

Prove of Maximum Power Transfer Theorem

$$0 \leq (R_{Th} - R_L)^2$$

$$\rightarrow 0 \leq R_{Th}{}^2 - 2.R_{Th}.R_L + R_L{}^2$$

$$\rightarrow 4.R_{Th}.R_L \leq R_{Th}{}^2 + 2.R_{Th}.R_L + R_L{}^2$$

$$\rightarrow 4.R_{Th}.R_L \leq (R_{Th} + R_L)^2$$

$$\rightarrow \frac{R_{Th}}{(R_{Th} + R_L)^2} \leq \frac{1}{4.R_L}$$

$$\rightarrow \frac{E^2 R_{Th}}{(R_{Th}+R_L)^2} \leq \frac{E^2}{4.R_L}$$

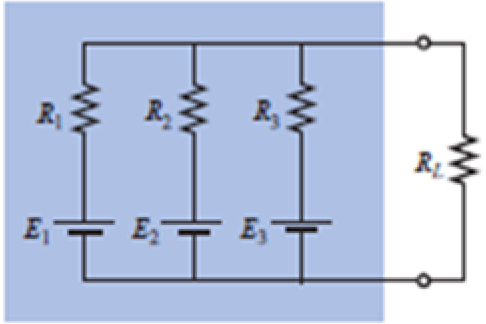
The dc operating efficiency is defined as the ratio of the power delivered to the load (P_L) to the power delivered by the source (P_s). That is,

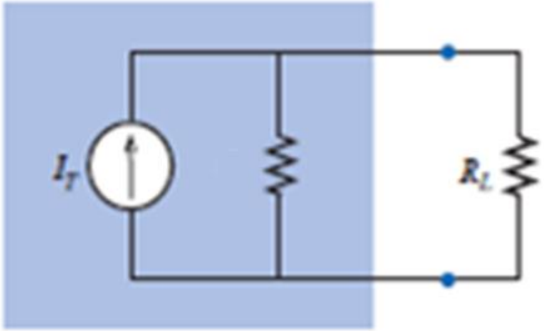
$$\boxed{\eta\% = \frac{P_L}{P_s} \times 100\%} \tag{9.4}$$

For the situation where $R_L = R_{Th}$,

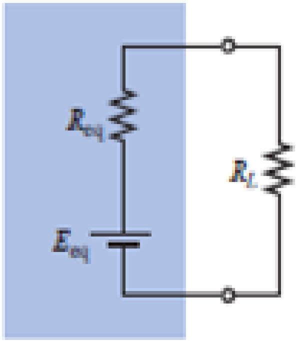
$$\begin{aligned}\eta\% &= \frac{I_L^2 R_L}{I_L^2 R_T} \times 100\% = \frac{R_L}{R_T} \times 100\% = \frac{R_{Th}}{R_{Th} + R_{Th}} \times 100\% \\ &= \frac{R_{Th}}{2R_{Th}} \times 100\% = \frac{1}{2} \times 100\% = \mathbf{50\%}\end{aligned}$$

5 Millman's Theorem





$$I_T = (\pm)\, I_1\, (\pm)\, I_2\, (\pm)\, I_3 = (\pm)\, E_1G_1\, (\pm)\, E_2G_2\, (\pm)\, E_3G_3 = (\pm)\,\frac{E_1}{R_1}\,(\pm)\,\frac{E_2}{R_2}\,(\pm)\,\frac{E_3}{R_3}$$

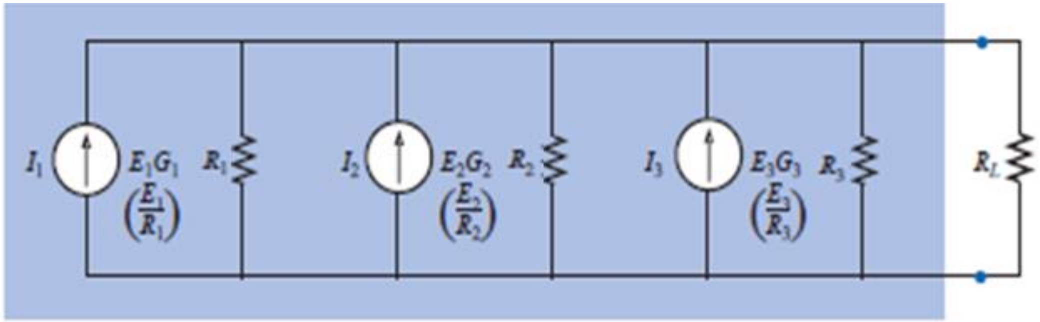


$R_T = R_1 \,||\, R_2 \,||\, R_3$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$G_T = G_1 + G_2 + G_3$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}}$$



$$I_1 = (\pm)E_1G_1 = (\pm)\frac{E_1}{R_1}$$

$$\boxed{I_n = (\pm)E_nG_n = (\pm)\frac{E_n}{R_n}}$$

$$E_{eq} = \frac{\pm\frac{E_1}{R_1} \pm \frac{E_2}{R_2} \pm \frac{E_3}{R_3} \pm \cdots \pm \frac{E_N}{R_N}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}}$$

5 Millman's Theorem

At first we have to convert all current sources to voltage source then it's easy to solve

The dual of Millman's theorem (Fig. 9.91) appears in Fig. 9.101. It can be shown that I_{eq} and R_{eq}, as in Fig. 9.101, are given by

$$I_{eq} = \frac{\pm I_1R_1 \pm I_2R_2 \pm I_3R_3}{R_1 + R_2 + R_3} \tag{9.12}$$

and

$$R_{eq} = R_1 + R_2 + R_3 \tag{9.13}$$

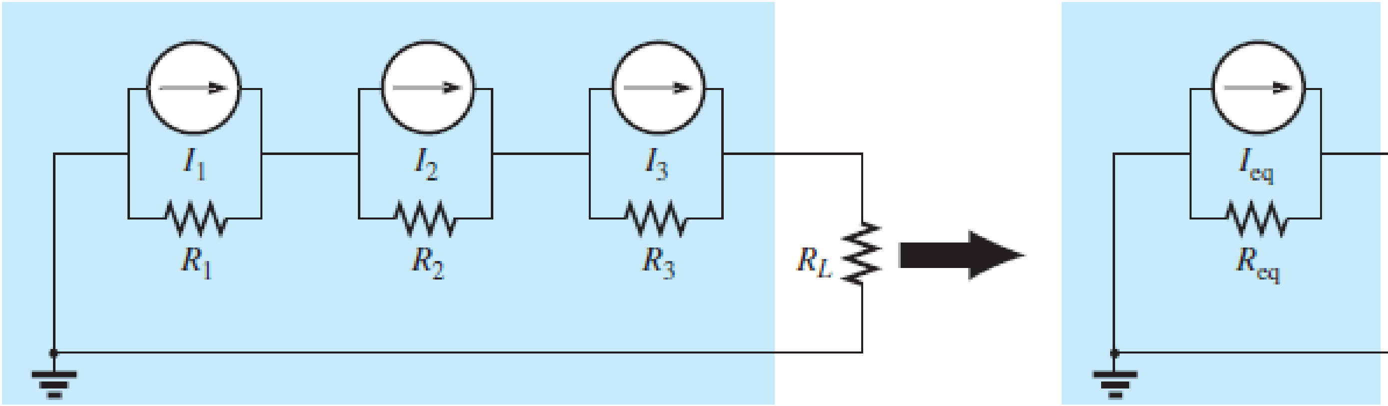


FIG. 9.101
The dual effect of Millman's theorem.

6. Substitution Theorem

- More simply, the theorem states that for branch equivalence, the terminal voltage and current must be the same.
- Consider the circuit in which the voltage across and current through the branch *a*-*b* are determined. Using the substitution theorem, a number of equivalent *a*-*b* branches are shown in Fig. 9.103.

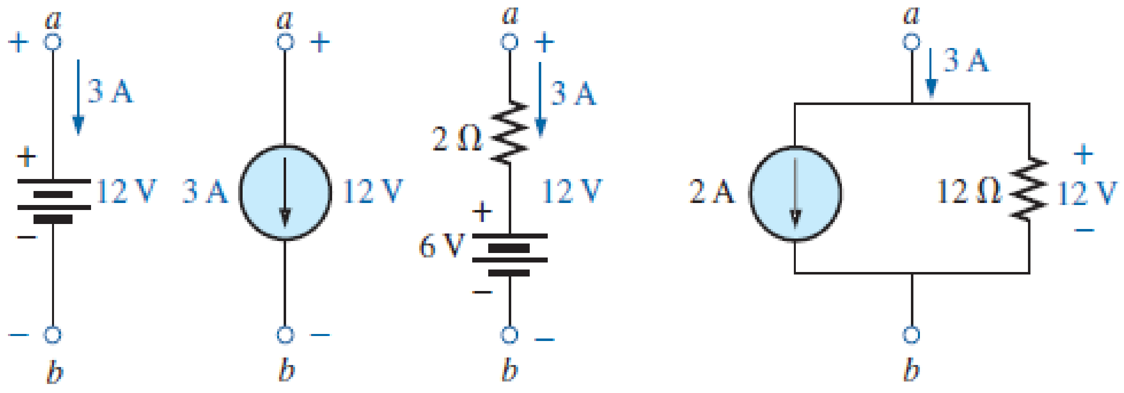


FIG. 9.103

Equivalent branches for the branch a-b in Fig. 9.102.

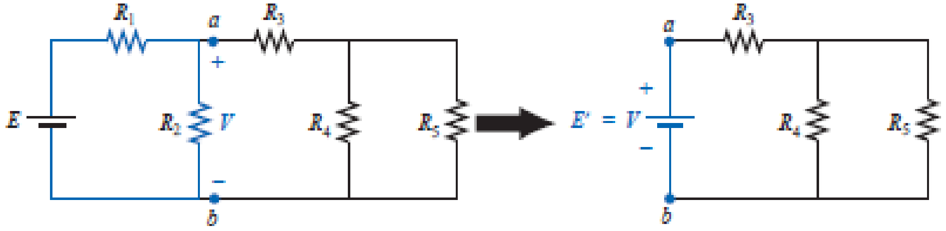


FIG. 9.105

Demonstrating the effect of knowing a voltage at some point in a complex network.

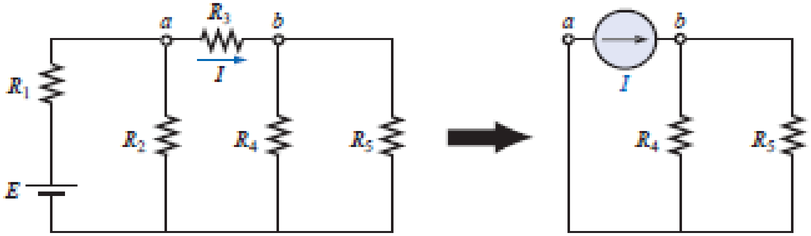


FIG. 9.106

Demonstrating the effect of knowing a current at some point in a complex network.

7. Reciprocity Theorem

- In the representative network of Fig. 9.107(a), the current *I* due to the voltage source *E* was determined. If the position of each is interchanged as shown in Fig. 9.107(b), the current *I* will be the same value as indicated.

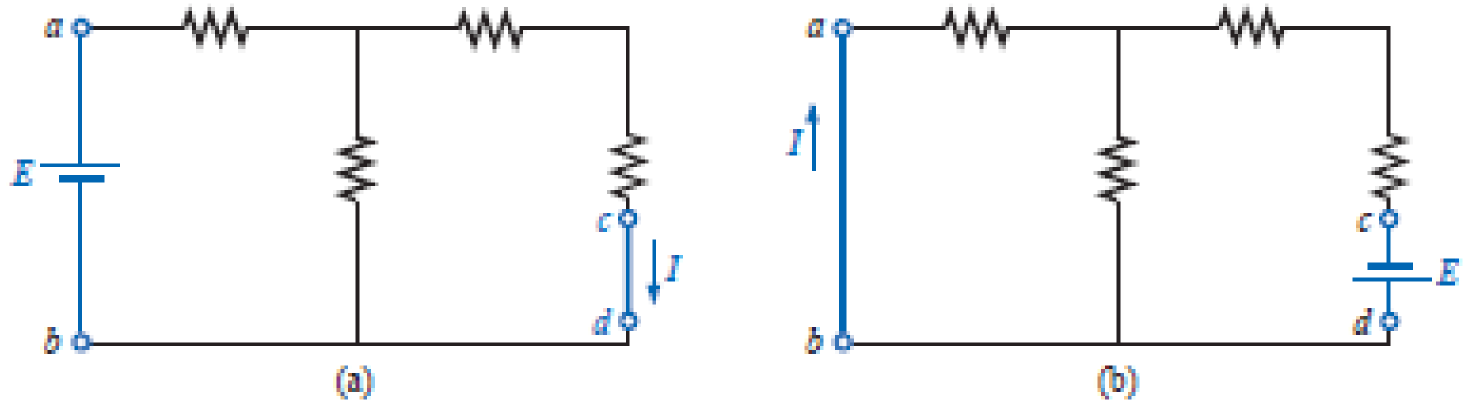


FIG. 9.107
Demonstrating the impact of the reciprocity theorem.

7. Reciprocity Theorem

- To demonstrate the validity of this statement and the theorem, consider the network of Fig. 9.108.

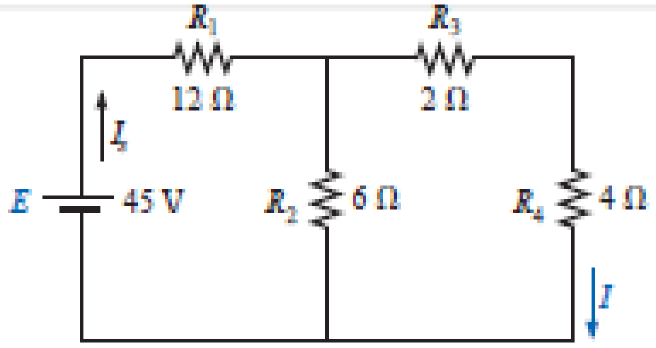


FIG. 9.108
Finding the current I due to a source E.

$$R_T = R_1 + R_2 \parallel (R_3 + R_4) = 12\ \Omega + 6\ \Omega \parallel (2\ \Omega + 4\ \Omega)$$
$$= 12\ \Omega + 6\ \Omega \parallel 6\ \Omega = 12\ \Omega + 3\ \Omega = 15\ \Omega$$

$$I_s = \frac{E}{R_T} = \frac{45\text{ V}}{15\ \Omega} = 3\text{ A}$$

$$I = \frac{3\text{ A}}{2} = \boxed{1.5\text{ A}}$$

For the network of Fig. 9.109, which corresponds to that of Fig. 9.107(b), we find

$$R_T = R_4 + R_3 + R_1 \parallel R_2$$
$$= 4\ \Omega + 2\ \Omega + 12\ \Omega \parallel 6\ \Omega = 10\ \Omega$$

and $$I_s = \frac{E}{R_T} = \frac{45\text{ V}}{10\ \Omega} = 4.5\text{ A}$$

so that $$I = \frac{(6\ \Omega)(4.5\text{ A})}{12\ \Omega + 6\ \Omega} = \frac{4.5\text{ A}}{3} = \boxed{1.5\text{ A}}$$

which agrees with the above.

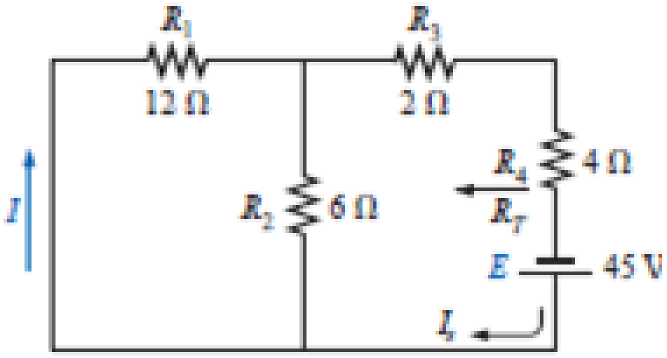


FIG. 9.109
Interchanging the location of E and I of Fig. 9.108 to demonstrate the validity of the reciprocity theorem.

THANK YOU